1. Introduction

Fluid flow around a heated cylinder and heat transfer from the cylinder into the fluid due to both free and forced convection possess a great amount of interest among the researchers these days because of such situations in a variety of applications [1–5] such as in the process of extrusion via tubes and cylinders, heat exchangers, drying of textiles, and other materials, processes of purification and steam engines, and so on. In some of the cases, heat transfers due to free convection, i.e., molecules of fluid move due to difference in density and temperature, and in some cases, it is due to forced convection, i.e., fluid molecules are forced to move by applying external forces. When free and forced convections are of comparable magnitudes, the heat flow is said to be with mixed convection. In this article, we have investigated heat transfer from the heated cylinder placed at inlet of the pipe due to forced convection in a power law fluid varying the Reynolds number and power law index. Many researchers have investigated heat flow past cylinder in isolation or many scattered cylinders experimentally or analytically. A brief survey of such literature is provided below.
transfer coefficient and analysed them for a wide range of Reynolds number and Prandtl number. Panda [7] studied the hydrodynamics of power law fluids past a pair of cylinders fixed in the domain of flow in side-by-side manner. He carried out a parametric study by taking power law index in the range of $0.2 \leq n \leq 1.8$, the Reynolds number in the range of $0.1 \leq Re \leq 100$, and the gap between two cylinders in the range of $1.2 \leq G \leq 4$ and observed the influence of Re, $n$, and $G$ upon stream lines, surface pressure, and drag and drift coefficients. The flow over and heat transfer from an isolated heated cylinder is the simplest model to investigate hydrodynamics, pressure on the surface of cylinder, and heat transfer. In this case, the flow and heat transfer are influenced by flow behaviour index, the Reynolds number, and the cylinder radius [8–13]. Sanjal and Dhiman [14] studied hydrodynamics of shear-thinning fluids flowing past a pair of square cylinders with mixed convection heat flow. The side-by-side gap between cylinders is parameterized in the range from 1 to 5; Reynolds number is in the range Re of 1–40 and Pr = 40. They found that the leading-edge flow separation from the cylinders disturbs the wake structures and vortex shedding patterns in case of shear-thinning fluids which was not earlier observed in case of Newtonian fluids [15].

Another numerical study has been carried out by Haider [16] to analyse the heat flow characteristics of Newtonian fluid past clusters of isothermal cylinders fixed within the flow domain. Cylinders were placed in-line or scattered manner and found that the heat flow from the scattered cylinders is slightly higher than when the cylinders are placed in-line. Kumar et al. [17] have performed a remarkable numerical study using commercial software FLUENT to analyse the forced convection around a heated half cylinder placed in the flow domain with 25% blockage ratio, Pr = 50, and. Re = 1–40. It was found that the drag coefficient magnitude is higher in shear-thickening fluids in comparison with shear-thinning fluids. It was also found by them that the heat transfer rate increases with increase in Reynolds number Re as an overall result. The average Nusselt number was found to have greater values in case of shear-thinning fluids as compared with shear-thickening and Newtonian fluids. There is always a complex interplay between kinematic and other fluid properties. Mishra [18] investigated forced convection heat transfer from a pair of heated cylinders numerically using COMSOL Multiphysics. Base fluid is taken to be a power law fluid with flow behaviour index in the range of $0.2 \leq n \leq 2$. A detailed parametric study with values in the range of $5 \leq Re \leq 200$, $0.7 \leq Pr \leq 100$, and $0.1 \leq D/L \leq 0.3$ (diameter to length ratio) reveals that a higher value of heat transfer is observed for higher values of the Reynolds number and Prandtl number in case of shear-thinning fluid. There are different but remarkable contribution by different researchers studying flow past and heat transfer from heated cylinder under different parametric considerations in literatures [19–27].

In current research, we are focused to study the flow of power law fluid through a rectangular channel and heat transfer from a heated cylinder of variable radius fixed near the inlet of the channel. The flow behaviour index is assumed to be in the range of $0.8 \leq n \leq 1.2$, the Reynolds number in the range from 1000 to 10000, and the blockage ratio (radius to height ratio) is taken to be 0.1, 0.2, or 0.3. The values of the local Nusselt number found in our case are in a great agreement with the correlation values provided by [28]. Flow variables will be further investigated in the parametric study using COMSOL Multiphysics by changing the values of the parameters listed above. In section 2, we have given the problem statement together with the domain discretization and governing nonlinear partial differential equations and boundary conditions. In section 3, a validation study has been carried out in order to compare the results with empirical correlation. Section 4 is dedicated for results and discussion, and finally in section 5, an overall summary of the work has been put and we give concluding remarks.

2. Problem Formulation

Consider the laminar flow of a power law fluid (time-independent non-Newtonian fluid) through a rectangular channel (of length $l = 4$ m and height $h = 1$ m) in which a heated circular cylinder has been fixed in the first half of the channel near inlet as shown in Figure 1. Assume that $r$ denotes radius of the channel and the cylinder has been maintained at a constant temperature $T_h = 293$ K. It is further assumed that

(i) Initial or reference temperature of the fluid is assumed to be $T_{ref} = 250$ K
(ii) Walls of the channel are insulated, i.e., heat flux through the walls is zero
(iii) Fluid velocity at the boundary is assumed to be nonzero, i.e., the slip boundary conditions will be used
(iv) The ratio of radius of the cylinder to the height of the channel $r/h$ is taken to be equal to either 1: 10, 2: 10, or 3: 10

2.1. Domain Discretization and Mesh Statistics. Finite element methods require the domain of interest be divided into small pieces called elements. Here, the domain of the problem is divided into small irregular triangles. Initially, we considered six different discretizations with number of elements $N = 5156$, $N = 10344$, $N = 12836$, $N = 25320$, $N = 58726$, and $N = 115152$. The pictorial representations for each of these discretizations corresponding to these $N$-values are put in Figure 2.

At the first stage, we will try to obtain solutions for different number of elements (mesh sizes) in order to obtain $N$-value (say $N_0$) such that all the solutions become mesh-independent, i.e., we can select any number of elements greater than $N_0$. It is well-known fact that increasing number of elements improves the solution to higher accuracy, a point will reach when number no further improvement is visible, and we say solutions have become mesh-independent. In Figure 3, we have presented the computed magnitude of velocity of fluid for different mesh sizes. The peak in the graph of velocity is achieved due to the presence of cylinder near the inlet of channel. The
convergence of solution is clearly observable from this figure as for \( N = 58726 \) and \( N = 115152 \) the velocity curves are overlapping. Extra finer meshes will be used for further computations. In case of extra finer meshes, number of triangular elements is \( N = 58726 \) and complete mesh statistics in case of extra finer meshes is presented in Table 1.

3. Governing Equations and Boundary Conditions

To analyse the fluid flow and heat transfer due to convection and conduction from the heated circular cylinder into the fluid domain \( \Omega \), the coupled system of equations involving momentum, continuity, and energy balance equations is used as the model. Let \( \mathbf{V} = (v_x, v_y) \) denote the velocity of the fluid, where \( v_x, v_y \) are \( x, y \)-components of velocity, respectively. The flow is governed by the following set of equations.

Momentum balance is as follows:

\[
\rho (\nabla \cdot \mathbf{V}) \mathbf{V} = \nabla \cdot \left( -p \mathbf{I} + \mu \left( \nabla \mathbf{V} + (\nabla \mathbf{V})^T \right) \right) + \mathbf{F}. \tag{1}
\]

Mass balance is as follows:

\[
\rho \mathbf{V} \cdot \mathbf{V} = 0, \tag{2}
\]

where \( \rho \) denotes density which is constant in case of incompressible flows and \( p \) denotes the hydrostatic pressure field. In the current problem, we use the power law model (time-independent Newtonian fluid model) according to which the apparent effective viscosity is represented as follows:
In this section, the solutions are validated by comparing them with empirical correlation. The empirical correlations express the Nusselt number as a function of the Reynolds number and the Prandtl number: \( Nu = f(Re, Pr) \). In case of the current simulations, we have compared the values of the local Nusselt number \( Nu_x \) of the fluid flow and heat transfer past a heated circular cylinder with the empirical correlation values given by Lienard [28]; accordingly, the local Nusselt number \( Nu_x \) is expressed as follows:

\[
Nu_x = 0.032Re_x^{0.8}Pr^{0.43}.
\]

These comparisons are shown in Figures 4–6 for different values of the local Reynolds number, cylinder radius, and the flow behaviour index. From these comparisons, it can be deduced that our results agree with the empirical correlation [28] to a good extent specially when we increase the cylinder to height ratio; see Figures 5 and 6.

### 5. Results and Discussion

The problem of non-isothermal flow of a power law fluid through a rectangular channel with a heated circular cylinder at temperature \( T = 293 \text{ K} \) fitted within the fluid near inlet of the channel has been modelled. The values of the Reynolds number are taken in the range of \( Re=1000–10000 \) flow behaviour index in the range of \( n=0.8–1.2 \) and radius of the fixed cylinder in the range of \( r = 0.1 \text{ m}–0.3 \text{ m} \) in order to perform parametric studies. An arbitrary power law fluid is inquired which has flux density, \( \kappa \) is material’s heat conductivity, \( \nabla T \) is the temperature gradient, and \( Q_0 \) is the volumetric heat source. In the current problem, there are no heat sources \( Q_0 = 0 \).

### 3.1. Boundary Conditions

Assume that \( \Omega \) denotes the domain of solution and \( \partial \Omega = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_4 \cup \Gamma_5 \) denote the whole boundary where \( \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \text{ and } \Gamma_5 \), respectively, denote the upper wall, lower wall, inlet, outlet, and the surface of the heated circular cylinder. As we are not interested to see the viscous effects near the walls and at the outer surface of the heated cylinder, therefore, slip conditions are chosen. The governing equations (1)–(6) will be discretized and solved subject to the following conditions using Galerkin finite element method implemented using COMSOL Multiphysics 5.4.

\[
\nabla \cdot \vec{n} = 0, \quad \text{on } \Gamma_1 \cup \Gamma_2 \cup \Gamma_5,
\]

\[
u = u_{in} = f(Re), \quad p \neq 0 \text{ on } \Gamma_3,
\]

\[
p = 0, \quad \text{on } \Gamma_5,
\]

\[
T = T_{ref} = 250 \text{ K}, \quad \text{in } \Omega,
\]

\[
-\nabla \cdot q = 0, \quad \text{on } \Gamma_1 \cup \Gamma_2.
\]

### 4. Comparison with Empirical Correlation for Validation of the Solutions

In this section, the solutions are validated by comparing them with empirical correlation. The empirical correlations express the Nusselt number as a function of the Reynolds number and the Prandtl number: \( Nu = f(Re, Pr) \). In case of the current simulations, we have compared the values of the local Nusselt number \( Nu_x \) of the fluid flow and heat transfer past a heated circular cylinder with the empirical correlation values given by Lienard [28]; accordingly, the local Nusselt number \( Nu_x \) is expressed as follows:

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Nu_x = 0.032Re_x^{0.8}Pr^{0.43}.
\]

The Nusselt number \( Nu_x \) is expressed as follows:

\[
Nu_x = 0.032Re_x^{0.8}Pr^{0.43}.
\]
initial temperature $T = 250$ K. The governing equations (1), (2), and (5) are discretized using the finite element method implemented with COMSOL Multiphysics and solved subject to the conditions given in equations (7)–(11) for pressure distribution, velocity profile, and temperature distribution. Consequently, many different simulations are generated for different values of the parameters listed above. Extra fine meshes are used to discretize the domain of the problem.

5.1. Pressure Distribution. Pressure contour plots for the flow past the cylinder with radius $r = 0.1$ m are displayed in Figure 7 for different values of the Reynolds number and the flow behaviour index. In Figure 8, we put into view the similar graphs by setting $r = 0.2$ m. Impact of the three parameters over pressure distribution on the front and back surface of the cylinder is very clear from these two figures. Pressure increases with the increasing value of the Reynolds
number whereas by increasing flow behaviour index from $n = 0.8$ to $n = 1.2$, we observe that pressure at the front and back surfaces decreases. It can be further deduced that in case of Pseudoplastic fluids, the pressure is lesser at the surface of the circular cylinder in comparison with the case when fluid is dilatant. We further can interpret that increasing radius also increases pressure on the cylinder surface. In Table 2, we present the maximum numerical values of pressure over the surface of the cylinder for different parametric changes. For further analysis, we have plotted maximum pressure graphs on surface of the cylinder against flow behaviour index $n$ for different Reynolds number Re and cylinder radius $r$ (see Figure 9). It can be deduced from these graphs that maximum value of pressure

![Graphs showing pressure variations](image-url)
on surface of the cylinder is decreasing with increasing value of the flow behaviour index.

5.2. Temperature Distribution. Temperature contour plots for the flow of power law fluid past the heated cylinder of radii range \( r = 0.1 \text{ m}, 0.2 \text{ m}, \) and \( 0.3 \text{ m} \) are displayed in Figures 10–12, respectively. These figures show that there is significant decrease in thickness of thermal layer along the horizontal line through the center of heated cylinder if the value of the Reynolds number increases from \( \text{Re} = 1000 \) to \( \text{Re} = 4000 \) and finally to \( \text{Re} = 10000 \). This type of investigations has also been reported by [16]. They have put different cylinders in in-line settings and scattered way and observed temperature changes in the fluid domain. We are confined to determine the changes in
Figure 7: Pressure distribution near the heated cylinder with radius $r = 0.1$ m.

Figure 8: Pressure distribution near the heated cylinder with radius $r = 0.2$ m.
Table 2: Numerical values of maximum pressure over the surface of heated cylinder under different conditions.

<table>
<thead>
<tr>
<th>Re</th>
<th>r (m)</th>
<th>N = 0.8, pressure (Pa)</th>
<th>N = 0.9, pressure (Pa)</th>
<th>N = 1.0, pressure (Pa)</th>
<th>N = 1.1, pressure (Pa)</th>
<th>N = 1.2, pressure (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>0.1</td>
<td>4.01E−04</td>
<td>1.64E−04</td>
<td>6.69E−05</td>
<td>2.70E−05</td>
<td>1.08E−05</td>
</tr>
<tr>
<td>1000</td>
<td>0.2</td>
<td>5.14E−04</td>
<td>2.09E−04</td>
<td>8.52E−05</td>
<td>3.45E−05</td>
<td>1.38E−05</td>
</tr>
<tr>
<td>1000</td>
<td>0.3</td>
<td>9.32E−04</td>
<td>3.72E−04</td>
<td>1.49E−04</td>
<td>5.97E−05</td>
<td>2.38E−05</td>
</tr>
<tr>
<td>2000</td>
<td>0.1</td>
<td>0.0015343</td>
<td>6.24E−04</td>
<td>2.53E−04</td>
<td>1.03E−04</td>
<td>4.13E−05</td>
</tr>
<tr>
<td>2000</td>
<td>0.2</td>
<td>0.0019974</td>
<td>8.04E−04</td>
<td>3.24E−04</td>
<td>1.31E−04</td>
<td>5.25E−05</td>
</tr>
<tr>
<td>2000</td>
<td>0.3</td>
<td>0.0037289</td>
<td>0.0014834</td>
<td>5.91E−04</td>
<td>2.35E−04</td>
<td>9.38E−05</td>
</tr>
<tr>
<td>4000</td>
<td>0.1</td>
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<td>0.0024174</td>
<td>9.78E−04</td>
<td>3.95E−04</td>
<td>1.59E−04</td>
</tr>
<tr>
<td>4000</td>
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<td>0.0078832</td>
<td>0.0031606</td>
<td>0.0012676</td>
<td>5.09E−04</td>
<td>2.04E−04</td>
</tr>
<tr>
<td>4000</td>
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<td>0.014932</td>
<td>0.0059432</td>
<td>0.0023639</td>
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</tr>
<tr>
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<td>0.1</td>
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<td>0.0094843</td>
<td>0.003822</td>
<td>0.0015426</td>
<td>6.22E−04</td>
</tr>
<tr>
<td>8000</td>
<td>0.2</td>
<td>0.031304</td>
<td>0.012518</td>
<td>0.0050127</td>
<td>0.0020069</td>
<td>8.03E−04</td>
</tr>
<tr>
<td>8000</td>
<td>0.3</td>
<td>0.059715</td>
<td>0.023781</td>
<td>0.0094685</td>
<td>0.003767</td>
<td>0.0014984</td>
</tr>
<tr>
<td>10000</td>
<td>0.1</td>
<td>0.037098</td>
<td>0.014785</td>
<td>0.0059351</td>
<td>0.002395</td>
<td>9.65E−04</td>
</tr>
<tr>
<td>10000</td>
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<td>0.048873</td>
<td>0.019513</td>
<td>0.0078118</td>
<td>0.0031264</td>
<td>0.0012505</td>
</tr>
<tr>
<td>10000</td>
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<td>0.037152</td>
<td>0.014795</td>
<td>0.005889</td>
<td>0.0023424</td>
</tr>
</tbody>
</table>

![Figure 9: Continued.](image-url)
temperature in the fluid domain due to changes in flow behaviour index, Reynolds number, and radius of the heated cylinder as well. The thickness of thermal layer along the horizontal line through the center of heated cylinder increases with the increasing value of the flow behaviour index. This means that if the fluid is dilatant, the temperature gradient has larger value. This larger value will be responsible of more flow of heat within the fluid. However, increasing radius does not seem to have any significant impact on temperature contours as shown in these figures.

5.3. Fluid Viscosity. Apparent viscosity contour plots for flow of power law fluid past the heated cylinder of radii in the range from $r = 0.1$ m to $r = 0.3$ m for different values of the fluid. However, increasing radius does not seem to have any significant impact on temperature contours as shown in these figures.

Figure 9: Maximum pressure around the circle: (a) Re = 1000, (b) Re = 4000, and (c) Re = 10,000.

Figure 10: Contours of temperature for $r = 0.1$. 
Figure 11: Contours of temperature for $r = 0.2$.

Figure 12: Contours of temperature for $r = 0.3$. 
Reynolds number are displayed in Figures 13 and 14. The viscosity dissipations can be clearly observed through these figures, and the graphs are plotted for different values of the cylinder radius $r$.

6. Conclusion

In the paper, we have discussed the non-isothermal flow through the rectangular channel fitted with a heated circular cylinder at constant temperature $T_h = 293$ K with different radii to height ratios of 1: 10, 2: 10, and 3: 10 near the inlet of the channel. Assuming a power law fluid flowing through the channel at initial reference temperature $T_{\text{ref}} = 250$ K, we were focused to analyse the temperature distribution within the fluid and its dependence upon various factors including the flow behaviour index $n$ and the Reynolds number $Re$. The flow behaviour index was taken in the range $T_h = 293$ K whereas the Reynolds number was assumed to be in the range $T_h = 293$ K for parametric study. Solutions were obtained using Galerkin’s finite element method implemented in COMSOL Multiphysics. Satisfactory results were achieved when compared our solutions with empirical correlation. It is further concluded that [25–27]

(i) It was found that the local Nusselt number obtained down the stream in this problem is in a very good agreement with that given by the correlation [28] for the current geometry of the problem.

(ii) It is found that the pressure increases with the increasing value of the Reynolds number whereas by increasing flow behaviour index from $n = 0.8$ to $n = 1.2$, we observe that pressure at the front and back surfaces decreases.

(iii) For pseudoplastic fluids, the pressure is lesser at the surface of the circular cylinder in comparison with the case when fluid is dilatant.

(iv) Increasing radius also increases pressure on the cylinder surface.

(v) Maximum value of pressure on surface of the cylinder is decreasing with increasing value of the flow behaviour index.

(vi) A significant decrease in thickness of thermal layer is observed along the horizontal line through the center of heated cylinder if the value of the Reynolds number increases.
The thickness of thermal layer along the horizontal line through the center of heated cylinder has been found to increase with the increasing value of the flow behaviour index.

It is found that the increasing radius does not seem to have any significant impact on temperature contours.

6.1. Future Work. This research can be further extended by adding the multiple scattered and inline heated cylinders and comparing the flow of heat in the two cases.

Nomenclature

- $\Omega$: Fluid domain
- $v_x$: Horizontal component of velocity
- $v_y$: Vertical component of velocity
- $p$: Hydrostatic pressure
- $K$: Flow consistency index
- $\dot{\gamma}$: Shear rate normal to the plane of shear
- $T(x, y)$: Temperature field
- $c_p$: Specific heat capacity
- $T_h$: Cylinder temperature
- $T_{\text{ref}}$: Initial or reference temperature of the fluid
- $q$: Local conductive heat flux
- $\kappa$: Material heat conductivity
- $N_u$: Nusselt number
- $Pr$: Prandtl number
- $Re$: Reynolds number
- $n$: Power law index
- $l$: Length of the channel
- $h$: Height of the channel
- $r$: Radius of the channel
- $N$: Number of elements in discretization.

Data Availability

No data were required to perform this research.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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