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Research Article

Computation of Metric Dimension of Certain Subdivided Convex Polytopes

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The distance $d(z_1, z_2)$ from vertex $z_1 \in V(G)$ to $z_2 \in V(G)$ is minimum length of (z_1, z_2)-path in a given connected graph G having $E(G)$ and $V(G)$ edges and vertices'/nodes' sets, respectively. Suppose $Z = \{z_1, z_2, z_3, \ldots, z_m\} \subseteq V(G)$ is an order set and $c \in V(G)$, and the code of c with reference to Z is the m-tuple $\{d(c, z_1), d(c, z_2), d(c, z_{13}), \ldots, d(c, z_k)\}$. Then, Z is named as the locating set or resolving set if each node of G has unique code. A locating set of least cardinality is described as a basis set for the graph G, and its cardinal number is referred to as metric dimension symbolized by $\dim(G)$. Metric dimension of certain subdivided convex polytopes ST_n has been computed, and it is concluded that just four vertices are sufficient for unique coding of all nodes belonging to this family of convex polytopes.

1. Introduction

In the discipline of computer science and mathematics, graph theory [1] is the survey of graphs that considers the link between edges and vertices. This is the most celebrated discipline these days that has applications [2] in computer science, information technology, biosciences, mathematics, social sciences, physics, chemistry, and linguistics. To illustrate pairwise relationship of objects, graph theory analysis is very important [3, 4].

Formally, a graph is the collection of vertices and edges. Among several types of different graphs, we will analyze a particular class of graph known as convex polytopes [5]. Convex polytopes are the principal geometric structures which are under investigation since antiquity. The charm of this concept is nowadays complemented by their significance for various mathematical fields, extending from algebraic geometry, linear programming, integration, and combinatorial optimization. Convex polytope is the simplest kind of polytopes [6] which satisfies the property of convex set in k-dimensional Euclidean space R^k. Convex polytopes play a vital part in enormous areas of mathematics as well as in applied disciplines, but its role in linear programming is most influential [7, 8].

Moreover, subdividing is a process in which we add an extra vertex on each edge of the graph in such a way such that each will be splitted into two edges, and the resulting graph is called subdivided graph of the original graph G. Since a couple of years, the variables associated with distances in graphs have enchanted the focus of various researchers, but in the recent years, the phenomenon that has centered certain surveys is termed as metric dimension [9]. The distance $d(z_1, z_2)$ from vertex $z_1 \in V(G)$ to $z_2 \in V(G)$ is minimum length of (z_1, z_2)-path in a given connected graph G having $E(G)$ and $V(G)$ edges and vertices'/nodes' sets, respectively. Suppose $Z = \{z_1, z_2, z_3, \ldots, z_m\} \subseteq V(G)$ is an order set and $c \in V(G)$; the code of c with reference to Z is the m-tuple $\{d(c, z_1), d(c, z_2), d(c, z_{13}), \ldots, d(c, z_k)\}$. Then, Z is named as the locating set or resolving set if each node of G has a unique code. A locating set of least cardinality is described as a basis set for the graph G, and its cardinal number is referred as metric dimension symbolized by $\dim(G)$. Moreover, if every member of a family of graphs possess the same metric dimension, then it is known as family with constant metric dimension [10, 11].

To find intruder's location in a network, Slater, in [12, 13], described metric dimension. Harary and Melter [14] are also the founder of metric dimension. The concept of constant metric dimension of certain class of convex polytopes is studied in [15, 16]. Generalization of Petersen graphs with its Bounden metric dimension has been studied in [3]. It is given in [17] that if we remove the join of the nodes b_i and b_{i+1}, the resulting class of convex polytopes is T_n, as given in Figure 1, where both the categories of convex polytopes have identical metric dimension 3.

Metric dimension of a specific class of convex polytope has been studied in this paper. The scope of metric dimension is broadened, the generalized family of convex polytope undergoes subdivision of T_n, and the metric dimension of subdivided convex polytope ST_n has been determined. The set of vertices that belongs to the graph of subdivided convex polytope ST_n consists of $\{f_i; n \geq i \geq 1 \text{ and } n = 2k\} \cup \{g_i; n \geq i \geq 1 \text{ and } n = 2k\} \cup \{h_i; 1 \leq i \leq k\} \cup \{l_i; 1 \leq i \leq k\} \cup \{m_i; 1 \leq i \leq k \text{ and } n = 2k\}$. The family of vertices on inner cycle is $\{f_i; n \geq i \geq 1 \text{ and } n = 2k\}$ and collection of vertices of outer cycle is $\{m_i; 1 \leq i \leq k \text{ and } n = 2k\}$. The collection of vertices assigned to central cycles is $\{h_i; 1 \leq i \leq k\} \cup \{l_i; 1 \leq i \leq k\}$. This study comprises of three main theorems which define its scope. Minimal cardinal number for the locating set of subdivided convex polytope ST_n is greater than two and is the topic of first theorem. In the second theorem, the work is extended to prove that minimal cardinality of the resolving set of convex polytope is greater than 3. Finally, it is demonstrated in the third theorem that the subdivided convex polytope ST_n has metric dimension equal to 4, and the formulation for the subdivided convex polytope ST_n is presented, as given in Figure 2. Lemma 1 and Theorem 1 presented in [15] were useful in finding the results of this paper.

Lemma 1. *Let Z be a locating set for a connected graph G and $z_1, z_2 \in V(G)$. If $d(z_1, z) \neq d(z_2, z), \forall z \in V(G) \setminus \{z_1, z_2\}$, then $\{z_1, z_2\} \cap Z \neq \phi$.*

Theorem 1. *Let G be a graph with metric dimension 2, and let the basis set in G be $\{z_1, z_2\} \subset V(G)$. Then, the following are true:*

(a) There is a unique path between z_1 and z_2

(b) The degree of each z_1 and z_2 is at most 3

2. Main Results

Theorem 2. *Prove that minimal cardinality for the locating set of subdivided convex polytope ST_n is greater than 2.*

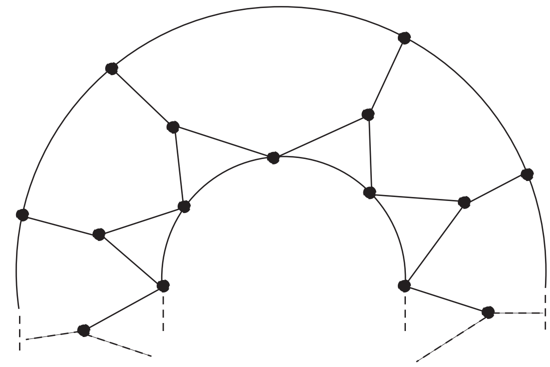

FIGURE 1: Convex polytope T_n.

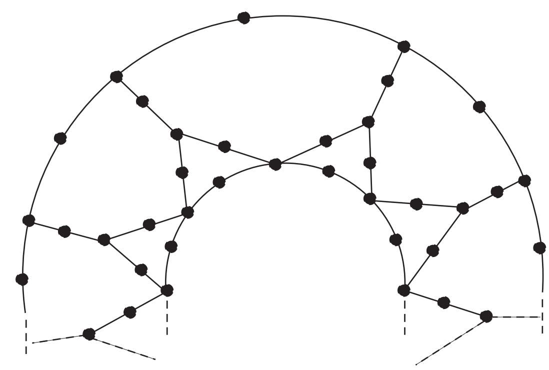

FIGURE 2: Subdivided convex polytope ST_n.

Proof. To prove that minimal cardinality for the locating set of subdivided convex polytope ST_n is greater than 2, the following cases exist.

Case I: both locating nodes have a place in the inner cycle. Without loss of generality, we can assume that the first locating node is f_1. Assume that the other locating node is $f_t\,(k+1 \geq t \geq 2)$. Thus, $Z = \{f_1, f_t\}$ is the possible locating set. If t varies from 2 to k, then $r(f_n/\{f_1, f_t\}) = (1, t) = r(g_n/\{f_1, f_t\})$ and $r(g_1/\{f_1, f_t\}) = r(g_n/\{f_1, f_t\}) = (1, t)$, for $t = k+1$, which contradict unique representation.

Case II: one vertex has a place on the inner cycle, and the other has a place with the family of points whose distance from the inward cycle is 1. With no loss of universality, we can assume that the first locating node is f_1. Assume the other node is $g_t\,(k+1 \geq t \geq 1)$. Thus, $Z = \{f_1, g_t\}$ is the possible locating set. If t varies from 1 to $k-1$ when t is odd, we get $r(g_{n-2}/\{f_1, g_t\}) = r(g_{n-1}/\{f_1, g_t\}) = (3, t+3)$, and if $2 \leq t \leq k$, t is even, we get $r(g_{n-2}/\{f_1, g_t\}) = r(g_{n-1}/\{f_1, g_t\}) = (3, t+4)$ and $r(g_{n-2}/\{f_1, g_t\}) = r(g_{n-1}/\{f_1, g_t\}) = (3, t-1)$ for $t = k+1$, which contradict unique representation.

Case III: when one vertex has a place on the inner cycle and the other has a place with the family of points whose distance from the inner cycle is 2. Without loss of universality, we can assume that the first locating node is f_1 and the other locating node is $h_t\,(k/2+1 \geq t \geq 1)$. Thus, $Z=\{f_1,h_t\}$ is the possible locating set. If $1 \leq t \leq k/2$, where t is odd, then we get $r(g_{n-2}/\{f_1,h_t\}) = r(g_{n-1}/\{f_1,h_t\}) = (3,t+4)$, and if $2 \leq t \leq k/2+1$, where t is even, we get $r(g_1/\{f_1,h_t\}) = r(g_n/\{f_1,h_t\}) = (1,t+3)$, which contradict unique representation.

Case IV: when one vertex has a place on the inner cycle and the other has a place with the family of points whose distance from the inner cycle is 3. Without loss of universality, we are able to assume that the first locating node is f_1 and the other locating node is $l_t\,(k/2+1 \geq t \geq 1)$. Thus, $Z=\{f_1,l_t\}$ is the possible locating set. If t varies from 1 to $k/2$ when t is odd, we get $r(g_{n-2}/\{f_1,l_t\}) = r(g_{n-1}/\{f_1,l_t\}) = (3,t+5)$, and if $2 \leq t \leq k/2+1$, where t is even, then we get $r(g_1/\{f_1,l_t\}) = r(g_n/\{f_1,l_t\}) = (1,t+4)$, which contradict unique representation.

Case V: one vertex has a place on the inner cycle and the other has a place with the family of points whose distance from the inner cycle is 4. Without loss of universality, we are able to assume that the first locating node is f_1 and the other locating node is $m_t\,(k+1 \geq t \geq 1)$. Thus, $Z=\{f_1,m_t\}$ is the possible locating set. If t varies from 1 to $k-1$, we get $r(f_n/\{f_1,m_t\}) = (1,t+4) = r(g_n/\{f_1,m_t\})$ and $r(g_1/\{f_1,m_t\}) = (1,t+2) = r(g_n/\{f_1,m_t\})$ when $t=k$, $k+1$ which contradict unique representation.

Case VI: both vertices have a place with the family of points whose distance from the inner cycle is 1. Without loss of universality, we are able to assume that the first locating node is g_1 and the other node is $g_t\,(k+1 \geq t \geq 2)$. Thus, $Z=\{g_1,g_t\}$ is the possible locating set. If t varies from 2 to k when t is even, we get $r(g_{n-2}/\{g_1,g_t\}) = r(g_{n-1}/\{g_1,g_t\}) = (4,t+4)$, and if $k/2 \leq t \leq k-1$, where t is odd, we get $r(g_{n-2}/\{g_1,g_t\}) = r(g_{n-1}/\{g_1,g_t\}) = (4,t+3)$, and if $t=k+1$, then $r(g_{n-2}/\{g_1,g_t\}) = r(g_{n-1}/\{g_1,g_t\}) = (4,t-1)$, which contradict unique representation.

Case VII: family of vertices whose distance from the inward cycle is 1 and 2, respectively. Without loss of universality, we are able to assume that the first locating node is g_1 and the other node is $h_t\,(1 \leq t \leq k/2+1)$. Thus, $Z=\{g_1,h_t\}$ is the possible locating set. If t varies from 1 to $k/2$ when t is odd, we get $r(g_{n-2}/\{g_1,h_t\}) = r(g_{n-1}/\{g_1,h_t\}) = (4,t+4)$, and if $2 \leq t \leq k/2+1$, where t is even, then we get $r(m_{n-8}/\{g_1,h_t\}) = r(g_{n-7}/\{g_1,h_t\}) = (6,t+1)$, which contradict unique representation.

Case VIII: family of vertices whose distance from the inner cycle is 1 and 3, respectively. Without loss of universality, we are able to assume that the first locating node is g_1 and the other locating node is $l_t\,(k/2+1 \geq t \geq 1)$. Thus, $Z=\{g_1,l_t\}$ is the possible locating set. If t varies from 1 to $k/2$ when t is odd, we get $r(g_{n-2}/\{g_1,l_t\}) = r(g_{n-1}/\{g_1,l_t\}) = (4,t+5)$, and if $2 \leq t \leq k/2+1$, where t is even, then we get $r(f_{n-8}/\{g_1,l_t\}) = r(m_n/\{g_1,l_t\}) = (4,t+2)$, which contradict unique representation.

Case IX: family of vertices whose distance from the inner cycle is 1 and 4, respectively. Without loss of universality, we are able to assume that the first locating node is g_1 and the other node is $m_t\,(k+1 \geq t \geq 1)$. Thus, $Z=\{g_1,m_t\}$ is the possible locating set. If t varies from 1 to $k-1$, we get $r(f_n/\{g_1,m_t\}) = (2,t+4) = r(g_n/\{g_1,m_t\})$ and $r(f_2/\{g_1,m_t\}) = r(g_n/\{g_1,m_t\}) = (2,t+2)$ when $t=k$, $k+1$, which contradict unique representation.

Case X: both vertices have a place with the family of points whose distance from the inner cycle is 2. Without loss of universality, we are able to assume that the first locating node is h_1 and the other node is $h_t\,(k/2+1 \geq t \geq 2)$. Thus, $Z=\{h_1,h_t\}$ is the possible locating set. If t varies from 2 to $k/2$ when t is odd, we get $r(h_{14}/\{h_1,h_t\}) = r(m_{n-5}/\{h_1,h_t\}) = (8,t+9)$, and if $2 \leq t \leq k/2$, where t is even, we get $r(g_6/\{h_1,h_t\}) = r(g_7/\{h_1,h_t\}) = (7,t+3)$, and if $t=k/2+1$, we get $r(h_{14}/\{h_1,h_t\}) = r(m_{n-5}/\{h_1,h_t\}) = (8,t+3)$, which contradict unique representation.

Case XI: family of vertices whose distance from the inner cycle is 2 and 3, respectively. Without loss of universality, we are able to assume that the first locating node is h_1 and the other node is $l_t\,(1 \leq t \leq k/2+1)$. Thus, $Z=\{h_1,l_t\}$ is the possible locating set. If t varies from 1 to $k/2$ when t is odd, we get $r(f_2/\{h_1,l_t\}) = r(g_3/\{h_1,l_t\}) = (3,t+3)$, and if $k/2-1 \leq t \leq k/2+1$, where t is even, then we get $r(f_{n-8}/\{h_1,l_t\}) = r(m_n/\{h_1,l_t\}) = (3,t+2)$, which contradict unique representation.

Case XII: family of vertices whose distance from the inner cycle is 2 and 4, respectively. Without loss of universality, we are able to assume that the first locating node is h_1 and the other node is $m_t\,(k+1 \geq t \geq 1)$. Thus, $Z=\{h_1,m_t\}$ is the possible locating set. If t varies from 1 to $k+1$, we get $r(g_1/\{h_1,m_t\}) = (1,t+2) = r(g_2/\{h_1,m_t\})$, which contradict unique representation.

Case XIII: both the vertices have a place with the family of points whose distance from the inner cycle is 3. Without loss of universality, we are able to assume that the first locating node is l_1 and the other node is $l_t\,(k/2+1 \geq t \geq 2)$. Thus, $Z=\{l_1,l_t\}$ is the possible locating set. If t varies from 2 to $k/2$, where t is odd, we get $r(f_{n-4}/\{l_1,l_t\}) = r(l_{14}/\{l_1,l_t\}) = (8,t+9)$, and if 2 $2 \leq t \leq k/2$, where t is even, we get $r(g_6/\{l_1,l_t\}) = r(g_7/\{l_1,l_t\}) = (8,t+4)$, and if $t=k/2+1$, then we get $r(f_{n-4}/\{l_1,l_t\}) = r(l_{14}/\{l_1,l_t\}) = (8,t+3)$, which contradict unique representation.

Case XIV: family of vertices whose distance from the inner cycle is 3 and 4 respectively. Without loss of universality, we are able to assume that the first locating node is l_1 and the other node is $m_t\,(k+1 \geq t \geq 1)$. Thus, $Z=\{l_1,m_t\}$ is the possible locating set. If t varies from 1 to $k+1$, we get $r(g_1/\{l_1, m_t\}) = (2, t+2) = r(g_2/\{l_1, m_t\})$, which contradict unique representation.

Case XV: both vertices have a place on the outer cycle. Without loss of universality, we are able to assume that the first locating node is m_1 and the other node is $m_t\,(k+1 \geq t \geq 2)$. Thus, $Z = \{m_1, m_t\}$ is the possible locating set. If t varies from 2 to k, we get $r(l_1/\{m_1, m_t\}) = (1, t) = r(m_n/\{m_1, m_t\})$ and $r(m_2/\{m_1, m_t\}) = r(m_n/\{m_1, m_t\}) = (1, t-2)$ when $t = k+1$, which contradict unique representation. From all the above cases, it is concluded that, for unique representation of each node, cardinality of the locating set is greater than two. Hence, the result is obtained. □

Theorem 3. *Prove that minimal cardinality for the locating set of subdivided convex polytope ST_n is greater than 3.*

Proof. To prove that minimal cardinality for the locating set of subdivided convex polytope ST_n is greater than 3, the following cases exist.

Case I: all the three vertices have a place on the inner cycle. Without loss of universality, we are able to assume that the first resolving vertex is f_1. Let the second resolving vertex is $f_r\,(2 \leq r \leq k)$. Suppose the third resolving vertex is $f_t\,(r+1 \leq t \leq k+1)$. Hence, $Z = \{f_1, f_r, f_t\}$. If $2 \leq r \leq k-1$ and $r+1 \leq t \leq k-1$, we get $r(g_{n-2}/\{f_1, f_r, f_t\}) = r(g_{n-1}/\{f_1, f_r, f_t\}) = (3, r+2, t+2)$, and if $2 \leq r \leq k-1$, $t = k$, we get $r(g_{n-2}/\{f_1, f_r, f_t\}) = r(g_{n-1}/\{f_1, f_r, f_t\}) = (3, r+2, t)$, and if $r = k$, $t = k+1$, we get $r(g_{n-2}/\{f_1, f_r, f_t\}) = r(g_{n-1}/\{f_1, f_r, f_t\}) = (3, r, t-2)$, which contradict unique representation.

Case II: one vertex has a place on the inner cycle and the other two have a place with the family of points whose distance from the inner cycle is 1. Without loss of universality, we are able to assume that the first resolving vertex is f_1. Let the second resolving vertex is $g_r\,(1 \leq r \leq k)$. Suppose the third resolving vertex is $g_t\,(r+1 \leq t \leq k+1)$. Hence, $Z = \{f_1, g_r, g_t\}$. If $1 \leq r \leq k-1$, where r is even and $r+1 \leq t \leq k-1$, where t is odd, we get $r(g_{n-2}/\{f_1, g_r, g_t\}) = r(g_{n-1}/\{f_1, g_r, g_t\}) = (3, r+4, t+3)$; if $1 \leq r \leq k-1$, where r is odd and $r+1 \leq t \leq k-1$, where t is even, we get $r(g_{n-2}/\{f_1, g_r, g_t\}) = r(g_{n-1}/\{f_1, g_r, g_t\}) = (3, r+3, t+4)$; if $1 \leq r \leq k-1$, where r is even and $t = k$, we get $r(g_{n-2}/\{f_1, g_r, g_t\}) = r(g_{n-1}/\{f_1, g_r, g_t\}) = (3, r+4, t)$; if $1 \leq r \leq k-1$, where r is odd and $t = k$, we get $r(g_{n-2}/\{f_1, g_r, g_t\}) = r(g_{n-1}/\{f_1, g_r, g_t\}) = (3, r+3, t)$; if $1 \leq r \leq k-1$, where r is even and $t = k+1$, we get $r(g_{n-2}/\{f_1, g_r, g_t\}) = r(g_{n-1}/\{f_1, g_r, g_t\}) = (3, r+4, t-1)$; if $1 \leq r \leq k-1$, r is odd and $t = k+1$, we get $r(g_{n-2}/\{f_1, g_r, g_t\}) = r(g_{n-1}/\{f_1, g_r, g_t\}) = (3, r+3, t-1)$; if $r = k$ and $t = k+1$, we get $r(g_{n-2}/\{f_1, g_r, g_t\}) = r(g_{n-1}/\{f_1, g_r, g_t\}) = (3, r, t-1)$, which contradict unique representation.

Case III: one vertex has a place on the inner cycle and the other two have a place with the family of points whose distance from the inner cycle is 2. Without loss of universality, we are able to assume that the first resolving vertex is f_1. Let the second resolving vertex be $h_r\,(1 \leq r \leq k/2)$. Suppose the third resolving vertex is $h_t\,(r+1 \leq t \leq k/2+1)$. Hence, $Z = \{f_1, h_r, h_t\}$. If $1 \leq r \leq k/2$, where r is odd and $r+1 \leq t \leq k/2-1$, we get $r(g_{n-2}/\{f_1, h_r, h_t\}) = r(g_{n-1}/\{f_1, h_r, h_t\}) = (3, r+4, t+5)$; if $1 \leq r \leq k/2$, where r is even and $t = k/2$, we get $(3, r+5, t+4)$; if $1 \leq r \leq k/2$, where r is odd and $t = k/2$, we get $(3, r+4, t+4)$; if $1 \leq r \leq k/2$, where r is odd and $t = k/2+1$, we get $r(g_{n-2}/\{f_1, h_r, h_t\}) = r(g_{n-1}/\{f_1, h_r, h_t\}) = (3, r+4, t+1)$; if $1 \leq r \leq k/2$, where r is even and $t = k/2+1$, we get $r(g_{n-2}/\{f_1, h_r, h_t\}) = r(g_{n-1}/\{f_1, h_r, h_t\}) = (3, r+5, t+1)$, which contradict unique representation.

Case IV: one vertex has a place on the inner cycle and the other two have a place with the family of points whose distance from the inner cycle is 3. Without loss of universality, we are able to assume that the first resolving vertex is f_1. Let the second resolving vertex be $l_r\,(1 \leq r \leq k/2)$. Suppose the third resolving vertex is $l_t\,(r+1 \leq t \leq k/2+1)$. Hence, $Z = \{f_1, l_r, l_t\}$. If $1 \leq r \leq k/2$, where r is odd and $r+1 \leq t \leq k/2-1$, we get $r(g_{n-2}/\{f_1, l_r, l_t\}) = r(g_{n-1}/\{f_1, l_r, l_t\}) = (3, r+5, t+6)$; if $1 \leq r \leq k/2$, where r is even and $t = k/2$, we get $r(g_{n-2}/\{f_1, l_r, l_t\}) = r(g_{n-1}/\{f_1, l_r, l_t\}) = (3, r+6, t+5)$; if $1 \leq r \leq k/2$, where r is odd and $t = k/2$, we get $r(g_{n-2}/\{f_1, l_r, l_t\}) = r(g_{n-1}/\{f_1, l_r, l_t\}) = (3, r+5, t+5)$; if $1 \leq r \leq k/2$, where r is odd and $t = k/2+1$, we get $r(g_{n-2}/\{f_1, l_r, l_t\}) = r(g_{n-1}/\{f_1, l_r, l_t\}) = (3, r+5, t+2)$; if $1 \leq r \leq k/2$, where r is even and $t = k/2+1$, then we get $r(g_{n-2}/\{f_1, l_r, l_t\}) = r(g_{n-1}/\{f_1, l_r, l_t\}) = (3, r+6, t+2)$, which contradict unique representation.

Case V: one vertex has a place on the inner cycle and the other two have a place with the family of points whose distance from the inner cycle is 4. Without loss of universality, we are able to assume that the first resolving vertex is f_1. Let the second resolving vertex is $m_r\,(1 \leq r \leq k)$. Suppose the third resolving vertex is $m_t\,(r+1 \leq t \leq k+1)$. Hence, $Z = \{f_1, m_r, m_t\}$. If $1 \leq r \leq k-1$ and $r+1 \leq t \leq k-1$, we get $r(f_n/\{f_1, m_r, m_t\}) = r(g_n/\{f_1, m_r, m_t\}) = (1, r+4, t+4)$; if $1 \leq r \leq k-1$ and $t = k$, we get $r(f_n/\{f_1, m_r, m_t\}) = r(g_n/\{f_1, m_r, m_t\}) = (1, r+4, t+2)$; if $1 \leq r \leq k-1$ and $t = k+1$, we get $r(f_n/\{f_1, m_r, m_t\}) = r(g_n/\{f_1, m_r, m_t\}) = (1, r+4, t)$, and if $r = k$ and $t = k+1$, we get $r(f_n/\{f_1, m_r, m_t\}) = r(g_n/\{f_1, m_r, m_t\}) = (1, r+2, t)$, which contradict unique representation.

Case VI: all the three vertices have a place with the family of points whose distance from the inner cycle is 1. Without loss of universality, we are able to assume that the first resolving vertex is g_1. Let the second resolving vertex be $g_r\,(2 \leq r \leq k)$. Suppose the third resolving vertex is $g_t\,(r+1 \leq t \leq k+1)$. Hence, $Z = \{g_1, g_r, g_t\}$. If $2 \leq r \leq k-1$, where r is even and

$r+1 \leq t \leq k-1$, where t is odd, we get $r(g_{n-2}/\{g_1, g_r, g_t\}) = r(g_{n-1}/\{g_1, g_r, g_t\}) = (4, r+4, t+3)$; if $2 \leq r \leq k-1$, where r is odd and $r+1 \leq t \leq k-1$, where t is even, we get $r(g_{n-2}/\{g_1, g_r, g_t\}) = r(g_{n-1}/\{g_1, g_r, g_t\}) = (4, r+3, t+4)$; if $2 \leq r \leq k-1$, where r is even and $t=k$, we get $r(g_{n-2}/\{g_1, g_r, g_t\}) = r(g_{n-1}/\{g_1, g_r, g_t\}) = (4, r+4, t)$; if $2 \leq r \leq k-1$, where r is odd and $t=k$, we get $r(g_{n-2}/\{g_1, g_r, g_t\}) = r(g_{n-1}/\{g_1, g_r, g_t\}) = (4, r+3, t)$; if $2 \leq r \leq k-1$, where r is even and $t=k+1$, we get $r(g_{n-2}/\{g_1, g_r, g_t\}) = r(g_{n-1}/\{g_1, g_r, g_t\}) = (4, r+4, t-1)$; if $2 \leq r \leq k-1$, where r is odd and $t=k+1$, we get $r(g_{n-2}/\{g_1, g_r, g_t\}) = r(g_{n-1}/\{g_1, g_r, g_t\}) = (4, r+3, t-1)$; if $r=k$ and $t=k+1$, we get $r(g_{n-2}/\{g_1, g_r, g_t\}) = r(g_{n-1}/\{g_1, g_r, g_t\}) = (4, r, t-1)$, which contradict unique representation.

Case VII: family of vertices whose distance from the inner cycle is 1 and 2, respectively. Without loss of universality, we are able to assume that the first resolving vertex is g_1. Let the second resolving vertex be $h_r (1 \leq r \leq k/2)$. Suppose the third resolving vertex is $h_t (r+1 \leq t \leq k/2+1)$. Hence, $Z = \{g_r, h_r, h_t\}$. If $1 \leq r \leq k/2$, where r is odd and $r+1 \leq t \leq k/2-1$, we get $r(g_{n-2}/\{g_r, h_r, h_t\}) = r(g_{n-1}/\{g_r, h_r, h_t\}) = (4, r+4, t+5)$; if $1 \leq r \leq k/2$, where r is even and $t=k/2$, we get $r(g_{n-2}/\{g_r, h_r, h_t\}) = r(g_{n-1}/\{g_r, h_r, h_t\}) = (4, r+5, t+4)$; if $1 \leq r \leq k/2$, where r is odd and $t=k/2$, we get $(4, r+4, t+4)$; if $1 \leq r \leq k/2$, where r is odd and $t=k/2+1$, we get $r(g_{n-2}/\{g_r, h_r, h_t\}) = r(g_{n-1}/\{g_r, h_r, h_t\}) = (4, r+4, t+1)$; if $1 \leq r \leq k/2$, where r is even and $t=k/2+1$, we get $r(g_{n-2}/\{g_r, h_r, h_t\}) = r(g_{n-1}/\{g_r, h_r, h_t\}) = (4, r+5, t+1)$, which contradict unique representation.

Case VIII: family of vertices whose distance from the inner cycle is 1 and 3, respectively. Without loss of universality, we are able to assume that the first resolving vertex is g_1. Let the second resolving vertex be $l_r (1 \leq r \leq k/2)$. Suppose the third resolving verte is $l_t (r+1 \leq t \leq k/2+1)$. Hence, $Z = \{g_1, l_r, l_t\}$. If $1 \leq r \leq k/2$, where r is odd and $r+1 \leq t \leq k/2-1$, we get $r(g_{n-2}/\{g_1, l_r, l_t\}) = r(g_{n-1}/\{g_1, l_r, l_t\}) = (4, r+5, t+6)$; if $1 \leq r \leq k/2$, where r is even and $t=k/2$, we get $r(g_{n-2}/\{g_1, l_r, l_t\}) = r(g_{n-1}/\{g_1, l_r, l_t\}) = (4, r+6, t+5)$; if $1 \leq r \leq k/2$, where r is odd and $t=k/2$, we get $r(g_{n-2}/\{g_1, l_r, l_t\}) = r(g_{n-1}/\{g_1, l_r, l_t\}) = (4, r+5, t+5)$; if $1 \leq r \leq k/2$, where r is odd and $t=k/2+1$, we get $r(g_{n-2}/\{g_1, l_r, l_t\}) = r(g_{n-1}/\{g_1, l_r, l_t\}) = (4, r+5, t+2)$; if $1 \leq r \leq k/2$, where r is even and $t=k/2+1$, then we get $r(g_{n-2}/\{g_1, l_r, l_t\}) = r(g_{n-1}/\{g_1, l_r, l_t\}) = (4, r+6, t+2)$, which contradict unique representation.

Case IX: family of vertices whose distance from the inner cycle is 1 and 4, respectively. Without loss of universality, we are able to assume that the first resolving vertex is g_1. Let the second resolving vertex be $m_r (1 \leq r \leq k)$. Suppose the third resolving vertex is $m_t (r+1 \leq t \leq k+1)$. Hence, $Z = \{g_1, m_r, m_t\}$. If $1 \leq r \leq k-1$ and $r+1 \leq t \leq k-1$, we get $r(f_n/\{g_1, m_r, m_t\}) = r(g_n/\{g_1, m_r, m_t\}) = (2, r+4, t+4)$; if $1 \leq r \leq k-1$ and $t=k$, we get $r(f_n/\{g_1, m_r, m_t\}) = r(g_n/\{g_1, m_r, m_t\}) = (2, r+4, t+2)$; if $1 \leq r \leq k-1$ and $t=k+1$, we get $r(f_n/\{g_1, m_r, m_t\}) = r(g_n/\{g_1, m_r, m_t\}) = (2, r+4, t)$, and if $r=k$ and $t=k+1$, we get $r(f_n/\{g_1, m_r, m_t\}) = r(g_n/\{g_1, m_r, m_t\}) = (2, r+2, t)$, which contradict unique representation.

Case X: all the three vertices have a place with the family of points whose distance from the inner cycle is 2. Without loss of universality, we are able to assume that the first resolving vertex is h_1. Let the second resolving vertex is $h_r (2 \leq r \leq k/2)$. Suppose the third resolving vertex is $h_t (r+1 \leq t \leq k/2+1)$. Hence, $Z = \{h_1, h_r, h_t\}$. If $2 \leq r \leq k/2$, where r is even and $r+1 \leq t \leq k/2+1$, where t is odd, we get $r(g_{n-2}/\{h_1, h_r, h_t\}) = r(g_{n-1}/\{\{h_1, h_r, h_t\}\}) = (4, r+5, t+4)$; if $2 \leq r \leq k/2$, where r is even and $r+1 \leq t \leq k/2+1$, where t is even, we get $r(g_{n-2}/\{h_1, h_r, h_t\}) = r(g_{n-1}/\{\{h_1, h_r, h_t\}\}) = (4, r+5, t+1)$; if $2 \leq r \leq k/2$, where r is odd and $r+1 \leq t \leq k/2+1$, where t is even, then we get $r(g_{n-2}/\{h_1, h_r, h_t\}) = r(g_{n-1}/\{\{h_1, h_r, h_t\}\}) = (4, r+4, t+1)$, which contradict unique representation.

Case XI: family of vertices whose distance from the inner cycle is 2 and 3, respectively. Without loss of universality, we are able to assume that the first resolving vertex is h_1. Let the second resolving vertex be $l_r (1 \leq r \leq k/2)$. Suppose the third resolving vertex is $l_t (r+1 \leq t \leq k/2+1)$. Hence, $Z = \{h_1, l_r, l_t\}$. If $1 \leq r \leq k/2$, where r is odd and $r+1 \leq t \leq k/2-1$, we get $r(g_{n-2}/\{h_1, l_r, l_t\}) = r(g_{n-1}/\{\{h_1, l_r, l_t\}\}) = (5, r+5, t+6)$; if $1 \leq r \leq k/2$, where r is even and $t=k/2$, we get $r(g_{n-2}/\{h_1, l_r, l_t\}) = r(g_{n-1}/\{\{h_1, l_r, l_t\}\}) = (5, r+6, t+5)$; if $1 \leq r \leq k/2$, where r is odd and $t=k/2$, we get $r(g_{n-2}/\{h_1, l_r, l_t\}) = r(g_{n-1}/\{\{h_1, l_r, l_t\}\}) = (5, r+5, t+5)$; if $1 \leq r \leq k/2$, where r is odd and $t=k/2+1$, we get $r(g_{n-2}/\{h_1, l_r, l_t\}) = r(g_{n-1}/\{\{h_1, l_r, l_t\}\}) = (5, r+5, t+2)$; if $1 \leq r \leq k/2$, where r is even and $t=k/2+1$, then we get $r(g_{n-2}/\{h_1, l_r, l_t\}) = r(g_{n-1}/\{\{h_1, l_r, l_t\}\}) = (5, r+6, t+2)$, which contradict unique representation.

Case XII: family of vertices whose distance from the inward cycle is 2 and 4, respectively. With no loss of universality, we are able to assume that the first resolving vertex is h_1. Let the second resolving vertex be $m_r (1 \leq r \leq k)$. Suppose the third resolving vertex is $m_t (r+1 \leq t \leq k+1)$. Hence, $Z = \{h_1, m_r, m_t\}$. If $1 \leq r \leq k-1$ and $r+1 \leq t \leq k-1$, we get $r(f_n/\{h_1, m_r, m_t\}) = r(g_n/\{h_1, m_r, m_t\}) = (3, r+4, t+4)$; if $1 \leq r \leq k-1$ and $t=k$, we get $r(f_n/\{h_1, m_r, m_t\}) = r(g_n/\{h_1, m_r, m_t\}) = (3, r+4, t+2)$; if $1 \leq r \leq k-1$ and $t=k+1$, we get $r(f_n/\{h_1, m_r, m_t\}) = r(g_n/\{h_1, m_r, m_t\}) = (3, r+4, t)$, and if $r=k$ and $t=k+1$, we get $r(f_n/\{h_1, m_r, m_t\}) = r(g_n/\{h_1, m_r, m_t\}) = (3, r+2, t)$, which contradict unique representation.

Case XIII: all the three vertices have a place with the family of points whose distance from the inner cycle is 3. Without loss of universality, we are able to assume that the first resolving vertex is l_1. Let the second resolving vertex be $l_r\,(2\le r\le k/2)$. Suppose the third resolving vertex is $l_t\,(r+1\le t\le k/2+1)$. Hence, $Z=\{l_1,l_r,l_t\}$. If $2\le r\le k/2$, where r is even and $r+1\le t\le k/2+1$, where t is odd, we get $r(g_{n-2}/\{l_1,l_r,l_t\})=r(g_{n-1}/\{\{l_1,l_r,l_t\}\})=(6,r+6,t+5)$; if $2\le r\le k/2$, where r is even and $r+1\le t\le k/2+1$, where t is even, we get $r(g_{n-2}/\{l_1,l_r,l_t\})=r(g_{n-1}/\{\{l_1,l_r,l_t\}\})=(6,r+6,t+2)$; if $2\le r\le k/2$, where r is odd and $r+1\le t\le k/2+1$, where t is even, then we get $r(g_{n-2}/\{l_1,l_r,l_t\})=r(g_{n-1}/\{\{l_1,l_r,l_t\}\})=(6,r+5,t+2)$, which contradict unique representation.

Case XIV: family of vertices whose distance from the inner cycle is 3 and 4, respectively. Without loss of universality, we are able to assume that the first resolving vertex is l_1. Let the second resolving vertex be $m_r\,(1\le r\le k)$. Suppose the third resolving vertex is $m_t\,(r+1\le t\le k+1)$. Hence, $Z=\{l_1,m_r,m_t\}$. If $1\le r\le k-1$ and $r+1\le t\le k-1$, we get $r(f_n/\{l_1,m_r,m_t\})=r(g_n/\{l_1,m_r,m_t\})=(4,r+4,t+4)$; if $1\le r\le k-1$ and $t=k$, we get $r(f_n/\{l_1,m_r,m_t\})=r(g_n/\{l_1,m_r,m_t\})=(4,r+4,t+2)$; if $1\le r\le k-1$ and $t=k+1$, we get $r(f_n/\{l_1,m_r,m_t\})=r(g_n/\{l_1,m_r,m_t\})=(4,r+4,t)$, and if $r=k$ and $t=k+1$, we get $r(f_n/\{l_1,m_r,m_t\})=r(g_n/\{l_1,m_r,m_t\})=(4,r+2,t)$, which contradict unique representation.

Case XV: all the three vertices have a place on the outer cycle. Without loss of universality, we are able to assume that the first resolving vertex is m_1. Let the second resolving vertex be $m_r\,(2\le r\le k)$. Suppose the third resolving vertex is $m_t\,(r+1\le t\le k+1)$. Hence, $Z=\{m_1,m_r,m_t\}$. If $2\le r\le k-1$ and $r+1\le t\le k-1$, we get $r(g_{n-2}/\{m_1,m_r,m_t\})=r(g_{n-1}/\{\{m_1,m_r,m_t\}\})=(5,r+4,t+4)$, and if $2\le r\le k-1$ and $t=k$, we get $r(g_{n-2}/\{m_1,m_r,m_t\})=r(g_{n-1}/\{\{m_1,m_r,m_t\}\})=(5,r+4,t+2)$, and if $r=k$ and $t=k+1$, we get $r(g_{n-2}/\{m_1,m_r,m_t\})=r(g_{n-1}/\{\{m_1,m_r,m_t\}\})=(5,r+4,t)$, which contradict unique representation. □

Theorem 4. *Prove that metric dimension of a subdivided convex polytope ST_n denoted by $d(ST_n)$ is 4.*

Proof. In this theorem, we take $n=2q, q\ge 3, q\in Z^+$. Assume that $Z=\{f_1,f_q,g_{q+1},m_{q+5}\}\subseteq V(ST_n)$; we will prove that Z is a locating set of ST_n which implies that dim $(ST_n)\le 4$. To prove that Z is the locating set, we will just prove that representation of each node with respect to Z is unique.

First of all, we find unique representations of the collection $\{f_1,f_2,f_3,\ldots,f_n\}$:

$$r\left(\frac{f_s}{Z}\right)=\begin{cases}\left(1-s+2q,-q+s,-q+s,2+s-\dfrac{3q}{2}\right), & 2q\ge s\ge\dfrac{3q}{2}+1,\\ (1-s+2q,-q+s,-q+s,-1-s+2q), & \dfrac{3q}{2}\ge s\ge q+1,\\ (-1+s,-s+q,2-s+q,-1-s+2q), & q\ge s\ge\dfrac{q}{2}+2,\\ (-1+s,-s+q,2-s+q,-3+q+s) & \dfrac{q}{2}+1\ge r\ge 2.\end{cases}\tag{1}$$

Representations of the family of vertices $\{g_1,g_2,g_3,\ldots,g_n\}$ are

Case I: when s is odd, then

$$r\left(\frac{g_s}{Z}\right)=\begin{cases}(2q-s+2,s-q+1,s-q+1,s-q-2), & 2q-1\ge s\ge q+7,\\ (2q-s+2,s-q+1,s-q+1,2q-s-2), & q+5\ge s\ge q+3,\\ (s,q-s+1,q-s+3,2q-s-2), & q+1\ge s\ge\dfrac{q}{2}+2,\\ (s,q-s+1,q-s+3,q+s-1), & \dfrac{q}{2}\ge s\ge 1.\end{cases}\tag{2}$$

Case II: when s is even, then

$$r\left(\frac{g_s}{Z}\right)=\begin{cases}(1+2q-s,q,-8+q,-3+s-q), & 2q\geq s\geq q+8,\\ (1+2q-s,-8+s,-8+s,-1+2q-s), & q+6\geq s\geq q+4,\\ (-1+q,-4+2q-s,2,-1+2q-s), & s=q+2,\\ (1+s,2,2+q-s,-1+2q-s), & q\geq s\geq \frac{q}{2}+3,\\ (1+s,q-s,2+q-s,-3+q+s). & \frac{q}{2}+1\geq s\geq 2.\end{cases} \tag{3}$$

Representations of the family of central vertices $\{h_1,h_2,h_3,\ldots,h_n\}$ are

$$r\left(\frac{h_s}{Z}\right)=\begin{cases}(2q-2s+2,2s-q+1,2s-q+1,2s-q-4), & q\geq s\geq \frac{q}{2}+4,\\ (2q-2s+2,2s-q+1,2s-q+1,2q-2s-2), & \frac{q}{2}+3\geq s\geq \frac{q}{2}+2,\\ (q,s-3,s-5,2q-2s-2), & i=\frac{q}{2}+1,\\ (2s,3,q+3-2s,2q-2s-2), & \frac{q}{2}\geq s\geq \frac{q}{2}-1,\\ (2s,q+1-2s,q+3-2s,q+2s-4), & \frac{q}{2}-2\geq s\geq 1.\end{cases} \tag{4}$$

Representations of the family of vertices $\{l_1,l_2,l_3,\ldots,l_n\}$ are

$$r\left(\frac{l_s}{Z}\right)=\begin{cases}(2q-2s+3,2s-q+2,2s-q+2,2s-q-5), & q\geq s\geq \frac{q}{2}+4,\\ (2q-2s+3,2s-q+2,2s-q+2,2q-2s-3), & \frac{q}{2}+3\geq s\geq \frac{q}{2}+2,\\ (q+1,s-2,s-4,2q-2s-3), & s=\frac{q}{2}+1,\\ (2s+1,4,q+4-2s,2q-2s-3), & \frac{q}{2}\geq s\geq \frac{q}{2}-1,\\ (2s+1,q+2-2s,q+4-2s,q+2s-5), & \frac{q}{2}-2\geq s\geq 1.\end{cases} \tag{5}$$

Representations of the family of vertices $\{m_1, m_2, m_3, \ldots, m_n\}$ on the outward cycle are

$$r\left(\frac{m_i}{Z}\right) = \begin{cases} (s-q-5, s-8, s-8, s-q-5), & s = 2q, \\ (2q-s+3, s-6, s-8, s-q-5), & 2q-1 \geq s \geq q+5, \\ (2q-s+3, s-6, s-8, 2q-s-5), & q+4 \geq s \geq q+2, \\ (2q-s+3, 2q-s-4, q+4-s, q+5-s), & q+1 \geq s \geq q, \\ (s+3, q+4-s, q+4-s, q+5-s), & q-1 \geq s \geq \frac{q}{2}+3, \\ (s+3, q+2-s, q+4-s, q+5-s), & \frac{q}{2}+2 \geq s \geq \frac{q}{2}+1, \\ (s+3, q+2-s, q+4-s, s+5), & \frac{q}{2} \geq s \geq 1. \end{cases} \quad (6)$$

Thus, with the help of this formulation, we can simply demonstrate that no two such vertices are available that possess identical representations which implies that $\dim(T_n) \leq 4$ in this case. Theorems 2 and 3 imply that neither $\dim(ST_n) \neq 2$ nor $\dim(ST_n) \neq 3$, respectively, so it can be concluded that dim $\dim(ST_n) \geq 4$. Hence, $\dim(ST_n) = 4$. □

3. Conclusion

Metric dimension of convex polytope ST_n after subdividing the convex polytope T_n has been studied. It is proved that the metric dimension of subdivided convex polytope ST_n is finite and independent of the count of vertices in all these graphs, and just four vertices which are selected properly are sufficient to locate all the vertices of this family of convex polytopes. The authors also proved that the metric dimension of subdivided convex polytope ST_n is neither 2 nor 3. The researcher gave the formulation of the representation of each vertex of convex polytope with respect to the resolving set and disproved lower dimensions by making different cases.

Open problem: determine edge metric dimension of the convex polytopes ST_n.

Data Availability

The data used to support the working are cited within the article as references.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors contributed equally to this work.

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