Hindawi
Journal of Mathematics
Volume 2022, Article ID 3567824, 8 pages
https://doi.org/10.1155/2022/3567824

Research Article

Topological Indices of Families of Bistar and Corona Product of Graphs

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Received 8 August 2021; Revised 23 October 2021; Accepted 14 March 2022; Published 26 April 2022

Academic Editor: Ewa Rak

Topological indices are graph invariants that are used to correlate the physicochemical properties of a chemical compound with its (molecular) graph. In this study, we study certain degree-based topological indices such as Randić index, Zagreb indices, multiplicative Zagreb indices, Narumi–Katayama index, atom-bond connectivity index, augmented Zagreb index, geometric-arithmetic index, harmonic index, and sum-connectivity index for the bistar graphs and the corona product $K_m o K_n'$, where K_n' represents the complement of complete graph K_n.

1. Introduction

Graph theory techniques [1–4] have applications in chemistry, biology, physics, and computer science. Topological indices are graph invariants that are used to study graph topology. Along with computer networks, graph theory is a powerful tool in other research areas such as coding theory, database management systems, circuit design, secret-sharing schemes, and theoretical chemistry. Cheminformatics is the combination of technology, graph theory, and chemistry. It links organic substances structure and physiochemical properties by using some useful graph invariants and their associated molecular graph. A molecular graph contains sets of points and lines which shows the atoms and covalent bond in the molecule. These points and lines are known as vertices and edges in graph theory, respectively [5]. Theoretical study of underlying chemical structure using useful graph invariants is an appealing area of research in mathematical chemistry due to its practical applications in QSAR/QSPR investigation [6]. Topological indices are used to estimate the physicochemical properties of the chemical compounds. A topological index can be considered as a function that maps a graph to a nonnegative real number [7].

Some degree-based topological indices are Randić index, Zagreb indices, Narumi–Katayama and multiplicative Zagreb indices, atom-bond connectivity index, augmented Zagreb index, geometric-arithmetic index, harmonic index, and sum-connectivity index [8].

Throughout this work, we use standard notations $G = (V, E)$ for graph, $V(G)$ set of vertices, $E(G)$ set of edges, and $d_w(G)$ degree of vertex w in G.

For any simple graph $G = (V, E)$, $w \sim x$ denotes the adjacent vertices w and x in graph G.

Definition 1. Randić index is the most popular topological index among all topological indices and is defined as [9]

$$R(G) = \sum_{w\sim x} \frac{1}{\sqrt{d_w(G)d_x(G)}}. \quad (1)$$

Definition 2. The first Zagreb index of a graph G is defined as

$$M_1(G) = \sum_{w\in V(G)} d_w(G)^2. \quad (2)$$

The second Zagreb index of a graph G is defined as

$$M_2(G) = \sum_{w\sim x} d_w(G)d_x(G). \quad (3)$$

The first and second Zagreb index are known as Zagreb indices [10].

Definition 3. The first multiplicative Zagreb index of a graph G is defined as

$$\prod_1 (G) = \prod_{w\in V(G)} d_w(G)^2, \quad (4)$$

with product going overall vertices of a graph G.

The second multiplicative Zagreb index of a graph G is defined as

$$\prod_2 (G) = \prod_{w\sim x} d_w(G)d_x(G). \quad (5)$$

The modified first multiplicative Zagreb index of a graph G is defined as

$$\prod_1^* (G) = \prod_{w\sim x} [d_w(G) + d_x(G)]. \quad (6)$$

The first, second, and modified first multiplicative Zagreb indices are known as Narumi–Katayama indices [8, 11].

Definition 4. The atom-bond connectivity index of a graph G is defined as [12]

$$ABC(G) = \sum_{w\sim x} \sqrt{\frac{d_w(G) + d_x(G) - 2}{d_w(G)d_x(G)}}. \quad (7)$$

Definition 5. The augmented Zagreb index of a graph G is defined as [13]

$$AZI(G) = \sum_{w\sim x} \left[\frac{d_w(G)d_x(G)}{d_w(G) + d_x(G) - 2}\right]^3. \quad (8)$$

Definition 6. The geometric-arithmetic index of a graph G is defined as [14]

$$GA(G) = \sum_{w\sim x} 2\left[\frac{\sqrt{d_w(G)d_x(G)}}{d_w(G) + d_x(G)}\right]. \quad (9)$$

Definition 7. The harmonic index of a graph G is defined as [15, 16]

$$H(G) = \sum_{w\sim x} \frac{2}{d_w(G) + d_x(G)}. \quad (10)$$

Definition 8. The sum-connectivity index of a graph G is defined as [17, 18]

$$SCI(G) = \sum_{w\sim x} \frac{1}{\sqrt{d_w(G) + d_x(G)}}. \quad (11)$$

Randić index was introduced by Milan Randić in 1975 [19]. After that, mathematicians did not pay attention to this index for nearly two decades. However, Pal, Erdos, and Bela Bollobas soon worked on this index and discovered the useful mathematics hidden within it. They published their first paper on this index in 1998. When the mathematical communities realized the value of the Randić index, they started to do research studies and soon a flood of publications on this descriptor were started. Randić also wrote two articles on this descriptor [8].

The work on Randić index encouraged theoretical chemists and mathematicians to discover other topological indices that depend on vertex degree.

During the study of total pi electron energy on molecular structure, some expressions which included the terms of the form

$$\begin{aligned} M_1(G) &= \sum_{w} d_w(G)^2, \\ M_2(G) &= \sum_{w\sim x} d_w(G)d_v(G), \end{aligned} \quad (12)$$

were occurred [8]. In the chemical theory, M_1 and M_2 are called first Zagreb index and second Zagreb index. These two indices were discovered by Trinajestic and Gutman in 1972 [8, 10]. At first, the first Zagreb index was also known as the Gutman index, but Balaban et al. did not want to introduce these descriptors by the names of the discoverers. So, they included M_1 and M_2 in the topological indices after 1982 and named them Zagreb group indices. However, soon, the name Zagreb group indices was changed to Zagreb indices. The second Zagreb index did not get any attention of mathematicians or mathematical chemists and that is why not a single property of M_2 was introduced in [8]. Narumi–Katayama [8] introduced the first product descriptor in 1984 and named it simple topological index. This index is defined as

$$NK(G) = \prod_{w\in V(G)} d_w(G). \quad (13)$$

This product index was later renamed the Narumi–Katayama index by Tomovic and Gutman. In 2010, Todeshine et al. proposed multiplicative versions of multiplicative Zagreb indices. These two graph invariants are known as the first and second multiplicative Zagreb indices, after Gutman. Eliasi et al. and Iranmanesh and Gutman proposed a multiplicative version of the first Zagreb index. The authors termed it as modified multiplicative Zagreb index, respectively [20].

The first geometric-arithmetic index also known as geometric-arithmetic index is the modified version of the Randić index. This index was proposed by Vukicevic and Furtula in 2009 [6, 8].

The atom-bond connectivity index is another topological index that is similar to the Randić index. This topological descriptor was introduced by Estrada et al. in 1998 [6, 13]. A recent study of this index has caught the attention of some researchers. Furtula et al. [8] introduced the augmented Zagreb index in 2009, a modified version of the atom-bond connectivity index. This descriptor has higher predictive power than the atom-bond connectivity index. Tosovic and Gutman investigated in 2013 that AZI produced nice effects in the formation of heat and also yielded the best results in boiling factors of octane isomers. Furtula et al. investigated the structure sensitivity of twelve topological indices using trees and found that the AZI has the best structure sensitivity [13].

In 1980, Siemion Fajtlowiez introduced another topological index. In 2012, Zhang worked on this index and named it as harmonic index [8]. In sharp construct to other topological indices, no chemical applications of this index were found, but in mathematical chemistry, such research studies are expected very much [8]. In the last few years, this index has attracted the great attention of theoreticians [15, 16].

Bo Zhou and Nenad Trinajstic discovered that the term $d_w d_x$ in the Randić index can be replaced by $d_w + d_x$ and named it sum-connectivity index [8]. This index got the attention of both applied and pure researchers. Some recent work on this index can be found in [21].

There are a number of studies of various topological indices of graphs establishing formulas for computing the indices and also providing upper and lower bounds on the values of such indices. In this study, certain vertex degree-based topological indices are studied. We have determined and computed the closed formulas of these indices for two special families of graphs of diameter three. A graph formed by joining the centers of two-star graphs of order m and n, i.e., $K_{1,m}$ and $K_{1,n}$ by an edge, is called the bistar graph and is denoted by $B(n;m)$. The corona product $K_m o K_n'$ of two graphs is defined as the graph obtained by taking one copy of K_m of order m and n copies of K_n' and then joining each vertex of the i^{th} copy of K_n' to the i^{th} vertex of K_m. These graphs are undirected having no loops and multiple edges. These results are novel and significant contributions in graph theory and network science, and they provide a good basis to understand the topology of these graphs and networks (Figures 1 and 2).

2. Topological Indices of Families of Bistar Graphs, i.e., $G = B(n;m)$

By looking at the earlier results for computing the topological indices for families of bistar graphs, here we introduce new degree-based topological indices to compute their values for these families of graphs.

Theorem 1. *The sum-connectivity index of families of bistar graphs $B(n;m)$ is*

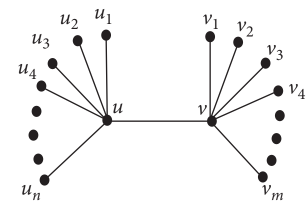

Figure 1: A representation of bistar graph $B(n;m)$ of order m and n.

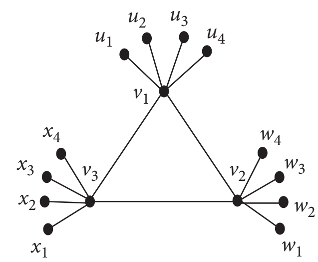

Figure 2: A representation of corona product $K_3 \circ K_4'$ of two graphs.

$$\frac{n}{\sqrt{n+2}} + \frac{1}{\sqrt{n+m+2}} + \frac{m}{\sqrt{m+2}}. \tag{14}$$

Proof. To find the sum-connectivity index of $B(n;m)$, firstly, we select a vertex u on $B(n;m)$ of degree $n+1$. There are n vertices $u_1, u_2, \ldots, u_n$ of degree 1 which are adjacent to u. For the vertices u and u_i, where $i = 1, 2, 3, \ldots, n$, the sum is obtained as

$$\begin{aligned}\sum_{u\sim u_i} \frac{1}{\sqrt{d_u(G) + d_{u_i}(G)}} &= \frac{n}{\sqrt{1+(n+1)}} \\ &= \frac{n}{\sqrt{n+2}}, \quad i = 1, 2, 3, \ldots, n.\end{aligned} \tag{15}$$

Since the degree of u is $n+1$, the other vertex which is adjacent to u is v of degree $m+1$. For the vertices u and v, we have

$$\begin{aligned}\frac{1}{\sqrt{d_u(G) + d_v(G)}} &= \frac{1}{\sqrt{(n+1)+(m+1)}} \\ &= \frac{1}{\sqrt{n+m+2}}.\end{aligned} \tag{16}$$

Now, we select a vertex v of degree $m+1$ on $B(n;m)$; there are m vertices $v_1, v_2, v_3, \ldots, v_m$ of degree 1 which are adjacent to v. For the vertices v and v_j, the sum is obtained as

$$\begin{aligned}\sum_{v\sim v_j} \frac{1}{\sqrt{d_v(G) + d_{v_j(G)}}} &= \frac{m}{\sqrt{1+m+1}} \\ &= \frac{m}{\sqrt{m+2}}, \quad j = 1, 2, 3, \ldots, m.\end{aligned} \tag{17}$$

Adding above equations, we have $SCI(G)=$ sum-connectivity index of $B(n;m)$:

$$\frac{n}{\sqrt{n+2}}+\frac{1}{\sqrt{n+m+2}}+\frac{m}{\sqrt{m+2}}. \tag{18}$$

□

Theorem 2. *The Zagreb indices of the families of bistar graphs $B(n;m)$ are*

$$\begin{aligned} M_1(G) &= (n+1)^2+(m+1)^2+n+m,\\ M_2(G) &= n(n+1)+(n+1)(m+1)+m(m+1). \end{aligned} \tag{19}$$

Proof. To find the first Zagreb index $M_1(G)$ of $B(n;m)$, we select a vertex u on $B(n;m)$ of degree $n+1$. For the vertex u, we have

$$\deg(u)^2=(n+1)^2. \tag{20}$$

Since there are n vertices $u_1,u_2,u_3,\ldots,u_n$ of $B(n;m)$ of degree 1 which are adjacent to u, thus, we have

$$\sum_{u_i}\deg(u_i)^2=n,\quad i=1,2,3,\ldots,n. \tag{21}$$

As the degree of u is $n+1$, the other vertex which is adjacent to u is v of degree $m+1$. For the vertex v, we have

$$\deg(v)^2=(n+1)^2. \tag{22}$$

Since the degree of v is $m+1$, the other vertices which are adjacent to v are m in numbers of degree 1:

$$\sum_{v_j}\deg(v_j)^2=m,\quad j=1,2,3,\ldots,m. \tag{23}$$

Adding equations (20) to (23), we have $M_1(G)=$ first Zagreb index of $B(n;m)$:

$$M_1(G)=(n+1)^2+(m+1)^2+n+m. \tag{24}$$

Similarly, $M_2(G)=$ second Zagreb index of $B(n;m)$ is

$$M_2(G)=n.(n+1)+(n+1)(m+1)+m(m+1). \tag{25}$$

□

Theorem 3. *The atom-bond connectivity index and augmented Zagreb index of families of bistar graphs $B(n;m)$ are*

$$\begin{aligned} &n\sqrt{\frac{n}{n+1}}+\sqrt{\frac{n+m}{(n+1)(m+1)}}+m\sqrt{\frac{m}{m+1}},\\ &n\left[\frac{n+1}{n}\right]^3+\left[\frac{(n+1)(m+1)}{n+m}\right]^3+m\left[\frac{m+1}{m}\right]^3. \end{aligned} \tag{26}$$

The proof of this Theorem is the same as Theorem 1.

Theorem 4. *The geometric-arithmetic index and harmonic index of families of bistar graphs $B(n;m)$ are*

$$\begin{aligned} &\frac{2n\sqrt{n+1}}{n+2}+\frac{2\sqrt{(n+1)(m+1)}}{n+m+2}+\frac{2m\sqrt{m+1}}{m+2},\\ &\frac{2n}{n+2}+\frac{2}{n+m+2}+\frac{2m}{m+2}. \end{aligned} \tag{27}$$

The proof of this theorem is the same as Theorem 1.

Theorem 5. *The Narumi–Katayama and multiplicative Zagreb indices of families of bistar graphs $B(n;m)$ are*

$$\begin{aligned} \prod_1(G) &= (n+1)^2(m+1)^2,\\ \prod_2(G) &= (n+1)^{n+1}(m+1)^{m+1},\\ \prod_1^*(G) &= (n+2)^n(n+m+2)(m+2)^m. \end{aligned} \tag{28}$$

Proof. To find the first multiplicative Zagreb index of $B(n;m)$, we select a vertex u on $B(n;m)$ of degree $n+1$. For the vertex u, we have

$$\deg(u)^2=(n+1)^2. \tag{29}$$

Since there are n vertices $u_1,u_2,u_3,\ldots,u_n$ of $B(n;m)$ of degree 1, thus, we have

$$\prod_{u_i}\deg(u_i)^2=\deg(u_1)^2\deg(u_2)^2,\ldots,\deg(u_n)^2=1. \tag{30}$$

Similarly, for the vertex v, we have

$$\deg(v)^2=(m+1)^2. \tag{31}$$

Also, for the vertices $v_1,v_2,v_3,\ldots,v_m$ of $B(n;m)$ of degree 1, we have

$$\prod_{v_j}\deg(v_j)^2=\deg(v_1)^2\deg(v_2)^2,\ldots,\deg(v_m)^2=1. \tag{32}$$

Multiplying equations (29) to (32), we have $\prod_1(G)=$ first multiplicative Zagreb index of families of bistar graphs $B(n;m)$:

$$\prod_1(G)=(n+1)^2(m+1)^2. \tag{33}$$

Similarly, second multiplicative Zagreb index and modified first multiplicative Zagreb index of families of bistar graphs are

$$\begin{aligned} \prod_2(G) &= (n+1)^{n+1}(m+1)^{m+1},\\ \prod_1^*(G) &= (n+2)^n(n+m+2)(m+2)^m. \end{aligned} \tag{34}$$

□

Example 1. Topological indices of bistar graph $B(5;6)$ are shown in Table 1.

Table 1: Topological indices of $G = B(5; 6)$.

$SCI(G)$	4.288
$R(G)$	4.463
$GA(G)$	8.465
$ABC(G)$	10.631
$M_1(G)$	96
$\prod_1(G)$	1764

3. Topological Indices of Families of Corona Product of Graphs, i.e., $G = K_m \circ \mathbf{K}_\mathbf{n}'$

Based on previous results for computing topological indices for families of corona product of graphs, we present new degree-based topological indices to compute their values for these families of graphs.

Theorem 6. *The Randić index and sum-connectivity index of families of corona product of graphs $K_m \circ K_n'$ are*

$$R(G) = \frac{mn}{\sqrt{m+n-1}} + \frac{m(m-1)}{2(m+n-1)},$$
$$SCI(G) = \frac{mn}{\sqrt{m+n}} + \frac{m(m-1)}{2\sqrt{2(m+n-1)}}. \quad (35)$$

Proof. To find the Randić index of the family of graphs $K_m \circ \mathrm{K}_\mathrm{n}'$ obtained by taking the corona product of complete graph K_m of order m and the complement of K_n of order n, firstly, we select a vertex v_1 on $K_m \circ \mathrm{K}_\mathrm{n}'$ as v_1 is adjacent to $m-1$ vertices $v_2, v_3, v_4, \ldots, v_m$ and n vertices $u_1, u_2, u_3, \ldots, u_n$. Therefore, the degree of v_1 is $m+n-1$.

For the vertices v_1 and u_i, where $i = 1, 2, 3, \ldots, n$, the sum is obtained as

$$\sum_{v_1 \sim u_i} \frac{1}{\sqrt{\deg_{v_1}(G)\deg_{u_i(G)}}} = \frac{n}{\sqrt{m+n-1}}. \quad (36)$$

As the graph is symmetric, the same result is obtained for the remaining $(m-1)$ vertices $v_2, v_3, \ldots, v_m$. So, combining all the vertices $v_1, v_2, v_3, \ldots, v_m$ equation (36) becomes

$$\sum_{v_1 \sim u_i} \frac{1}{\sqrt{\deg_{v_1}(G)\deg_{u_i(G)}}} + \sum_{v_2 \sim w_i} \frac{1}{\sqrt{\deg_{v_2}(G)\deg_{w_i(G)}}}$$
$$+ \cdots + \sum_{v_m \sim x_i} \frac{1}{\sqrt{\deg_{v_m}(G)\deg_{x_i(G)}}} \quad (37)$$
$$= \frac{mn}{\sqrt{(m+n-1)}} m \text{ times}.$$

Since v_1 is also adjacent to $m-1$ vertices $v_2, v_3, v_4, \ldots, v_m$ of degree $m+n-1$, for the vertices v_1 and v_j, where $j = 2, 3, 4, \ldots, m$, the sum is obtained as

$$\sum_{v_1 \sim v_j} \frac{1}{\sqrt{d_{v_1}(G)d_{v_j}(G)}} = \frac{m-1}{m+n-1}. \quad (38)$$

Since K_m is symmetric, the same result is obtained for remaining $m-1$ vertices $v_2, v_3, \ldots, v_m$. Combining the result for all m vertices $v_1, v_2, v_3, \ldots, v_m$, we have

$$= \frac{m-1}{m+n-1} + \frac{m-2}{m+n-1} + \frac{m-3}{m+n-1} + \cdots + \frac{m-(m-1)}{m+n-1}$$
$$= \frac{1}{m+n-1}[(m-1) + (m-2)$$
$$+ (m-3) + \cdots + (m-(m-1))]$$
$$= \frac{1}{m+n-1}[(m+m+m+, \ldots, +m)(m-1)\text{times}$$
$$-(1+2+3+, \ldots, +m-1)]$$
$$= \frac{1}{m+n-1}\left[m(m-1) - \frac{(m-1)(m-1+1)}{2}\right]$$
$$= \frac{1}{m+n-1}\left[m(m-1) - \frac{m(m-1)}{2}\right]$$
$$= \frac{1}{m+n-1}\left[\frac{m(m-1)}{2}\right]. \quad (39)$$

Adding equations (37) and (39), we have $R(G) =$ Randić index of $K_m \circ \mathrm{K}_\mathrm{n}'$:

$$R(G) = \frac{mn}{\sqrt{m+n-1}} + \frac{m(m-1)}{2(m+n-1)}. \quad (40)$$

Similarly, $SCI(G) =$ sum-connectivity index of $K_m \circ \mathrm{K}_\mathrm{n}'$ is

$$SCI(G) = \frac{mn}{\sqrt{m+n}} + \frac{m(m-1)}{2\sqrt{2(m+n-1)}}. \quad (41)$$

□

Theorem 7. *The atom-bond connectivity index and augmented Zagreb index of families of corona product of graphs $K_m \circ K_n'$ are*

$$ABC(G) = mn\sqrt{\frac{m+n-2}{m+n-1}}$$
$$+ \frac{m(m-1)}{2(m+n-1)}\sqrt{2m+2n-4},$$
$$AZI(G) = mn\left(\frac{m+n-1}{m+n-2}\right)^3 \quad (42)$$
$$+ \frac{m(m-1)(m+n-1)^6}{2(2m+2n-4)^3}.$$

The proof of this theorem is the same as Theorem 6.

Theorem 8. *The geometric-arithmetic index and harmonic index of families of corona product of graphs $K_m \circ K_n'$ are*

$$GA(G) = \frac{2mn\sqrt{m+n-1}}{m+n} + \frac{m(m-1)(m+n-1)}{2m+2n-2},$$
$$H(G) = \frac{2mn}{m+n} + \frac{m(m-1)}{2m+2n-2}. \quad (43)$$

The proof of this theorem is the same as Theorem 6.

Theorem 9. *The Zagreb indices of families of corona product of graphs $K_m \circ K_n'$ are*

$$M_1(G) = mn + m(m+n-1)^2,$$
$$M_2(G) = mn(m+n-1) + \frac{m(m-1)(m+n-1)^2}{2}. \quad (44)$$

Proof. For the families of corona product of graphs there are mn vertices of degree one and m vertices $v_1, v_2, v_3, \ldots, v_m$ of degree $m+n-1$, respectively. For the vertices $u_1, u_2, u_3, \ldots, u_n$, we have

$$\begin{aligned}\sum_{u_i} \deg(u_i)^2 &= \deg(u_1)^2 + \deg(u_2)^2 + \cdots + \deg(u_n)^2, \quad i = 1, 2, 3, \ldots, n \\ &= (1)^2 + (1)^2, \ldots, + (1)^2\, n \text{ times} = n.\end{aligned} \quad (45)$$

Since there are mn vertices of degree 1, combining the above result for all mn vertices, thus, the above equation becomes

$$= n + n + n + \cdots + n\, m \text{ times} = mn. \quad (46)$$

Also, for the vertex v_1, we have

$$\deg(v_1)^2 = (m+n-1)^2. \quad (47)$$

Since the graph is symmetric, the same result is obtained for remaining $(m-1)$ vertices $v_2, v_3, \ldots, v_m$:

$$\begin{aligned}\sum_{v_j} \deg(v_j)^2 &= \deg(v_1)^2 + \deg(v_2)^2 + \cdots + \deg(v_m)^2, \quad j = 1, 2, 3, \ldots, m \\ &= (m+n-1)^2 + (m+n-1)^2 + \cdots + (m+n-1)^2\, m \text{ times} \\ &= m(m+n-1)^2.\end{aligned} \quad (48)$$

Adding equations (46) and (48), we have $M_1(G)$ = first Zagreb index of families of corona product of graphs $K_m \circ K_n'$:

$$M_1(G) = mn + m(m+n-1)^2. \quad (49)$$

Similarly, $M_2(G)$ = second Zagreb index of families of corona product of graphs $K_m \circ K_n'$:

$$M_2(G) = mn(m+n-1) + \frac{m(m-1)(m+n-1)^2}{2}. \quad (50)$$

□

Theorem 10. *The Narumi–Katayma and multiplicative Zagreb indices of families of corona product of graphs $K_m \circ K_n'$ are*

$$\prod_1 (G) = (m+n-1)^{2m},$$
$$\prod_2 (G) = (m+n-1)^{m^2-m+mn},$$
$$\prod_1^* (G) = (m+n)^{mn} (2m+2n-2)^{m^2-m/2}. \quad (51)$$

Proof. For the families of corona product of graphs, there are mn vertices of degree one and m vertices $v_1, v_2, v_3, \ldots, v_m$ of degree $m+n-1$. For the vertices $u_1, u_2, u_3, \ldots, u_n$, we have

$$\begin{aligned}\prod_{u_i} \deg(u_i)^2 &= \deg(u_1)^2 \deg \\ &(u_2)^2, \ldots, \deg(u_n)^2, \quad i = 1, 2, 3, \ldots, n \\ &= (1)^2 (1)^2, \ldots, (1)^2\, n \text{ times} = (1)^n = 1.\end{aligned} \quad (52)$$

Since there are mn vertices of degree 1, so the same result is obtained for all mn vertices. Thus, the above equation becomes

$$(1)^{mn} = 1. \quad (53)$$

Also, for the vertex v_1, we have

$$\deg(v_1)^2 = (m+n-1)^2. \quad (54)$$

Since the graph is symmetric the same result is obtained for remaining $(m-1)$ vertices $v_2, v_3, \ldots, v_m$. Thus equation (54) becomes

Table 2: Topological indices of $G = K_6 \circ K_3'$.

$\prod_1 (G)$	6, 8710000000
$H(G)$	5.875
$SCI(G)$	9.750
$R(G)$	8.239
$GA(G)$	26.314
$ABC(G)$	23.853
$M_1(G)$	402

$$\begin{aligned}\prod_{v_j} \deg(v_j)^2 &= \deg(v_1)^2 \deg(v_2)^2 \deg(v_3)^2, \ldots, \deg(v_m)^2, \quad j = 1, 2, 3, \ldots, m \\ &= (m+n-1)^2 (m+n-1)^2 (m+n-1)^2, \ldots, (m+n-1)^2 m \text{ times} = (m+n-1)^{2m}.\end{aligned} \tag{55}$$

Multiplying equations (53) to (55), we have $\prod_1 (G)$ = first Zagreb index of families of corona product of graphs $K_m \circ K_n'$

$$\prod_1 (G) = (1).(m+n-1)^{2m} = (m+n-1)^{2m}. \tag{56}$$

Similarly, second multiplicative Zagreb index and modified first multiplicative Zagreb index of $K_m \circ K_n'$ are

$$\begin{aligned}\prod_2 (G) &= (m+n-1)^{m^2 - m + mn}, \\ \prod_1^* (G) &= (m+n)^{mn} (2m + 2n - 2)^{m^2 - m/2}.\end{aligned} \tag{57}$$

□

Example 2. Topological indices of corona product of graphs $K_6 \circ K_3'$ are shown in Table 2.

4. Concluding Remarks

There are several articles published on calculating topological indices for different families of graphs. Some have found applications, but others were devoted to the mathematical side to throw more light on the relationship between these concepts. This study introduces Randić index, Zagreb indices, Narumi–Katayama, and multiplicative Zagreb indices. We further discussed the atom-bond connectivity index, augmented Zagreb index, geometric-arithmetic index, harmonic index, and sum-connectivity index for two particular families of graphs for the first time. We have determined and computed the closed formulas of these families of graphs. We had checked that all vertex degree-based topological indices of families of bistar graphs and corona product of graphs remain the same for all values of m and n. We have also determined the values of some topological indices for $B(5; 6)$ and $K_6 \circ K_3'$ graphs, as shown in Tables 1 and 2. This work will give new directions for considering and computing topological indices of several other families of graphs. In the future, we are interested in investigating and calculating some other topological indices of two more special families of graphs whose diameters are greater than three.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

Authors' Contributions

All authors have contributed equally in the preparation of this manuscript.

Acknowledgments

The authors express their appreciation to the Deanship of Scientific Research at King Khalid University for funding this work through the public research project, under Grant no. R.G.P.1/258/43.

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