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Research Article

Exponentiated Gull Alpha Exponential Distribution with Application to COVID-19 Data

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In this paper, the main aim is to define a statistical distribution that can be used to model COVID-19 data in Mexico and Canada. Using the method of exponentiation on the gull alpha exponential distribution introduces a new distribution with three parameters called the exponentiated gull alpha power exponential (EGAPE) distribution. The distribution has the benefit of being able to represent monotonic and nonmonotonic failure rates, both of which are often seen in dependability issues. It is possible to determine the quantile function as well as the skewness, kurtosis, and order statistics of the suggested distribution. The approach of maximum likelihood is used in order to calculate the parameters of the model, and the RMSE and average bias are utilised in order to evaluate how successful the strategy is. In conclusion, the flexibility of the new distribution is demonstrated by modeling COVID-19 data. From the practical application, we can conclude that the proposed model outperformed the competing models and therefore can be used as a better option for modeling COVID-19 and other related datasets.

1. Introduction

Over the last decade, modeling using probability distributions has attracted the attention of most researchers. There are several ways to modify a distribution, and the most versatile approach is the introduction of an extra shape parameter so as to increase the flexibility of the distribution. Creating a new family of distributions and basing a new distribution off of an already existing baseline distribution is still another technique that may be used. Many families have been developed in the literature, for instance, Mudholkar and Srivastava1 developed the exponentiated Weibull family of distributions, Nadarajah and Kotz 2 developed the exponentiated types of distributions, the logistic-X family was proposed by Tahir et al. [3], the exponentiated TX family was proposed by Alzaghal et al.4, and Kumaraswamy–Marshall–Olkin family was proposed by Alizadeh et al. 5 (for further reading on other families of distributions, the reader is referred to [6–12]).

The exponential distribution has been widely used to model survival analysis data. However, the shortcoming of the exponential distribution is that it only models data with constant hazard function. As a consequence of this, numerous adjustments to the exponential distributions have been produced in the scientific literature. For instance, Bhati et al. 13 were the ones who first recommended using the Lindley exponential distribution. and extended new generalized exponential distribution was proposed by Eghwerido et al. 14.

For further reading on the extensions of the exponential distribution, the reader is referred to [15–21].

Exponentiated gull alpha power exponential distribution is a highly innovative and simple model introduced in this study. In Section 2, we give the details of the brand-new design. Section 3 derives a few additional distributional features. The maximum likelihood approach of parameter estimation is described. It is used to examine the performance of the estimators in Section 4. Section 5 focuses on applying the theory to real-world data. Section 6 focuses on the findings.

2. The New EGAPE Distribution

The gull alpha power exponential distribution has been widely used in survival analysis. The CDF and the PDF of the GAPE distribution can be written and formulated as shown below:

$$F(x) = \frac{\alpha\left(1 - e^{-(\lambda x)}\right)}{\alpha^{\left(1-e^{-(\lambda x)}\right)}},$$
$$f(x) = \lambda e^{-(\lambda x)}\alpha^{e^{-(\lambda x)}}\left[1 - \left(1 - e^{-(\lambda x)}\right)\log \alpha\right]. \quad (1)$$

X is assumed to be distributed with the EGAPE distribution with three parameters α, λ, a if the CDF and PDF, respectively, are given as

$$F(x) = \left[\frac{\alpha\left(1 - e^{-(\lambda x)}\right)}{\alpha^{\left(1-e^{-(\lambda x)}\right)}}\right]^{a}. \quad (2)$$

The following is the formula for the equivalent PDF to equation (2):

$$f(x) = a\lambda e^{-(\lambda x)}\alpha^{e^{-(\lambda x)}}\left[1 - \left(1 - e^{-(\lambda x)}\right)\log \alpha\right]\left[\frac{\alpha\left(1 - e^{-(\lambda x)}\right)}{\alpha^{\left(1-e^{-(\lambda x)}\right)}}\right]^{a-1}. \quad (3)$$

The following formula may be used to describe the survival function of the EGAPE distribution:

$$S(x) = 1 - \left[\frac{\alpha\left(1 - e^{-(\lambda x)}\right)}{\alpha^{\left(1-e^{-(\lambda x)}\right)}}\right]^{a}. \quad (4)$$

The hazard function is given as

$$h(x) = \frac{a\lambda e^{-(\lambda x)}\alpha^{e^{-(\lambda x)}}\left[1 - \left(1 - e^{-(\lambda x)}\right)\log \alpha\right]\left[\alpha\left(1 - e^{-(\lambda x)}\right)/\alpha^{\left(1-e^{-(\lambda x)}\right)}\right]^{a-1}}{1 - \left[\alpha\left(1 - e^{-(\lambda x)}\right)/\alpha^{\left(1-e^{-(\lambda x)}\right)}\right]^{a}}. \quad (5)$$

Equations (6) and (7) give the reversed and cumulative hazard functions, respectively.

$$\tau(x) = \frac{a\lambda e^{-(\lambda x)}\alpha^{e^{-(\lambda x)}}\left[1 - \left(1 - e^{-(\lambda x)}\right)\log \alpha\right]\left[\alpha\left(1 - e^{-(\lambda x)}\right)/\alpha^{\left(1-e^{-(\lambda x)}\right)}\right]^{a-1}}{\left[\alpha\left(1 - e^{-(\lambda x)}\right)/\alpha^{\left(1-e^{-(\lambda x)}\right)}\right]^{a}}, \quad (6)$$

$$H(x) = -\ln\left(1 - \left[\frac{\alpha\left(1 - e^{-(\lambda x)}\right)}{\alpha^{\left(1-e^{-(\lambda x)}\right)}}\right]^{a}\right). \quad (7)$$

Figures 1 and 2 depict the different contours of the probability density function (PDF) as well as the hazard rate curves, respectively. As observed, the PDF can exhibit shapes as unimodal, decreasing, or right skewed.

2.1. EGAPE Submodels. The EGAPE distribution includes some well-known submodels which include

(a) If $a = 1$, then we obtain gull alpha power exponential distribution.

(b) If $a = \alpha = 1$, we have exponential distribution.

(c) If $\alpha = 1$, then we obtain exponentiated exponential distribution.

3. Important Mathematical Properties

In this section, all the formulae derived here including moments, incomplete moments, entropies, and order statistics can be handled by many computational software programs.

3.1. Quantile Function. Random samples from the EGAPE can be generated by inverting equation (2).

$$x_q = -\frac{1}{\lambda}\log\left(\frac{\log \alpha + W_{-1}\left(-\log \alpha \times e^{\log u/a}/\alpha\right)}{\log \alpha}\right). \quad (8)$$

To obtain the median of EGAPE, put $u = 0.5$, and we have

$$x_{0.5} = -\frac{1}{\lambda}\log\left(\frac{\log \alpha + W_{-1}\left(-\log \alpha \times e^{\log 0.5/a}/\alpha\right)}{\log \alpha}\right). \quad (9)$$

Some quantile values for the EGAPE are displayed in Table 1.

3.2. Moments. The EGAPE distribution's r_{th} moment is defined as

$$E(x^r) = \int_0^\infty x^r f(x)\mathrm{d}x,$$
$$E(x^r) = \int_0^\infty x^r a\lambda e^{-(\lambda x)}\alpha^{e^{-(\lambda x)}}\left[1 - \left(1 - e^{-(\lambda x)}\right)\log \alpha\right]\left[\frac{\alpha\left(1 - e^{-(\lambda x)}\right)}{\alpha^{\left(1-e^{-(\lambda x)}\right)}}\right]^{a-1}\mathrm{d}x. \quad (10)$$

The above equation does not have an explicit expression, and therefore we can find the moments numerically by using R version 4.1.2. Table 2 gives the moments for the EGAPE for selected parameter values given as I: $\alpha = 1.9$, $\lambda = 1.0$, $a = 0.9$, II: $\alpha = 1.4, \lambda = 1.8, a = 0.2$, III: $\alpha = 2.4, \lambda = 1.5$, $\beta = 1.3, a = 0.9$, and IV: $\alpha = 1.4, \lambda = 0.5, a = 0.9$.

3.3. Order Statistics. For an ordered random sample $X_1, X_2, \ldots\ldots, X_n$, from EGAPE distribution, the PDF of the i^{th} minimum and maximum order statistics are provided by the following equations:

$$f_{X(1)}(x) = nf(x)(1 - F(x))^{n-1},$$
$$f_{X(n)}(x) = nf(x)(F(x))^{n-1}. \quad (11)$$

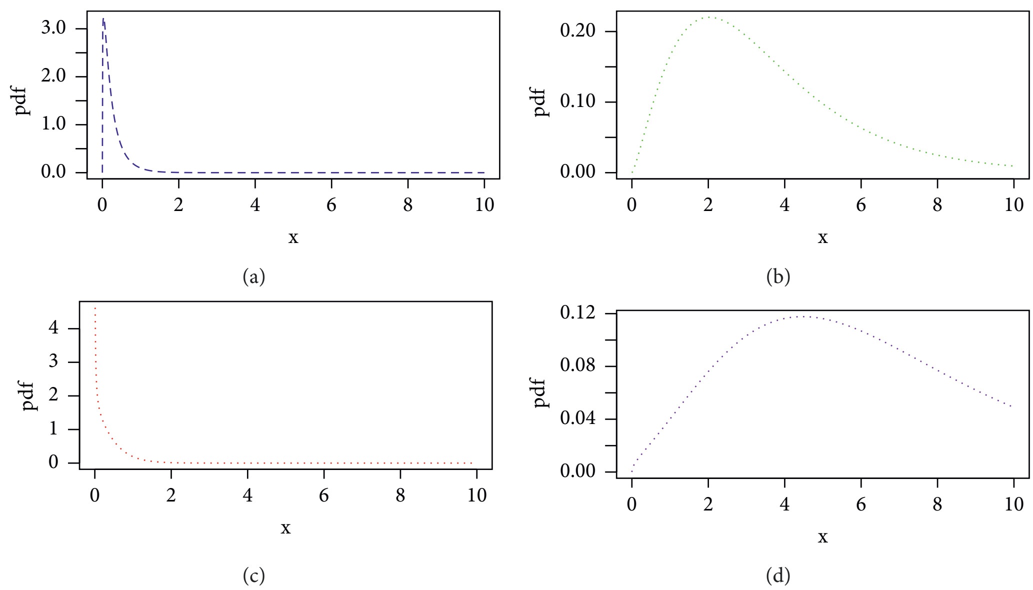


Figure 1: EGAPE distribution PDF plots. (a) $\alpha = 1.5$, $\lambda = 3.2$, $a = 1.2$. (b) $\alpha = 0.8$, $\lambda = 0.5$, $a = 2.3$. (c) $\alpha = 0.3$, $\lambda = 2.7$, $a = 0.5$. (d) $\alpha = 0.2$, $\lambda = 0.3$, $a = 1.5$.

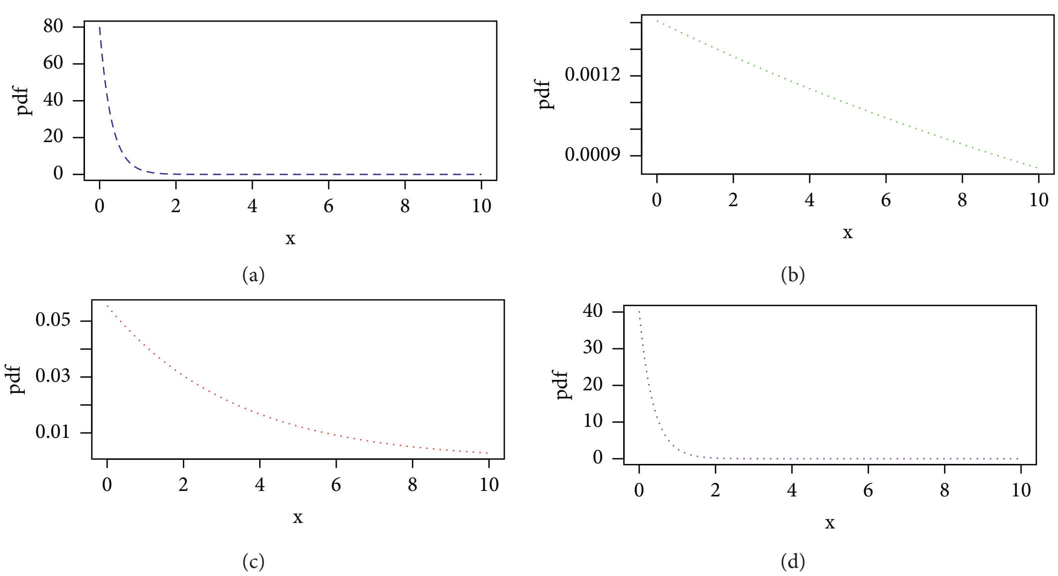


Figure 2: EGAPE distribution hrf plots. (a) $\alpha = 1.5$, $\lambda = 3.2$, $a = 1.2$. (b) $\alpha = 0.8$, $\lambda = 0.05$, $a = 2.3$. (c) $\alpha = 0.03$, $\lambda = 2.7$, $a = 0.05$. (d) $\alpha = 0.2$, $\lambda = 0.3$, $a = 1.5$.

Table 1: Quantile values for EGAPE distribution.

Quantile	(0.4, 0.3, 0.8)	(0.5, 0.8, 0.3)	(1.2, 0.7, 2.3)	(0.9, 0.7, 0.5)
0.1	0.004	0.143	0.045	0.059
0.2	0.038	0.352	0.122	0.163
0.3	0.145	0.607	0.223	0.298
0.4	0.359	0.913	0.348	0.464
0.5	0.706	1.281	0.501	0.668
0.6	1.207	1.734	0.690	0.921
0.7	1.908	2.320	0.936	1.248
0.8	2.929	3.144	1.281	1.708
0.9	4.687	4.546	1.867	2.489

Table 2: Moments for EGAPE distribution.

	I	II	III	IV
μ_1	1.890	0.287	0.598	0.674
μ_2	7.009	0.255	0.514	1.121
μ_3	40.516	0.388	0.779	3.038
μ_4	318.216	0.876	1.663	11.513
SD	1.852	0.416	0.395	0.816
CV	0.979	1.458	0.661	1.209
CS	2.247	2.996	4.586	2.543
CK	10.513	17.839	21.100	12.977

Thus,

$$f_{X(1)}(x) = na\lambda e^{-(\lambda x)}\alpha^{e^{-(\lambda x)}}\left[1-\left(1-e^{-(\lambda x)}\right)\log\alpha\right]\left[\frac{\alpha\left(1-e^{-(\lambda x)}\right)}{\alpha^{\left(1-e^{-(\lambda x)}\right)}}\right]^{a-1}\left(1-\left[\frac{\alpha\left(1-e^{-(\lambda x)}\right)}{\alpha^{\left(1-e^{-(\lambda x)}\right)}}\right]^{a}\right)^{n-1},$$

$$f_{X(n)}(x) = na\lambda e^{-(\lambda x)}\alpha^{e^{-(\lambda x)}}\left[1-\left(1-e^{-(\lambda x)}\right)\log\alpha\right]\left[\frac{\alpha\left(1-e^{-(\lambda x)}\right)}{\alpha^{\left(1-e^{-(\lambda x)}\right)}}\right]^{a-1}\left(\left[\frac{\alpha\left(1-e^{-(\lambda x)}\right)}{\alpha^{\left(1-e^{-(\lambda x)}\right)}}\right]^{a}\right)^{n-1}. \tag{12}$$

3.4. Skewness and Kurtosis. In the context of the EGAPE distribution, the definitions of Galton skewness and Moors kurtosis are stated as follows:

$$Z_K = \frac{Q(1/8)+Q(3/4)-Q(1/4)-Q(2/4)}{Q(3/4)-Q(1/4)},$$
$$Z_M(x) = \frac{Q(1/8)+Q(3/8)-Q(5/8)-Q(1/8)}{Q(3/4)-Q(1/4)}, \tag{13}$$

where Q describe different quartile values. Clearly, the extra shape parameters α and a have an effect on the skewness and kurtosis values Figures 3 and 4.

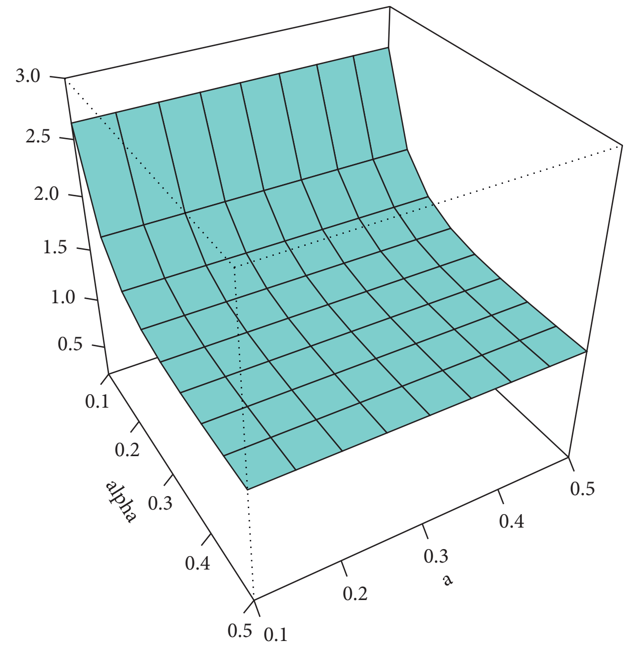


Figure 3: Kurtosis values for EGAPE distribution.

3.5. Entropy. The Renyi entropy of EGAPE distribution:

$$R_H(x) = \frac{1}{1-p}\log\int_0^\infty f^p(x)\mathrm{d}x. \tag{14}$$

From equation (11), the Renyi entropy $R_H(x)$ becomes

$$R_H(x) = \frac{1}{1-p}\log\int_0^\infty\left[a\lambda e^{-(\lambda x)}\alpha^{e^{-(\lambda x)}}\left[1-\left(1-e^{-(\lambda x)}\right)\log\alpha\right]\left[\frac{\alpha\left(1-e^{-(\lambda x)}\right)}{\alpha^{\left(1-e^{-(\lambda x)}\right)}}\right]^{a-1}\right]^p\mathrm{d}x. \tag{15}$$

4. Parameter Estimation

To determine the MLEs of the given parameter estimation, for the model parameters λ, α, a, we use the log likelihood function, which may be expressed as

$$\ln L = 2n\ln\alpha\lambda + \sum_{i=1}^{n}2e^{-\lambda x_i}\ln\alpha - 2\sum_{i=1}^{n}\lambda x_i + \ln\ln\alpha - \sum_{i=1}^{n}\ln\left(1-e^{-\lambda x_i}\right) + 2(a-1)\sum_{i=1}^{n}\ln\frac{\alpha\left(1-e^{-\lambda x_i}\right)}{\alpha^{\left(1-e^{-\lambda x_i}\right)}}. \tag{16}$$

The equations of the EGAPE that give the maximum likelihood are provided by

$$\frac{\partial l}{\partial a} = 2\sum_{i=1}^{n}\ln\frac{\alpha\left(1-e^{-\lambda x_i}\right)}{\alpha^{1-e^{-\lambda x_i}}}, \tag{17}$$

$$\frac{\partial l}{\partial a} = \frac{2n}{\alpha} + \frac{\alpha}{\ln\alpha} + \sum_{i=1}^{n}\frac{2(a-1)\left(\left(\left(1-e^{-\lambda x_i}\right)/\alpha^{1-e^{-\lambda x_i}}\right)-\left(\left(1-e^{-\lambda x_i}\right)^2/\alpha^{1-e^{-\lambda x_i}}\right)\right)\alpha^{1-e^{-\lambda x_i}}}{\alpha\left(1-e^{-\lambda x_i}\right)}, \tag{18}$$

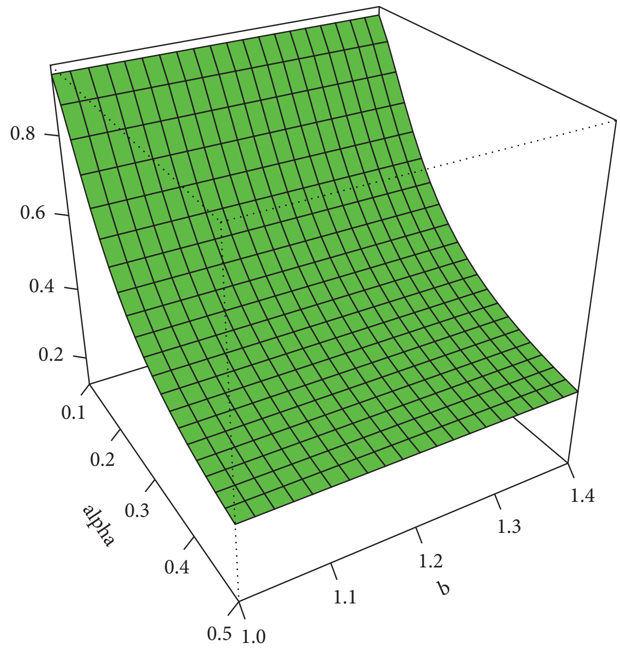


FIGURE 4: Skewness values for EGAPE distribution.

$$\frac{\partial l}{\partial \lambda} = \frac{2n}{\lambda} - \sum_{i=1}^{n} 2x_i - \sum_{i=1}^{n} \frac{x_i e^{-\lambda x_i}}{1 - e^{-\lambda x_i}} + \sum_{i=1}^{n} \frac{2(a-1)\left(\left(\alpha x_i e^{-\lambda x_i}/\alpha^{1-e^{-\lambda x_i}}\right) - \left(\lambda\left(1 - e^{-\lambda x_i}\right) x_i e^{-\lambda x_i \ln\alpha}/\alpha^{1-e^{-\lambda x_i}}\right)\right)\alpha^{1-e^{-\lambda x_i}}}{\alpha\left(1 - e^{-\lambda x_i}\right)}. \quad (19)$$

Equating equations (17)–(19) to zero and solving simultaneously, the maximum likelihood estimators of the parameters are obtained.

4.1. Monte Carlo Simulation. The effectiveness of the maximum likelihood method of estimate is examined by means of a simulation study, with attention given to both the root mean squared errors and the average bias. This evaluation is carried out with respect to the maximum likelihood technique. The simulation was carried out using version 4.1.2 of the R program, and the following is the technique that was followed for it:

(a) 2000 random samples from different sample sizes $n = 100, 200, 300, \ldots, 2000$ are generated using the quantile function.

(b) Two different sets of parameter values are considered (Set I: $\lambda = 0.2, \alpha = 0.3, a = 0.5$ and Set II $\lambda = 0.3$, $\alpha = 0.9, a = 0.4$).

(c) The calculated root mean squared errors as well as the average bias are then presented when the result has been obtained.

4.1.1. Remarks on Simulation. From Tables 2 and 3, we have recognized that by increasing the size of the samplethe average bias of the parameter estimates gets smaller. The same observation is made on the root mean squared error. This shows clearly that the maximum likelihood method performs well as a method of estimating the parameters from Tables 3 and 4.

5. Applications

This section presents the effectiveness and the flexibility of the EGAPE distribution by two real datasets. The datasets are on the COVID-19 mortality rates for different countries. In addition, we provide the evaluation of the goodness of fit of the distribution and comparison to other competing models. The measures include the BIC, AIC, HQIC, and CAIC. The smaller the values of these statistics, the better the model. The proposed distribution is compared to the following:

(i) Exponential distribution (E).
The CDF is given by $F(x) = 1 - e^{-\lambda x}$.

(ii) Gull alpha power exponential (GAPE).
The CDF is given by $F(x) = \alpha(1 - e^{-\lambda x})/\alpha^{1-e^{-\lambda x}}$.

(iii) The exponentiated generalized exponential distribution (EGE).
The CDF is given as $F(x) = (1 - (e^{-\lambda x})^a)^b$.

(iv) The Marshall–Olkin generalized exponential distribution (MOGE).
The CDF is given by

$$F(x) = \frac{\left(1 - e^{-\lambda x}\right)^a}{\theta + (1-\theta)\left(1 - e^{-\lambda x}\right)^{\alpha}}. \quad (20)$$

(v) Marshall–Olkin alpha power inverted exponential (MOAPIE) distribution.
The CDF is given as

Table 3: RMSE and AB for Set I: $\lambda = 0.2, \alpha = 0.3, a = 0.5$.

	RMSE			Average bias		
n	λ	α	a	λ	α	a
100	0.04006	0.312	0.1965	0.00421	0.0864	−0.1897
200	0.0274	0.2006	0.1968	0.00123	0.0481	−0.1935
300	0.0223	0.1567	0.1977	0.00156	0.0246	−0.1954
400	0.0189	0.1284	0.1969	0.0000571	0.02462	−0.1947
500	0.0178	0.1147	0.1972	0.000527	0.0180	−0.1952
600	0.0156	0.1053	0.1973	0.000338	0.0175	−0.1958
700	0.0142	0.1005	0.1961	0.0001486	0.0207	−0.1942
800	0.0136	0.0903	0.1961	0.000376	0.01686	−0.1941
900	0.0125	0.0853	0.1965	0.000285	0.01359	−0.1944
1000	0.0121	0.0817	0.1965	0.000278	0.01333	−0.1927
1100	0.0118	0.0762	0.1961	0.0000742	0.0123	−0.1937
1200	0.0109	0.0737	0.1961	−0.0000982	0.01197	−0.1933
1300	0.0108	0.0692	0.1965	−0.00004508	0.01061	−0.1930
1400	0.0100	0.0678	0.1965	0.00000505	0.00975	−0.1941
1500	0.0092	0.0625	0.1960	0.000210	0.00785	−0.1932
1600	0.00950	0.0619	0.1955	−0.0000425	0.00938	−0.1922
1700	0.00929	0.0604	0.1952	0.000259	0.00756	−0.1914
1800	0.0090	0.0579	0.1945	0.000239	0.00793	−0.1903
1900	0.0086	0.05658	0.1958	0.000101	0.0068	−0.1926
2000	0.0084	0.0562	0.1942	0.0001023	0.00719	−0.1896

Table 4: RMSE and AB for Set I: $\lambda = 0.3, \alpha = 0.4, a = 0.6$.

	RMSE			Bias		
n	λ	α	a	λ	α	a
100	0.06053	0.3741	0.2046	0.00356	0.0861	−0.1909
200	0.0421	0.2557	0.2031	0.00185	0.0402	−0.1964
300	0.0331	0.1962	0.2010	0.000364	0.0335	−0.1963
400	0.0283	0.1612	0.2002	0.000179	0.0236	−0.1969
500	0.0255	0.1437	0.2001	0.000587	0.0193	−0.1972
600	0.0235	0.1304	0.2009	0.00135	0.01124	−0.1986
700	0.0216	0.1196	0.2002	0.00107	0.01107	−0.1983
800	0.0199	0.1108	0.2000	0.0000244	0.01322	−0.1983
900	0.0189	0.1048	0.2000	0.0004689	0.01069	−0.1986
1000	0.0178	0.0989	0.1999	0.000533	0.00965	−0.1985
1100	0.0174	0.0927	0.2000	0.000591	0.00608	−0.1989
1200	0.0164	0.0906	0.1996	0.0000385	0.008727	−0.1988
1300	0.0158	0.0873	0.1999	−0.000253	0.008357	−0.1989
1400	0.0149	0.0844	0.1999	−0.0000364	0.00895	−0.1986
1500	0.0144	0.0809	0.1995	0.000309	0.00701	−0.1985
1600	0.0141	0.0786	0.1998	0.000270	0.00606	−0.1989
1700	0.0135	0.0737	0.2000	0.000518	0.00355	−0.1993
1800	0.0133	0.0714	0.1997	0.000434	0.00407	−0.1990
1900	0.0126	0.0702	0.1997	0.0000924	0.003956	−0.1989
2000	0.0125	0.0687	0.1996	0.000438	0.004439	−0.1989

$$F(x) = \frac{\alpha^{e^{-\lambda x^{-1}}} - 1}{(\alpha - 1)\theta - (\theta - 1)\left(\alpha^{e^{-\lambda x^{-1}}} - 1\right)}. \quad (21)$$

(vi) Alpha power inverted exponential (APIE) distribution.

The CDF is given as

$$F(x) = \frac{\alpha^{e^{-\lambda x^{-1}}} - 1}{(\alpha - 1)}. \quad (22)$$

(vii) Weibull moment exponential distribution (WME) distribution.

The CDF is given as

$$F(x) = 1 - \exp\left\{-a\left[\frac{1 - (1 + (x/\beta))e^{-x/\beta}}{(1 + (x/\beta))e^{-x/\beta}}\right]^b\right\}. \quad (23)$$

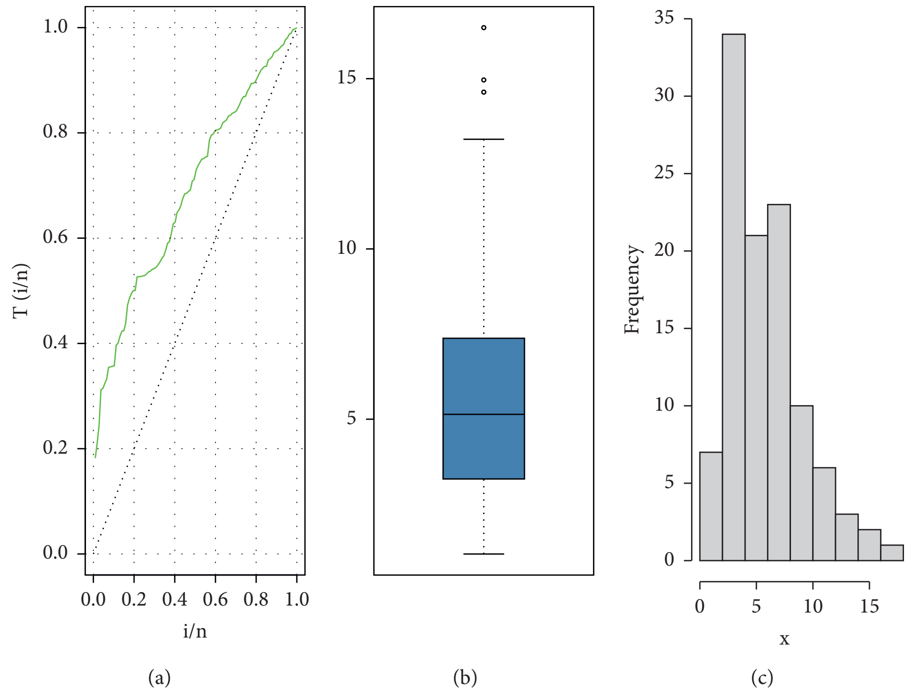


FIGURE 5: (a) TTT plot, (b) boxplot, and (c) histogram for Mexico data.

TABLE 5: Estimates and SE (in parentheses).

Distribution	Estimates (SE in parenthesis)		
EGAPE (λ, α, a)	5.284 (2.476)	1.456 (0.768)	0.339 (0.059)
GAPE (λ, α)	0.400 (0.035)	0.013 (0.011)	
E (λ)	0.174 (0.016)		
EE (λ, α)	0.360 (0.034)	3.982 (0.672)	
MOAPIE (λ, θ, a)	0.003 (0.002)	1.254 (0.381)	8.887 (0.917)
MOGE (λ, θ, a)	0.360 (0.072)	4.109 (0.784)	0.956 (0.677)
APIE (λ, α)	9.247 (1.018)	0.007 (0.008)	

TABLE 6: Goodness of fit measures for Mexico mortality rate data.

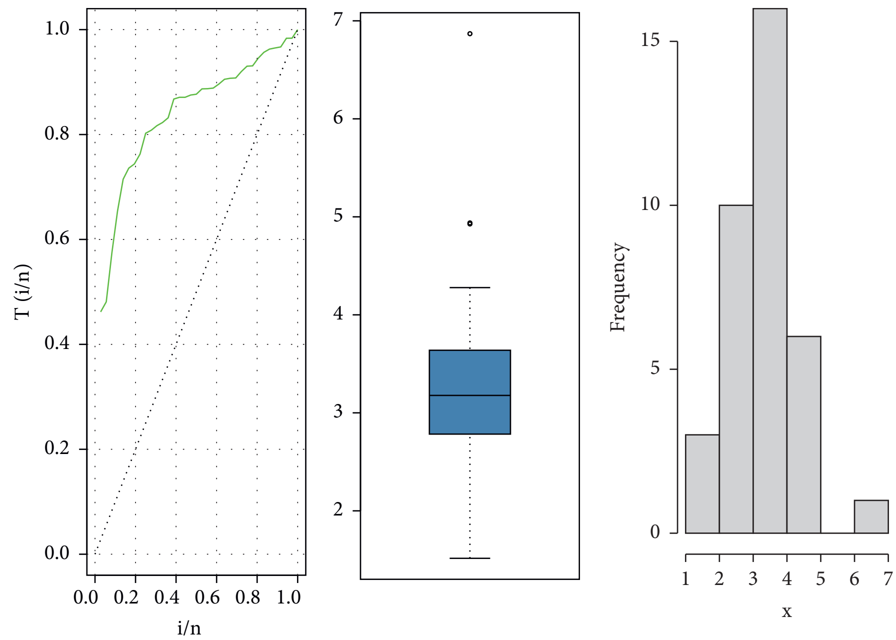
Distribution	AIC	BIC	CAIC	HQIC	KS	W	A
EGAPE	537.983	546.034	538.216	541.247	0.069	0.056	0.308
GAPE	539.358	544.720	539.350	541.533	0.079	0.083	0.491
EGE	538.356	546.403	538.580	541.619	0.072	0.059	0.329
E	596.131	598.81	596.160	597.218	0.241	0.062	0.363
MOAPIE	551.267	559.314	551.267	554.530	0.118	0.177	1.069
MOGE	538.355	546.140	538.586	541.618	0.074	0.059	0.328
APIE	550.634	555.998	550.748	552.809	0.089	0.195	1.212

5.1. Dataset I: Mexico Mortality Data. The first dataset depicts the Mexico mortality rate obtained from the following link: https://covid19.who.int/. Figure 5 shows the TTT plot for the Mexico mortality rate data. The data have an increasing hazard rate. Results for the Mexico application data are displayed in Tables 5 and 6.

5.2. Dataset II: Canada Data. The second dataset gives the COVID-19 mortality rate data for Canada for a length of 36 days obtained from the following link: https://covid19.who.int/. As depicted in Figure 6, the data are characterized by an increasing hazard rate. The results for dataset I are presented in Tables 7 and 8.

5.3. Concluding Remarks on the Two Applications. Based on the application from the two datasets, the following conclusions are drawn:

(1) Referring to dataset I, it can be observed that EGAPE provides the lowest values for the Kolmogorov–Smirnov and the W∗ and A∗ distances.



(a) (b) (c)

Figure 6: (a) TTT plot, (b) boxplot, and (c) histogram for Canada data.

Table 7: Estimates and SE (in parentheses).

Distribution	Estimates (SE in parenthesis)		
EGAPE (λ, α, a)	5.420 (5.267)	0.007 (0.041)	1.219 (0.152)
E (λ)	1.058 (0.0507)		
EE (λ, α)	1.058 (0.122)	17.977 (5.931)	
MOAPIE (λ, θ, a)	8.396 (7.967)	0.005 (0.003)	11.984 (1.856)
EGE (λ, θ, a)	0.654 (0.400)	1.522 (0.950)	16.799 (0.023)
APIE (λ, α)	2.719 (0.672)	0.211 (0.185)	

Table 8: Goodness of fit measures for Canada mortality rate data.

Distribution	AIC	BIC	CAIC	HQIC	KS	W	A
EGAPE	102.746	107.49	103.496	104.404	0.121	0.106	0.636
EGE	104.634	109.386	105.385	106.293	0.131	0.104	0.614
E	159.559	161.112	159.559	160.112	0.409	0.099	0.574
MOAPIE	112.433	117.183	113.183	114.091	0.409	0.099	1.022
EE	102.341	105.508	102.705	103.447	0.151	0.104	0.622
APIE	160.153	163.320	160.517	161.258	0.390	0.139	0.843

(2) With regard to dataset II, we can deduce that EGAPE provides the lowest values for the Kolmogorov–Smirnov and the W∗ and A∗ distances.

(3) Since the datasets are on COVID-19 mortality rates, the EGAPE model is the best model with comparison to the competing models to handle this set of data.

(4) In future, the study may be extended to consider other estimation methods of parameters like maximum product spacing and weighted least squares, among others.

(5) Since the study considered complete samples, future research may incorporate censored data.

(6) For future research, the regression framework may be considered to incorporate covariates.

6. Conclusion

In this paper, the main purpose was to develop a new three-parameter distribution called the EGAPE distribution which can be useful in modeling datasets that exhibit both monotone and nonmonotone hazard shapes. Several mathematical properties of the EGAPE distribution were derived. The estimation of the parameters of the distribution was estimated through the maximum likelihood method. Simulation study was performed to investigate the effectiveness of the method of estimation, and it was found that the root mean squared error and the average bias decrease with increase in sample size. Two real datasets were used to demonstrate the flexibility of the EGAPE distribution against its competing distribution. By use of goodness of fit tests, we demonstrated that the EGAPE distribution provides a better fit compared to that of competing distributions considered. With regard to application of the proposed model, other domains apart from COVID-19-related data can be considered like reliability engineering and financial sciences. Also, we will extend our work in the future by making regression analyses of the COVID-19 infections in future infections. This may help researchers and scientists to prepare suitable amounts of vaccines and enough space in hospitals. Also in our future work, we will study the vaccination rate and its effect on the mortality rate. Another approach that could be done is to use machine learning to predict future infections. Also, we will work on censored samples to avoid time-consuming and reduce the costs of the experiments. We will use different censoring schemes such as type-I and type-II censored schemes.

Data Availability

The essential documentation that was needed to support the findings of this study may be located within the entire article that was written on the study.

Conflicts of Interest

The writers warrant that they do not have any conflicts of interest to disclose.

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