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*Research Article*

# Various Uses of Topological Invariants in Jahangir Graph $J_{\beta,\alpha}$

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Topological indices are analytical indicators of a graph's topology. Graph theory has diverse applications in chemistry and conducts research on molecular structures, and its popularity has steadily increased since then. Many physicochemical attributes of a molecular substance can be identified using topological indices. Topological indices based on these chemical molecular structures can assist researchers in better understanding the physical properties, chemical reactivity, and biological activity. It is a way to fulfill the lack of experiments and give a theoretical basis for the manufacture of many chemical products. The objective of this research is to discuss bidistance degree-based topological invariants in the Jahangir graph.

## 1. Introduction

Chemical graph theory is a field of mathematical chemistry that uses graph theory to model chemical events numerically. In the computational and theoretical aspects of chemistry, topological indices play a critical role in estimating mechanical characteristics [1–7]. In chemistry, a lot of algebraic polynomials are helpful [8, 9]. Graph theory analyses the different properties of graphs [10]. The topological indices in graphs exhibit the graphical structure as well as a variety of other aspects. They frequently rely on the distances between the vertices, the degrees of the vertices, or the network portrayed by a matrix. Many networks' topological indices are crucial in computer-aided modeling, mathematical chemistry, and medicinal research.

Weiner [11] effectively brought the theory of topological indices based on graph distance in chemistry in 1947. To correlate aspects of aromatic aldehydes with the geometries of their molecular graphs, he proposed a distance-based topological index known as the wiener index.

Weiner was the first to provide a topological index assessment for graph $G$ in 1947 [12, 13]. He created the first index, the Weiner index, and since then, many further indices have been discovered. Few of them are discussed in the investigation of physicochemical properties of various substances in G. A-index demonstrates a high degree of predictability [14]. Kulli invented the K-Banhatti and K-hyper-Banhatti indices in 2016 [15]. In 2017, Kulli proposed new further topological indices, the modified K-Banhatti index and the harmonic K-Banhatti index [16]. The Banhatti indices are to some extent inspired by the Zagreb indices given by Gutman in 1972.

Baig et al. examine Revan and hyper-Revan indices for octahedral networks [17], [18]. Recently, Zhao et al. discussed the B-indices, R-index, and hyper-R-index for the very important structure of silicon carbide isomer [19]. As hydrocarbons are the main part of organic chemistry, Kulli et al. studied the multiplicative version of these indices in chemistry [20]. Revan polynomials are also introduced to get more information about the molecular structure of benzene [21]. Due to great importance of the Revan index, different forms of it were introduced, like Revan weighted Szeged index was introduced by Kandan in 2018 [22]. More advanced information about graphs and topological indices is discussed in references [23–34].

## 2. Jahangir Graph $J_{\beta,\alpha}$

The Jahangir graph, abbreviated as $J_{\beta,\alpha}$, is a graph with $\beta\alpha+1$ vertices and $\alpha(\beta+1)$ edges $\forall\beta\geq 2$ and $\alpha\geq 3$. $J_{\beta,\alpha}$ is made up of a cycle $C_{\beta,\alpha}$ plus one additional vertex that is at a distance from the vertices of $C_{\beta,\alpha}$. We will explore the different topological indices of the Jahangir graph $J_{\beta,\alpha}$. Our analyses will give a wide range of information about the Jahangir graph $J_{\beta,\alpha}$.

## 3. Preliminaries

First and second K-Banhatti indices: Kulli established the first and second K-Banhatti indices in [35] as follows:

$$\begin{aligned} B_1(J_{\beta,\alpha}) &= \sum_{ue\in E(G)} [d(u)+d(e)],\\ B_2(J_{\beta,\alpha}) &= \sum_{ue\in E(G)} [d(u)\times d(e)]. \end{aligned} \tag{1}$$

Modified first and second K-Banhatti indices are as follows:

$$\begin{aligned} {}^{m}B_1(J_{\beta,\alpha}) &= \sum_{ue\in E(G)} \frac{1}{d(u)+d(e)},\\ {}^{m}B_2(J_{\beta,\alpha}) &= \sum_{ue\in E(G)} \frac{1}{d(u)\times d(e)}. \end{aligned} \tag{2}$$

First and second K-hyper-Banhatti indices are as follows:

$$\begin{aligned} HB_1(J_{\beta,\alpha}) &= \sum_{ue\in E(G)} [d(u)+d(e)]^2,\\ HB_2(J_{\beta,\alpha}) &= \sum_{ue\in E(G)} [d(u)\times d(e)]^2. \end{aligned} \tag{3}$$

First and second hyper-Revan indices are as follows:

$$\begin{aligned} HR_1(J_{\beta,\alpha}) &= \sum_{uv\in E(G)} [r_G(u)+r_G(v)]^2,\\ HR_2(J_{\beta,\alpha}) &= \sum_{uv\in E(G)} [r_G(u)\times r_G(v)]^2. \end{aligned} \tag{4}$$

First and second Revan indices are as follows:

$$\begin{aligned} R_1(J_{\beta,\alpha}) &= \sum_{uv\in E(G)} [r_G(u)+r_G(v)],\\ R_2(J_{\beta,\alpha}) &= \sum_{uv\in E(G)} [r_G(u)r_G(v)]. \end{aligned} \tag{5}$$

Third and F-Revan indices are as follows:

$$\begin{aligned} \breve{R}_3(J_{\beta,\alpha}) &= \sum_{uv\in E(G)} |r_G(u)-r_G(v)|,\\ FR(J_{\beta,\alpha}) &= \sum_{uv\in E(G)} (r_G(u)^2-r_G(v)^2). \end{aligned} \tag{6}$$

Modified first and second Revan indices are as follows:

$$\begin{aligned} {}^{\alpha}R_1(J_{\beta,\alpha}) &= \sum_{uv\in E(G)} \frac{1}{r_G(u)+r_G(v)},\\ {}^{\alpha}R_2(J_{\beta,\alpha}) &= \sum_{uv\in E(G)} \frac{1}{r_G(u)r_G(v)}. \end{aligned} \tag{7}$$

Sum and product connectivity Revan indices are as follows:

$$\begin{aligned} SR(J_{\beta,\alpha}) &= \sum_{uv\in E(G)} \frac{1}{\sqrt{r_G(u)+r_G(v)}},\\ PR(J_{\beta,\alpha}) &= \sum_{uv\in E(G)} \frac{1}{\sqrt{r_G(u)r_G(v)}}. \end{aligned} \tag{8}$$

General first and second Revan indices are as follows:

$$\begin{aligned} R_1^{\vartheta}(J_{\beta,\alpha}) &= \sum_{uv\in E(G)} (r_G(u)+r_G(v))^2,\\ R_2^{\vartheta}(J_{\beta,\alpha}) &= \sum_{uv\in E(G)} (r_G(u)\times r_G(v))^2. \end{aligned} \tag{9}$$

Harmonic and symmetric division Revan indices are as follows:

$$\begin{aligned} HR(J_{\beta,\alpha}) &= \sum_{uv\in E(G)} \frac{2}{(r_G(u)+r_G(v))},\\ SDR(J_{\beta,\alpha}) &= \sum_{uv\in E(G)} \frac{r_G(u)}{r_G(v)}+\frac{r_G(v)}{r_G(u)}. \end{aligned} \tag{10}$$

Throughout this study, we will use the finite connected graph $G$, as shown in Figure 1. We denoted the vertices set by $V(G)$ and the edges set by $E(G)$. Suppose $u, v\in V(G)$, then we denote the degree of $u$ by $d(u)$, the distance between $u$ and $v$ by $d(u,v)$, an edge between the vertices $u$ and $v$ by $e=uv$, the degree of an edge $e$ by $d(e)$, where $d(e)=d(u)+d(v)-2$, and the maximum and minimum degrees in a graph by $\Delta(G)$ and $\delta(G)$ respectively. The nodes and lines are also called the elements of a graph.

## 4. Bidistance Edges

We have introduced a new way to find the TIs. It is an approach named bi-distance edges. In bi-distance edges, there are two single edges and three vertices. The concept is very beneficial for estimating the physio-chemical properties of chemical compounds. This concept gives very good correlation with many properties of graphs. The single-distance edge partition and the bidistance edge partition are

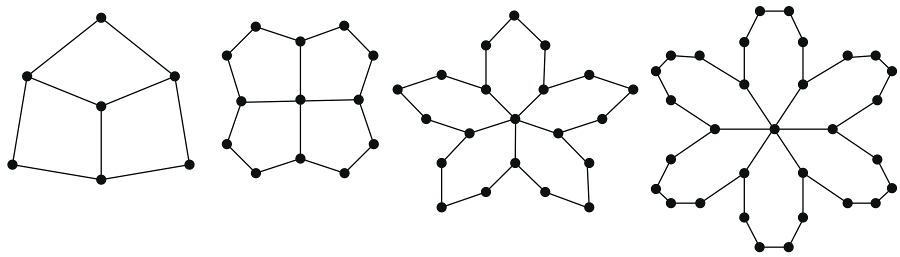


FIGURE 1: Jahangir graphs.

listed in Tables 1 and 2. The connected Jahangir graph $J_{5,3}$ is shown in Figure 2.

The single distance Banhatti index for Jahangir graph $J_{5,3}$ is given as follows:

$$\begin{aligned} B_1(J_{5,3}) &= \sum_{ue\in E(G)} [d(u) + d(e)] \\ &= 9(2+2) + 6(3+3) + 3(3+4) \\ &= 36 + 36 + 21 = 93. \end{aligned} \tag{11}$$

The bidistance first Banhatti index for the Jahangir graph $J_{5,3}$ is computed as follows:

$$\begin{aligned} B_1(J_{5,3}) &= \sum_{ue\in E(G)} [d(u) + d(e)] \\ &= 9(2+2) + 12(3+3) + 3(3+4) \\ &= 36 + 72 + 21 = 93. \end{aligned} \tag{12}$$

TABLE 1: The single distance edge partition.

| $(d(u), d(v))$ | Frequency | $d_G(e)$ |
|---|---|---|
| (2, 2) | 9 | 2 |
| (3, 2) | 6 | 3 |
| (3, 3) | 3 | 4 |

TABLE 2: The bidistance edge partition.

| $(d(u), d(v))$ | Frequency | $d_G(e)$ |
|---|---|---|
| (2, 2) | 9 | 2 |
| (3, 2) | 12 | 3 |
| (3, 3) | 3 | 4 |

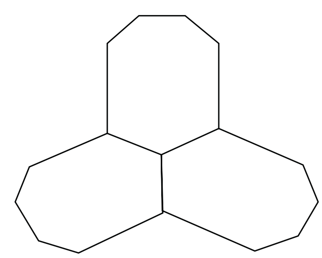

FIGURE 2: Jahangir graph $J_{5,3}$.

## 5. Main Results

In this section, we study twenty important results related to TIs and also check out their results in graph form. First, we have edges and vertex divisions, as given in Table 3. The edges are partitioned into different categories on the basis of degree. In Table 3, we have Revan vertices $r_G(u), r_G(v)$. The Revan indices are defined as follows:

$$\begin{aligned} r_G(u) &= \Delta(G) + \delta(G) - d_G(u), \\ r_G(v) &= \Delta(G) + \delta(G) - d_G(v), \end{aligned} \tag{13}$$

where terms $\Delta(G)$ and $\delta(G)$ are the maximum and minimum degrees among the vertices respectively, and the degree of Banhatti edge is computed as follows:

$$d(e) = d_G(u) + d_G(v) - 2. \tag{14}$$

We are dealing with the division of edges and various terms derived from this division. The Jahangir graph has four types of different edge parcels, as listed in Table 3.

**Theorem 1.** *Consider $J_{\beta,\alpha}$ represents the Jahangir graph, then*

$$\begin{aligned} B_1(J_{\beta,\alpha}) &= 8\alpha\beta + 13\alpha^2 + 19\alpha - 16\beta, \\ B_2(J_{\beta,\alpha}) &= 8\alpha\beta + 16\alpha^2 + 2\alpha^3 + 18\alpha - 16\beta. \end{aligned} \tag{15}$$

*Proof.* Let $J_{\beta,\alpha}$ be the Jahangir graph. The first K-Banhatti index of $J[\beta, \alpha]$ is then calculated by using Table 3:

$$\begin{aligned} B_1(J_{\beta,\alpha}) &= \sum_{ue\in E(G)} [d(u) + d(e)] \\ &= \sum_{uv=e} [d(u) + d(e) + d(v) + d(e)] \\ &= 2\alpha[(3+3) + (2+3)] + (\alpha\beta - 2\beta)[(2+2) \\ &\quad + (2+2)] + \frac{1}{2}\alpha^2 - \frac{1}{2}\alpha[(3+4) + (3+4)] \\ &\quad + 2\alpha[(2+\alpha) + (\alpha+\alpha)] \\ &= 2\alpha(11) + (\alpha\beta - 2\beta)(8) \\ &\quad + \frac{1}{2}\alpha^2 - \frac{1}{2}\alpha(14) + 2\alpha(2+3\alpha) \\ &= 8\alpha\beta + \frac{13}{2}\alpha^2 + 19\alpha - 16\beta. \end{aligned} \tag{16}$$

Table 3: Degree-based partition of edges.

| $d(u), d(v)$ | Frequency | $d(e)$ | $r_G(u)$ | $r_G(v)$ |
|---|---|---|---|---|
| (3, 2) | $2\alpha$ | 3 | $\alpha - 1$ | $\alpha$ |
| (2, 2) | $\alpha\beta - 2\beta$ | 2 | $\alpha$ | $\alpha$ |
| (3, 3) | $(1/2)\,\alpha^2 - (1/2)\,\alpha$ | 4 | $\alpha - 1$ | $\alpha - 1$ |
| (2, $\alpha$) | $2\alpha$ | $\alpha$ | $\alpha$ | $2\alpha + 2$ |

Second K-Banhatti index of $J[\beta, \alpha]$ is determined as follows:

$$
\begin{aligned}
\mathcal{B}_2\left(J_{\beta,\alpha}\right) &= \sum_{ue\in E(G)} [d(u) \times d(v)] \\
&= \sum_{uv=e} [d(u) \times d(e) + d(v) \times d(e)] \\
&= 2\alpha[(3 \times 3) + (2 \times 3)] \\
&\quad + (\alpha\beta - 2\beta)[(2 \times 2) + (2 \times 2)] \\
&\quad + \frac{1}{2}\alpha^2 - \frac{1}{2}\alpha[(3 \times 4) + (3 \times 4)] \\
&\quad + 2\alpha[(2 \times \alpha) + (\alpha \times \alpha)] \\
&= 2\alpha(15) + (\alpha\beta - 2\beta)(8) + \frac{1}{2}\alpha^2 \\
&\quad - \frac{1}{2}\alpha(24) + 2\alpha\left(2\alpha + \alpha^2\right) \\
&= 8\alpha\beta + 16\alpha^2 + 2\alpha^3 + 18\alpha - 16\beta.
\end{aligned}
\tag{17}
$$

□

**Theorem 2.** *Consider $J_{\beta,\alpha}$ represents the Jahangir graph, then*

$$
\begin{aligned}
{}^{m}\mathcal{B}_1\left(J_{\beta,\alpha}\right) &= \frac{1}{2}\alpha\beta + \frac{1}{7}\alpha^2 + \frac{62}{105}\alpha - \beta + \left[\frac{3\alpha + 2}{(2 + \alpha)}\right], \\
{}^{m}\mathcal{B}_2\left(J_{\beta,\alpha}\right) &= \frac{1}{2}\alpha\beta + \frac{1}{12}\alpha^2 + \frac{1}{36}\alpha - \frac{2}{\alpha}\beta + 1.
\end{aligned}
\tag{18}
$$

*Proof.* Let $J_{\beta,\alpha}$ be the Jahangir graph. The modified first K-Banhatti index of $J[\beta, \alpha]$ is then determined using Table 3 as follows:

$$
\begin{aligned}
{}^{m}\mathcal{B}_1\left(J_{\beta,\alpha}\right) &= \sum_{ue\in E(G)} \frac{1}{d(u) + d(e)} \\
&= \sum_{uv=e} \frac{1}{d(u) + d(e)} + \frac{1}{d(v) + d(e)} \\
&= 2\alpha\left[\frac{1}{6} + \frac{1}{5}\right] + (\alpha\beta - 2\beta)\left[\frac{1}{4} + \frac{1}{4}\right] + \frac{1}{2}\alpha^2 \\
&\quad - \frac{1}{2}\alpha\left[\frac{1}{7} + \frac{1}{7}\right] + 2\alpha\left[\frac{1}{2 + \alpha} + \frac{1}{2\alpha}\right] \\
&= 2\alpha\left(\frac{11}{30}\right) + (\alpha\beta - 2\beta)\left(\frac{1}{2}\right) + \frac{1}{2}\alpha^2 \\
&\quad - \frac{1}{2}\alpha\left(\frac{2}{7}\right) + 2\alpha\left(\frac{3\alpha + 2}{4\alpha + 2\alpha^2}\right) \\
&= \frac{1}{2}\alpha\beta + \frac{1}{7}\alpha^2 + \frac{62}{105}(\alpha - \beta) + \left(\frac{3\alpha + 2}{(2 + \alpha)}\right).
\end{aligned}
\tag{19}
$$

From the definition of second K-Banhatti index (${}^{m}\mathcal{B}_2(J_{\beta,\alpha})$), we have

$$
\begin{aligned}
{}^{m}\mathcal{B}_2\left(J_{\beta,\alpha}\right) &= \sum_{ue\in E(G)} \frac{1}{d(u) \times d(e)} \\
&= \sum_{uv=e} \frac{1}{d(u) \times d(e)} + \frac{1}{d(v) \times d(e)} \\
&= 2\alpha\left[\frac{1}{9} + \frac{1}{6}\right] + (\alpha\beta - 2\beta)\left[\frac{1}{4} + \frac{1}{4}\right] \\
&\quad + \frac{1}{2}\alpha^2 - \frac{1}{2}\alpha\left[\frac{1}{12} + \frac{1}{12}\right] + 2\alpha\left[\frac{1}{2\alpha} + \frac{1}{\alpha^2}\right] \\
&= \frac{1}{2}\alpha\beta + \frac{1}{12}\alpha^2 + \frac{1}{36}\alpha + \frac{2}{\alpha} - \beta + 1.
\end{aligned}
\tag{20}
$$

□

**Theorem 3.** *Consider $J_{\beta,\alpha}$ represents the Jahangir graph, then*

$$
\begin{aligned}
H\mathcal{B}_1\left(J_{\beta,\alpha}\right) &= 32\alpha\beta + 10\alpha^3 + 57\alpha^2 + 81\alpha - 64\beta, \\
H\mathcal{B}_2\left(J_{\beta,\alpha}\right) &= 32\alpha\beta + 2\alpha^5 + 8\alpha^3 + 144\alpha^2 + 90\alpha - 64\beta.
\end{aligned}
\tag{21}
$$

*Proof.* Let $J_{\beta,\alpha}$ be the Jahangir graph. The first K-hyper-Banhatti index of $J[\beta, \alpha]$ is evaluated using Table 3:

$$
\begin{aligned}
H\mathcal{B}_1\left(J_{\beta,\alpha}\right) &= \sum_{ue} [d(u) + d(e)]^2 \\
&= \sum_{uv=e\in E} \left[(d(u) + d(e))^2 + (d(v) + d(e))^2\right] \\
&= 2\alpha\left[(3 + 3)^2 + (2 + 3)^2\right] \\
&\quad + (\alpha\beta - 2\beta)\left[(2 + 2)^2 + (2 + 2)^2\right] \\
&\quad + \frac{1}{2}\alpha^2 - \frac{1}{2}\alpha\left[(3 + 4)^2 + (3 + 4)^2\right] \\
&\quad + 2\alpha\left[(2 + \alpha)^2 + (2\alpha)^2\right] \\
&= 32\alpha\beta + 10\alpha^3 + 57\alpha^2 + 81\alpha - 64\beta.
\end{aligned}
\tag{22}
$$

From the definition of second K-hyper-Banhatti index $H\mathcal{B}_2(J_{\beta,\alpha})$, we have

$$
\begin{aligned}
\mathrm{HB}_2\left(J_{\beta,\alpha}\right) &= \sum_{ue} [d(u)\times d(v)]^2 \\
&= \sum_{uv=e\in E}\left[(d(u)\times d(e))^2+(d(v)\times d(e))^2\right] \\
&= 2\alpha\left[(3\times 3)^2+(2\times 3)^2\right] \\
&\quad +(\alpha\beta-2\beta)\left[(2\times 2)^2+(2\times 2)^2\right] \\
&\quad +\frac{1}{2}\alpha^2-\frac{1}{2}\alpha\left[(3\times 4)^2+(3\times 4)^2\right] \\
&\quad +2\alpha\left[(2\times\alpha)^2+(\alpha\times\alpha)^2\right] \\
&= 32\alpha\beta+2\alpha^5+8\alpha^3+144\alpha^2+90\alpha-64\beta.
\end{aligned} \tag{23}
$$

□

**Theorem 4.** *Consider $J_{\beta,\alpha}$ represents the Jahangir graph, then*

$$
\begin{aligned}
\mathrm{HR}_1\left(J_{\beta,\alpha}\right) &= 2\alpha^4+20\alpha^3+22\alpha^2+8\alpha+4\alpha^3\beta-8\alpha^2\beta, \\
\mathrm{HR}_2\left(J_{\beta,\alpha}\right) &= 4\alpha^5+8\alpha^4+16\alpha^3+8\alpha+2\alpha^3\beta-4\alpha^2\beta.
\end{aligned} \tag{24}
$$

*Proof.* Let $J_{\beta,\alpha}$ be the Jahangir graph. The first hyper-Revan index of $J[\beta,\alpha]$ is then calculated using Table 3 as follows:

$$
\begin{aligned}
\mathrm{HR}_1\left(J_{\beta,\alpha}\right) &= \sum_{uv\in E(G)}\left[r_G(u)+r_G(v)\right]^2 \\
&= 2\alpha[\alpha-1+\alpha]^2+\alpha\beta-2\beta[\alpha+\alpha]^2 \\
&\quad +\frac{1}{2}\alpha^2-\frac{1}{2}\alpha[\alpha-1+\alpha-1]^2+2\alpha[\alpha+2\alpha+2]^2 \\
&= 2\alpha[2\alpha-1]^2+\alpha\beta-2\beta[2\alpha]^2 \\
&\quad +\frac{1}{2}\alpha^2-\frac{1}{2}\alpha[2\alpha-2]^2+2\alpha[\alpha+2\alpha+2]^2 \\
&= 2\alpha\left(4\alpha^2-4\alpha+1\right)+\alpha\beta-2\beta\left(4\alpha^2\right) \\
&\quad +\frac{1}{2}\alpha^2-\frac{1}{2}\alpha\left(4\alpha^2-8\alpha+4\right)^2 \\
&\quad +2\alpha\left(9\alpha^2+12\alpha+4\right) \\
&= 2\alpha^4+20\alpha^3+22\alpha^2+8\alpha+4\alpha^3\beta-8\alpha^2\beta.
\end{aligned} \tag{25}
$$

By the definition of second hyper-Revan index ($\mathrm{HR}_2(J_{\beta,\alpha})$), we have

$$
\begin{aligned}
\mathrm{HR}_2\left(J_{\beta,\alpha}\right) &= \sum_{uv\in E(G)}\left[r_G(u)\times r_G(v)\right]^2 \\
&= 2\alpha(2\alpha\times 1)^2+\alpha\beta-2\beta(2\alpha)^2+\frac{1}{2}\alpha^2 \\
&\quad -\frac{1}{2}\alpha(2\alpha\times 2)^2+2\alpha(\alpha\times 2\alpha+2)^2 \\
&= 2\alpha(2\alpha)^2+\alpha\beta-2\beta(2\alpha)^2+\frac{1}{2}\alpha^2 \\
&\quad -\frac{1}{2}\alpha(4\alpha)^2+2\alpha\left(2\alpha^2+2\right)^2 \\
&= 4\alpha^5+8\alpha^4+16\alpha^3+8\alpha+2\alpha^3\beta-4\alpha^2\beta.
\end{aligned} \tag{26}
$$

□

**Theorem 5.** *Consider $J_{\beta,\alpha}$ represents the Jahangir graph, then*

$$
\begin{aligned}
\breve{R}_3\left(J_{\beta,\alpha}\right) &= 3\alpha^2+5\alpha, \\
\mathrm{FR}\left(J_{\beta,\alpha}\right) &= 2\alpha^3\beta-4\alpha^2\beta+11\alpha^3+16\alpha^2+9\alpha.
\end{aligned} \tag{27}
$$

*Proof.* Let $J_{\beta,\alpha}$ be the Jahangir graph. The Revan index of $J[\beta,\alpha]$ is then calculated using Table 3 as follows:

$$
\breve{R}_3\left(J_{\beta,\alpha}\right) = \sum_{uv\in E(G)}\left|r_G(u)-r_G(v)\right|,
$$

$$
\begin{aligned}
&= 2\alpha(1)+(\alpha\beta-2\beta)(0) \\
&\quad +\frac{1}{2}\alpha^2-\frac{1}{2}\alpha(2)+2\alpha(\alpha+2) \\
&= 2\alpha+\alpha^2-\alpha+2\alpha^2+4\alpha = 3\alpha^2+5\alpha.
\end{aligned} \tag{28}
$$

By the definition of F-Revan index ($\mathrm{FR}(J_{\beta,\alpha})$), we have

$$
\begin{aligned}
\mathrm{FR}\left(J_{\beta,\alpha}\right) &= \sum_{uv\in E(G)}\left(r_G(u)^2-r_G(v)^2\right) \\
&= 2\alpha\left[(\alpha-1)^2+\alpha^2\right]+\alpha\beta-2\beta\left[\alpha^2+\alpha^2\right] \\
&\quad +\frac{1}{2}\alpha^2-\frac{1}{2}\alpha\left[(\alpha-1)^2+(\alpha-1)^2\right] \\
&\quad +2\alpha\left[\alpha^2+(2\alpha+2)^2\right] \\
&= 2\alpha\left[\alpha^2+1-2\alpha+\alpha^2\right]+\alpha\beta-2\beta\left(2\alpha^2\right) \\
&\quad +\frac{1}{2}\alpha^2-\frac{1}{2}\alpha\left[\alpha^2+1-2\alpha+\alpha^2+1-2\alpha\right] \\
&\quad +2\alpha\left(\alpha^2+4\alpha^2+4+8\alpha\right) \\
&= 2\alpha\left(2\alpha^2-2\alpha+1\right)+\alpha\beta-2\beta\left(2\alpha^2\right)+\frac{1}{2}\alpha^2 \\
&\quad -\frac{1}{2}\alpha\left(2\alpha^2-4\alpha+2\right)^2+2\alpha\left(5\alpha^2+8\alpha+4\right) \\
&= 2\alpha^3\beta-4\alpha^2\beta+11\alpha^3+16\alpha^2+9\alpha.
\end{aligned} \tag{29}
$$

□

**Theorem 6.** *Consider $J_{\beta,\alpha}$ represents the Jahangir graph, then*

$$
\begin{aligned}
R_1\left(J_{\beta,\alpha}\right) &= 2\alpha^2\beta-2\alpha\beta+\alpha^3+8\alpha^2+3\alpha, \\
R_2\left(J_{\beta,\alpha}\right) &= \alpha^3\beta-2\alpha^2\beta+\frac{1}{2}\alpha^4+\frac{9}{2}\alpha^3+\frac{7}{2}\alpha^2-\frac{1}{2}\alpha.
\end{aligned} \tag{30}
$$

*Proof.* Let $J_{\beta,\alpha}$ be the Jahangir graph. The first Revan index of $J\,[\beta,\,\alpha]$ is then determined using Table 3 as follows:

$$\begin{aligned}
R_1\left(J_{\beta,\alpha}\right) &= \sum_{uv\in(G)} \left[r_G(u) + r_G(v)\right] \\
&= 2\alpha(\alpha-1+\alpha) + (\alpha\beta - 2\beta)(\alpha+\alpha) \\
&\quad + \frac{1}{2}\alpha^2 - \frac{1}{2}\alpha(\alpha-1+\alpha-1) + 2\alpha(\alpha+2\alpha+2) \\
&= 2\alpha(2\alpha-1) + \alpha\beta - 2\beta(2\alpha)^2 + \frac{1}{2}\alpha^2 \\
&\quad - \frac{1}{2}\alpha(2\alpha-2) + 2\alpha(3\alpha+2) \\
&= 2\alpha^2\beta - 2\alpha\beta + \alpha^3 + 8\alpha^2 + 3\alpha.
\end{aligned} \tag{31}$$

From the definition of second Revan index ($R_2\,(J_{\beta,\alpha})$), we have

$$\begin{aligned}
R_2\left(J_{\beta,\alpha}\right) &= \sum_{uv\in(G)} \left[r_G(u) r_G(v)\right] \\
&= 2\alpha[(\alpha-1)\times\alpha] + \alpha\beta - 2\beta[\alpha\times\alpha] \\
&\quad + \frac{1}{2}\alpha^2 - \frac{1}{2}\alpha[(\alpha-1)\times(\alpha-1)] \\
&\quad + 2\alpha[\alpha\times(2\alpha+2)] \\
&= 2\alpha\left(\alpha^2-\alpha\right) + \alpha\beta - 2\beta(\alpha)^2 + \frac{1}{2}\alpha^2 \\
&\quad - \frac{1}{2}\alpha\left(\alpha^2 - 2\alpha + 1\right) + 2\alpha\left(2\alpha^2 + 2\alpha\right) \\
&= \alpha^3\beta - 2\alpha^2\beta + \frac{1}{2}\alpha^4 + \frac{9}{2}\alpha^3 + \frac{7}{2}\alpha^2 + \frac{1}{2}\alpha.
\end{aligned} \tag{32}$$

□

**Theorem 7.** *Consider $J_{\beta,\alpha}$ represents the Jahangir graph, then*

$$\begin{aligned}
{}^{\alpha}R_1\left(J_{\beta,\alpha}\right) &= \frac{2\alpha}{2\alpha-1} + \frac{\alpha\beta-2\beta}{2\alpha} + \frac{\alpha^2}{4(\alpha-1)} \\
&\quad - \frac{\alpha}{4(\alpha-1)} + \frac{2\alpha}{3\alpha+2}, \\
{}^{\alpha}R_2\left(J_{\beta,\alpha}\right) &= \frac{2}{\alpha-1} + \frac{\alpha\beta-2\beta}{\alpha^2} + \frac{\alpha^2}{2\left(1-2\alpha+\alpha^2\right)} \\
&\quad - \frac{\alpha}{2\left(1-2\alpha+\alpha^2\right)} + \frac{\alpha}{1+\alpha^2}.
\end{aligned} \tag{33}$$

*Proof.* Let $J_{\beta,\alpha}$ be the Jahangir graph. The modified first Revan index of $J\,[\beta,\,\alpha]$ is then calculated using Table 3 as follows:

$$\begin{aligned}
{}^{\alpha}R_1\left(J_{\beta,\alpha}\right) &= \sum_{uv\in E(G)} \left[r_G(u) + r_G(v)\right] \\
&= 2\alpha\left[\frac{1}{\alpha-1+\alpha}\right] + (\alpha\beta-2\beta)\left[\frac{1}{\alpha+\alpha}\right] \\
&\quad + \frac{1}{2}\alpha^2 - \frac{1}{2}\alpha\left[\frac{1}{\alpha-1+\alpha-1}\right] + 2\alpha\left[\frac{1}{\alpha+2\alpha+2}\right] \\
&= 2\alpha\left[\frac{1}{2\alpha-1}\right] + (\alpha\beta-2\beta)\left[\frac{1}{2\alpha}\right] + \frac{1}{2}\alpha^2 \\
&\quad - \frac{1}{2}\alpha\left[\frac{1}{2\alpha-2}\right] + 2\alpha\left[\frac{1}{3\alpha+2}\right] \\
&= \frac{2\alpha}{2\alpha-1} + \frac{\alpha\beta-2\beta}{2\alpha} + \frac{\alpha^2}{4(\alpha-1)} - \frac{\alpha}{4(\alpha-1)} + \frac{2\alpha}{3\alpha+2}.
\end{aligned} \tag{34}$$

From the definition of modified second Revan index (${}^{\alpha}R_2\,(J_{\beta,\alpha})$), we have

$$\begin{aligned}
{}^{\alpha}R_2\left(J_{\beta,\alpha}\right) &= \sum_{uv\in E(G)} \frac{1}{r_G(u) r_G(v)} \\
&= 2\alpha\left[\frac{1}{(\alpha-1)(\alpha)}\right] + (\alpha\beta-2\beta)\left[\frac{1}{2\alpha^2}\right] \\
&\quad + \frac{1}{2}\alpha^2 - \frac{1}{2}\alpha\left[\frac{1}{(\alpha-1)^2}\right] + 2\alpha\left[\frac{1}{\alpha(2\alpha+2)}\right] \\
&= 2\alpha\left[\frac{1}{-\alpha+\alpha^2}\right] + (\alpha\beta-2\beta)\left[\frac{1}{\alpha^2}\right] + \frac{1}{2}\alpha^2 \\
&\quad - \frac{1}{2}\alpha\left[\frac{1}{\left(1-2\alpha+\alpha^2\right)}\right] + 2\alpha\left[\frac{1}{2+2\alpha^2}\right] \\
&= \frac{2}{\alpha-1} + \frac{\alpha\beta-2\beta}{\alpha^2} + \frac{\alpha^2}{2\left(1-2\alpha+\alpha^2\right)} \\
&\quad - \frac{\alpha}{2\left(1-2\alpha+\alpha^2\right)} + \frac{\alpha}{\left(1+\alpha^2\right)}.
\end{aligned} \tag{35}$$

□

**Theorem 8.** *Consider $J_{\beta,\alpha}$ represents the Jahangir graph, then*

$$\begin{aligned}
\mathrm{SR}\left(J_{\beta,\alpha}\right) &= \frac{2\alpha}{\sqrt{2\alpha-1}} + \frac{\alpha\beta-2\beta}{\sqrt{2\alpha}} + \frac{\alpha^2}{2\sqrt{2(\alpha-1)}} \\
&\quad - \frac{\alpha}{2\sqrt{2(\alpha-1)}} + \frac{2\alpha}{\sqrt{3\alpha+2}}, \\
\mathrm{PR}\left(J_{\beta,\alpha}\right) &= 2\alpha\left[\frac{1}{\sqrt{\alpha(\alpha-1)}}\right] + \beta - 2\frac{\beta}{\alpha} + \frac{\alpha^2}{2(\alpha-1)} \\
&\quad - \frac{\alpha}{2(\alpha-1)} + 2\alpha\left[\frac{1}{\sqrt{\alpha(2\alpha+2)}}\right].
\end{aligned} \tag{36}$$

*Proof.* Let $J_{\beta,\alpha}$ be the Jahangir graph. The sum connectivity Revan index of $J\,[\beta,\,\alpha]$ is then calculated using Table 3 as follows:

$$\begin{aligned}
\mathrm{SR}\left(J_{\beta,\alpha}\right) &= \sum_{uv\in E(G)} \frac{1}{\sqrt{r_G(u)+r_G(v)}} \\
&= 2\alpha\left[\frac{1}{\sqrt{\alpha-1+\alpha}}\right] + (\alpha\beta-2\beta)\left[\frac{1}{\sqrt{\alpha+\alpha}}\right] \\
&\quad + \frac{1}{2}\alpha^2 - \frac{1}{2}\alpha\left[\frac{1}{\sqrt{\alpha-1+\alpha-1}}\right] \\
&\quad + 2\alpha\left[\frac{1}{\sqrt{\alpha+2\alpha+2}}\right] \\
&= \frac{2\alpha}{\sqrt{2\alpha-1}} + \frac{\alpha\beta-2\beta}{\sqrt{2\alpha}} \\
&\quad + \frac{\alpha^2}{2\sqrt{2(\alpha-1)}} - \frac{\alpha}{2\sqrt{2(\alpha-1)}} + \frac{2\alpha}{\sqrt{3\alpha+2}}.
\end{aligned} \tag{37}$$

From the definition of product connectivity Revan index (PR $(J_{\beta,\alpha})$), we have

$$\begin{aligned}
\mathrm{PR}\left(J_{\beta,\alpha}\right) &= \sum_{uv\in E(G)} \frac{1}{\sqrt{r_G(u)r_G(v)}} \\
&= 2\alpha\left[\frac{1}{\sqrt{\alpha(\alpha-1)}}\right] + (\alpha\beta-2\beta)\left[\frac{1}{\sqrt{\alpha^2}}\right] \\
&\quad + \frac{1}{2}\alpha^2 - \frac{1}{2}\alpha\left[\frac{1}{\sqrt{(\alpha-1)^2}}\right] + 2\alpha\left[\frac{1}{\sqrt{\alpha(2\alpha+2)}}\right] \\
&= 2\alpha\left[\frac{1}{\sqrt{\alpha(\alpha-1)}}\right] + \beta - 2\frac{\beta}{\alpha} + \frac{\alpha^2}{2(\alpha-1)} \\
&\quad - \frac{\alpha}{2(\alpha-1)} + 2\alpha\left[\frac{1}{\sqrt{\alpha(2\alpha+2)}}\right].
\end{aligned} \tag{38}$$

□

**Theorem 9.** *Consider $J_{\beta,\alpha}$ represents the Jahangir graph, then*

$$\begin{aligned}
R_1^{\vartheta}\left(J_{\beta,\alpha}\right) &= 2\alpha^2\beta - 2\alpha\beta + \alpha^3 + 8\alpha^2 + 3\alpha, \\
R_2^{\vartheta}\left(J_{\beta,\alpha}\right) &= \alpha^5\beta - 2\beta\alpha^4 + \frac{1}{2}\alpha^6 + \frac{23}{2}\alpha^5 \\
&\quad + 5\alpha^4 + 3\alpha^3 - \frac{3}{2}\alpha^2 - \frac{1}{2}\alpha.
\end{aligned} \tag{39}$$

*Proof.* Let $J_{\beta,\alpha}$ be the Jahangir graph. The general first Revan index of $J\,[\beta,\,\alpha]$ is then calculated using Table 3 as follows:

$$\begin{aligned}
R_1^{\vartheta}\left(J_{\beta,\alpha}\right) &= \sum_{uv\in E(G)} \left(r_G(u)+r_G(v)\right)^2 \\
&= 2\alpha(\alpha-1+\alpha)^{\vartheta} + \alpha\beta - 2\beta(\alpha+\alpha)^{\vartheta} \\
&\quad + \frac{1}{2}\alpha^2 - \frac{1}{2}\alpha(\alpha-1+\alpha-1)^{\vartheta} + 2\alpha(\alpha+2\alpha+2)^{\vartheta} \\
&= 2\alpha(2\alpha-1)^{\vartheta} + \alpha\beta - 2\beta(2\alpha)^{\vartheta} + \frac{1}{2}\alpha^2 \\
&\quad - \frac{1}{2}\alpha(2\alpha-2)^{\vartheta} + 2\alpha(3\alpha+2)^{\vartheta} \\
\therefore \vartheta &= 2 \\
&= 2\alpha^2\beta - 2\alpha\beta + \alpha^3 + 8\alpha^2 + 3\alpha.
\end{aligned} \tag{40}$$

From the definition of general second Revan index ($R_2^{\vartheta}$ $(J_{\beta,\alpha})$), we have

$$\begin{aligned}
R_2^{\vartheta}\left(J_{\beta,\alpha}\right) &= \sum_{uv\in E(G)} \left(r_G(u)\times r_G(v)\right)^2 \\
&= 2\alpha((\alpha-1)\alpha)^{\vartheta} + \alpha\beta - 2\beta\left(\alpha^2\right)^{\vartheta} + \frac{1}{2}\alpha^2 \\
&\quad - \frac{1}{2}\alpha\left((\alpha-1)^2\right)^{\vartheta} + 2\alpha((2\alpha+2)\alpha)^{\vartheta} \\
&= 2\alpha\left(\alpha^2-\alpha\right)^{\vartheta} + \alpha\beta - 2\beta\left(\alpha^2\right)^{\vartheta} \\
&\quad + \frac{1}{2}\alpha^2 - \frac{1}{2}\alpha\left((\alpha-1)^2\right)^{\vartheta} + 2\alpha\left(2\alpha^2+2\alpha\right)^{\vartheta} \\
\therefore \vartheta &= 2 \\
&= 2\alpha\left(\alpha^2-\alpha\right)^2 + \alpha\beta - 2\beta\left(\alpha^2\right)^2 \\
&\quad + \frac{1}{2}\alpha^2 - \frac{1}{2}\alpha\left((\alpha-1)^2\right)^2 + 2\alpha\left(2\alpha^2+2\alpha\right)^2 \\
&= 2\alpha\left(\alpha^4+\alpha^2-2\alpha^3\right) + \alpha^5\beta - 2\beta\alpha^4 \\
&\quad + \frac{1}{2}\alpha^2 - \frac{1}{2}\alpha\left(\alpha^4+4\alpha^3+6\alpha^2+4\alpha+1\right) \\
&\quad + 2\alpha\left(4\alpha^4+\alpha^2+4\alpha^3\right) \\
&= \alpha^5\beta - 2\beta\alpha^4 + \frac{1}{2}\alpha^6 + \frac{23}{2}\alpha^5 \\
&\quad + 5\alpha^4 + 3\alpha^3 - \frac{3}{2}\alpha^2 - \frac{1}{2}\alpha.
\end{aligned} \tag{41}$$

□

**Theorem 10.** *Consider $J_{\beta,\alpha}$ represents the Jahangir graph, then*

Table 4: Numerical comparison.

| Banhatti indices | $(J_{\beta,\alpha})=(2,3)$ | $(J_{\beta,\alpha})=(3,4)$ | $(J_{\beta,\alpha})=(4,5)$ | $(J_{\beta,\alpha})=(5,6)$ | $(J_{\beta,\alpha})=(6,7)$ |
|---|---|---|---|---|---|
| Ɓ$_1$ $(J_{\beta,\alpha})$ | 190 | 332 | 516 | 742 | 1010 |
| Ɓ$_2$ $(J_{\beta,\alpha})$ | 268 | 504 | 836 | 1276 | 1836 |
| $^{m}$Ɓ$_1$ $(J_{\beta,\alpha})$ | 6.2567 | 9.9806 | 14.9516 | 21.1850 | 28.689 |
| $^{m}$Ɓ$_2$ $(J_{\beta,\alpha})$ | 3.5003 | 5.944 | 9.6219 | 14.4997 | 21.1344 |
| HƁ$_1$ $(J_{\beta,\alpha})$ | 1090 | 2068 | 3464 | 5338 | 7686 |
| HƁ$_2$ $(J_{\beta,\alpha})$ | 2332 | 5416 | 11684 | 23644 | 45004 |
| $\mathrm{HR}_1$ $(J_{\beta,\alpha})$ | 996 | 2560 | 554 | 10632 | 18676 |
| $\mathrm{HR}_2$ $(J_{\beta,\alpha})$ | 2112 | 7392 | 20140 | 46416 | 9492 |
| $\breve{R}_3$ $(J_{\beta,\alpha})$ | 42 | 68 | 100 | 138 | 182 |
| FR $(J_{\beta,\alpha})$ | 504 | 1188 | 2420 | 4446 | 7560 |
| $R_1$ $(J_{\beta,\alpha})$ | 132 | 276 | 500 | 822 | 1260 |
| $R_2$ $(J_{\beta,\alpha})$ | 192 | 566 | 1260 | 2463 | 4382 |
| $^{\alpha}R_1$ $(J_{\beta,\alpha})$ | 2.829 | 3.464 | 4.149 | 4.858 | 5.578 |
| $^{\alpha}R_2$ $(J_{\beta,\alpha})$ | 2.272 | 1.944 | 1.797 | 1.718 | 1.669 |
| SR $(J_{\beta,\alpha})$ | 6.809 | 9.733 | 13.089 | 16.818 | 20.882 |
| PR $(J_{\beta,\alpha})$ | 5.8409 | 7.0743 | 8.4271 | 9.8335 | 11.2688 |
| $R_1^{\vartheta}$ $(J_{\beta,\alpha})$ | 132 | 276 | 500 | 822 | 1260 |
| $R_2^{\vartheta}$ $(J_{\beta,\alpha})$ | 3792 | 16806 | 5471 | 145743 | 337092 |
| HR $(J_{\beta,\alpha})$ | 5.6576 | 6.6429 | 8.2987 | 9.7152 | 11.1570 |
| SDR $(J_{\beta,\alpha})$ | 91.334 | 161.134 | 249.917 | 358.772 | 487.667 |

$$\begin{aligned}\mathrm{HR}\left(J_{\beta,\alpha}\right) &= \left[\frac{4\alpha}{2\alpha-1}\right]+\beta-2\frac{\beta}{\alpha}+\frac{\alpha^2}{2(\alpha-1)}\\ &\quad -\frac{\alpha}{2(\alpha-1)}+\left[\frac{4\alpha}{3\alpha+2}\right],\\ \mathrm{SDR}\left(J_{\beta,\alpha}\right) &= 2\left(\frac{2\alpha^2+1-2\alpha}{(\alpha-1)}\right)+2\alpha\beta\\ &\quad -4\beta+\alpha^2-\alpha+\left(\frac{5\alpha^2+8\alpha+4}{\alpha+1}\right).\end{aligned} \tag{42}$$

*Proof.* Let $J_{\beta,\alpha}$ be the Jahangir graph. The harmonic Revan index of $J[\beta,\alpha]$ is then calculated using Table 3 as follows:

$$\begin{aligned}\mathrm{HR}\left(J_{\beta,\alpha}\right) &= \sum_{uv\in E(G)}\frac{2}{r_G(u)+r_G(v)}\\ &= 2\alpha\left[\frac{2}{\alpha-1+\alpha}\right]+(\alpha\beta-2\beta)\left[\frac{2}{2\alpha}\right]\\ &\quad +\frac{1}{2}\alpha^2-\frac{1}{2}\alpha\left[\frac{2}{2\alpha-2}\right]+2\alpha\left[\frac{2}{\alpha+2\alpha+2}\right]\\ &= \left[\frac{4\alpha}{2\alpha-1}\right]+\beta-2\frac{\beta}{\alpha}+\frac{1}{2}\alpha^2\\ &\quad -\frac{1}{2}\alpha\left[\frac{1}{(\alpha-1)}\right]+\left[\frac{4\alpha}{3\alpha+2}\right]\\ &= \left[\frac{4\alpha}{2\alpha-1}\right]+\beta-2\frac{\beta}{\alpha}+\frac{\alpha^2}{2(\alpha-1)}\\ &\quad -\frac{\alpha}{2(\alpha-1)}+\left[\frac{4\alpha}{3\alpha+2}\right].\end{aligned} \tag{43}$$

From the definition of symmetric division Revan index ($\mathrm{SDR}\,(J_{\beta,\alpha})$), we have

$$\begin{aligned}\mathrm{SDR}\left(J_{\beta,\alpha}\right) &= \sum_{uv\in E(G)}\frac{r_G(u)}{r_G(v)}+\frac{r_G(v)}{r_G(u)}\\ &= 2\alpha\left(\frac{\alpha-1}{\alpha}+\frac{\alpha}{\alpha-1}\right)+\alpha\beta-2\beta\left(\frac{\alpha}{\alpha}+\frac{\alpha}{\alpha}\right)+\frac{1}{2}\alpha^2\\ &\quad -\frac{1}{2}\alpha\left(\frac{\alpha-1}{\alpha-1}+\frac{\alpha-1}{\alpha-1}\right)+2\alpha\left(\frac{\alpha}{2\alpha+2}+\frac{2\alpha+2}{\alpha}\right)\\ &= 2\alpha\left(\frac{\alpha^2+(\alpha-1)^2}{\alpha(\alpha-1)}\right)+\alpha\beta-2\beta(2)+\frac{1}{2}\alpha^2\\ &\quad -\frac{1}{2}\alpha(2)+2\alpha\left(\frac{\alpha^2+(2\alpha+2)^2}{(2\alpha+2)(\alpha)}\right)\\ &= 2\left(\frac{2\alpha^2+1-2\alpha}{(\alpha-1)}\right)+2\alpha\beta-4\beta\\ &\quad +\alpha^2-\alpha+\left(\frac{5\alpha^2+8\alpha+4}{(\alpha+1)}\right).\end{aligned} \tag{44}$$

□

## 6. Discussion and Graphical Representation

Graphs are a type of perspective drawing that uses lines and points to depict a precise sequence of data and information. Network theory, molecular chemistry, and many disciplines of mathematics, as well as medicines and organic chemistry, all utilize it. In this section, we put the value of $\beta=2,3,4,5,6$ and $\alpha=3,4,5,6,7$. Then, we evaluate all the values of $\beta$ and $\alpha$ in the topological indices, which are discussed above. The numerical representation of the above calculated topological

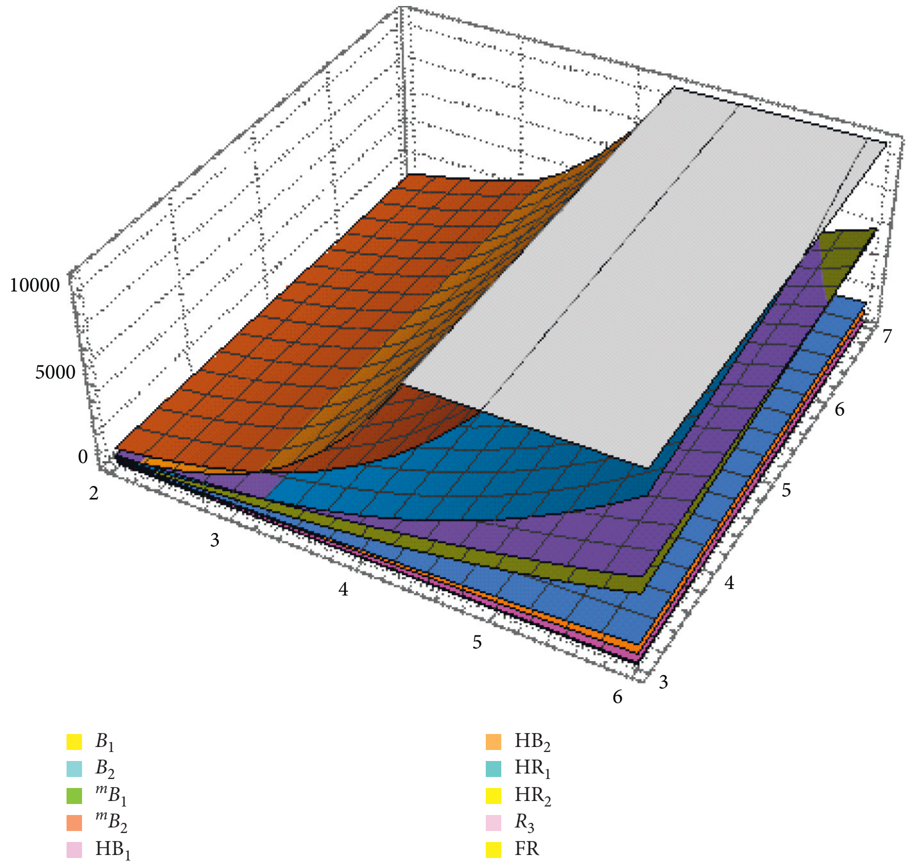


Figure 3: Comparison of topological indices.

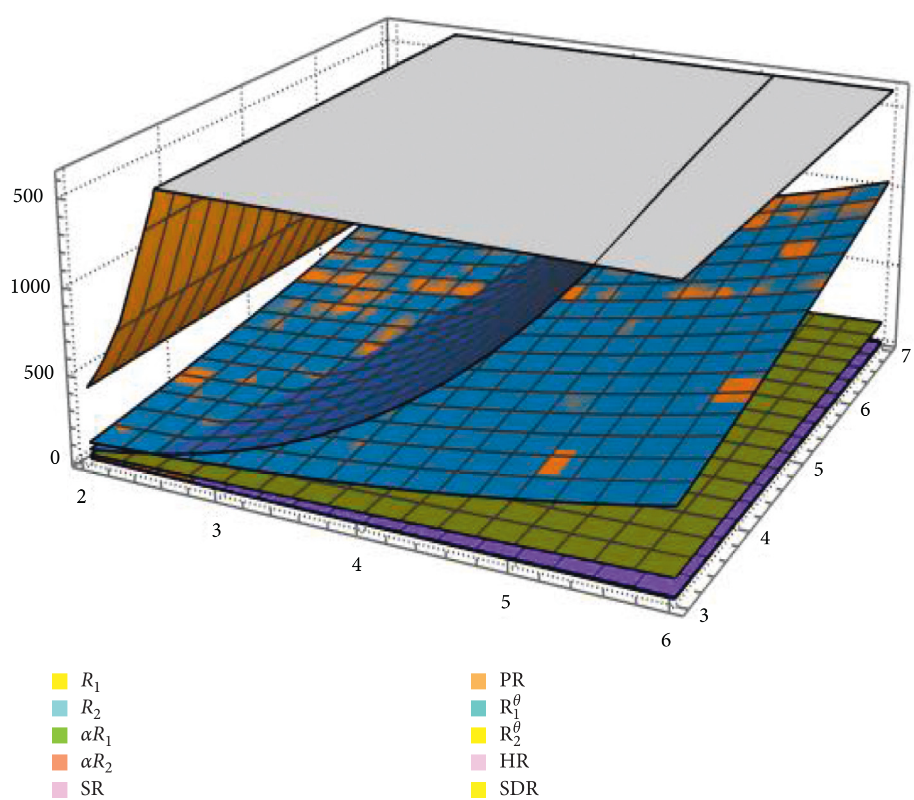


Figure 4: Comparison of topological indices.

indices is listed in Table 4 and the graphical representations are shown in Figures 3 and 4.

## 7. Applications

The Banhatti and Revan indices have a strong relationship with the physical and chemical characteristics of various structures. Asthma drugs are also useful for the treatment of COVID-19. The first Banhatti index has a correlation coefficient $r = 0.974$ with molar volume, and the second Banhatti index is correlated with the boiling point ($r = 0.9192$) and enthalpy ($r = 0.9125$) of these drugs. Hyper-Revan indices are effective to describe the molar volume of asthma drugs. Both these indices have a correlation with the different properties of silicon carbide structures discussed in this manuscript. These indices also tell us about the structural properties of linear alkanes.

## 8. Conclusion

In this article, we discuss some topological invariants such as $\mathsf{B}_1$ $(J_{\beta,\alpha})$, $\mathsf{B}_2$ $(J_{\beta,\alpha})$, ${}^{\mathfrak{m}}\mathsf{B}_1$ $(J_{\beta,\alpha})$, ${}^{\mathfrak{m}}\mathsf{B}_2$ $(J_{\beta,\alpha})$, $\mathrm{HB}_1$ $(J_{\beta,\alpha})$, $\mathrm{HB}_2$ $(J_{\beta,\alpha})$, $\mathrm{HR}_1$ $(J_{\beta,\alpha})$, $\mathrm{HR}_2$ $(J_{\beta,\alpha})$, $\check{R}_3$ $(J_{\beta,\alpha})$, FR $(J_{\beta,\alpha})$, $R_1$ $(J_{\beta,\alpha})$, $R_2$ $(J_{\beta,\alpha})$, ${}^{\alpha}R_1$ $(J_{\beta,\alpha})$, ${}^{\alpha}R_2$ $(J_{\beta,\alpha})$, SR $(J_{\beta,\alpha})$, PR $(J_{\beta,\alpha})$, $R_1^{\vartheta}$ $(J_{\beta,\alpha})$, $R_2^{\vartheta}$ $(J_{\beta,\alpha})$, HR $(J_{\beta,\alpha})$, and SDR $(J_{\beta,\alpha})$ for the Jahangir graph. These topological indices aid in the decoding of the graph's hidden information storage. Isomorphism does not affect these results. These findings could have a significant impact on industrial applications and pharmaceuticals.

## Data Availability

No data were used in this manuscript.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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