Hindawi
Journal of Mathematics
Volume 2022, Article ID 4689857, 8 pages
https://doi.org/10.1155/2022/4689857

Research Article

Modulation Instability Analysis, Solitary Wave Solutions, Dark Soliton Solutions, and Complexitons for the (3 + 1)-Dimensional Nonlinear Schrödinger Equation

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Received 1 July 2022; Revised 29 August 2022; Accepted 31 August 2022; Published 22 September 2022

Academic Editor: Kenan Yildirim

This paper addresses the (3 + 1)-dimensional nonlinear Schrödinger equation, which could be utilized to express many physical media the envelope of the wave amplitude. With the help of He's semi-inverse method, the solitary wave solutions are explicitly constructed to the equation. The dark soliton solutions of the equation are also strictly constructed by making use of the solitary ansatz method; in order to guarantee the existence of solitons, some conditions are given. Furthermore, by employing the tan h method, we also present complexitons of the equation. Finally, with the aid of linear stability analysis, an effective and straightforward method is presented to analyze modulation instability of the equation.

1. Introduction

As everyone knows, the nonlinear Schrödinger equations (NLSEs) have been attractive, which has the vital role in various fields, such as applied mathematics [1], theoretical physics [2], and engineering [3] [4]. NLSEs arises from the research of plasma physics, fluid dynamics, elastic media, modelling of deep water, nonlinear optics, and many others. In the field of nonlinear optics, optical soliton is one of the important research areas. In the past many years, great progress has been made in the area of research, and the optical solitons are successfully used in the long distance telecommunication. As mathematical models for this phenomena, the study of exact solutions of NLSEs can help us better understand this phenomena. In past years, to construct the exact solutions of these equations, various straightforward and effective approaches have been used, for example, the sine-cosine method [5, 6], the (G'/G)-expansion method [7–9], the first integral method [10], and the soliton ansatz method [11–14].

In the present paper, we study the (3 + 1)-dimensional nonlinear Schrödinger equation [15–18]:

$$iu_t + c\left(u_{xx} + u_{yy} + u_{zz}\right) + bu + a|u|^2 u = 0, \tag{1}$$

where u is the envelope amplitude and a, b, and c are real constants. When $b = 0$, it can be reduced to (3 + 1)-dimensional time dependent NLSE [19], which occurs in lots of areas of electromagnetic wave propagation, physics quantum mechanics, and optoelectronic devices [20]. In [19], Gagnon and Winternitz studied symmetry reductions of the (3 + 1)-dimensional nonlinear Schrödinger equation.

To the best of our knowledge, optical soliton solutions by applying the He's semi-inverse method and the solitary ansatz method have not been discussed of Equation (1). The main result of this paper is to derive the solitary wave solitons, dark soliton solutions, and complexitons; the modulation instability analysis of the model is also studied.

The structure of the present paper is given as follows. In Section 2, a variable transformation could be applied to obtain the solitary wave solutions of Equation (1). Its dark soliton solutions are constructed on the basis of the solitary ansatz method [21–23] and symbol calculation methods [24, 25] in Section 3. In Section 4, according to the tan h method, its complexitons are also provided in a rational method. In Section 5, modulation instability of the equation is also studied.

2. Solitary Wave Solution

We will employ He's semi-inverse method [26, 27] to construct solitary wave solutions for Equation (1). Firstly, we seek solution of the form:

$$u(x,y,z,t)=g(\xi)e^{i\phi},\xi=x+y+z-mt,\phi=\alpha x+\beta y+\gamma z-nt, \tag{2}$$

where m is constant wave speed to be decided later and α, β, and γ, n are also constants. Thus, one obtains

$$\begin{aligned} u_t&=\left(-mg'-ing\right)e^{i\phi},\\ u_{xx}&=\left(g''+2i\alpha g'-\alpha^2 g\right)e^{i\phi},\\ u_{yy}&=\left(g''+2i\beta g'-\beta^2 g\right)e^{i\phi},\\ u_{zz}&=\left(g''+2i\gamma g'-\gamma^2 g\right)e^{i\phi},\\ |u|^2u&=g^3e^{i\phi}. \end{aligned} \tag{3}$$

If we substitute these into Equation (1), then from the imaginary part of the obtained equation, we have

$$m=2c(\alpha+\beta+\gamma). \tag{4}$$

Therefore, we arrive at the following ordinary differential equation from the obtained equation

$$3cg''-\left(c\alpha^2+c\beta^2+c\gamma^2-b-n\right)g+ag^3=0. \tag{5}$$

According to [26–28], we could establish the following variational formulation:

$$J=\int_0^\infty\left[\frac{3}{2}c\left(g'\right)^2+\frac{1}{2}\left(c\alpha^2+c\beta^2+c\gamma^2-b-n\right)g^2-\frac{1}{4}ag^4\right]d\xi. \tag{6}$$

Based on the Ritz method, we consider the solitary wave solution in the form

$$g(\xi)=p\ \sec h\,(q\xi), \tag{7}$$

where p represents the amplitude of the soliton and q is the inverse width of the soliton. Putting (7) into (6), we can get

$$\begin{aligned} J&=\int_0^\infty\left\{\frac{3}{2}c\left[p^2q^2\ \sec h^2(q\xi)\tan h^2(q\xi)\right]\right.\\ &\quad+\frac{1}{2}\left(c\alpha^2+c\beta^2+c\gamma^2-b-n\right)p^2\sec h^2(q\xi)\\ &\quad\left.-\frac{1}{4}ap^4\sec h^4(q\xi)\right\}d\xi\\ &=\frac{3}{2}cp^2q^2\int_0^\infty\left[\sec h^2(q\xi)\tan h^2(q\xi)\right]d\xi\\ &\quad+\frac{1}{2}\left(c\alpha^2+c\beta^2+c\gamma^2-b-n\right)p^2\int_0^\infty\sec h^2(q\xi)d\xi\\ &\quad-\frac{1}{4}ap^4\int_0^\infty\sec h^4(q\xi)d\xi\\ &=\frac{1}{2}cp^2q+\frac{p^2}{2q}\left(c\alpha^2+c\beta^2+c\gamma^2-b-n\right)-\frac{ap^4}{6q}. \end{aligned} \tag{8}$$

For guaranteeing J stationary with respect to p and q leads to the following formulation:

$$\begin{aligned} \frac{\partial J}{\partial p}&=cpq+\frac{p}{q}\left(c\alpha^2+c\beta^2+c\gamma^2-b-n\right)-\frac{2}{3}a\frac{p^3}{q}=0,\\ \frac{\partial J}{\partial q}&=\frac{1}{2}cp^2-\frac{p^2}{2q^2}\left(c\alpha^2+c\beta^2+c\gamma^2-b-n\right)+\frac{ap^4}{6q^2}=0, \end{aligned} \tag{9}$$

which is equivalent to

$$\begin{aligned} &3cq^2+3\left(c\alpha^2+c\beta^2+c\gamma^2-b-n\right)-2ap^2=0,\\ &3cq^2-3\left(c\alpha^2+c\beta^2+c\gamma^2-b-n\right)+ap^2=0. \end{aligned} \tag{10}$$

Solving the above equation about p and q, we have

$$p=\sqrt{\frac{2\left(c\alpha^2+c\beta^2+c\gamma^2-b-n\right)}{a}},q=\sqrt{\frac{c\alpha^2+c\beta^2+c\gamma^2-b-n}{3c}}. \tag{11}$$

Therefore, we can get the solitary wave solution

$$g(\xi)=\sqrt{\frac{2\left(c\alpha^2+c\beta^2+c\gamma^2-b-n\right)}{a}}\sec h\left(\sqrt{\frac{c\alpha^2+c\beta^2+c\gamma^2-b-n}{3c}}\xi\right). \tag{12}$$

In terms of (2), it is straightforward to find out the solitary wave solution of the $(3+1)$-dimensional nonlinear Schrödinger equation:

$$\begin{aligned} u&=\exp\left[i(\alpha x+\beta y+\gamma z-nt)\right]\times\sqrt{\frac{2\left(c\alpha^2+c\beta^2+c\gamma^2-b-n\right)}{a}}\sec h\\ &\quad\left[\sqrt{\frac{c\alpha^2+c\beta^2+c\gamma^2-b-n}{3c}}(x+y+z-2c(\alpha+\beta+\gamma)t)\right]. \end{aligned} \tag{13}$$

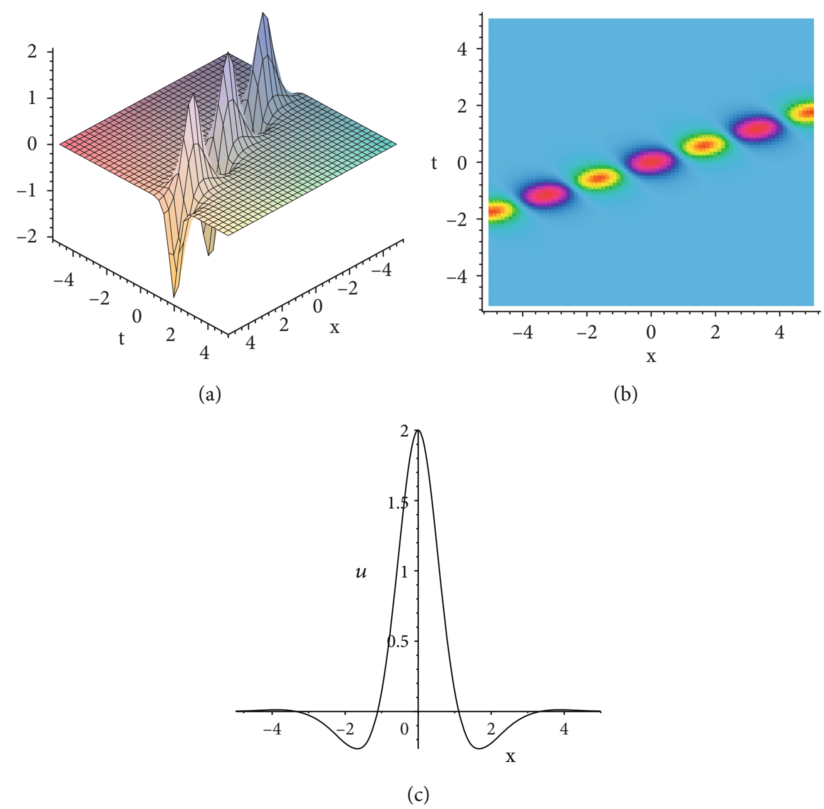


FIGURE 1: (Color online) The solitary wave solution (13) of Equation (1) with suitable parameters: $a = 3$, $b = \sqrt{2} - 4$, $c = 1$, $\alpha = \sqrt{2}$, $\beta = \gamma = 0$, $y = z = 0$, and $n = -\sqrt{2}$. (a) Perspective view of the solitary wave solution of the real part. (b) The overhead view (density plot) of the solitary wave solution of the real part. (c) The wave propagation pattern of the wave along the x-axis of the real part.

With the help of software Maple, we present Figures 1 and 2 by taking suitable parameters.

3. Dark Soliton Solution

We utilize the solitary ansatz method to Equation (1) for constructing the dark soliton solution in this section. To begin with, we consider the following hypothesis [21, 29]:

$$u = P(x, y, z, t)e^{i\phi(x,y,z,t)}, \tag{14}$$

where $P(x, y, z, t)$ denotes the shape of the pulse and $\phi(x, y, z, t) = -k_1 x - k_2 y - k_3 z + \omega t + \theta$ represents the phase portion of the soliton; k_1, k_2, and k_3 are the wave numbers; ω is the frequency of the soliton; and θ denotes the center of the phase. From (14), we can get the following form:

$$\begin{aligned} iu_t &= (iP_t - \omega P)e^{i\phi}, \\ u_{xx} &= \left(P_{xx} - 2ik_1 P_x - k_1^2 P\right)e^{i\phi}, \\ u_{yy} &= \left(P_{yy} - 2ik_2 P_x - k_2^2 P\right)e^{i\phi}, \\ u_{zz} &= \left(P_{zz} - 2ik_3 P_x - k_3^2 P\right)e^{i\phi}, \\ |u|^2 u &= P^3 e^{i\phi}. \end{aligned} \tag{15}$$

Then, substituting (15) into Equation (1) and separating real and imaginary parts of the obtained equation yields

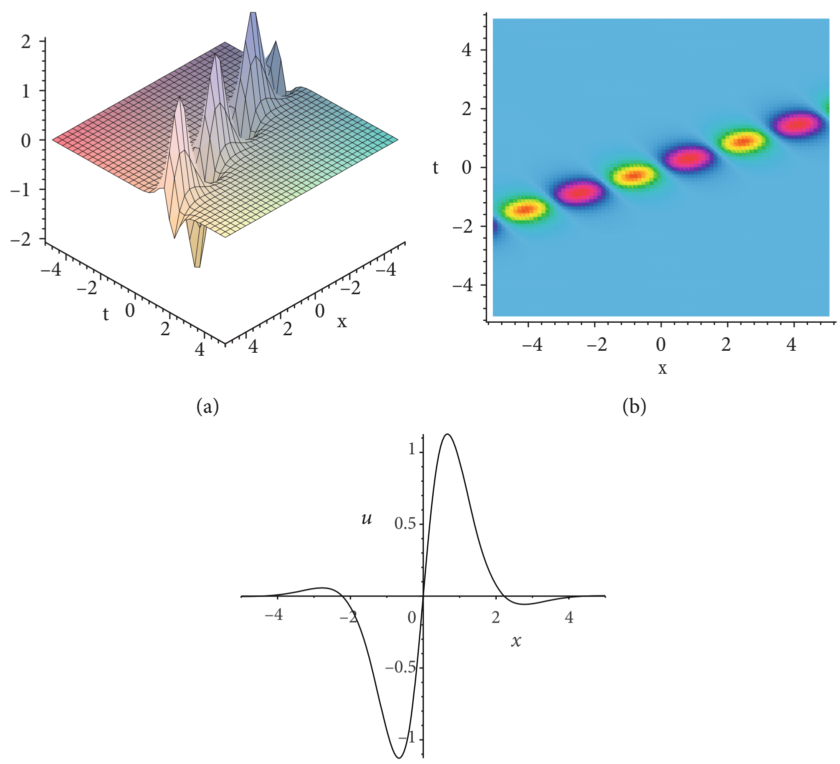
$$\begin{aligned} -\omega P + c\left(P_{xx} - k_1^2 P + P_{yy} - k_2^2 P + P_{zz} - k_3^2 P\right) + bP + aP^3 = 0, \\ P_t + c\left(-2k_1 P_x - 2k_2 P_y - 2k_3 P_z\right) = 0. \end{aligned} \tag{16}$$

For getting the dark soliton solution of Equation (1), we assume

$$P = A \tan h^p \tau, \tag{17}$$

in which $\tau = B_1 x + B_2 y + B_3 z - vt$; A, B_1, B_2, B_3 and the exponent p are unknown free parameters; and v is the velocity of the soliton. Therefore, one obtains

$$\begin{aligned} P_t &= -pvA\left(\tan h^{p-1}\tau - \tan h^{p+1}\tau\right), \\ P_x &= pAB_1\left(\tan h^{p-1}\tau - \tan h^{p+1}\tau\right), \\ P_y &= pAB_2\left(\tan h^{p-1}\tau - \tan h^{p+1}\tau\right), \\ P_z &= pAB_3\left(\tan h^{p-1}\tau - \tan h^{p+1}\tau\right), \\ P_{xx} &= pAB_1^2\left[(p-1)\tan h^{p-2}\tau - 2p\tan h^{p}\tau + (p+1)\tan h^{p+2}\tau\right], \\ P_{yy} &= pAB_2^2\left[(p-1)\tan h^{p-2}\tau - 2p\tan h^{p}\tau + (p+1)\tan h^{p+2}\tau\right], \\ P_{zz} &= pAB_3^2\left[(p-1)\tan h^{p-2}\tau - 2p\tan h^{p}\tau + (p+1)\tan h^{p+2}\tau\right], \\ P^3 &= A^3 \tan h^{3p}\tau. \end{aligned} \tag{18}$$



(a)

(b)

(c)

Figure 2: (Color online) The solitary wave solution (13) of Equation (1) with suitable parameters: $a = 3$, $b = \sqrt{2} - 4$, $c = 1$, $\alpha = \sqrt{2}$, $\beta = \gamma = 0$, $y = z = 0$, and $n = -\sqrt{2}$. (a) Perspective view of the solitary wave solution of the imaginary part. (b) The overhead view (density plot) of the solitary wave solution of the imaginary part. (c) The wave propagation pattern of the wave along the x-axis of the imaginary part.

The substitution of (18) into (16) yields

$$\begin{aligned}&-\omega A \tan h^{p}\tau + cpAB_1^2\left[(p-1)\tan h^{p-2}\tau - 2p\tan h^{p}\tau + (p+1)\tan h^{p+2}\tau\right]\\&- ck_1^2 A\tan h^{p}\tau + cpAB_2^2\left[(p-1)\tan h^{p-2}\tau - 2p\tan h^{p}\tau + (p+1)\tan h^{p+2}\tau\right]\\&- ck_2^2 A\tan h^{p}\tau + cpAB_3^2\left[(p-1)\tan h^{p-2}\tau - 2p\tan h^{p}\tau + (p+1)\tan h^{p+2}\tau\right]\\&- ck_3^2 A\tan h^{p}\tau + bA\tan h^{p}\tau + aA^3\tan h^{3p}\tau = 0,\end{aligned} \tag{19}$$

$$\begin{aligned}&-pvA\left(\tan h^{p-1}\tau - \tan h^{p+1}\tau\right) - 2k_1 cpAB_1\left(\tan h^{p-1}\tau - \tan h^{p+1}\tau\right)\\&-2k_2 cpAB_2\left(\tan h^{p-1}\tau - \tan h^{p+1}\tau\right) - 2k_3 cpAB_3\left(\tan h^{p-1}\tau - \tan h^{p+1}\tau\right) = 0.\end{aligned} \tag{20}$$

According to (20), it is obvious that

$$v = -2k_1 cB_1 - 2k_2 cB_2 - 2k_3 cB_3, \tag{21}$$

from (19), equating the exponents of $\tan h^{3p}\tau$ and $\tan h^{p+2}\tau$ yields $3p = p + 2$ so that $p = 1$. Then, setting their respective coefficients to zero results in

$$\begin{aligned}&-\omega A - 2cAB_1^2 - ck_1^2 A - 2cAB_2^2 - ck_2^2 A - 2cAB_3^2 - ck_3^2 A + bA = 0,\\&2cAB_1^2 + 2cAB_2^2 + 2cAB_3^2 + aA^3 = 0.\end{aligned} \tag{22}$$

After some calculation with (22), we get

$$\begin{aligned}&A = \sqrt{\frac{2c\left(B_1^2 + B_2^2 + B_3^2\right)}{-a}},\ (ac < 0),\\&\omega = -2cB_1^2 - ck_1^2 - 2cB_2^2 - ck_2^2 - 2cB_3^2 - ck_3^2 + b.\end{aligned} \tag{23}$$

Finally, the dark soliton solution to Equation (1) is presented by

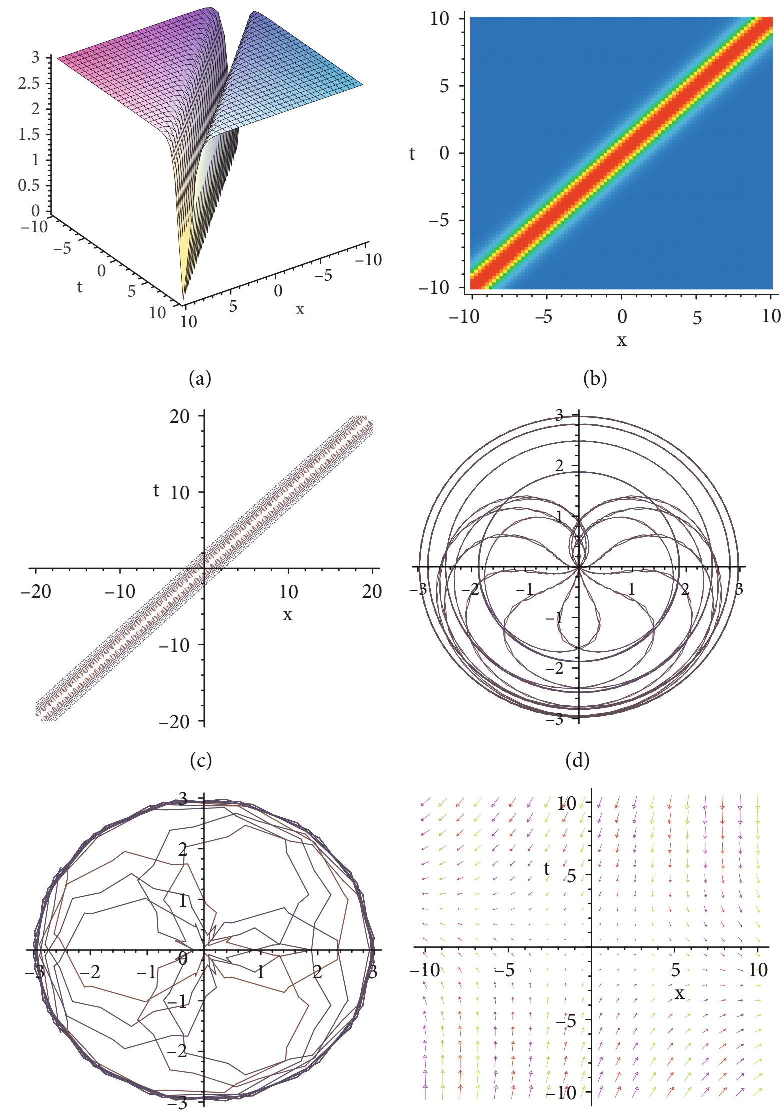
$$u = A\tan h\left(B_1 x + B_2 y + B_3 z - vt\right)e^{i(-k_1 x - k_2 y - k_3 z + \omega t + \theta)}. \tag{24}$$

Here, A and B_1, B_2, and B_3 are restricted by the first expression in (23); v is given by (21); and the second expression of (23) provides ω. All of these conditions guarantee the existence of the dark soliton solutions. In the following, we also present Figure 3 of the dark soliton solution of Equation (1) by selecting suitable parameters.

4. Complexitons

In this part, the tan h method will be used to construct the complexitons [29] of Equation (1). Meanwhile, it can be utilized to carry out the integration to Equation (1). Firstly, consider the following hypothesis [30]:

$$u(x, y, z, t) = f(\zeta)e^{i\mu}, \tag{25}$$



(a) (b) (c) (d) (e) (f)

FIGURE 3: (Color online) Dark soliton solution (24) for Eq. (1) with suitable parameters: $a = -1$, $b = 3/2$, $c = 1/2$, $k_1 = k_3 = 1$, $k_2 = -1$, $\theta = 0$, $B_1 = B_2 = 1$, $B_3 = -1$, and $y = z = 1$. ($|u|^2$) (a) Perspective view of the dark soliton solution. (b) The overhead view (density plot) of the solution. (c) The corresponding contour plot. (d) The contour plot in spherical coordinates. (e) The contour plot in cylindrical coordinates. (f) Field plot.

where $\zeta = k_0(x + y + z - 2l_1 t + \chi)$; $f(\zeta)$ is a real valued function; $\mu = l_1 x + l_2 y + l_3 z + \beta_1 t + \theta_0$; and the constants l_1, l_2, l_3, β_1, and θ_0 are to be determined later. By putting (25) into Equation (1), the following equation is revealed:

$$M_1 f'' + M_2 f' + M_3 f + a f^3 = 0. \tag{26}$$

Here, $M_1 = 3ck_0^2$, $M_2 = -2ik_0 l_1 + 2icl_1 k_0 + 2icl_2 k_0 + 2icl_3 k_0$, and $M_3 = b - \beta_1 - cl_1^2 - cl_2^2 - cl_3^2$. According to balancing principle, the linear term of highest order derivative f'' is balanced with the highest order nonlinear terms f^3. Then by the tanh method, we suppose that Equation (26) has the following solution:

$$f = s_0 + s_1 Y, \tag{27}$$

where $Y = \tan h\zeta$; substituting (27) into (26) yields the following algebra equations:

$$\begin{aligned} Y^0 &: M_2 s_1 + M_3 s_0 + a s_0^3 = 0, \\ Y^1 &: -2M_1 s_1 + M_3 s_1 + 3a s_0^2 s_1 = 0, \\ Y^2 &: -M_2 s_1 + 3a s_0 s_1^2 = 0, \\ Y^3 &: 2M_1 s_1 + a s_1^3 = 0. \end{aligned} \tag{28}$$

From (28), it is easy to get the following results:

$$\begin{aligned} &s_0 = 0,\\ &M_2 = 0,\\ &s_1 = \pm\sqrt{\frac{-6ck_0^2}{a}}, ac < 0,\\ &M_3 = 2M_1,\\ &\beta_1 = b - c\left(l_1^2 + l_2^2 + l_3^2\right) - 6ck_0^2. \end{aligned} \tag{29}$$

Finally, Equation (1) admits the following complexitons:

$$u = \pm\sqrt{\frac{-6ck_0^2}{a}} \tan h\left[k_0\left(x + y + z - 2l_1 t + \chi\right)\right] e^{i\left(l_1 x + l_2 y + l_3 z + \beta_1 t + \theta_0\right)}. \tag{30}$$

5. Modulation Instability Analysis

In the previous sections, solitary wave solutions, dark soliton solutions, and complexitons of Equation (1) have been investigated. Now, in this section, we will employ linear stability analysis to study the modulation instability of Equation (1). Modulation instability analysis can be utilized to analyze whether the modulated envelopes are modulationally stable or not [31–33]. Firstly, we suppose that Equation (1) has the stationary solutions of the following form:

$$u = u_0 e^{i\left(r_1 x + r_2 y + r_3 z + wt\right)}, \tag{31}$$

where u_0, r_1, $r_{2,}$ and r_3 are the real constants. Substituting (31) into Equation (1), we can get

$$w = -c\left(r_1^2 + r_2^2 + r_3^2\right) + b + au_0^2. \tag{32}$$

Based on the concept of linear stability analysis, we set the stationary solutions (31) with a small perturbation as follows:

$$u = \left(u_0 + \varepsilon\tilde{u}\right) e^{i\left(r_1 x + r_2 y + r_3 z + wt\right)}, \tag{33}$$

where ε is a perturbation parameter; as for $\tilde{u}$, we consider it has the following form:

$$\tilde{u} = R_1 e^{i\left(\widetilde{r_1}x + \widetilde{r_2}y + \widetilde{r_3}z - \varpi t\right)} + R_2 e^{-i\left(\widetilde{r_1}x + \widetilde{r_2}y + \widetilde{r_3}z - \varpi t\right)}, \tag{34}$$

where $\widetilde{r_j}$, (j =1,2,3) and ϖ are the normalized wave numbers and real disturbance frequency, respectively. Taking Equations (33) and (34) into Equation (1) yields two linear homogeneous equations about R_1 and R_2 as follows:

$$\begin{aligned} &\Gamma_{11}R_1 + \Gamma_{12}R_2 = 0,\\ &\Gamma_{21}R_1 + \Gamma_{22}R_2 = 0, \end{aligned} \tag{35}$$

with

$$\begin{aligned} \Gamma_{11} &= \varpi - w - c\left(\widetilde{r_1}^2 + 2r_1\widetilde{r_1} + r_1^2 + \widetilde{r_2}^2 + 2r_2\widetilde{r_2} + r_2^2 + \widetilde{r_3}^2 + 2r_3\widetilde{r_3} + r_3^2\right)\\ &\quad + b + 2au_0^2,\\ \Gamma_{12} &= au_0^2,\\ \Gamma_{21} &= au_0^2,\\ \Gamma_{22} &= -\varpi - w + c\left(-\widetilde{r_1}^2 + 2r_1\widetilde{r_1} - r_1^2 - \widetilde{r_2}^2 + 2r_2\widetilde{r_2} - r_2^2 - \widetilde{r_3}^2 + 2r_3\widetilde{r_3} - r_3^2\right)\\ &\quad + b + 2au_0^2. \end{aligned} \tag{36}$$

For making (35) has the nontrivial solutions: the determinant of coefficients of Equation (35) need to satisfy

$$\begin{vmatrix} \Gamma_{11} & \Gamma_{12}\\ \Gamma_{21} & \Gamma_{22} \end{vmatrix} = 0. \tag{37}$$

Substituting (36) into (37), we get the following dispersion relation:

$$\varpi = c\left(2r_1\widetilde{r_1} + 2r_2\widetilde{r_2} + 2r_3\widetilde{r_3}\right) \pm \sqrt{Y}, \tag{38}$$

with

$$\begin{aligned} Y = &\left[-w - c\left(\widetilde{r_1}^2 + r_1^2 + \widetilde{r_2}^2 + r_2^2 + \widetilde{r_3}^2 + r_3^2\right) + b + 3au_0^2\right]\\ &\left[-w - c\left(\widetilde{r_1}^2 + r_1^2 + \widetilde{r_2}^2 + r_2^2 + \widetilde{r_3}^2 + r_3^2\right) + b + au_0^2\right]. \end{aligned} \tag{39}$$

If $Y \geq 0$, ϖ is always real, then the steady state for Equation (1) is stable against the small perturbation. On the other hand, the steady-state solution becomes unstable in the case of $Y < 0$ due to the value of ϖ will be complex.

6. Conclusions and Discussions

In this paper, we have investigated the $(3 + 1)$-dimensional nonlinear Schrödinger equation. With the help of the He's semi-inverse method, we have constructed a solitary wave solution of Equation (1). It could be seen that He's semi-inverse method is straightforward. The profile of the solitary wave solution that was obtained numerically has been shown in Figures 1 and 2. Subsequently, based on the solitary ansatz method, we have established dark soliton solution of Equation (1). In Figure 3, we have shown the dynamical characteristics of the dark soliton solution, in which amplitude and shape of dark soliton solution maintain unchange during the propagation. It needs to be noted that the $(3 + 1)$-dimensional nonlinear Schrödinger equation can be integrated by the inverse scattering transform method that is one of the most powerful methods to carry out the integration of NLEEs. However, this paper has carried out the integration of the $(3 + 1)$-dimensional nonlinear Schrödinger equation by using a couple of simpler methods that are known as the ansatz method and the tanh method. These two methods are a few of the several methods of integrability that was developed in the past decade. What's more, the tan h method have been applied to study the complexitons solutions of Equation (1). Finally, the modulation instability analysis for the $(3+1)$-dimensional nonlinear Schrödinger equation has been presented. We hope that the obtained solutions will be indeed valuable for the future research, and our results are going to be helpful to a variety of fields of applied mathematics.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by Foundation of Anhui Provincial Education Department (Grant Nos. 2020jxtd286, 2020xsxxkc488, KJ2021A1102, and KJ2021ZD0136) and the Foundation of Suzhou University (Grant Nos. 2020BS011, 2020yzd06, szxy2020xxkc03, szxy2021jyxm16, and 2021XJPT40).

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