Hindawi
Journal of Mathematics
Volume 2022, Article ID 4789155, 5 pages
https://doi.org/10.1155/2022/4789155

Research Article

Generalized Cut Functions and *n*-Ary Block Codes on UP-Algebras

Ali N. A. Koam, Azeem Haider, and Moin A. Ansari

Department of Mathematics, College of Science, Jazan University, Post Box 2097 New Campus, Jazan, Saudi Arabia

Correspondence should be addressed to Moin A. Ansari; maansari@jazanu.edu.sa

Received 8 October 2021; Accepted 8 April 2022; Published 29 April 2022

Academic Editor: M. T. Rahim

In this paper, the work is comprised of n-ary block codes for UP-algebras and their interrelated properties. n-ary block codes for a known UP-algebra is constructed and further it is shown that for each n-ary block code U, it is easy to associate a UP-algebra U in such a way that the newly constructed n-ary block codes generated by U, i.e., U_x, contain the code U as a subset. We define a UP-algebra valued function on a set say X, then we prove that for every n-ary block-code U, a generalized UP-valued cut function exists that determines U. We have also proved that the UP-algebras associated to an n-ary block code are not unique up to isomorphism.

1. Introduction

Logical algebras like BCI/BCK, BE, KU-algebras, and many others with their fuzzy, intuitionistic, and more related concepts have been interesting topics of study for researchers in recent years and have been widely considered as a strong tool for information systems and many other branches of computer sciences including fuzzy informatics with rough and soft concepts. Imai and Iseki [1] introduced BCK/BCI algebras as a generalization of the concept of set-theoretic difference and proportional calculi. BCI/BCK algebras form an important class of logical algebras. They have numerous applications to different domains of mathematics, e.g., sets theory, semigroup theory, group theory, derivational algebras, etc. As per the requirement to establish certain rational logic systems as a logical foundation for uncertain information processing, different types of logical systems are felt to be established. For this reason, researchers introduced and studied many types of logical algebras by using the concepts of BCI/BCK algebras.

A block code is related to channel coding that is one of the main types of it. Block code adds redundancy to a message so that, at the receiver end, one can easily decode the message with a minimum number of errors, where it is already provided that the information rate would not exceed the channel capacity. The task of a block code is to encode the strings that are formed by an alphabet set say $\mathscr{C}$ into code words by encoding each letter of $\mathscr{C}$ separately. As per the importance block of codes, they can be source codes used in data compression or channel codes used for detection and correction of channel errors [2]. Codes based on a family of algorithms were constructed by Lempel and Ziv [3], which are applicable for real-world problems and sequences. A detailed terminology based on codes and decoding through graphs is discussed in [4]. Ali et al. introduced the concept of n-ary block codes related to KU-algebras in [5].

Many researchers have made their studies based on block codes in the past few years considering different branches and different directions. One of them is logical algebra. Surdive et al. studied coding theory in hyper BCK-algebras [6]. Jun and Song [7] defined and studied codes based on BCK-algebras. Further Fu and Xin [8] introduced the concept of block codes in lattices.

Iampan introduced the concept of UP-algebras [9]. Iampan contributed on different aspects related to UP-algebras in [10]. Senapati et al. [11] represented UP-algebras in an intervalued intuitionistic fuzzy environment. Moin et al. [12] introduced graphs of UP-algebras and studied related results. The binary block codes associated to UP-algebras were discussed by Moin et al. [13]. Wajsberg algebras arising from binary block codes were studied by Flaut and Vasile [14].

In this paper, we have introduced and investigated generalized UP-valued cut functions and their several properties. Also, we have established n-ary block-codes for UP-algebras by using the notion of generalized UP-valued cut functions. We show that every finite UP-algebra determines a block-code.

Section 2 contains preliminaries and related definitions with some examples. Section 3 is based on the main results.

2. Preliminaries

This section is comprises with the concepts of UP-algebras, UP-subalgebras, UP-ideals, UP-valued function (cut function), and other important terminologies with examples and some related results.

Definition 1 (see [9]). A UP-algebra is a structure $(U, *, \varnothing)$ of type $(2, 0)$ with a single binary operation $*$ that satisfies the following identities: for any $x, y, z \in U$,

(UP-1): $(y * z) * [(x * y) * (x * z)] = \varnothing$

(UP-2): $\varnothing * x = x$

(UP-3): $x * \varnothing = \varnothing$

(UP-4): $x * y = y * x = \varnothing$ implies $x = y$

For a commutative UP-algebras U we have the condition for commutativity as $x * (x * y) = y * (y * x)$.

We define a partial order relation in a UP-algebra U as $y \leq x$ if and only if $x * y = \varnothing$. If $(U, *, \varnothing)$ and $(V, \circ, \varnothing)$ are two UP-algebras, then a map $f: U \longrightarrow V$ with the property $f(x * y) = f(x) \circ f(y)$, for all $x, y \in U$, is called a UP-algebra morphism. If f is one-one and onto map, then f is simply called isomorphism of U.

Example 1. Let $U = \{\varnothing, a, b, c\}$ be a set in which $*$ is defined by the following Cayley table

$*$	$\varnothing$	a	b	c
$\varnothing$	$\varnothing$	a	b	c
a	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
b	$\varnothing$	a	$\varnothing$	c
c	$\varnothing$	a	b	$\varnothing$

We observe here that $U = \{\varnothing, a, b, c\}$ is a UP-algebra.

Example 2. Let $U = \{a_n | n = 1, 2, 3, \ldots, 9\}$ and define a binary operation $*$ on U as $a_i * a_j = a_k, \forall a_i, a_j, a_k \in U$ where $k = (\mathrm{lcm}(i, j)/j)$. Then $(U, *, a_1)$ is a UP-algebra. The following table represents this operation:

$*$	a_1	a_2	a_3	a_4	a_5	a_6
a_1	a_1	a_2	a_3	a_4	a_5	a_6
a_2	a_1	a_1	a_3	a_2	a_5	a_3
a_3	a_1	a_2	a_1	a_4	a_5	a_2
a_4	a_1	a_1	a_3	a_1	a_5	a_3
a_5	a_1	a_2	a_3	a_4	a_1	a_6
a_6	a_1	a_1	a_1	a_2	a_5	a_1

Lemma 1 (see [10]). *In a UP-algebra U the following properties hold for any $a, b, c \in U$:*

(UP-5) $a * a = \varnothing$

(UP-6) $a * b = \varnothing$ *and* $b * c = \varnothing \Rightarrow a * c = \varnothing$

(UP-7) $a * b = \varnothing \Rightarrow (c * a) * (c * b) = \varnothing$

(UP-8) $a * b = \varnothing \Rightarrow (b * c) * (a * c) = \varnothing$

(UP-9) $a * (b * a) = \varnothing$

(UP-10) $(b * a) * a = \varnothing \Leftrightarrow a = b * a$

(UP-11) $a * (b * b) = \varnothing$

Lemma 2. *Let $U = (A, *, \varnothing)$ be UP-algebras, then define a binary relation $\leq$ on U as follows: for all $a, b, c \in A$*

(UP-12) $a \leq a$

(UP-13) $\varnothing \leq a$

(UP-14) $b * a \leq a$

(UP-15) $a \leq b$ *and* $b \leq a \Rightarrow a = b$

(UP-16) $b \leq a$ *and* $c \leq b \Rightarrow c \leq a$

(UP-17) $b \leq a \Rightarrow c * b \leq c * a$

(UP-18) $b \leq a \Rightarrow a * c \leq b * c$

(UP-19) $(a * b) * (a * c) \leq b * c$

Definition 2 (see [9]). A nonempty subset A of a UP-algebra U is called a UP-ideal of U if it satisfies the following conditions:

(1) $\varnothing \in A$

(2) $a * (b * c) \in A$, $b \in A$ implies $a * c \in A$, for all $a, b, c \in U$

Proposition 1. *An algebra $(U, *, \varnothing)$ of type $(2, 0)$ is a UP-algebra if and only if the given conditions are satisfied:*

(1) $(c * a) * ((b * c) * (b * a)) = \varnothing$ *for all* $a, b, c \in U$

(2) $(b * \varnothing) * a = a$ *for all* $a, b \in U$

(3) For all $a, b, c \in U$ *such that* $a * b = \varnothing, b * a = \varnothing \Rightarrow a = b$

Proof. If $(U, *, \varnothing)$ is a UP-algebra. Then, (1) follows from (UP-1).

Next, (3) follows from (UP-4).

By using (UP-2) and (UP-3) we get (2) as $(b * 1) * a = 1 * a = a$.

Indirectly we consider $(U, *, \varnothing)$ satisfies given conditions, then (UP-1) and (UP-4) follows from (1) and (2), respectively. Next, replace b by a, a by 1 and c by 1 in (1) and using (3) we get, $(\varnothing * \varnothing) * [(a * \varnothing) * (a * \varnothing)] = \varnothing \Rightarrow (a * \varnothing) * (a * \varnothing) = \varnothing \Rightarrow a * \varnothing = \varnothing$ which shows (UP-3). Further, using $a * \varnothing = \varnothing$ in (2) we get, $\varnothing * a = a$ for all $a \in U$. Hence $(U, *, \varnothing)$ is a UP-algebra.

Let $(U, *, \varnothing)$ be a finite UP-algebra with n elements and $\mathfrak{U}$ be a finite nonempty set. A map $f\colon \mathfrak{U} \longrightarrow U$ is called a UP-function. Let $\mathfrak{U}_n = \{0, 1, 2, \ldots, n-1\}$ be a finite set. In the following, we will consider UP-algebra U and the set $\mathfrak{U}$, where $U = \{l_0, l_1, \ldots, l_{n-1}\}$, $\mathfrak{U} = \{u_0, u_1, \ldots, u_{m-1}\} m \leq n$. A generalized cut function of f is a map $f_{l_j}\colon \mathfrak{U} \longrightarrow \mathfrak{U}_n, l_j \in U$, such that $f_{l_j}(u_i) = u$ if and only if $l_j * f(u_i) = l_u$, for all $l_j, l_k \in U, u_i \in \mathfrak{U}$, and $i, j, u \in \{0, 1, 2, \ldots, n-1\}$.

For such each UP-function $f\colon \mathfrak{U} \longrightarrow U$, it is easy to define an n-ary block code with codewords having length m. For this purpose, we suppose that for each element $l \in U$ the generalized cut function $f_l\colon \mathfrak{U} \longrightarrow \mathfrak{U}_n$. For every such function, there will be corresponding a codeword w_r, having symbols taken from the set $\mathfrak{U}_n$. So, we get $w_l = w_0, w_1, \ldots, w_{m-1}$, with $w_i = j, j \in K_n$, if and only if $f_l(xi) = j$, that means $l * f(xi) = l_j$. We denote this new code by U_X. Hence, it is easy to associate an n-ary block code for every such UP-algebra. □

Example 3. We take the UP-algebra $U = \{1, 2, 3, 4\}$ having $\circ$ where $\circ$ is defined by the following table:

$\circ$	1	a	b	c
1	1	a	b	c
a	1	1	a	c
b	1	1	1	c
c	1	a	b	1

We can easily show that $\mathfrak{U} = \mathfrak{U}_4 = \{1, 2, 3, 4\}$. We consider the generalized cut function $f\colon \mathfrak{U} \longrightarrow U$, $f(1) = 2$, $f(2) = a, f(3) = b, f(4) = c$ and $f_l\colon \mathfrak{U}_4 \longrightarrow \mathfrak{U}_4$, $l \in U$. In this way $r = 1$, returns the codeword $w_1 = 0000$. For $l = a$, we get the codeword 1001. In fact, $f_a(1) = 2$, since $a \circ f(1) = a \circ 1 = a = f(1)$; $f_a(a) = 1$ since $a \circ f(2) = a \circ a = 1 = f(0)$; $f_a(b) = 1$ and $a \circ f(2) = a \circ b = 1 = f(1)$; $f_a(c) = 1$, also $a \circ f(4) = a \circ c = a = f(2)$.

The following result investigates about the existence of the converse part whether it is true or not.

3. Main Results

We consider a finite set $\mathfrak{U}_n' = \{1, 2, \ldots, n-1\}$ and its n-ary codewords $U = \{v_1, v_2, \ldots, v_m\}$, of length $h, h \geq n-2$, ascending ordered after lexicographic order. We consider $v_i = v_{i1} v_{i2}, \ldots, v_{ih}, v_{ij} \in \mathfrak{U}_n', j \in \{1, 2, \ldots, h\}$, with v_{ij} descending ordered such that $v_{i_{v_{ik}}} \leq u, i \in \{1, 2, \ldots, m\}$, $u \in \{1, 2, \ldots, \min\{n-1, h\}\}$ and $v_{ij} = 1$ in the rest.

Definition 3. Let $U = \{v_1, v_2, \ldots, v_m\}$ be an n-ary code. Further we suppose that $v_i = v_{i1}, v_{i2} \ldots, v_{ih}, v_{ij} \in \mathfrak{U}_n'$. $j \in \{1, 2, \ldots, h\}, q \geq n-2$, as above. We now associate a matrix $A = (\alpha_{st})_{s,t \in \{0,1,\ldots,l-1\}}, A \in \mathscr{A}_l(\mathfrak{U}_n)$, to this code where $l = m + h + 1$. Let $l = m + h + 1$. We define $\alpha_{ss} = 0$, $\alpha_{s0} = s, \alpha_{0s} = 0, s \in \{0, 1, 2, \ldots, l-1\}$. For $1 \leq s \leq h$, let $\alpha_{st} = 1$, if $t < s$, and $\alpha_{st} = 0$, if $t \geq s$. For $s > h$, we put $\alpha_{st} = v_{it}$, for $t \in \{1, 2, \ldots, h\}$ and $\alpha_{s(h+j)} = 1$, for $h + j < s$. We suppose that $\alpha_{st} = 0$, for $t \geq s$.

Here, A is the lower triangular matrix, and it is known as the matrix associated with the n-ary block code $U = \{v_1, v_2, \ldots, v_m\}$.

Definition 4. Consider $A \in \mathscr{A}_i(\mathfrak{U}_n)$ is associated to the n-ary block code $U = \{k_1, k_2, \ldots, k_m\}$ defined on $\mathfrak{U}_n'$. Suppose that $\mathfrak{U}_l = \{0, 1, \ldots, l-1\}$ is a nonempty set. The multiplication $i \circ j = \alpha_{ij}$ is defined on $\mathfrak{U}_l$.

Theorem 1. *The set $(\mathfrak{U}_\varnothing, \circ, 0)$ is a UP-algebra.*

Proof. We see here that Proposition 1 (2), (3) are well defined. From Definition 1, we need to show that $(b * c) * ((c * a) * (c * a)) = 1$, for all $a, b, c \in \{0, 1, \ldots, l-1\}$. For the elements a, b, c we have 3 situations here that are given as follows:

Case 1: $c = 0, b \neq 0$. We get $b * a \leq a$, which implies $a * (b * a) = 0$.

Case 2: $b = 0, c \neq 0$. We need to show that $c * ((c * a) * a) = 0$. Thus for $a = 0$, it is obvious and for $c = 0$, we obtain $(0 * a) * (0 * a) = 0 * (a * a) = 0$. For $c \neq 0, a \geq l - m, c \in \{1, 2, \ldots, h\}$, we have $((a * c) * a) = w_x w_{ac} \leq c$, therefore $c * (c * a) * a = 0$. For $c \neq 0, a \geq l - m$, $c \geq h + 1$, we obtain $c * ((c * a) * a) * c = 0$. Next, if $c * a = 1$, since $1 * a \leq n - 1 < h + 1 \leq c$, in returns we get $c * ((c * a) * a) = c * (1 * a) = 0$. If $c * a = 0$, then $c * ((c * a) * a) = c * (0 * a) = c * a = 0$. For $a < l - m$, $c \leq h + 1$, we have $c * ((c * a) * a) = 0$, since $c * a = 1$, $1 * a = 1$ and $c * 1 = 0$. For $a < l - m, c > h + 1$, it results $c * ((c * a) * a) = 0$, since $c * a = 0$, it yield $c * (0 * a) = c * a = 0$.

Case 3: $c \neq 0, b \neq 0$. Here, we have to prove that $(c * a) * ((b * k) * (b * a)) = 0$. Hence, it is shown for $a = 0$. Furthermore, let $a \neq 0$. For $a \geq l - m$ and $b, c < l - m, b < k$, we get $n - 1 \geq (b * a) \geq (c * a)$, hence $((c * a) * (b * a)) = 1$. We also get $b * c = 1$, hence $(c * a) * ((c * b) * (b * a)) = 1 * 1 = 0$. For $a \geq l - m$ and $b, c < l - m, c < b$, we get $n - 1 \geq (c * a) \geq (b * a)$, then $((c * a) * (b * a)) = 0$. It results that $(c * a) * ((b * c) * (b * a)) = 0$. For $a \geq l - m$ and $b, c \geq l - m$, $b < c$, we can get that $b * a = 1$ and $c * a = 1$, so $(c * a) * (b * a) = 0$. We also obtain $b * a = 1$, $c * a = 0$, and $b * c = 1$, since $b < c$. It yield $(c * a) * ((b * k) * (b * a)) = 1 * (0 * 1) = 1 * 1 = 0$.

Or, we can have $b * a = 0, c * a = 0$,; hence $(c * a) *$

$((b*k)*(b*a))=0$. For $a \geq l-m$ and $b,k \geq l-m$, $c<b$, we can have $b*a=1$ and $c*a=1$, hence $(c*a)*(b*a)=0$. Or, we have $c*a=1$, $b*a=0$, and $b*c=0$, as a result we get zero. We also can have $b*a=0, c*a=0$,; hence the relation is 0. For $a \leq l-m$ and $c<l-m<b$, if $b*a=0$, it shows that the asked relation is 0. If $b*a=1$, then $(c*a)*((b*k)*(b*a))=0*((c*a)*1)=\beta*1$, with $\beta \geq 1$, and $c<a$.

For $a \geq l-m$ and $b<l-m<c$, we have that $b*a=1$. If $c*a=1$, we obtain 0. If $c*a=0$, hence we find $(c*a)*((b*c)*(b*a))=(b*c)*(0*1)=(b*c)*1=0$, since $b*c \geq 1$.

For $a<l-m$ and $b,c<l-m, b<c$, we have $b*a=1, c*a=1$,; therefore, we obtain the result as zero. For $a<l-m$ and $b,c<l-m, c \leq b$, we can obtain $(c*a)*((b*c)*(b*a))=1*(0*1)=0$. Or, we find $(b*a)=0$; thus, we can say that obtained result is 0. For $a<l-m$ and $b,c<l-m, b<c$, since $b<c$, it returns $b*c=1$. We can get $b*a=1, c*a=0$ and $b*c=1$, therefore $(c*a)*((b*c)*(b*a))=0*(1*1)=1*1=0$. For $a<l-m$ and $c<l-m \leq b$, we can get $(c*a)*((b*k)*(b*a))=1*(0*1)=0$. Or, if $(b*a)=0$; thus we obtain 0 and that is required. For $a<l-m$ and $b,c \geq l-m, b<c$, we have $(b*a)=0$; then, we get zero. For $a<l-m$ and $b,c \geq l-m, b>c$, it results $(b*a)=0$, hence the asked relation is 0. □

Note.

(1) We find that a UP-algebra $(\mathfrak{U}_l, *, 0)$ from Theorem 1 is extracted by using the matrix $\mathscr{A}$, which is uniquely determined by an n-ary code, say U, given as per Definition 1; thus, we can say that $(\mathfrak{U}_r, *, 0)$ is a uniquely determined algebra.

(2) By Theorem 1, we suppose that $(C_l, *, 0)$ is the resulted UP-algebra, with $\mathfrak{U}_l=\{0,1,2,\ldots,l-1\}$. If $U=\{a_0=1, a_1, a_2, \ldots a_{l-1}\}$ with multiplication "$*$" given by the relation $a_i * a_j = a_c$ if and only if $a*b=c$, for $a,b,c \in \{0,1,2,\ldots,l-1\}$, then $(U, *, 1)$ is a UP-algebra.

(3) If we suppose that $C_h=\{0,1,2,\ldots h-1\}$, the map $f\colon C_h \longrightarrow U, f(a)=a_i$, returns a code U_X, that can be associated to the above UP-algebra $(U, *, 1)$, that contains the code U as a subset.

We consider U as an n-ary block code. Then, from Theorem 1 and above Note, we can have a UP-algebra U in such a way that the obtained n-ary block code U_X contains the n-ary block code U as of its subset. Suppose that U is a binary block code with m code words of length h. By using the abovementioned notations, consider X is the associated UP-algebra and $W=\{1, w_1, \ldots, w_r\}$ is the associated n-ary block codes that contains the code U. Next consider $w_a = a_1 a_2, \ldots, a_h$ and $w_b = b_1 b_2, \ldots, b_h$ are two codewords that belong to W. Here, we define an order relation $\leq_c$ on W by the following logic $w_a \leq w_b$ if and only if $b_i \leq a_i$, for all $i \in \{1,2,\ldots,h\}$. On $U=W$, with the order relation $\leq_c$, we define the following multiplication:

(1) $a \circ 1 = 1$ and $a \circ a = 1, \forall a \in U$

(2) $b \circ a = 1$ if $a \leq_c b, \forall a, b \in U$

(3) $b \circ a = a$ if $b \leq_c a, \forall a, b \in U$

This order relations give UP-algebra structure. It is clear that $w_l \leq_c \cdots \leq_c w_1 \leq_c 1$. □

Proposition 2. *$V=\{1, w_{l-m}, w_{l-m+1}, \ldots, w_l\}$ gives an UP-algebra ideal in the U.*

Proof. Considering $V=\{1, w_{l-m}, w_{l-m+1}, \ldots, w_l\}$. We will show that $b \in V, a \in U$, and $b \circ a \in V$, implies $a \in V$. By using multiplication rule in the UP-algebra U and chosen n-ary codes, we get for $a \in U-V, b \circ a = a \in U-V$. If $a, b \in V$, then $b \circ a = a \in V$ or $b \circ a = 1 \in V$. □

Example 4. Consider $K_5=\{0,1,2,3,4\}, n=5, q=4, m=3$, $l=8, V=\{w_1, w_2, w_3\}$, with $w_1=3211, w_2=4221, w_3=4321$.

Entries of the matrix $\mathscr{A}$ associated with the n-ary code U, are $a_{ij}=0, \forall i \leq j$, $a_{i1}=i-1, a_{62}=3, a_{63}=2, a_{72}=4$, $a_{73}=2, a_{74}=2, a_{82}=4, a_{83}=3, a_{84}=2$ and $a_{ij}=1$ for the rest of i and j.

The corresponding UP-algebra, $(U, \circ, 1)$, where $U=\{a_0=1, a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$, is shown with the following multiplication table.

$\circ$	1	a_1	a_2	a_3	a_4	a_5	a_6	a_7
1	1	a_1	a_2	a_3	a_4	a_5	a_6	a_7
a_1	1	1	a_1	a_1	a_1	a_3	a_4	a_4
a_2	1	1	1	a_1	a_1	a_2	a_2	a_3
a_3	1	1	1	1	a_1	a_1	a_2	a_2
a_4	1	1	1	1	1	a_1	a_1	a_1
a_5	1	1	1	1	1	1	a_1	a_1
a_6	1	1	1	1	1	1	1	a_1
a_7	1	1	1	1	1	1	1	1

Considering $\mathfrak{U}=\{1,2,3,4\}$. The map $f\colon \mathfrak{U} \longrightarrow U$, $f(1)=a_1, f(2)=a_2, f(3)=a_3, f(4)=a_4$ gives us the following block code $U'=\{0000, 1000, 1100, 1110, 1111, 3211, 4221, 4321\}$, that contains U as a subset.

$$0000-1000-1100-1110-1111-3211-4221-4321. \tag{1}$$

Clearly it is a noncommutative UP-algebra as $(a_6 \circ a_7) \circ a_7 = a_1 \circ a_7 = a_4$ and $(a_7 \circ a_6) \circ a_6 = 1 \circ a_6 = a_6$. This clarifies that $\mathfrak{U}$ is not an implicative UP-algebra. Also we note that it is not a positive implicative UP-algebra. Since $(a_6 \circ a_7) \circ a_6 = a_1 \circ a_6 = a_3 \neq a_6$ and $a_3 \circ (a_6 \circ a_7) = a_3 \circ a_1 = 1 \neq (a_3 \circ a_6)(a_3 \circ a_7) = a_1 \circ a_2 = a_1$.

Example 5. Consider $K_4=\{0,1,2,3\}, n=4, q=5, m=3$, $l=9, U=\{w_1, w_2, w_3\}$, with $w_1=21111, w_2=32111, w_3=33111$.

Entries of the matrix $\mathscr{A}$ associated with the n-ary code U, are $a_{ij} = 0 \forall, i \leq j,\ a_{i1} = i - 1, a_{72} = 2, a_{82} = 3, a_{92} = 3,\ a_{83} = 2, a_{93} = 3$ and $a_{ij} = 1$ for the rest of i and j.

The corresponding UP-algebra $(X, \circ, 1)$, where $X = \{a_0 = 1, a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ is shown with the following multiplication table.

$\circ$	1	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
1	1	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
a_1	1	1	a_1	a_1	a_1	a_1	a_2	a_3	a_3
a_2	1	1	1	a_1	a_1	a_1	a_1	a_2	a_3
a_3	1	1	1	1	a_1	a_1	a_1	a_2	a_1
a_4	1	1	1	1	1	a_1	a_1	a_1	a_1
a_5	1	1	1	1	1	1	a_1	a_1	a_1
a_6	1	1	1	1	1	1	1	a_1	a_1
a_7	1	1	1	1	1	1	1	a_1	a_1
a_8	1	1	1	1	1	1	1	1	1

Let $K = \{1, 2, 3, 4, 5\}$. Then, $f\colon K \longrightarrow X, f(1) = a_1, f(2) = a_2, f(3) = a_3, f(4) = a_4, f(5) = a_5$ returns the given block code $U' = \{00000, 10000, 11000, 11100, 11110, 21111, 32211, 33111\}$, where U is contained in it as a subset. The diagram of this generated code is given as

```
                                                32211
                                               /
00000 - 10000 - 11000 - 11100 - 11110 - 21111
                                               \
                                                33111
```

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] Y. Imai and K. Iski, "On axiom systems of propositional calculi. XIV," *Proceedings of the Japan Academy*, vol. 42, no. 1, pp. 19–22, 1966.

[2] S. Lin and D. J. Costello, *Error Control Coding: Fundamentals and Applications*, Prentice Hall, Hoboken, NJ, USA, 1983.

[3] J. Ziv and A. Lempel, "Compression of individual sequences via variable-rate coding," *IEEE Transactions on Information Theory*, vol. 24, no. 5, pp. 530–536, 1978.

[4] N. Wiberg, *Codes and Decoding on General Graphs*Linkping University, Linköping, Sweden, 1996.

[5] A. N. A. Koam, M. A. Ansari, and A. Haider, "Ansari and Azeem Haider, *n*-ary block codes related to KU-algebras," *Journal of Taibah University For Science*, vol. 14, no. 1, pp. 172–176, 2020.

[6] A. T. Surdive, N. Slestin, and L. Clestin, "Coding theory and hyper BCK-algebras," *Journal of Hyperstructures*, vol. 7, no. 2, pp. 82–93, 2018.

[7] Y. B. Jun and S. Z. Song, "Codes based on BCK-algebras," *Information Sciences*, vol. 181, Article ID 51025109, 2011.

[8] Y. Fu and X. L. Xin, "Lattices and block codes," *UPB Scientific Bulletin, Series A: Applied Mathematics and Physics*, vol. 79, no. 3, 2017.

[9] A. Iampan, "A new branch of the logical algebra: UP-algebras," *Journal of Algebra and Related Topics*, vol. 5, no. 1, pp. 35–54, 2017.

[10] A. Iampan, "The UP-isomorphism theorems for UP-algebras," *Discussiones Mathematicae-General Algebra and Applications*, vol. 39, no. 1, pp. 113–123, 2019.

[11] T. Senapati, G. Muhiuddin, and K. P. Shum, "Representation of UP-Algebras in intervalued-valued intuitionistic fuzzy environment," *Italian Journal of Pure and Applied Mathematics*, vol. 38, pp. 497–518, 2017.

[12] M. Ansari, A. Haidar, and A. N. A. Koam, "On a graph associated to UP-algebras," *Mathematical and Computational Applications*, vol. 23, no. 4, p. 61, 2018.

[13] M. A. Ansari, A. N. A. Koam, and A. Haider, "On binary block codes associated to UP-algebras," *Italian Journal of Pure and Applied Mathematics*, vol. 47, pp. 205–220, 2022.

[14] C. Flaut and R. Vasile, "Wajsberg algebras arising from binary block codes," 2019, https://arxiv.org/abs/1904.07169.