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Research Article

Computing Topological Indices and Polynomials of the Rhenium Trioxide

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In the study of mathematical chemistry and chemical graph theory, a topological index, also known as a connectivity index, is the arithmetical framework of a graph that specifies its topology and also graph invariant. These topological indices are used to model quantitative structure relationships (*QSARs*), which are connections between the work of biological or other molecular structures and the chemical structures. This study computed the first, second, and Hyper Zagreb indices, as well as Zagreb polynomials, Redefined Zagreb indices, Randic index, *ABC* index, and *GA* index of chemical structure of Rhenium Trioxide.

1. Introduction

Graph theory is the branch of mathematics dealing with graphs which are basically the structures used to model pairwise relationship in different things. A graph consists of vertices that are affixed to edges [1]. A graph G is formed of $V(G) \neq \phi$ of elements known as vertices, having finite numbers $E(G)$ of unorganized pairings for $V(G)$ units known as edges [2]. $V(G)$ and $E(G)$ were assigned to the vertex and edge sets of G, respectively. In chemical graph theory, atoms are represented by vertices and bonds are represented by edges [3, 4]. Chemical graph theory (GT) is the discipline of mathematical chemistry where we quantitatively implement devices of the graph premise to a material [5, 6]. Now a days, the proposal makes a significant contribution to computational sciences. As a result, we can define a subatomic diagram as a constrained network in which the nodes representing atoms and edges represent covalent bonds in concealed complex formation [7]. Symmetric key is a analytical standard associated to the composition that indicates the relationship between the concentration of the element and a wide range of physiosynthetic qualities, composite sensitivity, and organic action [8]. Molecular graphs are all chemical structures having structural formulas that comprise covalently bond compounds or molecules. Chemical graph theory tools were previously used for a variety of reasons, including numeration, systemization of the topic being discussed, and nomenclature [9, 10]. It also describes the process of assembling rules or regulations including a framework or strategy, as well as computer programming [11, 12]. The existence of isomerism, that is justified by the component graph hypothesis reinforces the necessity of graph theory techniques of science [13, 14]. The computational geometry of molecules according to well-defined rules is the essence of chemistry [15]. A topological graph index is a mathematical formula that may be applied to any graph that describes a molecule structure. The corresponding counting corresponding polynomials are some most common topological indices [16, 17]. On the other way, topological indices (TIs) are analytical principles related to chemical compositions for determining the relationship between chemical structure and various attributes. Physical qualities such as boiling temperatures, molar heats of formation, thermodynamic data, as well as chemical reactivity such as octane values as well as reactivity data, and biological activity are among these features [8, 18]. Topological indices, in particular, appear to hold promise for last mentioned goal, specifically $QSAR$ [19, 20]. The quantitative structure-activity relationship ($QSAR$) is a computational modelling technique for determining connections between chemical compound structural features and biological activities [21]. In this research, we discuss about the inorganic compound ReO_3 rhenium trioxide or rhenium(VI) oxide. It is a bright red solid with a metallic lustre that looks like copper. It is the Group 7th element, only stable trioxide. Having average estimation absorption of 1 part per billion, the most infrequent element on Earth crust is rhenium. From any stable elements, rhenium trioxide is the only element that has the 3rd highest melting point of $5903K$. In this research, we talk about different topological indices and polynomials of rhenium trioxide.

2. Preliminaries

In the world of chemical graph theory, wiener index (WI) which is known as wiener number presented by Henry wiener [22]. Actually, wiener index is the topological index of a molecule, stated nonhydrogen atoms in a molecule are denoted by total lengths of the shortest path in all pairs of vertices in a chemical graph.

The number of edges linking a vertex determines its degree in graph theory. If $'r'$ and $'r'$ are the vertices of a graph, then $\Psi(t), \Psi(r)$ are the degree of $'t'$ and $'r'$ vertices, respectively.

Gutman and Trinajstic introduced in his study 1st, 2nd Zegreb indices as

$$M_1(G) = \sum_{tr\in E(G)} [\Psi(t) + \Psi(r)],$$
$$M_2(G) = \sum_{tr\in E(G)} [\Psi(t) \times \Psi(r)]. \quad (1)$$

Shirdel et al. established "hyper Zagreb index" as follows:

$$HM(G) = \sum_{tr\in E(G)} [\Psi(t) + \Psi(r)]^2. \quad (2)$$

Atom bond connectivity index is the most important and well known connectivity topological indices, presented by Estrada et al.:

$$ABC(G) = \sum_{tr\in E(G)} \sqrt{\frac{\Psi(t) + \Psi(r) - 2}{\Psi(t) \times \Psi(r)}}. \quad (3)$$

"Geometrically arithmetic index (GA)" is another important topological figure of connectivity which is presented by Vukicevic et al.:

$$GA(G) = \sum_{tr\in E(G)} \frac{2\sqrt{\Psi(t) \times \Psi(r)}}{\Psi(t) + \Psi(r)}. \quad (4)$$

Gutman and Trinajstic presented 1st and 2nd Zagreb polynomial as

$$M_1(G, x) = \sum_{tr\in E(G)} x^{[\Psi(t)+\Psi(r)]},$$
$$M_2(G, x) = \sum_{tr\in E(G)} x^{[\Psi(t)\times\Psi(r)]}. \quad (5)$$

Redefined 1st, 2nd, and 3rd Zagreb indices are the most significant indices presented by Usha et al. stated below as

$$\mathrm{Re}ZM_1(G) = \sum_{tr\in E(G)} \frac{[\Psi(t) + \Psi(r)]}{[\Psi(t) \times \Psi(r)]},$$
$$\mathrm{Re}ZM_2(G) = \sum_{tr\in E(G)} \frac{[\Psi(t) \times \Psi(r)]}{[\Psi(t) + \Psi(r)]}, \quad (6)$$
$$\mathrm{Re}ZM_3(G) = \sum_{tr\in E(G)} [\Psi(t) + \Psi(r)][\Psi(t) \times \Psi(r)].$$

There is also well known topological index called Randic in 1975 presented as

$$RI(G) = \sum_{tr\in E(G)} \frac{1}{\sqrt{\Psi(t) \times \Psi(r)}}. \quad (7)$$

SCI was introduced by Zhou et al. and defined as

$$SCI(G) = \sum_{tr\in E(G)} \frac{1}{\sqrt{\Psi(t) + \Psi(r)}}. \quad (8)$$

3. Main Results of Rhenium Trioxide

Rhenium is the derived from the Latin word Rhenus which means "Rhine" is the second last element that is discovered and having stable isotope. Rhenium is a chemical element that is denoted by $'\mathrm{Re}'$ with atomic number 75. It is silver gray colored, heavy, and 3rd row transition metal in 7th group of periodic table. Rhenium look like manganese and technetium chemically in nature; it is acquired by the consequence of ancestry and purification copper ores and molybdenum. Rhenium expresses the number of oxidation states in the wide limitation of −1 to +7.As compared others, the oxide ReO_3 is the best electrical conductor at room temperature resistively that is close to copper, as shown in Figure 1. The cubic crystal of ReO_3 identifies particular perovskite-type crystal structure that is related to $O_1^h(Pm3m)$ having lattice parameter $a_0 = 3.7504A^0$. The crystal lattice of ReO_6 contains octahedra joined from edges and the unit cell of Bravais that holds f.u. of ReO_3. The red colored rhenium trioxide shows metallic conductivity, it is also called covalent metal (see Figure 1).

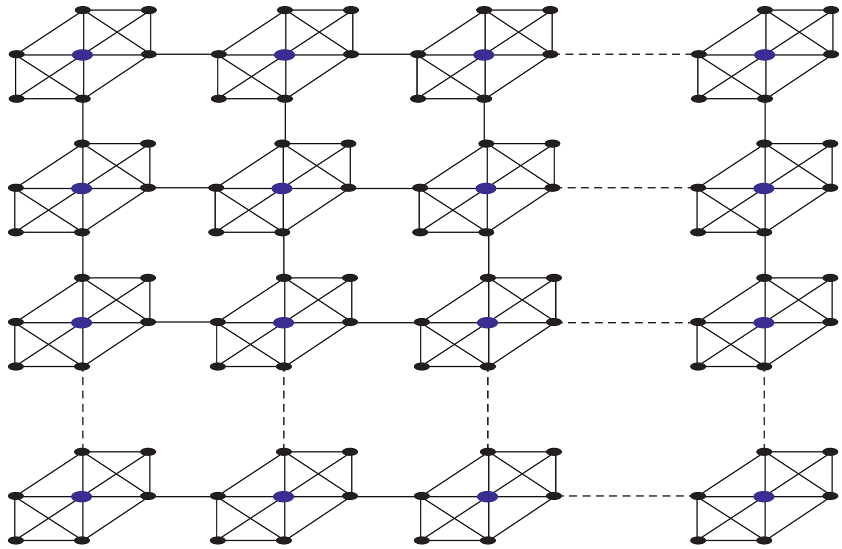

Figure 1: Structure of rhenium trioxide.

Theorem 1. *If R is the graph of rhenium trioxide as illustrated in Figure 1, then*

$$
\begin{aligned}
M_1(R) &= 154mn + 44m - 26n + 20,\\
M_2(R) &= 366mn + 87m - 63n,\\
HM(R) &= 1502mn + 352m - 410n,\\
PM_1(R) &= 2^{18mn-3m-9n+10} \times 3^{4mn+8m+8n-16} \times 5^{6mn+3m-3n-2} \times 7^{2m+2n} \times 11^{4mn-2m-2n},\\
PM_2(R) &= 2^{6mn+16m+16n} \times 3^{12mn} \times 5^{24mn+2m-10n-12},\\
M_1(G,x) &= (2m+2n)x^7 + (4mn-2m-2n+4)x^8 + 2mn + 4m + 4n - 8x)^9\\
&\quad + (6mn+3m-3n-2)x^{10} + (4mn-2m-2n)x^{11},\\
M_2(G,x) &= (2m+2n)x^{12} + (4mn-2m-2n)x^{15} + 4x^{16} + 2mnx^{18}\\
&\quad + (4m+4m-8)x^{20} + (2m+2n)x^{24} + (6mn+m-5n-2)x^{25} + (4mn-2m-2n)x^{30},\\
\mathrm{Re}ZM_1(R) &= \frac{27}{5}mn + \frac{21}{10}m - \frac{3}{10}n - \frac{12}{5},\\
\mathrm{Re}ZM_2(R) &= \frac{823}{22}mn + 10.4129m - 4.5871n - \frac{133}{9},\\
\mathrm{Re}ZM_3(R) &= 3624mn + 718m - 782n - 1428,\\
RI(R) &= 3.4345mn + 1.1985m - 0.0015n - 1.6833,\\
SCI(R) &= 5.1843mn + 1.7278m - 0.1696n - 1.8849,\\
HI(R) &= \frac{1669}{495}mn + \frac{6907}{6930}m - \frac{23}{6930}n - \frac{53}{45},\\
ABC(R) &= 9.3620mn + 3.0175m - 0.3767n - 3.4147,\\
GA(R) &= 15.7420mn + 4.9861m - 1.0139n - 5.9505,
\end{aligned}
\tag{9}
$$

Proof. Let G be isomorphic to rhenium trioxide. Then, the edge partition for the chemical graph of rhenium trioxide based on degree sum of vertices of each other is given by

$$\begin{aligned}
E_{34} &= 2(m+n) = 2m+2n,\\
E_{35} &= 4mn-2m-2n,\\
E_{36} &= 2mn,\\
E_{44} &= 4,\\
E_{45} &= 4m+4n-8,\\
E_{46} &= 2(m+n) = 2m+2n,\\
E_{55} &= 6m+m-5n-2\\
&\quad m<2,\\
&\quad n<5,\\
&\quad m\\
&\quad n\in Z^{+},\\
E_{56} &= 4mn-2m-2n.
\end{aligned} \tag{10}$$

□

3.1. First Zagreb Index

$$\begin{aligned}
M_1(G) &= \sum_{tr\in E(G)}[\Psi(t)+\Psi(r)],\\
M_1(R) &= \sum_{tr\in E_{34}(R)}[\Psi(t)+\Psi(r)] + \sum_{tr\in E_{35}(R)}[\Psi(t)+\Psi(r)] + \sum_{tr\in E_{36}(R)}[\Psi(t)+\Psi(r)] + \sum_{tr\in E_{44}(R)}[\Psi(t)+\Psi(r)]\\
&\quad + \sum_{tr\in E_{45}(R)}[\Psi(t)+\Psi(r)] + \sum_{tr\in E_{46}(R)}[\Psi(t)+\Psi(r)] + \sum_{tr\in E_{55}(R)}[\Psi(t)+\Psi(r)] + \sum_{tr\in E_{56}(R)}[\Psi(t)+\Psi(r)]\\
&= 7|E_{34}(R)| + 8|E_{35}(R)| + 9|E_{36}(R)| + 8|E_{44}(R)| + 9|E_{45}(R)| + 10|E_{46}(R)| + 10|E_{55}(R)| + 11|E_{56}(R)|\\
&= 7\{2(m+n)\} + 8(4mn-2m-2n) + 9(2mn) + 8(4) + 9(4m+4n-8)\\
&\quad + 10\{2(m+n)\} + 10(6mn+m-5n-2) + 11(4mn-2m-2n),\\
M_1(R) &= 154mn + 44m - 26n + 20.
\end{aligned} \tag{11}$$

3.2. Second Zagreb Index

$$\begin{aligned}
M_2(G) &= \sum_{tr\in E(G)}[\Psi(t)\times\Psi(r)]\\
&= 12|E_{34}(R)| + 15|E_{35}(R)| + 18|E_{36}(R)| + 16|E_{44}(R)| + 20|E_{45}(R)| + 24|E_{46}(R)| + 25|E_{55}(R)| + 30|E_{56}(R)|\\
&= 12\{2(m+n)\} + 15(4mn-2m-2n) + 18(2mn) + 16(4) + 20(4m+4n-8)\\
&\quad + 24\{2(m+n)\} + 25(6mn+m-5n-2) + 30(4mn-2m-2n),\\
M_2(R) &= 366mn + 87m - 63n.
\end{aligned} \tag{12}$$

3.3. Hyper Zagreb Index

$$
\begin{aligned}
HM_1(G) &= \sum_{tr\in E(G)} [\Psi(t)+\Psi(r)]^2\\
&= 49|E_{34}(R)| + 64|E_{35}(R)| + 81|E_{36}(R)| + 64|E_{44}(R)| + 81|E_{45}(R)| + 100|E_{46}(R)| + 100|E_{55}(R)| + 121|E_{56}(R)|\\
&= 49\{2(m+n)\} + 64(4mn-2m-2n) + 81(2mn) + 64(4) + 81(4m+4n-8)\\
&\quad + 100\{2(m+n)\} + 100(6mn+m-5n-2) + 121(4mn-2m-2n),\\
HM_1(R) &= 1502mn + 352m - 410n.
\end{aligned}
\tag{13}
$$

The comparison between $M_1(G)$, $M_2(G)$, and $HM(G)$ is shown in Table 1 and Figure 2.

3.4. First Zagreb Polynomial

$$
\begin{aligned}
M_1(G_1,x) &= \sum_{tr\in E(G)} x^{[\Psi(t)+\Psi(r)]}\\
&= |E_{34}(R)|x^7 + |E_{35}(R)|x^8 + |E_{36}(R)|x^9 + |E_{44}(R)|x^8 + |E_{45}(R)|x^9\\
&\quad + |E_{46}(R)|x^{10} + |E_{55}(R)|x^{10} + |E_{56}(R)|x^{11},\\
M_1(R_1,x) &= (2m+2n)x^7 + (4mn-2m-2n)x^8 + (2mn)x^9 + 4x^8 + (4m+4n-8)x^9\\
&\quad + (2m+2n)x^{10} + (6mn+m-5n-2)x^{10} + (4mn-2m-2n)x^{11}\\
&\quad + (6mn+3m-3n-2)x^{10} + (4mn-2m-2n)x^{11}.
\end{aligned}
\tag{14}
$$

3.5. Second Zagreb Polynomial

$$
\begin{aligned}
M_2(G_1,x) &= \sum_{tr\in E(G)} x^{[\Psi(t)\times\Psi(r)]}\\
&= |E_{34}(R)|x^{12} + |E_{35}(R)|x^{15} + |E_{36}(R)|x^{18} + |E_{44}(R)|x^{16} + |E_{45}(R)|x^{20}\\
&\quad + |E_{46}(R)|x^{24} + |E_{55}(R)|x^{25} + |E_{56}(R)|x^{30},\\
M_2(R_1,x) &= (2m+2n)x^{12} + (4mn-2m-2n)x^{15} + (2mn)x^{18} + 4x^{16} + (4m+4n-8)x^{20}\\
&\quad + (2m+2n)x^{24} + (6mn+m-5n-2)x^{25} + (4mn-2m-2n)x^{30}.
\end{aligned}
\tag{15}
$$

The comparison between first and second Zagreb polynomials is represented in Table 2 and Figure 3.

3.6. Redefined First Zagreb Index

$$
\begin{aligned}
\mathrm{Re}ZM_1(G) &= \sum_{tr\in E(G)} \left[\frac{\Psi(t)+\Psi(r)}{\Psi(t)\times\Psi(r)}\right]\\
&= \frac{7}{12}|E_{34}(R)| + \frac{8}{15}|E_{35}(R)| + \frac{9}{18}|E_{36}(R)| + \frac{8}{16}|E_{44}(R)| + \frac{9}{20}|E_{45}(R)|\\
&\quad + \frac{10}{24}|E_{46}(R)| + \frac{10}{25}|E_{55}(R)| + \frac{11}{30}|E_{56}(R)|\\
&= \frac{7}{12}\{2(m+n)\} + \frac{8}{15}(4mn-2m-2n) + \frac{1}{2}(2mn) + \frac{1}{2}(4) + \frac{9}{20}(4m+4n-8)\\
&\quad + \frac{5}{12}\{2(m+n)\} + \frac{2}{5}(6mn+m-5n-2) + \frac{11}{30}(4mn-2m-2n),\\
\mathrm{Re}ZM_1(R) &= \frac{27}{5}mn + \frac{21}{10}m - \frac{3}{10}n - \frac{12}{5}.
\end{aligned}
\tag{16}
$$

Table 1: Comparison between $M_1(G)$, $M_2(G)$, and $HM(G)$.

(m,n)	$M_1(R)$	$M_2(R)$	$HM(R)$
$(1,1)$	192	390	1444
$(2,2)$	672	1512	5892
$(3,3)$	1460	3366	13344
$(4,4)$	2556	5952	23800
$(5,5)$	3960	9270	37260
$(6,6)$	5672	13320	53724
$(7,7)$	7692	18102	73192
$(8,8)$	10020	23616	95664
$(9,9)$	12656	29862	121140
$(10,10)$	15600	36840	149620

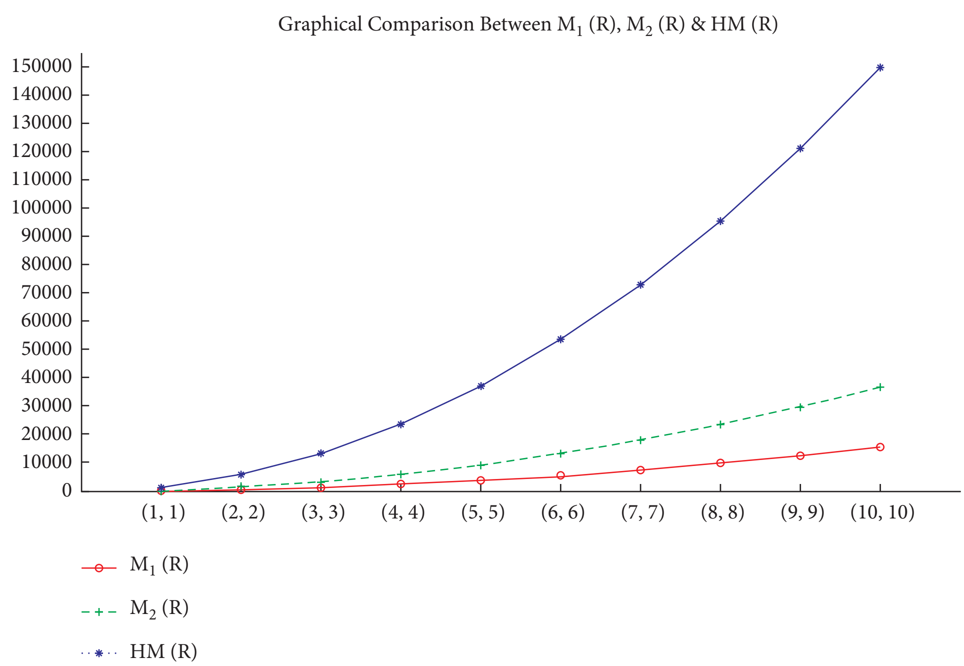

Figure 2: The graphical representation of Table 1.

Table 2: Connection between $M_1(G_1,x)$ and $M_2(G_1,x)$.

(m,n)	$M_1(G_1,x)$	$M_2(G_1,x)$
$(1,1,1)$	14	14
$(2,2,1)$	66	66
$(3,3,1)$	150	150
$(4,4,1)$	266	266
$(5,5,1)$	414	414
$(6,6,1)$	594	594
$(7,7,1)$	806	806
$(8,8,1)$	1050	1050
$(9,9,1)$	1326	1326
$(10,10,1)$	1634	1634

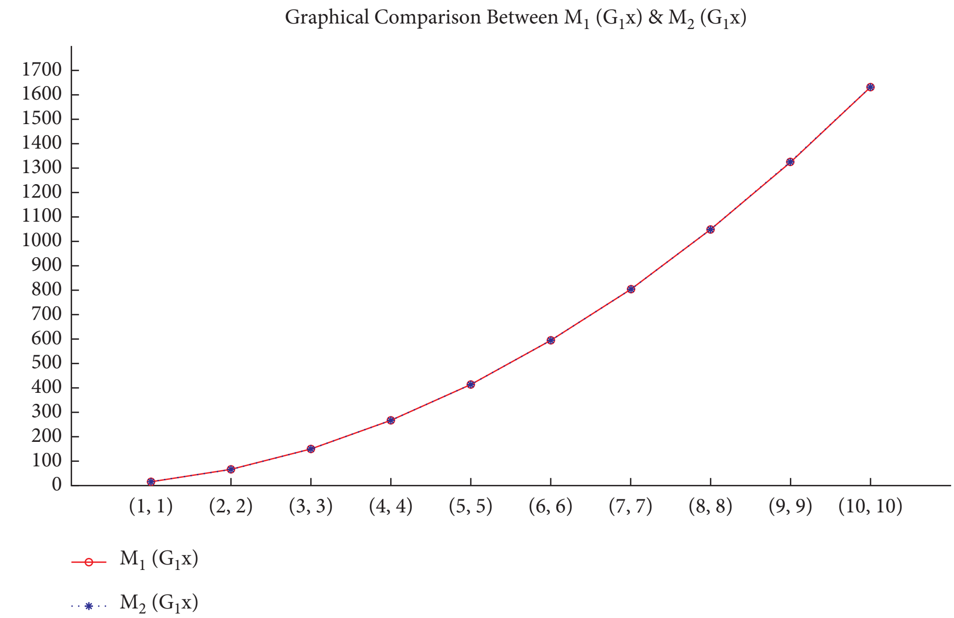

Figure 3: The graphical representation of Table 2.

3.7. Redefined Second Zagreb Index

$$\begin{aligned}
\mathrm{Re}ZM_2(G) &= \sum_{tr\in E(G)}\left[\frac{\Psi(t)\times\Psi(r)}{\Psi(t)+\Psi(r)}\right]\\
&= \frac{12}{7}\left|E_{34}(R)\right| + \frac{15}{8}\left|E_{35}(R)\right| + \frac{18}{9}\left|E_{36}(R)\right| + \frac{16}{8}\left|E_{44}(R)\right| + \frac{20}{9}\left|E_{45}(R)\right| + \frac{24}{10}\left|E_{46}(R)\right| + \frac{25}{10}\left|E_{55}(R)\right| + \frac{30}{11}\left|E_{56}(R)\right|\\
&= \frac{12}{7}\{2(m+n)\} + \frac{15}{8}(4mn-2m-2n) + \frac{18}{9}(2mn) + \frac{16}{8}(4) + \frac{20}{9}(4m+4n-8)\\
&\quad + \frac{24}{10}\{2(m+n)\} + \frac{25}{10}(6mn+m-5n-2) + \frac{30}{11}(4mn-2m-2n),\\
\mathrm{Re}ZM_2(R) &= \frac{823}{22}mn + 10.4129m - 4.5871n - \frac{133}{9}.
\end{aligned} \tag{17}$$

3.8. Redefined Third Zagreb Index

$$\begin{aligned}
\mathrm{Re}ZM_3(G) &= \sum_{tr\in E(G)}[\Psi(t)+\Psi(r)][\Psi(t)\times\Psi(r)]\\
&= 7\times 12\left|E_{34}(R)\right| + 8\times 15\left|E_{35}(R)\right| + 9\times 18\left|E_{36}(R)\right| + 8\times 16\left|E_{44}(R)\right|\\
&\quad + 9\times 20\left|E_{45}(R)\right| + 10\times 24\left|E_{46}(R)\right| + 10\times 25\left|E_{55}(R)\right| + 11\times 30\left|E_{56}(R)\right|\\
&= 84\{2(m+n)\} + 120(4mn-2m-2n) + 162(2mn) + 128(4) + 180(4m+4n-8)\\
&\quad + 240\{2(m+n)\} + 250(6mn+m-5n-2) + 330(4mn-2m-2n),\\
\mathrm{Re}ZM_3(R) &= 3624mn + 718m - 782n - 1428.
\end{aligned} \tag{18}$$

The comparison between redefined Zagreb indices is represented in Table 3 and Figure 4.

3.9. Randic Index

$$\begin{aligned} RI(G) &= \sum_{tr\in E(G)}\left[\frac{1}{\sqrt{\Psi(t)\times\Psi(r)}}\right] \\ &= \frac{1}{\sqrt{12}}\{2(m+n)\} + \frac{1}{\sqrt{15}}(4mn-2m-2n) + \frac{1}{\sqrt{18}}(2mn) + \frac{1}{\sqrt{16}}(4) \\ &\quad + \frac{1}{\sqrt{20}}(4m+4n-8) + \frac{1}{\sqrt{24}}\{2(m+n)\} + \frac{1}{\sqrt{25}}(6mn+m-5n-2) \\ &\quad + \frac{1}{\sqrt{30}}(4mn-2m-2n), \\ RI(R) &= 3.4345mn + 1.1985m - 0.0015n - 1.6833. \end{aligned} \tag{19}$$

3.10. Sum Connectivity Index

$$\begin{aligned} SCI(G) &= \sum_{tr\in E(G)}\left[\frac{1}{\sqrt{\Psi(t)+\Psi(r)}}\right] \\ &= \frac{1}{\sqrt{7}}\{2(m+n)\} + \frac{1}{\sqrt{8}}(4mn-2m-2n) + \frac{1}{\sqrt{3}}(2mn) + \frac{1}{\sqrt{2\sqrt{2}}}(4) \\ &\quad + \frac{1}{\sqrt{3}}(4m+4n-8) + \frac{1}{\sqrt{10}}\{2(m+n)\} + \frac{1}{\sqrt{10}}(6mn+m-5n-2) + \frac{1}{\sqrt{11}}(4mn-2m-2n), \\ SCI(R) &= 5.1843mn + 1.7278m - 0.1696n - 1.8849. \end{aligned} \tag{20}$$

3.11. Harmonic Index

$$\begin{aligned} HI(G) &= \sum_{tr\in E(G)}\left[\frac{2}{\sqrt{\Psi(t)\times\Psi(r)}}\right] \\ &\frac{2}{7}\{2(m+n)\} + \frac{1}{4}(4mn-2m-2n) + \frac{2}{9}(2mn) + \frac{1}{4}(4) + \frac{2}{9}4(m+4n-8) \\ &\quad + \frac{2}{10}\{2(m+n)\} + \frac{1}{5}(6mn+m-5n-2) + \frac{2}{11}(4mn-2m-2n), \\ HI(R) &= \frac{1669}{495}mn + \frac{6907}{6930}m - \frac{23}{6930}n - \frac{53}{45}. \end{aligned} \tag{21}$$

Table 3: Connection between $ReZG_1$, $ReZG_2$, and $ReZG_3$.

(m, n)	$ReZG_1(G)$	$ReZG_2(G)$	$ReZG_3(G)$
(1, 1)	4.8	28.4571	2132
(2, 2)	28.2	146.5102	12940
(3, 3)	51.6	339.3814	30996
(4, 4)	91.2	607.0709	56300
(5, 5)	141.6	949.5785	88852
(6, 6)	202.8	1366.9043	128652
(7, 7)	274.8	1859.0483	175700
(8, 8)	357.6	2426.0104	229996
(9, 9)	451.2	3067.7908	291540
(10, 10)	555.6	3784.3893	360332

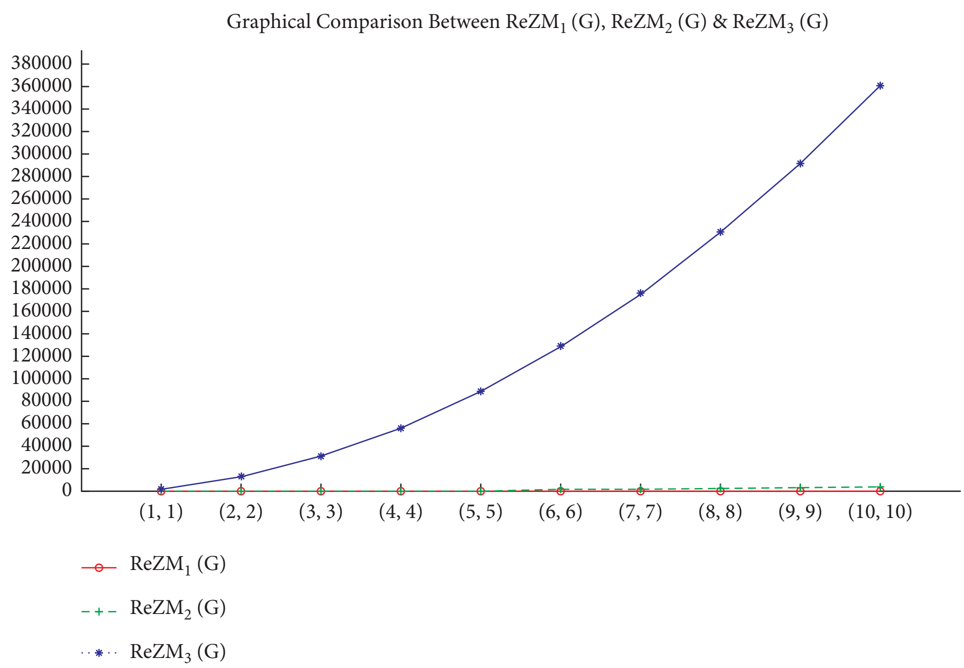

Figure 4: Graphical representation of Table 3.

The connection between the $RI(R)$, $SCI(R)$, and $HI(R)$ is shown in Table 4 and Figure 5.

3.12. ABC Index

$$ABC(G) = \sum_{tr \in E(G)} \sqrt{\frac{\Psi(t) + \Psi(r) - 2}{\Psi(t) \times \Psi(r)}}$$

$$= \frac{\sqrt{15}}{16}\{2(m+n)\} + \frac{\sqrt{10}}{5}(4mn - 2m - 2n) + \frac{\sqrt{14}}{6}(2mn) + \frac{\sqrt{6}}{4}(4) + \frac{\sqrt{35}}{10}(4m + 4n - 8) + \frac{\sqrt{3}}{3}\{2(m+n)\} + \frac{2\sqrt{2}}{5}(6mn + m - 5n - 2) + \frac{\sqrt{30}}{10}(4mn - 2m - 2n), \quad (22)$$

$$ABC(R) = 9.3620mn + 3.0175m - 0.3767n - 3.4147.$$

Table 4: Connection between $RI(R)$, $SCI(R)$, and $HI(R)$.

(m,n)	$RI(R)$	$SCI(R)$	$HI(R)$
(1, 1)	2.9482	4.8576	3.1873
(2, 2)	14.4487	21.9687	14.2958
(3, 3)	32.8182	49.4484	32.1478
(4, 4)	48.4687	87.2967	56.7431
(5, 5)	90.1642	135.5136	88.0820
(6, 6)	129.1407	194.0991	126.1642
(7, 7)	167.9862	263.0532	170.9899
(8, 8)	228.2157	342.3759	222.5590
(9, 9)	287.2842	432.0672	280.8716
(10, 10)	353.7367	532.1271	345.9276

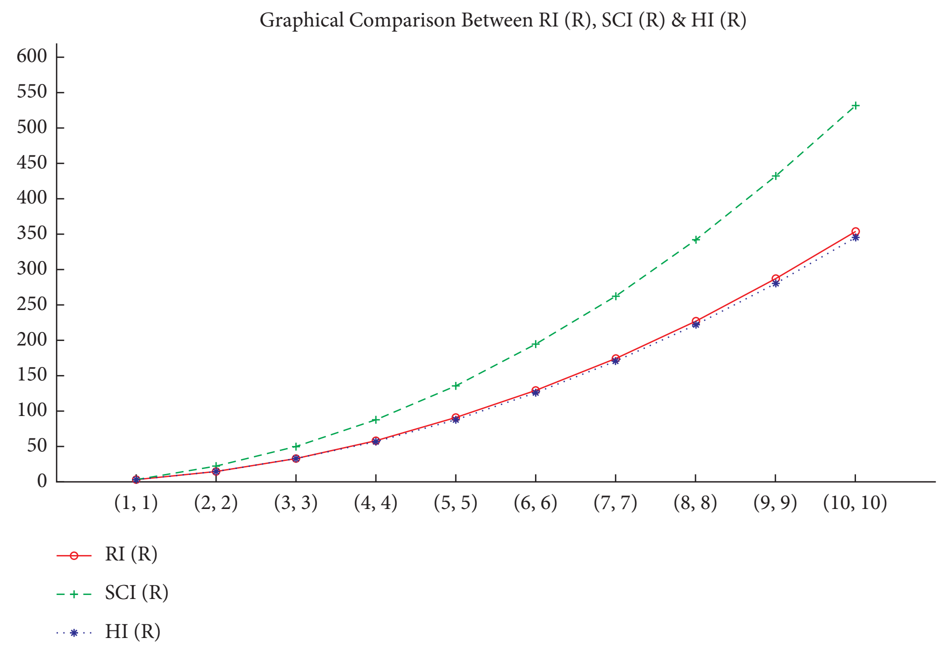

Figure 5: Graphical representation of Table 4.

Table 5: Connection between $ABC(R)$ and $GA(R)$

(m,n)	$ABC(R)$	$GA(R)$
(1, 1)	8.5881	13.7637
(2, 2)	39.3149	64.9619
(3, 3)	88.7657	147.6441
(4, 4)	156.9405	261.8103
(5, 5)	243.8393	407.4605
(6, 6)	349.4621	584.5947
(7, 7)	473.8089	793.2129
(8, 8)	616.8797	1033.3151
(9, 9)	778.6745	1304.9013
(10, 10)	959.1933	1607.9715

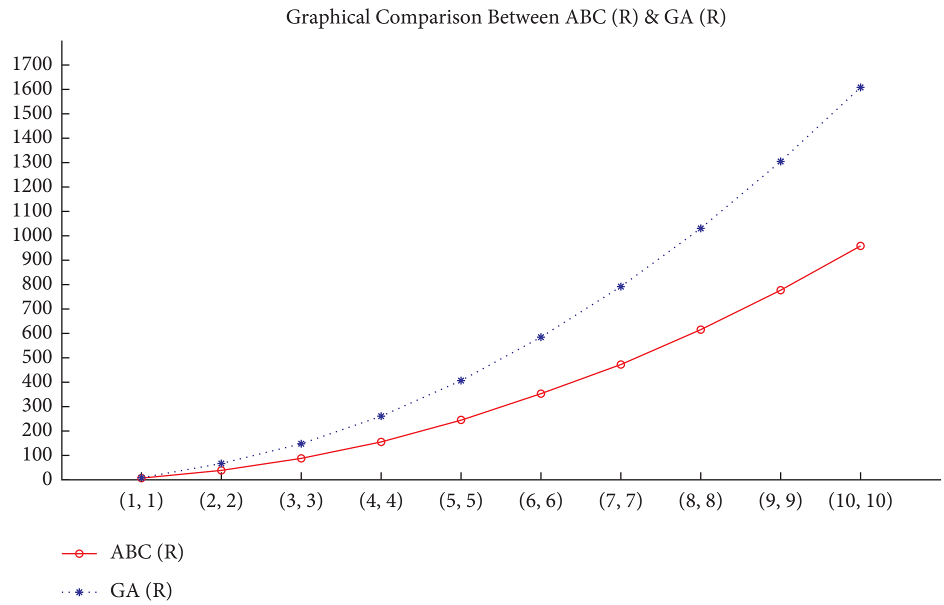

FIGURE 6: The graphical representation of Table 5.

3.13. GA Index

$$\begin{aligned} GA(G) &= \sum_{tr\in E(G)} \frac{2\sqrt{\Psi(t)\times\Psi(r)}}{\Psi(t)+\Psi(r)} \\ &= \frac{4\sqrt{3}}{7}\{2(m+n)\} + \frac{\sqrt{15}}{4}(4mn-2m-2n) + \frac{2\sqrt{2}}{3}(2mn) + 1(4) \\ &\quad + \frac{4\sqrt{5}}{9}(4m+4n-8) + \frac{2\sqrt{6}}{5}\{2(m+n)\} + 1(6mn+m-5n-2) + \frac{2\sqrt{30}}{11}(4mn-2m-2n), \end{aligned} \tag{23}$$

$$GA(R) = 15.7420mn + 4.9861m - 1.0139n - 5.9505.$$

The connection between $ABC(R)$ and $GA(R)$ is shown in Table 5 and Figure 6.

4. Conclusion

In this research work, researchers computed different topological indices such as Zagreb indices, redefined Zagreb indices, atom bond connectivity, harmonic index and sum connectivity index, and many other indices of chemical graph of rhenium trioxide (ReO_3). Rhenium trioxide also has been expanded by a researcher upto m and n cycles. Researchers also explained the comparison between the different versions of topological indices in a numerical way with the help of table and also expressed them in the graphical pattern. By comparing these topological indices with tables and graphs, the importance of indices shows that these indices results have significant relationship with each other.

Data Availability

No data were used to supoort this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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