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Research Article

Solution of Space-Time Fractional Differential Equations Using Aboodh Transform Iterative Method

Michael A. Awuya,[1] **Gbenga O. Ojo**,[2] **and Nazim I. Mahmudov**[1]

[1]*Department of Mathematics, Faculty of Arts and Sciences, Eastern Mediterranean University, Famagusta, T.R. North Cyprus via Mersin 10, Turkey*
[2]*Department of Information System Engineering, Faculty of Engineering Cyprus West University, Famagusta, T.R. North Cyprus via Mersin 10, Turkey*

Correspondence should be addressed to Nazim I. Mahmudov; nazim.mahmudov@emu.edu.tr

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A relatively new and efficient approach based on a new iterative method and the Aboodh transform called the Aboodh transform iterative method is proposed to solve space-time fractional differential equations, the fractional order is considered in the Caputo sense. This method is a combination of the Aboodh transform and the new iterative method and gives the solution in series form with easily computable components. The nonlinear term is easily handled by the new iterative method, to affirm the simplicity and performance of the proposed method, five examples were considered, and the solution plots were presented to show the effect of the fractional order. The outcome reveals that the approach is accurate and easy to implement.

1. Introduction

Fractional Calculus can be described as the field of mathematics that consists of ordinary and partial derivatives of positive noninteger order. It is the generalization of classical integral and differential equations [1, 2]. One major attractive property of fractional calculus is the nonlocal property.

Recently, various problems in Biology and Physics has been modeled with fractional order derivative, an analytical solution of the Fornberg–Whithan equation was presented in [3], fractional model of the Rosenau–Hyman equation which is a KdV-like equation was considered in [4], for application of fractional derivative to Biology population model see [5], the numerical study of HIV-1 infection of CD4+ T-cell was presented in [6], Caputo–Fabrizio fractional model of photocatalytic degradation of dyes was studied in [7], a wavelet based numerical scheme for fractional order SEIR epidemic of measles by using Genocchi polynomials was presented in [8], and the investigation of fractional order susceptible-infected-recovered epidemic model of childhood disease was presented in [9]. Therefore, it is extremely important to find an effective method of solving fractional differential equations, as only the solutions can give a better comprehension of the underlying problems. Many researchers have presented different methods for solving fractional differential equations such as reproducing kernel discretization method [10], Chebyshev wavelet collocation method, [11] Tichonov regularization method [12], Chebyshev collocation method, [13] q-homotopy analysis Shehu transform method [14], Fractional differential transform, [15] Fractional variational iterational method [16], and iterative Laplace transform method [17].

In 2016, the new iterative method was presented by Daftardar–Gejji and Jafari to solve functional equations [18], but now the iterative method has been used to solve many integral and fractional order differential equations. [5, 19, 20] But most of these methods considered a single term time-fractional order differential equations.

In this paper, the main objective is to extend the Aboodh transform iterative method to solve space-time fractional differential equations with more than a single term fractional derivative. The fractional derivative is considered in Caputo sense both for time and space, when $\alpha = \beta = 1$, the space-time fractional differential equations becomes the classical differential equations. The rest of this paper is arranged as follows: in Section 2, we gave some definitions and a preliminary concept of Aboodh transform. In Section 3, we described briefly the Aboodh transform iterative method for space-time fractional derivative while in Section 4, a few examples were considered to describe the efficiency of the method. Finally, we concluded in Section 5.

2. Definitions and Preliminaries

In this section, we give some definitions and notions about Aboodh transform.

Definition 1. Caputo time-fractional derivative of order $\alpha > 0$ for the function $Q(x,t)$ is defined as follows [1, 2]:

$$D_t^{\alpha} Q(x,t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} Q^{(n)}(x,\tau)\mathrm{d}\tau, \quad n-1 < \alpha \le n. \tag{1}$$

Similarly, the Caputo space fractional derivative of order $\beta > 0$ for the function $Q(x,t)$ is defined as follows:

$$D_x^{\beta} Q(x,t) = \frac{1}{\Gamma(n-\beta)} \int_0^x (x-t)^{n-\beta-1} Q^{(n)}(x,t)\mathrm{d}t, \quad n-1 < \beta \le n. \tag{2}$$

Remark 1. $D_t^{\alpha} Q(x,t) = D_x^{\beta} Q(x,t) = 0$, whenever $Q(x,t)$ is a constant.

Remark 2. $D_t^{\alpha} t^b = \{ (\Gamma(b+1)/\Gamma(b-\alpha+1)) \ t^{b-\alpha}$, if $n-1 < \alpha \le n, b > \alpha - 1, 0, n-1 < \alpha \le n, b \le \alpha - 1$.

Definition 2. One parameter Mittag-Leffler function is given as follows [5]:

$$E_{\alpha}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(1+k\alpha)}, \quad \alpha, z \in C\mathrm{Re}(\alpha) \ge 0. \tag{3}$$

Definition 3. The Aboodh transform of $Q(t)$ is defined as follows [5]:

$$\mathscr{A}[Q(t)] = \frac{1}{v} \int_0^{\infty} Q(t) e^{-vt} \mathrm{d}t = A(v), \quad t \ge 0. \tag{4}$$

The inverse Aboodh transform of function $Q(t)$ if $\mathscr{A}[Q(t)] = A(v)$ is defined as follows:

$$Q(t) = \mathscr{A}^{-1}[A(v)]. \tag{5}$$

Remark 3. The Aboodh transform of the function $Q(t)$ satisfy the linearity property [5].

Definition 4. The Aboodh transform for Caputo time-fractional derivative of order β is given as follows [5]:

$$\mathscr{A}\left[D_t^{\beta} Q(x,t); v\right] = v^{\beta} \mathscr{A}[Q(x,t)] - \sum_{k=0}^{m-1} \frac{Q^{(k)}(x,0)}{v^{2-\beta+k}}, \quad m-1 < \beta \le m. \tag{6}$$

3. Basic Idea of the Proposed Method

Consider the space-time fractional partial differential equation of the form.

$$D_t^{\alpha} Q(x,t) = \Phi\left(Q(x,t), D_x^{\beta} Q(x,t), D_x^{2\beta} Q(x,t), D_x^{3\beta} Q(x,t)\right), \quad 0 < \alpha, \beta \le 1, \tag{7}$$

with the initial conditions

$$Q^{(k)}(x,0) = h_k, \quad k = 0, 1, \ldots, m-1, \tag{8}$$

$Q(x,t)$ is the unknown function to be determine and $\Phi(Q(x,t), D_x^{\beta} Q(x,t), D_x^{2\beta} Q(x,t), D_x^{3\beta} Q(x,t))$ can be linear or nonlinear operator of $Q(x,t)$, $D_x^{\beta} Q(x,t)$, $D_x^{2\beta} Q(x,t)$, and $D_x^{3\beta} Q(x,t)$ For convenience we represent $Q(x,t)$ with Q, so by applying the Aboodh transform to both sides of equation (7) we have the following equation:

$$\mathscr{A}[Q(x,t)] = \frac{1}{v^{\alpha}} \left(\sum_{k=0}^{m-1} \frac{Q^{(k)}(x,0)}{v^{2-\alpha+k}} + \mathscr{A}\left[\Phi\left(Q, D_x^{\beta} Q, D_x^{2\beta} Q, D_x^{3\beta} Q\right)\right] \right), \tag{9}$$

taking the inverse Aboodh transform, we get the following equation:

$$Q(x,t) = \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}} \left(\sum_{k=0}^{m-1} \frac{Q^{(k)}(x,0)}{v^{2-\alpha+k}} + \mathscr{A}\left[\Phi\left(Q, D_x^{\beta} Q, D_x^{2\beta} Q, D_x^{3\beta} Q\right)\right] \right)\right]. \tag{10}$$

The Aboodh transform iterative method gives the solution in form of an infinite series.

$$Q(x,t) = \sum_{i=0}^{\infty} Q_i. \tag{11}$$

Since $\Phi(Q, D_x^{\beta} Q, D_x^{2\beta} Q, D_x^{3\beta} Q)$ is either a linear or nonlinear operator which can be decomposed as follows:

$$\Phi\left(Q, D_x^{\beta}Q, D_x^{2\beta}Q, D_x^{3\beta}Q\right) = \Phi\left(Q_0, D_x^{\beta}Q_0, D_x^{2\beta}Q_0, D_x^{3\beta}Q_0\right) + \sum_{i=0}^{\infty}\left\{\Phi\left(\sum_{k=0}^{i}\left(Q_k, D_x^{\beta}Q_k, D_x^{2\beta}Q_k, D_x^{3\beta}Q\right)\right) - \Phi\left(\sum_{k=1}^{i-1}\left(Q_k, D_x^{\beta}Q_k, D_x^{2\beta}Q_k, D_x^{3\beta}Q\right)\right)\right\}. \tag{12}$$

Substituting equations (12) and (11) into equation (10) we obtain the following equation:

$$\begin{aligned}\sum_{i=0}^{\infty} Q_i(x,t) &= \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\sum_{k=0}^{m-1}\frac{Q^{(k)}(x,0)}{v^{2-\alpha+k}} + \mathscr{A}\left[\Phi\left(Q_0, D_x^{\beta}Q_0, D_x^{2\beta}Q_0, D_x^{3\beta}Q_0\right)\right]\right)\right] \\ &+ \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\mathscr{A}\left[\sum_{i=0}^{\infty}\left\{\Phi\left(\sum_{k=0}^{i}\left(Q_k, D_x^{\beta}Q_k, D_x^{2\beta}Q_k, D_x^{3\beta}Q_k\right)\right)\right\}\right]\right)\right] \\ &- \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\mathscr{A}\left[\left\{\Phi\left(\sum_{k=1}^{i-1}\left(Q_k, D_x^{\beta}Q_k, D_x^{2\beta}Q_k, D_x^{3\beta}Q_k\right)\right)\right\}\right]\right)\right].\end{aligned} \tag{13}$$

Now, recursively, we compute the terms.

$$\begin{aligned}Q_0(x,t) &= \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\sum_{k=0}^{m-1}\frac{Q^{(k)}(x,0)}{v^{2-\alpha+k}}\right], \\ Q_1(x,t) &= \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\mathscr{A}\left[\Phi\left(Q_0, D_x^{\beta}Q_0, D_x^{2\beta}Q_0, D_x^{3\beta}Q_0\right)\right]\right)\right], \\ &\vdots \\ Q_{m+1}(x,t) &= \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\mathscr{A}\left[\sum_{i=0}^{\infty}\left\{\Phi\left(\sum_{k=0}^{i}\left(Q_k, D_x^{\beta}Q_k, D_x^{2\beta}Q_k, D_x^{3\beta}Q_k\right)\right)\right\}\right]\right)\right] \\ &- \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\mathscr{A}\left[\sum_{i=0}^{\infty}\left\{\Phi\left(\sum_{k=1}^{i-1}\left(Q_k, D_x^{\beta}Q_k, D_x^{2\beta}Q_k, D_x^{3\beta}Q_k\right)\right)\right\}\right]\right)\right], \quad m = 1, 2, \ldots.\end{aligned} \tag{14}$$

The series converges rapidly, for convergence see [18, 21]. So the m-term analytically approximate solution of equation (7) is given by the following equation:

$$Q(x,t) \approx \sum_{i=0}^{m-1} Q_i. \tag{15}$$

4. Application

Here, the Aboodh transform iterative method is applied to solve five distinct space-time fractional differential equations with suitable initial conditions.

Example 1. Consider the fractional Airy's-like equation with an additional term [22].

$$D_t^{\alpha}Q(x,t) = D_x^{\beta}Q + Q, \quad 0 < \alpha, \beta \le 1. \tag{16}$$

With the initial condition,

$$Q(x,0) = x^3. \tag{17}$$

Applying the Aboodh transform on both sides of equation (16), we obtain the following equation:

$$\mathscr{A}[Q(x,t)] = \frac{1}{v^{\alpha}}\left(\sum_{k=0}^{m-1}\frac{Q^{(k)}(x,0)}{v^{2-\alpha+k}} + \mathscr{A}\left[D_x^{\beta}Q + Q\right]\right), \tag{18}$$

taking the inverse Aboodh transform on equation (18), we have the following equation:

$$Q(x,t) = \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\sum_{k=0}^{m-1}\frac{Q^{(k)}(x,0)}{v^{2-\alpha+k}} + \mathscr{A}\left[D_x^{\beta}Q + Q\right]\right)\right]. \tag{19}$$

Using the Aboodh transform iterative procedure, we obtain the following equation:

$$
\begin{aligned}
Q_0(x,t) &= \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\sum_{k=0}^{m-1}\frac{Q^{(k)}(x,0)}{v^{2-\alpha+k}}\right)\right] \\
&= \mathscr{A}^{-1}\left[\frac{Q(x,0)}{v^{2}}\right] \\
&= x^{3}, \\
Q_1(x,t) &= \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\mathscr{A}\left[D_x^{\beta}Q_0 + Q_0\right]\right)\right] \\
&= \mathscr{A}^{-1}\left[\frac{\Gamma(4)x^{3-\beta}}{v^{2+\alpha}\Gamma(4-\beta)} + \frac{x^{3}}{v^{2+\alpha}}\right] \\
&= \frac{\Gamma(4)x^{3-\beta}t^{\alpha}}{\Gamma(\alpha+1)\Gamma(4-\beta)} + \frac{x^{3}t^{\alpha}}{\Gamma(\alpha+1)}, \\
Q_2(x,t) &= \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\mathscr{A}\left[D_x^{\beta}(Q_0+Q_1) + (Q_0+Q_1)\right]\right)\right] - \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\mathscr{A}\left[D_x^{\beta}Q_0 + Q_0\right]\right)\right] \\
&= \mathscr{A}^{-1}\left[\frac{\Gamma(4)x^{3-\beta}}{v^{2+\alpha}\Gamma(4-\beta)} + \frac{\Gamma(4)x^{3-2\beta}}{v^{2+2\alpha}\Gamma(4-2\beta)} + \frac{2\Gamma(4)x^{3-\beta}}{v^{2+2\alpha}\Gamma(4-\beta)} + \frac{x^{3}}{v^{2+\alpha}} + \frac{x^{3}}{v^{2+2\alpha}}\right] - \mathscr{A}^{-1}\left[\frac{\Gamma(4)x^{3-\beta}}{v^{2+\alpha}\Gamma(4-\beta)} + \frac{x^{3}}{v^{2+\alpha}}\right] \\
&= \frac{\Gamma(4)x^{3-2\beta}t^{2\alpha}}{\Gamma(2\alpha+1)\Gamma(4-2\beta)} + \frac{2\Gamma(4)x^{3-\beta}t^{2\alpha}}{\Gamma(4-\beta)\Gamma(2\alpha+1)} + \frac{x^{3}t^{2\alpha}}{\Gamma(2\alpha+1)}, \\
&\vdots
\end{aligned}
\tag{20}
$$

and so on. The series solution is given by the following equation:

$$Q(x,t) = Q_0 + Q_1 + Q_2 + \cdots. \tag{21}$$

Figure 1 represent the solution plots of equation (16) when $\alpha = \beta = .02, .04, .06, .08, 2, .4, .6, .8$ at $x = 1$ and $t = 1$, respectively. While the remaining are the surface plots.

Example 2. Consider the nonlinear space-time fractional Fokker–Planck equation [23].

$$D_t^{\alpha}Q(x,t) = D_x^{\beta}\left(\frac{xQ}{3}\right) - \left(\frac{4}{x}Q^{2}\right)_x + \left(Q^{2}\right)_{xx}, \quad 0 < \alpha, \beta \leq 1. \tag{22}$$

With the initial condition,

$$Q(x,0) = x^{2}. \tag{23}$$

Applying the Aboodh transform on equation (22), we obtain the following equation:

$$\mathscr{A}[Q(x,t)] = \frac{1}{v^{\alpha}}\left(\sum_{k=0}^{m-1}\frac{Q^{(k)}(x,0)}{v^{2-\alpha+k}} + \mathscr{A}\left[D_x^{\beta}\left(\frac{x}{3}Q\right) - \left(\frac{4}{x}Q^{2}\right)_x + \left(Q^{2}\right)_{xx}\right]\right), \tag{24}$$

taking the inverse Aboodh transform, we have the following equation:

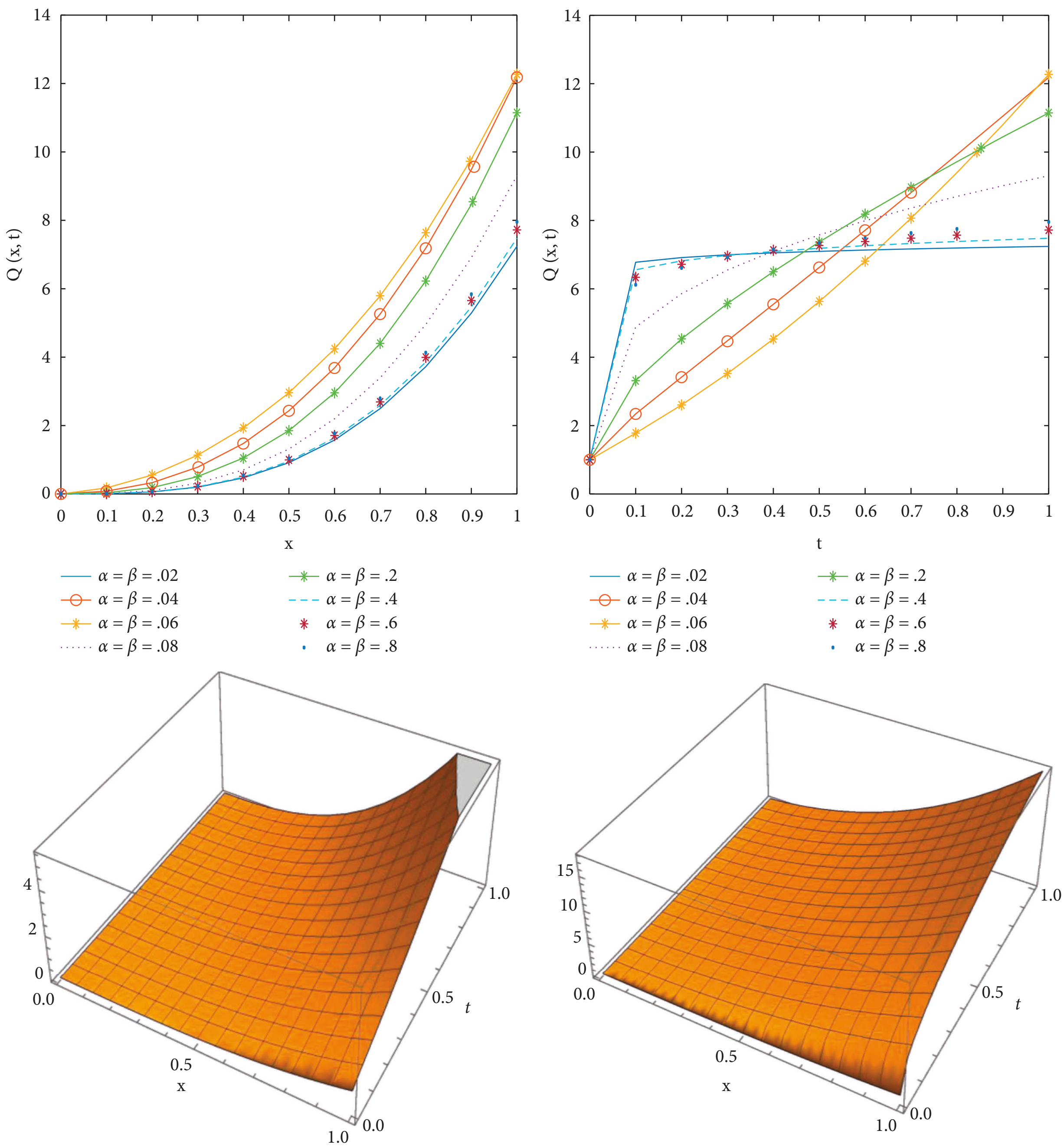

Figure 1: Comparison of the solution at various values of alpha and beta.

$$Q(x,t) = \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\sum_{k=0}^{m-1}\frac{Q^{(k)}(x,0)}{v^{2-\alpha+k}} + \mathscr{A}\left[D_x^{\beta}\left(\frac{x}{3}Q\right) - \left(\frac{4}{x}Q^2\right)_x + \left(Q^2\right)_{xx}\right]\right)\right]. \tag{25}$$

Using the Aboodh transform iterative method procedure, we obtain the following equation:

$$
\begin{aligned}
Q_0(x,t) &= \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\sum_{k=0}^{m-1}\frac{Q^{(k)}(x,0)}{v^{2-\alpha+k}}\right)\right]\\
&= \mathscr{A}^{-1}\left[\frac{Q(x,0)}{v^{2}}\right]\\
&= x^{2},\\
Q_1(x,t) &= \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\mathscr{A}\left[D_x^{\beta}\left(\frac{x}{3}Q_0\right)-\left(\frac{4}{x}Q_0^{2}\right)_x+\left(Q_0^{2}\right)_{xx}\right]\right)\right]\\
&= \mathscr{A}^{-1}\left[\frac{2x^{3-\beta}}{\Gamma(4-\beta)v^{2+\alpha}}\right]\\
&= \frac{2x^{3-\beta}t^{\alpha}}{\Gamma(4-\beta)\Gamma(\alpha+1)},\\
Q_2(x,t) &= \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\mathscr{A}\left[D_x^{\beta}\left(\frac{x}{3}(Q_0+Q_1)\right)-\left(\frac{4}{x}(Q_0+Q_1)^{2}\right)_x+(Q_0+Q_1)^{2}_{xx}\right]\right)\right]\\
&\quad -\mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\mathscr{A}\left[D_x^{\beta}\left(\frac{x}{3}Q_0\right)-\left(\frac{4}{x}Q_0^{2}\right)_x+\left(Q_0^{2}\right)_{xx}\right]\right)\right]\\
&= \mathscr{A}^{-1}\left[\frac{2x^{3-\beta}}{\Gamma(4-\beta)v^{2+\alpha}}+\frac{2\Gamma(3-\beta)x^{4-2\beta}}{3\Gamma(4-\beta)\Gamma(3-2\beta)v^{2+2\alpha}}-\frac{(4-\beta)16x^{3-\beta}}{\Gamma(4-\beta)v^{2+2\alpha}}\right]\\
&\quad -\mathscr{A}^{-1}\left[\frac{(5-2\beta)16x^{4-2\beta}}{[\Gamma(4-\beta)\Gamma(\alpha+1)]^{2}v^{2+3\alpha}}+\frac{4(5-\beta)(4-\beta)x^{3-\beta}}{\Gamma(4-\beta)v^{2+2\alpha}}\right]\\
&\quad +\mathscr{A}^{-1}\left[\frac{4(6-2\beta)(5-2\beta)\Gamma(2\alpha+1)x^{4-2\beta}}{[\Gamma(4-\beta)\Gamma(\alpha+1)]^{2}v^{2+3\alpha}}-\frac{2x^{3-\beta}}{\Gamma(4-\beta)v^{2+\alpha}}\right]\\
&= \frac{2\Gamma(5-\beta)x^{4-2\beta}t^{2\alpha}}{3\Gamma(4-\beta)\Gamma(5-2\beta)\Gamma(2\alpha+1)}+\frac{(4-4\beta)(4-\beta)x^{3-\beta}t^{2\alpha}}{\Gamma(4-\beta)\Gamma(2\alpha+1)}+\frac{(8-8\beta)(5-2\beta)\Gamma(2\alpha+1)x^{4-2\beta}t^{3\alpha}}{[\Gamma(4-\beta)\Gamma(\alpha+1)]^{2}\Gamma(3\alpha+1)},\\
&\;\;\vdots
\end{aligned}
\tag{26}
$$

and so on. The series solution is obtained as follows:

$$
\begin{aligned}
Q(x,t) &= Q_0+Q_1+Q_2+\cdots\\
&= x^{2}+\frac{2x^{3-\beta}t^{\alpha}}{\Gamma(4-\beta)\Gamma(\alpha+1)}\\
&\quad+\frac{2\Gamma(5-\beta)x^{4-2\beta}t^{2\alpha}}{3\Gamma(4-\beta)\Gamma(5-2\beta)\Gamma(2\alpha+1)}\\
&\quad+\frac{(4-4\beta)(4-\beta)x^{3-\beta}t^{2\alpha}}{\Gamma(4-\beta)\Gamma(2\alpha+1)}\\
&\quad+\frac{(8-8\beta)(5-2\beta)\Gamma(2\alpha+1)x^{4-2\beta}t^{3\alpha}}{[\Gamma(4-\beta)\Gamma(\alpha+1)]^{2}\Gamma(3\alpha+1)}+\cdots.
\end{aligned}
\tag{27}
$$

Setting $\beta=1$ in equation (27), we obtain the following equation:

$$
\begin{aligned}
Q(x,t) &= x^{2}\left(1+\frac{t^{\alpha}}{\Gamma(\alpha+1)}+\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}+\cdots\right)\\
&= x^{2}\sum_{i=0}^{\infty}\frac{t^{i\alpha}}{\Gamma(i\alpha+1)}.
\end{aligned}
\tag{28}
$$

The solution obtained in equation (28) converges to the exact solution in a closed form as $i\longrightarrow\infty$,

$$
\begin{aligned}
Q(x,t) &= x^{2}\lim_{i\longrightarrow\infty}\sum_{i=0}^{\infty}\frac{t^{i\alpha}}{\Gamma(i\alpha+1)}\\
&= x^{2}E_{\alpha}(t^{\alpha}).
\end{aligned}
\tag{29}
$$

So, by setting $\alpha = 1$, we obtain the following equation:

$$Q(x,t) = x^2 e^t. \tag{30}$$

Which is the same solution obtained in [23]. Figure 2 represents the solution plots of equation (22) when $\alpha = \beta = .02, .04, .06, .08, .2, .6, .8$ at $x = 1$ and $t = 1$ respectively. While the remaining are the surface plots.

Example 3. Consider the one-dimensional space-time diffusion equation [24].

$$D_t^{\alpha} = D_x^{2\beta} Q + D_x^{\beta}(xQ), \quad 0 < \alpha, \beta \le 1. \tag{31}$$

With the initial condition,

$$Q(x,0) = 1. \tag{32}$$

Applying the Aboodh transform on equation (31), we obtain the following equation:

$$\mathscr{A}[Q(x,t)] = \frac{1}{v^{\alpha}}\left(\sum_{k=0}^{m-1}\frac{Q^{(k)}(x,0)}{v^{2-\alpha+k}} + \mathscr{A}\left[D_x^{2\beta}Q + D_x^{\beta}(xQ)\right]\right). \tag{33}$$

Taking the Aboodh transform inverse of equation (33), we obtain the following equation:

$$Q(x,t) = \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\sum_{k=0}^{m-1}\frac{Q^{(k)}(x,0)}{v^{2-\alpha+k}} + \mathscr{A}\left[D_x^{2\beta}Q + D_x^{\beta}(xQ)\right]\right)\right]. \tag{34}$$

Using the Aboodh transform iterative procedure, we obtain the following equation:

$$\begin{aligned}
Q_0(x,t) &= \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\sum_{k=0}^{m-1}\frac{Q^{(k)}(x,0)}{v^{2-\alpha+k}}\right)\right]\\
&= \mathscr{A}^{-1}\left[\frac{Q(x,0)}{v^2}\right]\\
&= 1,\\
Q_1(x,t) &= \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\mathscr{A}\left[D_x^{2\beta}Q_0 + D_x^{\beta}(xQ_0)\right]\right)\right]\\
&= \mathscr{A}^{-1}\left[\frac{x^{1-\beta}}{v^{2+\alpha}\Gamma(2-\beta)}\right]\\
&= \frac{x^{1-\beta}t^{\alpha}}{\Gamma(\alpha+1)\Gamma(2-\beta)},\\
Q_2(x,t) &= \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\mathscr{A}\left[D_x^{2\beta}(Q_0+Q_1) + D_x^{\beta}(x(Q_0+Q_1))\right]\right)\right] - \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\mathscr{A}\left[D_x^{2\beta}Q_0 + D_x^{\beta}(xQ_0)\right]\right)\right]\\
&= \mathscr{A}^{-1}\left[\frac{x^{1-\beta}}{\Gamma(2-\beta)v^{2+\alpha}} + \frac{\Gamma(3-\beta)x^{2-2\beta}}{\Gamma(3-2\beta)\Gamma(2-\beta)v^{2+2\alpha}}\right] - \mathscr{A}^{-1}\left[\frac{x^{1-\beta}}{\Gamma(2-\beta)v^{2+\alpha}}\right]\\
&= \frac{(2-\beta)x^{2-2\beta}t^{2\alpha}}{\Gamma(3-2\beta)\Gamma(2\alpha+1)}.
\end{aligned} \tag{35}$$

The series solution is obtained as follows:

$$\begin{aligned}
Q(x,t) &= Q_0 + Q_1 + Q_2 + \cdots\\
&= 1 + \frac{x^{1-\beta}t^{\alpha}}{\Gamma(2-\beta)\Gamma(\alpha+1)}\\
&\quad + \frac{(2-\beta)x^{2-2\beta}t^{\alpha}}{\Gamma(3-2\beta)\Gamma(2\alpha+1)} + \cdots.
\end{aligned} \tag{36}$$

Setting $\beta = 1$ in equation (36), we obtain the following equation:

$$\begin{aligned}
Q(x,t) &= 1 + \frac{t^{\alpha}}{\Gamma(\alpha+1)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \cdots\\
&= \sum_{i=0}^{\infty}\frac{t^{i\alpha}}{\Gamma(i\alpha+1)}.
\end{aligned} \tag{37}$$

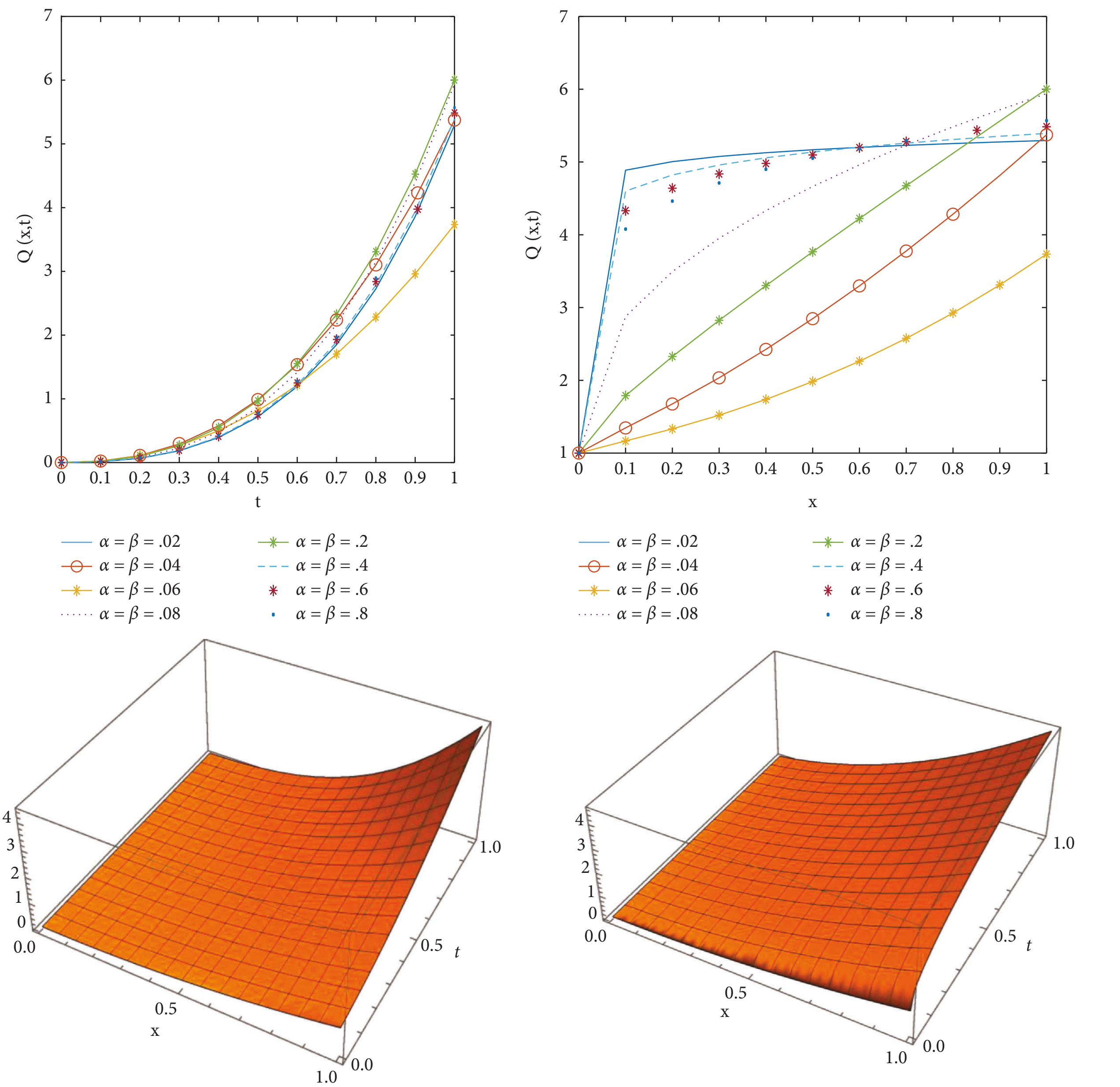

FIGURE 2: Comparison of the solution at various values of alpha and beta.

The solution obtained in equation (37) converges to the exact solution in a closed form as $i \longrightarrow \infty$,

$$
\begin{aligned}
Q(x,t) &= \lim_{i\longrightarrow\infty} \sum_{i=0}^{\infty} \frac{t^{i\alpha}}{\Gamma(i\alpha+1)} \\
&= E_{\alpha}(t^{\alpha}).
\end{aligned} \tag{38}
$$

So, by setting $\alpha = (1/2)$, we obtain the following equation:

$$
Q(x,t) = E_{(1/2)}\left(t^{(1/2)}\right). \tag{39}
$$

Which is the solution obtained in [24] using the natural transform method. Figure 3 represents the solution plots of equation (31) when $\alpha = \beta = .02, .04, .06, .08, .2, .4, .6, .8$ at $x = 1$ and $t = 1$, respectively. While the remaining are the surface plots.

Example 4. Consider the space-time fractional Airy's partial differential equations [22].

$$
D_t^{\alpha} Q(x,t) = D_x^{3\beta} Q, \quad 0 < \alpha, \beta \leq 1. \tag{40}
$$

With the initial condition,

$$
Q(x,0) = \frac{1}{6} x^3. \tag{41}
$$

Applying the Aboodh transform on equation (40), we get the following equation:

$$\mathscr{A}[Q(x,t)] = \frac{1}{v^{\alpha}}\left(\sum_{k=0}^{m-1}\frac{Q^{(k)}(x,0)}{v^{2-\alpha+k}} + \mathscr{A}\left[D_x^{3\beta}Q\right]\right), \tag{42}$$

taking the inverse Aboodh transform of equation (42), we get the following equation:

$$Q(x,t) = \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\sum_{k=0}^{m-1}\frac{Q^{(k)}(x,0)}{v^{2-\alpha+k}} + \mathscr{A}\left[D_x^{3\beta}Q\right]\right)\right]. \tag{43}$$

Using the Aboodh transform iterative method procedure, we obtain the following equation:

$$\begin{aligned}
Q_0(x,t) &= \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\sum_{k=0}^{m-1}\frac{Q^{(k)}(x,0)}{v^{2-\alpha+k}}\right)\right]\\
&= \mathscr{A}^{-1}\left[\frac{Q(x,0)}{v^2}\right]\\
&= \frac{1}{6}x^3,\\
Q_1(x,t) &= \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\mathscr{A}\left[D_x^{3\beta}Q_0\right]\right)\right]\\
&= \mathscr{A}^{-1}\left[\frac{x^{3-3\beta}}{\Gamma(4-3\beta)v^{2+\alpha}}\right]\\
&= \frac{x^{3-3\beta}t^{\alpha}}{\Gamma(4-3\beta)\Gamma(\alpha+1)},\\
Q_2(x,t) &= \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\mathscr{A}\left[D_x^{3\beta}(Q_1+Q_0)\right]\right)\right] - \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\mathscr{A}\left[D_x^{3\beta}Q_0\right]\right)\right]\\
&= \mathscr{A}^{-1}\left[\frac{\Gamma(4)x^{3-3\beta}}{6\Gamma(4-3\beta)v^{2+\alpha}}\right] - \mathscr{A}^{-1}\left[\frac{x^{3-3\beta}}{\Gamma(4-3\beta)v^{2+\alpha}}\right]\\
&= 0,\\
&\vdots
\end{aligned} \tag{44}$$

and so on. The series solution is obtained as follows:

$$\begin{aligned}
Q(x,t) &= Q_0 + Q_1 + Q_2 + \cdots\\
&= \frac{1}{6}x^3 + \frac{x^{3-3\beta}t^{\alpha}}{\Gamma(4-3\beta)\Gamma(\alpha+1)} + 0 + \cdots,
\end{aligned} \tag{45}$$

for all $i > 1$, $Q_i(x,t) = 0$. Setting $\beta = 1$ in equation (45), we obtain the following equation:

$$\begin{aligned}
Q(x,t) &= \frac{1}{6}x^3 + \frac{t^{\alpha}}{\Gamma(\alpha+1)} + 0 + 0 + \cdots\\
&= \frac{1}{6}x^3 + \frac{t^{\alpha}}{\Gamma(\alpha+1)}.
\end{aligned} \tag{46}$$

We obtain the exact solution when $\alpha = 1$,

$$Q(x,t) = \frac{1}{6}x^3 + t, \tag{47}$$

which is the solution obtained in [22]. Figure 4 represents the solution plots of equation (40) when $\alpha = \beta = .02, .04, .06, .08, .2, .4, .6, .8$ at $x = 1$ and $t = 1$ respectively. While Figure 5 is the surface plots.

Example 5. Consider the nonlinear space-time fractional Fokker–Planck equation which consists of a single term time-fractional order and three terms of space fractional order [23].

$$D_t^{\alpha}Q(x,t) = D_x^{\beta}\left(\frac{xQ}{3}\right) - D_x^{\beta}\left(\frac{4Q^2}{x}\right) + D_x^{2\beta}\left(Q^2\right),\quad 0<\alpha,\beta\leq 1. \tag{48}$$

Subject to the initial condition,

$$Q(x,0) = x^2. \tag{49}$$

Applying the Aboodh transform on equation (48), we obtain the following equation:

$$\mathscr{A}[Q(x,t)] = \frac{1}{v^{\alpha}}\left(\sum_{k=0}^{m-1}\frac{Q^{(k)}(x,0)}{v^{2-\alpha+k}} + \mathscr{A}\left[D_x^{\beta}\left(\frac{xQ}{3}\right) - D_x^{\beta}\left(\frac{4Q^2}{x}\right) + D_x^{2\beta}\left(Q^2\right)\right]\right), \tag{50}$$

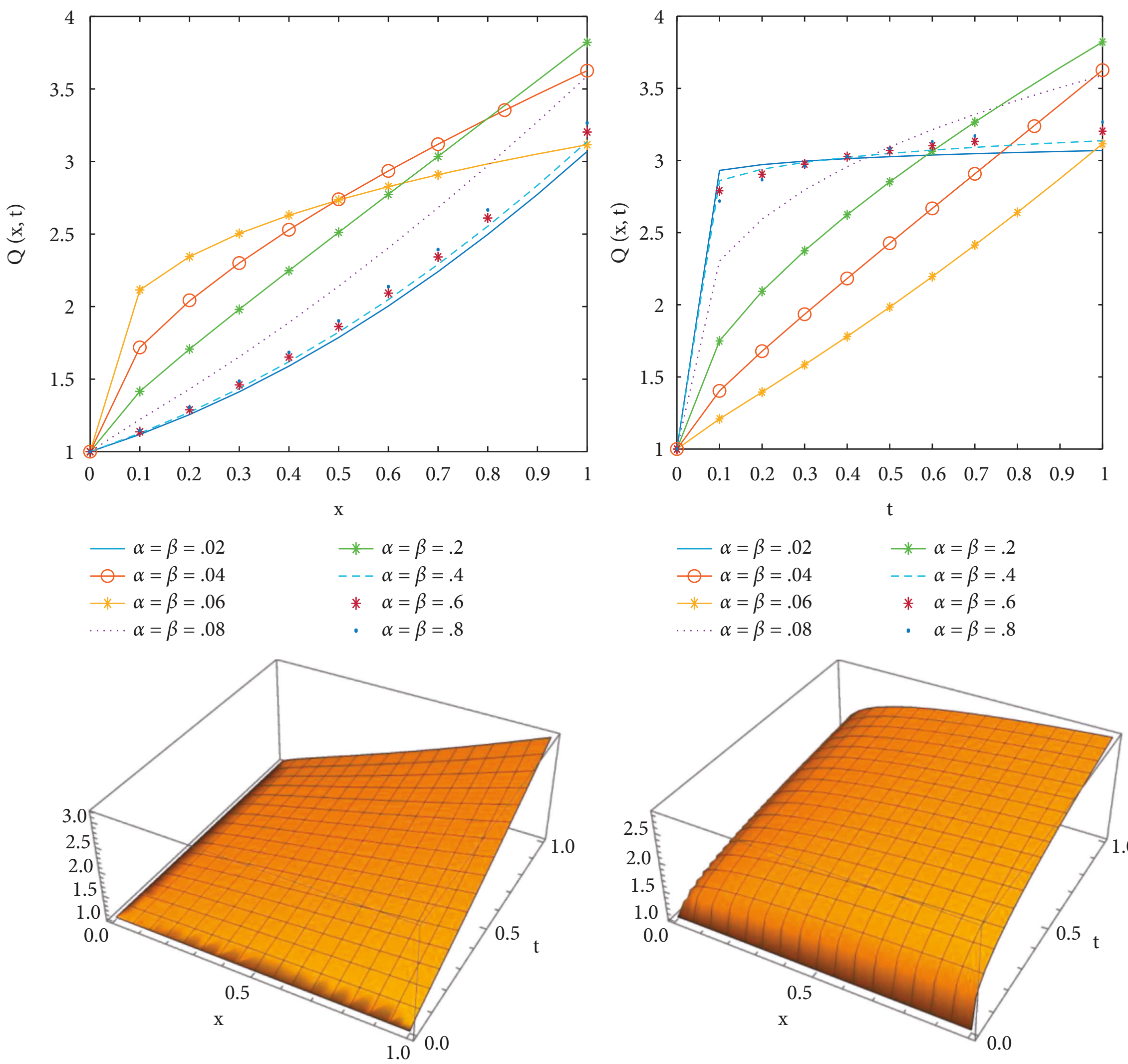

Figure 3: Comparison of the solution at various values of alpha and beta.

taking the inverse Aboodh transform, we get the following equation:

$$Q(x,t) = \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\sum_{k=0}^{m-1}\frac{Q^{(k)}(x,0)}{v^{2-\alpha+k}} + \mathscr{A}\left[D_x^{\beta}\left(\frac{xQ}{3}\right) - D_x^{\beta}\left(\frac{4Q^2}{x}\right) + D_x^{2\beta}\left(Q^2\right)\right]\right)\right]. \tag{51}$$

Using Aboodh transform iterative procedure, we get the following equation:

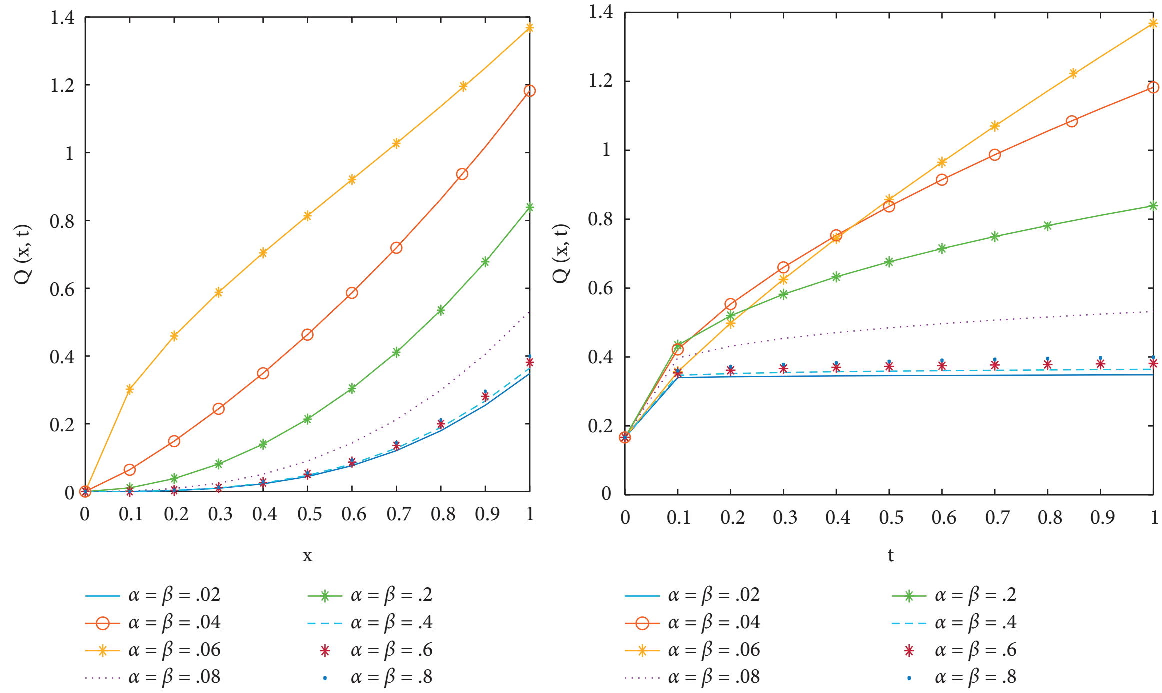

Figure 4: Comparison of the solution at various values of alpha and beta.

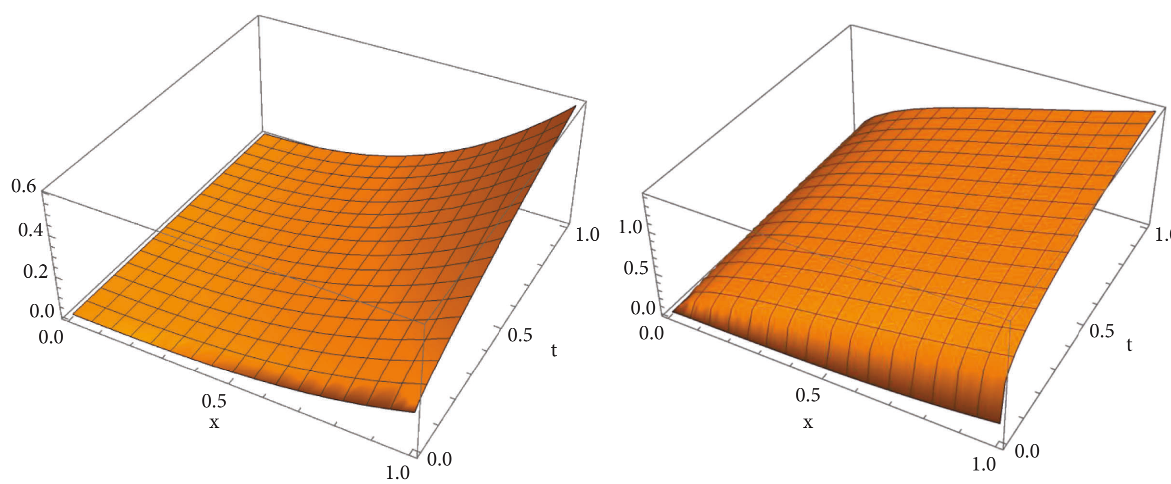

Figure 5: Comparison of the solution at various values of alpha and beta. (a) Early strength. (b) Early strength loss rate.

$$
\begin{aligned}
Q_0(x,t) &= \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\sum_{k=0}^{m-1}\frac{Q^{(k)}(x,0)}{v^{2-\alpha+k}}\right)\right]\\
&= \mathscr{A}^{-1}\left[\frac{Q(x,0)}{v^{2}}\right]\\
&= x^2,\\
Q_1(x,t) &= \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\mathscr{A}\left[D_x^{\beta}\left(\frac{xQ_0}{3}\right)-D_x^{\beta}\left(\frac{4Q_0^2}{x}\right)+D_x^{2\beta}\left(Q_0^2\right)\right]\right)\right]\\
&= \mathscr{A}^{-1}\left[\frac{24x^{4-2\beta}}{\Gamma(5-2\beta)v^{2+\alpha}}-\frac{22x^{3-\beta}}{\Gamma(4-\beta)v^{2+\alpha}}\right]\\
&= \frac{24x^{4-2\beta}t^{\alpha}}{\Gamma(5-2\beta)\Gamma(\alpha+1)}-\frac{22x^{3-\beta}t^{\alpha}}{\Gamma(4-\beta)\Gamma(\alpha+1)},\\
Q_2(x,t) &= \mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\mathscr{A}\left[D_x^{\beta}\left(\frac{x(Q_0+Q_1)}{3}\right)-D_x^{\beta}\left(\frac{4(Q_0+Q_1)^2}{x}\right)+D_x^{2\beta}(Q_0+Q_1)^2\right]\right)\right]\\
&\quad-\mathscr{A}^{-1}\left[\frac{1}{v^{\alpha}}\left(\mathscr{A}\left[D_x^{\beta}\left(\frac{xQ_0}{3}\right)-D_x^{\beta}\left(\frac{4Q_0^2}{x}\right)+D_x^{2\beta}\left(Q_0^2\right)\right]\right)\right]\\
&= \mathscr{A}^{-1}\left[\frac{8\Gamma(6-2\beta)x^{5-3\beta}}{\Gamma(5-2\beta)\Gamma(6-3\beta)v^{2+2\alpha}}-\frac{22\Gamma(5-\beta)x^{4-2\beta}}{3\Gamma(4-\beta)\Gamma(5-2\beta)v^{2+2\alpha}}+\frac{\Gamma(4)x^{3-\beta}}{3\Gamma(4-\beta)v^{2+\alpha}}\right]\\
&\quad+\mathscr{A}^{-1}\left[\frac{-1152\Gamma(8-4\beta)\Gamma(2\alpha+1)x^{7-5\beta}}{[\Gamma(5-2\beta)\Gamma(\alpha+1)]^2\Gamma(8-5\beta)v^{2+3\alpha}}-\frac{192\Gamma(6-2\beta)x^{5-3\beta}}{\Gamma(5-2\beta)\Gamma(6-3\beta)v^{2+2\alpha}}\right]\\
&\quad+\mathscr{A}^{-1}\left[\frac{-1936\Gamma(6-2\beta)\Gamma(2\alpha+1)x^{5-3\beta}}{[\Gamma(4-\beta)\Gamma(\alpha+1)]^2\Gamma(6-3\beta)v^{2+3\alpha}}+\frac{4,224\Gamma(7-3\beta)\Gamma(2\alpha+1)x^{6-4\beta}}{\Gamma(5-2\beta)\Gamma(\alpha+1)^2\Gamma(4-\beta)\Gamma(7-4\beta)v^{2+3\alpha}}\right]\\
&\quad+\mathscr{A}^{-1}\left[\frac{176\Gamma(5-\beta)x^{4-2\beta}}{\Gamma(4-\beta)\Gamma(5-2\beta)v^{2+2\alpha}}-\frac{4\Gamma(4)x^{3-\beta}}{\Gamma(4-\beta)v^{2+\alpha}}+\frac{576\Gamma(9-4\beta)\Gamma(2\alpha+1)x^{8-6\beta}}{[\Gamma(5-2\beta)\Gamma(\alpha+1)]^2\Gamma(9-6\beta)v^{2+3\alpha}}\right]\\
&\quad+\mathscr{A}^{-1}\left[\frac{-1056\Gamma(8-3\beta)\Gamma(2\alpha+1)x^{7-5\beta}}{[\Gamma(5-2\beta)\Gamma(\alpha+1)]^2\Gamma(4-\beta)\Gamma(8-5\beta)v^{2+3\alpha}}+\frac{48\Gamma(7-2\beta)x^{6-4\beta}}{\Gamma(5-2\beta)\Gamma(7-4\beta)v^{2+2\alpha}}\right]\\
&\quad+\mathscr{A}^{-1}\left[\frac{484\Gamma(7-2\beta)\Gamma(2\alpha+1)x^{6-4\beta}}{[\Gamma(4-\beta)\Gamma(\alpha+1)]^2\Gamma(7-4\beta)v^{2+3\alpha}}+\frac{44\Gamma(6-\beta)x^{5-3\beta}}{\Gamma(4-\beta)\Gamma(6-3\beta)v^{2+2\alpha}}+\frac{24x^{4-2\beta}}{\Gamma(5-2\beta)v^{2+\alpha}}\right]\\
&\quad-\mathscr{A}^{-1}\left[\frac{24x^{4-2\beta}}{\Gamma(5-2\beta)v^{2+\alpha}}-\frac{22x^{3-\beta}}{\Gamma(4-\beta)v^{2+\alpha}}\right]\\
&= \frac{8\Gamma(6-2\beta)x^{5-3\beta}t^{2\alpha}}{\Gamma(5-2\beta)\Gamma(6-3\beta)\Gamma(2\alpha+1)}-\frac{1152\Gamma(8-4\beta)\Gamma(2\alpha+1)x^{7-5\beta}t^{3\alpha}}{[\Gamma(5-2\beta)\Gamma(\alpha+1)]^2\Gamma(8-5\beta)\Gamma(3\alpha+1)}\\
&\quad-\frac{22\Gamma(5-\beta)x^{4-2\beta}t^{2\alpha}}{3\Gamma(4-\beta)\Gamma(5-2\beta)\Gamma(2\alpha+1)}+\frac{4224\Gamma(7-3\beta)\Gamma(2\alpha+1)x^{6-4\beta}t^{3\alpha}}{\Gamma(5-2\beta)\Gamma(\alpha+1)^2\Gamma(4-\beta)\Gamma(7-4\beta)\Gamma(3\alpha+1)}\\
&\quad-\frac{1436\Gamma(6-2\beta)\Gamma(2\alpha+1)x^{5-3\beta}t^{3\alpha}}{[\Gamma(4-\beta)\Gamma(\alpha+1)]^2\Gamma(6-3\beta)\Gamma(3\alpha+1)}-\frac{192\Gamma(6-2\beta)x^{5-3\beta}t^{2\alpha}}{\Gamma(5-2\beta)\Gamma(6-3\beta)\Gamma(2\alpha+1)}\\
&\quad+\frac{176\Gamma(5-\beta)x^{4-2\beta}x^{4-2\beta}t^{2\alpha}}{\Gamma(4-\beta)\Gamma(5-2\beta)\Gamma(2\alpha+1)}+\frac{576\Gamma(9-4\beta)\Gamma(2\alpha+1)x^{8-6\beta}t^{3\alpha}}{[\Gamma(5-2\beta)\Gamma(\alpha+1)]^2\Gamma(9-6\beta)\Gamma(3\alpha+1)}\\
&\quad-\frac{1056\Gamma(8-3\beta)\Gamma(2\alpha+1)x^{7-5\beta}t^{3\alpha}}{[\Gamma(5-2\beta)\Gamma(\alpha+1)]^2\Gamma(4-\beta)\Gamma(8-5\beta)\Gamma(3\alpha+1)}+\frac{48\Gamma(7-2\beta)x^{6-4\beta}t^{2\alpha}}{\Gamma(5-2\beta)\Gamma(7-4\beta)\Gamma(2\alpha+1)}\\
&\quad+\frac{484\Gamma(7-2\beta)\Gamma(2\alpha+1)x^{6-4\beta}t^{3\alpha}}{[\Gamma(4-\beta)\Gamma(\alpha+1)]^2\Gamma(7-4\beta)\Gamma(3\alpha+1)}+\frac{44\Gamma(6-\beta)x^{5-3\beta}t^{2\alpha}}{\Gamma(4-\beta)\Gamma(6-3\beta)\Gamma(2\alpha+1)},\\
&\vdots
\end{aligned}
\tag{52}
$$

and so on. The series solution is obtained as follows:

$$Q(x,t) = Q_0 + Q_1 + Q_2 + \cdots. \tag{53}$$

Setting $\alpha = \beta = 1$ we get the following equation:

$$\begin{aligned} Q(x,t) &= x^2 + x^2 t + \frac{x^2 t^2}{2} + \cdots \\ &= x^2\left(1 + t + \frac{t^2}{2} + \cdots\right). \end{aligned} \tag{54}$$

Hence,

$$\begin{aligned} Q(x,t) &= x^2 \sum_{i=0}^{\infty} \frac{t^i}{i!} \\ &= x^2 e^t, \end{aligned} \tag{55}$$

which agrees with the exact solution obtained in [23], also it is similar to the solution obtained in Example 2. The reason being that in Example 2, only one space fractional derivative term was considered while here three terms of space fractional derivative was considered.

5. Conclusion and Future Work

We proposed the Aboodh transform iterative method for the solution of space-time fractional differential equation with fractional order derivative in more than one term. The proposed method is efficient and effective, the method combined the Aboodh transform which is a modification of the Laplace transform with the new iterative method. To the best of our knowledge, no attempt has been recorded regarding the approximate analytical solution of space-time fractional differential equations using the Aboodh transform iterative method which is the novelty of this study.

The new iterative method decomposes the linear and nonlinear term. Some examples were considered, if $\alpha = \beta = 1$ the fractional differential equations becomes the classical differential equations. Aboodh transforms iterative method yields closed form solutions in this study and exact solutions in some cases. Also, the effect of the fractional orders α and β are displayed in Figures 1 to 5, this is left for the readers in different fields of study to transcribe for different applications.

In the future, we hope to extend the Aboodh transform iterative method to solve boundary value problems with consideration for other fractional order differential equations which till date have not been solved either analytically or numerically.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

Authors' Contributions

N.I.M. did the conceptualization of the idea; G.O.O.did the formal analysis; M.A.A did the formal analysis.

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