

## Research Article

# First General Zagreb Co-Index of Graphs under Operations

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Topological indices are graph-theoretic parameters which are widely used in the subject of chemistry and computer science to predict the various chemical and structural properties of the graphs respectively. Let  $G$  be a graph; then, by performing subdivision-related operations  $S$ ,  $Q$ ,  $R$ , and  $T$  on  $G$ , the four new graphs  $S(G)$  (subdivision graph),  $Q(G)$  (edge-semi-total),  $R(G)$  (vertex-semi-total), and  $T(G)$  (total graph) are obtained, respectively. Furthermore, for two simple connected graphs  $G$  and  $H$ , we define  $F$ -sum graphs (denoted by  $G_{+F}H$ ) which are obtained by Cartesian product of  $F(G)$  and  $H$ , where  $F \in \{S, R, Q, T\}$ . In this study, we determine first general Zagreb co-index of graphs under operations in the form of Zagreb indices and co-indices of their basic graphs.

## 1. Introduction

Graph theory has given different valuable tools in which likely the best tool is known as topological index (TI) that is used to predict structural and chemical properties of graphs such as connectivity, solubility, freezing point, boiling point, critical temperature, and molecular mass, see [1]. The medical behaviors and drugs' particles of the different compounds are discussed with the help of various TIs in the pharmaceutical industries, see [2]. In addition, for the study of molecules, the quantitative structures' activity relationships (QSAR) and quantitative structures' property relationships (QSPR) are very useful techniques which are mostly performed with the help of TIs [3].

There are three basic types of TIs depending on the parameters of degree, distance, and polynomial. According to recent review [4], the degree-based TIs are mostly studied. First of all, Wiener calculated the boiling point of paraffin with the help of a degree-based TI, see [5]. Gutman and

Trinajsti introduced Zagreb indices and used them to compute the different structure-based characteristics of the molecular graphs [6].

Later on, Shenggui and Huiling characterized the graphs for the first general Zagreb index [7]. Bedratyuk and Savenko calculated the ordinary generating function and linear recurrence relation for the sequence of the general first Zagreb index [8].

Recently, Ashrafi et al. defined Zagreb co-index and computed it for graphs which are formed using various operations, see [9, 10]. Kinkar et al. computed the first Zagreb co-indices of trees under different conditions, see [11]. Mansour and Song established relationship between Zagreb indices and co-indices of graphs [12]. Huaa and Zhang computed sharp bounds on the first Zagreb co-index in terms of Wiener, eccentric distance sum, eccentric connectivity, and degree distance indices [13]. Gutman et al. calculated relations between the Zagreb indices and co-indices of a graph  $G$  and of its complement  $\bar{G}$  [14, 15].

In graph theory, the operations (union, intersection, complement, product, and subdivision) play an important role to develop new structure of graphs. Yan et al. computed the Wiener index for new graphs using five different operations  $L$ ,  $S$ ,  $Q$ ,  $R$ , and  $T$  on a graph  $G$  such as line graph  $L(G)$ , subdivided graph  $S(G)$ , line superposition graph  $Q(G)$ , triangle parallel graph  $R(G)$ , and total graph  $T(G)$ , respectively, see [16]. After that, Eliasi and Taeri computed Wiener indices of newly defined  $F$ -sum graphs represented as  $(G_{1+F}G_2)$ , where  $F \in \{S, R, Q, T\}$  [17]. Furthermore, Deng et al. calculated Zagreb indices [18], Ibraheem et al. [19] forgot co-index, Liu et al. obtained first general Zagreb (FGZ) index [20], and Javaid et al. [21] computed the bounds of first Zagreb co-index; furthermore, they also studied the connection-based Zagreb index and co-index [22] of these graphs.

In this study, we computed FGZ co-index of graphs under operations such as  $\overline{M}_1(G_{+S}H)$ ,  $\overline{M}_1(G_{+R}H)$ ,  $\overline{M}_1(G_{+Q}H)$ , and  $\overline{M}_1(G_{+T}H)$ . The rest of the study is settled as follows. Section 2 contains preliminaries. In Section 3, the main results of our work are discussed, and Section 4 has the conclusion of work.

### 2. Preliminaries

Let  $G$  be a simple and connected graph with vertex and edge set denoted by  $V(G)$  and  $E(G)$ , respectively. The degree of vertex any vertex  $v$  in  $G$  is the number of edges incident on it and denoted by  $d(v)$ . Let  $G$  be a graph; then, its complement is defined as  $|V(\overline{G})| = |V(G)|$  and  $uv \notin \overline{G}$  iff  $uv \in G$  denoted as  $\overline{G}$ . Gutman and Trinajsti introduced the first and second Zagreb indices as [6]

$$\begin{aligned}
 M_1(G) &= \sum_{p_1 p_2 \in E(G)} [d_G(p_1) + d_G(p_2)], \\
 M_2(G) &= \sum_{p_1 p_2 \in E(G)} [d_G(p_1)d_G(p_2)].
 \end{aligned}
 \tag{1}$$

Ashrafi et al. defined first Zagreb co-index  $\overline{M}_1(G)$  as follows, see [10]:

$$\overline{M}_1(G) = \sum_{y_1 y_2 \notin E(G)} [d_G(y_1) + d_G(y_2)].
 \tag{2}$$

Let  $G$  be a graph; then,  $S(G)$  is obtained by adding one vertex in every edge of  $G$ .

- (i)  $R(G)$  is obtained from  $S(G)$  by inserting an edge between the vertices that are adjacent in  $G$
- (ii)  $Q(G)$  is obtained from  $S(G)$  by inserting an edge between new vertices that adjacent edges of  $G$
- (iii) Apply both  $R(G)$  and  $Q(G)$  on  $S(G)$ ; then,  $T(G)$  is obtained

Suppose two connected graphs  $G$  and  $H$ ; then, their  $F$ -sum graph is represented by  $G_{+F}H$  having vertex set  $|V(G_{+F}H)| = V(G) \cup E(G) \times V(H)$  and  $(y_1, y_2) (z_1, z_2) \in E(G_{+F}H)$  iff  $y_1 = z_1 \in V(G)$  and  $y_2 \sim z_2 \in H$   $y_2 = z_2 \in V(H)$  and  $y_1 \sim z_1 \in F(G)$ , where  $F \in \{S, R, Q, T\}$ .

For details, see Figure 1 and 2.

### 3. Main Results

This section contains results about FGZ co-index of graphs under operations.

**Theorem 1.** *Let  $G_{+S}H$  be  $S$ -sum graph; then, its first general Zagreb co-index is given as*

$$\begin{aligned}
 \overline{M}_1^\theta(G_{+S}H) &= 2^\theta(n_2^2 e_1^2 - n_2 e_1) + +2^\theta e_1 [2n_1(e_2 + \overline{e}_2) + n_2(n_1 - 2)] \\
 &+ \sum_{i=0}^{\theta} \binom{\theta}{i} \left[ M_1^{\theta-i}(G) \overline{M}_1^{i+1}(H) + M_1^{\gamma-i}(G) M_1^i(H) + M_1^{i+1}(H) (M_1^{\theta-i}(G) + \overline{M}_1^{\gamma-i}(G)) + \overline{M}_1^{i+1}(H) \right. \\
 &\left. (M_1^{\gamma-i}(G) + \overline{M}_1^{\gamma-i}(G)) + M_1^i(H) (\overline{M}_1^{\theta-i}(G) + M_1^{\gamma-i}(G)) + M_1^{\gamma-i}(G) \overline{M}_1^i(H) + \overline{M}_1^{\gamma-i}(G) (M_1^i(H) + \overline{M}_1^i(H)) \right],
 \end{aligned}
 \tag{3}$$

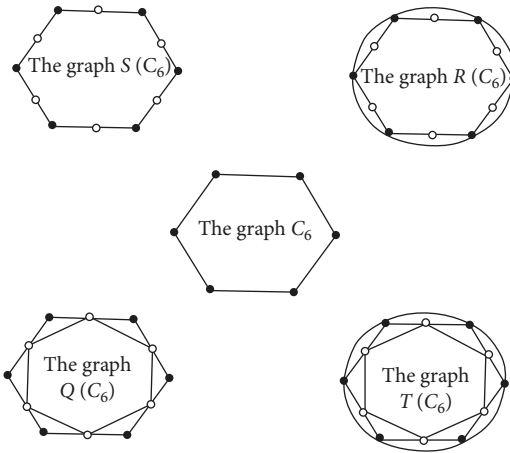


FIGURE 1: Subdivision of  $C_5$ .

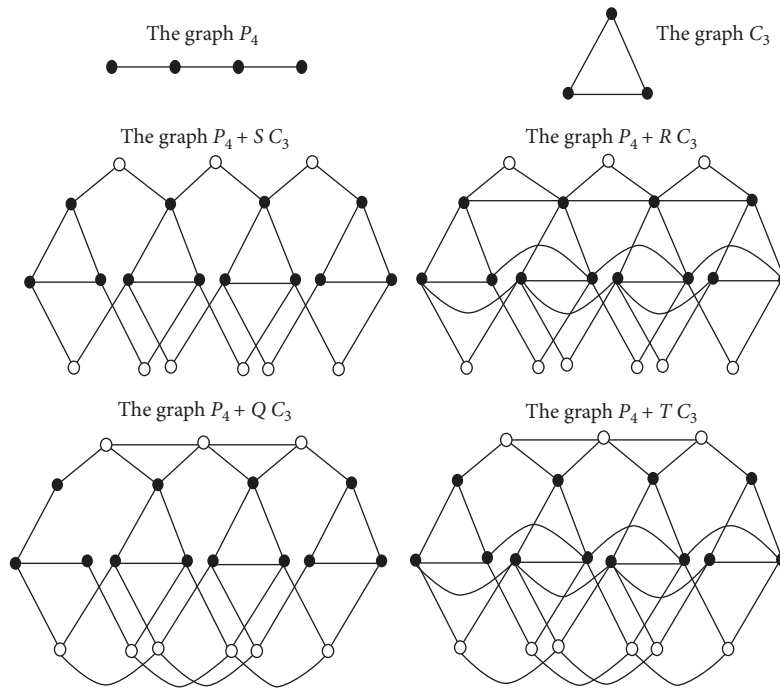


FIGURE 2:  $F$ -sum graphs for  $P_5$  and  $P_6$ .

where  $\theta = \gamma - 1$ .

*Proof.* Using equation (2), we have

$$\begin{aligned} \overline{M}(G_{+S}H) &= \sum_{(y_1, y_2) \mid (z_1, z_2) \notin EG_{+S}H} [d(y_1, z_1) + d(y_2, z_2)], \\ \overline{M}_1^\gamma(G_{+S}H) &= \sum_{z_1, z_2 \in H} \left[ \sum_{y_1, y_2 \in V(S(G)-G)} [d_{G_{+S}H}^\theta(y_1, z_1) + d_{G_{+S}H}^\theta(y_2, z_2)] + \sum_{y_1, y_2 \in V_G} [d_{G_{+S}H}^\theta(y_1, z_1) + d_{G_{+S}H}^\theta(y_2, z_2)] \right] \\ &\quad + \sum_{y_1, y_2 \in V(S(G)) \mid y_1 \in V(G), y_2 \in V(S(G)-G)} [d_{G_{+S}H}^\theta(y_1, z_1) + d_{G_{+S}H}^\theta(y_2, z_2)] \\ &= \sum A + \sum B + \sum C, \end{aligned}$$

$$\begin{aligned}
\sum A &= \sum_{y_1, y_2 \in V(S(G)-G)} \sum_{z_1, z_2 \in V_{G_2}} [d_{G+S}^\theta(y_1, z_1) + d_{G+S}^\theta(y_2, z_2)], \\
\sum A &= 2^\theta(n_2^2 e_1^2 - n_2 e_1), \\
\sum B &= \sum B_1 + \sum B_2 + \sum B_3 + \sum B_4 + \sum B_5 + \sum B_6, \\
\sum B_1 &= \sum_{y \in V(G)} \sum_{z_1, z_2 \notin E(H)} [d_{G+S}^\theta(y, z_1) + d_{G+S}^\theta(y, z_2)] \\
&= \sum_{y \in V(G)} \sum_{z_1, z_2 \notin E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [d_G^{\theta-i}(y) d_H^i(z_1) + d_G^{\theta-i}(y) d_H^i(z_2)] \\
&= \sum_{y \in V(G)} \sum_{z_1, z_2 \notin E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} d_G^{\theta-i}(y) [d_H^i(z_1) + d_H^i(z_2)], \\
\sum B_2 &= \sum_{y_1, y_2 \in E(G)} \sum_{z \in V(H)} [d_{G+S}^\theta(y_1, z) + d_{G+S}^\theta(y_2, z)] \\
&= \sum_{y_1, y_2 \in E(G)} \sum_{z \in V(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [d_G^{\theta-i}(y_1) d_H^i(z) + d_G^{\theta-i}(y_2) d_H^i(z)] \\
&= \sum_{y_1, y_2 \in V_G} \sum_{z \in H} \sum_{i=0}^{\theta} \binom{\theta}{i} d_H^i(z) [d_G^{\theta-i}(y_1) d_G^{\theta-i}(y_2)], \\
\sum B_3 &= \sum_{y_1, y_2 \in E(G)} \sum_{z_1, z_2 \in E(H)} [d_{G+S}^\theta(y_1, z_1) + d_{G+S}^\theta(y_2, z_2)] \\
&= \sum_{y_1, y_2 \in E(G)} \sum_{z_1, z_2 \in E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [d_G^{\theta-i}(y_1) d_H^i(z_1) + d_G^{\theta-i}(y_2) d_H^i(z_2)], \\
\sum B_4 &= \sum_{y_1, y_2 \notin E(G)} \sum_{z_1, z_2 \in E(H)} [d_{G+S}^\theta(y_1, z_1) + d_{G+S}^\theta(y_2, z_2)] \\
&= \sum_{y_1, y_2 \notin E(G)} \sum_{z_1, z_2 \in E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [d_G^{\theta-i}(y_1) d_H^i(z_1) + d_G^{\theta-i}(y_2) d_H^i(z_2)], \\
\sum B_5 &= \sum_{y_1, y_2 \in E(G)} \sum_{z_1, z_2 \notin E(H)} [d_{G+S}^\theta(y_1, z_1) + d_{G+S}^\theta(y_2, z_2)] \\
&= \sum_{y_1, y_2 \in E(G)} \sum_{z_1, z_2 \notin E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [d_G^{\theta-i}(y_1) d_H^i(z_1) + d_G^{\theta-i}(y_2) d_H^i(z_2)], \\
\sum B_6 &= \sum_{y_1, y_2 \notin E(G)} \sum_{z_1, z_2 \notin E(H)} [d_{G+S}^\theta(y_1, z_1) + d_{G+S}^\theta(y_2, z_2)] \\
&= \sum_{y_1, y_2 \notin E(G)} \sum_{z_1, z_2 \notin E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [d_G^{\theta-i}(y_1) d_H^i(z_1) + d_G^{\theta-i}(y_2) d_H^i(z_2)],
\end{aligned}$$

$$\begin{aligned}
 \sum B &= \sum_{i=0}^{\theta} \binom{\theta}{i} \left[ M_1^{\theta-i}(G) \overline{M}_1^{i+1}(H) + M_1^{\gamma-i}(G) M_1^i(H) \right. \\
 &\quad \left. + M_1^{i+1}(H) (M_1^{\gamma-i}(G) + \overline{M}_1^{\gamma-i}(G)) + \overline{M}_1^{i+1}(H) (M_1^{\gamma-i}(G) + \overline{M}_1^{\gamma-i}(G)) \right], \\
 \sum C &= \sum C_1 + \sum C_2 + \sum C_3 + \sum C_4 + \sum C_5, \\
 \sum C_1 &= \sum_{y_1 y_2 \notin E(S(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(S(G)-G)} \sum_{z \in H} [d_{G+S H}^{\theta}(y_1, z) + d_{G+S H}^{\theta}(y_2, z)] \\
 &= \sum_{y_1 y_2 \notin E(S(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(S(G)-G)} \sum_{z \in H} \sum_{i=0}^{\theta} \binom{\theta}{i} [d_G^{\theta-i}(y_1) d_H^i(z) + 2^{\theta}] \\
 &= \sum_{i=0}^{\theta} \binom{\theta}{i} \overline{M}_1^{\theta-i}(G) M_1^i(H) + 2^{\theta} n_2 e_1 (n_1 - 2), \\
 \sum C_2 &= \sum_{y_1 y_2 \in E(S(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(S(G)-G)} \sum_{z_1 z_2 \in E(H)} [d_{G+S H}^{\theta}(y_1, z_1) + d_{G+S H}^{\theta}(y_2, z_2)] \\
 &= \sum_{y_1 y_2 \in E(S(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(S(G)-G)} \sum_{z_1 z_2 \in E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [d_G^{\theta-i}(y_1) d_H^i(z_1) + 2^{\theta}], \\
 \sum C_3 &= \sum_{y_1 y_2 \in E(S(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(S(G)-G)} \sum_{z_1 z_2 \notin E(H)} [d_{G+S H}^{\theta}(y_1, z_1) + d_{G+S H}^{\theta}(y_2, z_2)] \\
 &= \sum_{y_1 y_2 \in E(S(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(S(G)-G)} \sum_{z_1 z_2 \notin E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [d_G^{\theta-i}(y_1) d_H^i(z_1) + 2^{\theta}], \\
 \sum C_4 &= \sum_{y_1 y_2 \notin E(S(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(S(G)-G)} \sum_{z_1 z_2 \in E(H)} [d_{G+S H}^{\theta}(y_1, z_1) + d_{G+S H}^{\theta}(y_2, z_2)] \\
 &= \sum_{y_1 y_2 \notin E(S(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(S(G)-G)} \sum_{z_1 z_2 \in E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [d_G^{\theta-i}(y_1) d_H^i(z_1) + 2^{\theta}], \\
 \sum C_5 &= \sum_{y_1 y_2 \notin E(S(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(S(G)-G)} \sum_{z_1 z_2 \notin E(H)} [d_{G+S H}^{\theta}(y_1, z_1) + d_{G+S H}^{\theta}(y_2, z_2)] \\
 &= \sum_{y_1 y_2 \notin E(S(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(S(G)-G)} \sum_{z_1 z_2 \notin E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [d_G^{\theta-i}(y_1) d_H^i(z_1) + 2^{\theta}], \\
 \sum C &= \sum_{i=0}^{\theta} \binom{\theta}{i} \left[ M_1^i(H) (\overline{M}_1^{\theta-i}(G) + M_1^{\gamma-i}(G)) + M_1^{\gamma-i}(G) \overline{M}_1^i(H) + \overline{M}_1^{\gamma-i}(G) (M_1^i(H) + \overline{M}_1^i(H)) \right] \\
 &\quad + 2^{\theta+1} e_1 n_1 (e_2 + \overline{e}_2). \tag{4}
 \end{aligned}$$

We arrived at desired result by putting the values in equation (4).  $\square$

**Theorem 2.** Let  $G_{+R}H$  be  $R$ -sum graph; then, its first general Zagreb co-index is given as

$$\begin{aligned} \overline{M}_1^\gamma(G_{+R}H) &= 2^\theta(n_2^2e_1^2 - n_2e_1) + 2^\theta e_1[2n_1(e_2 + \overline{e}_2) + n_2(n_1 - 2)]2 \sum_{i=0}^{\theta} \binom{\theta}{i} \\ &\quad \left[ M_1^{\theta-i}(G)\overline{M}_1^{i+1}(H) + (M_1^{\gamma-i}(G) + \overline{M}_1^{\gamma-i}(G)) + \overline{M}_1^{i+1}(H)(M_1^{\gamma-i}(G) + \overline{M}_1^{\gamma-i}(G)) \right] \\ &\quad + \overline{M}_1^{\theta-i}(G)M_1^i(H) + M_1^{\gamma-i}(G)(M_1^i(H) + \overline{M}_1^i(H)) + \overline{M}_1^{\gamma-i}(G)(M_1^i(H) + \overline{M}_1^i(H)), \end{aligned} \quad (5)$$

where  $\gamma = \theta - 1$ .

*Proof.* Using equation (2), we have

$$\begin{aligned} \overline{M}^\gamma G_{+R}H &= \sum_{(y_1, y_2)(z_1, z_2) \notin EG_{+R}H} [d^\theta(y_1, z_1) + d^\theta(y_2, z_2)], \\ \overline{M}_1^\gamma(G_{+R}H) &= \sum_{z_1, z_2 \in H} \left[ \sum_{y_1, y_2 \in (V(R(G)-G))} [d_{G_{+R}H}^\theta(y_1, z_1) + d_{G_{+R}H}^\theta(y_2, z_2)] + \sum_{y_1, y_2 \in V_G} [d_{G_{+R}H}^\theta(y_1, z_1) + d_{G_{+R}H}^\theta(y_2, z_2)] \right] \\ &\quad + \sum_{y_1, y_2 \in V(R(G_1))y_1 \in V(G_1)y_2 \in V(R(G)-G)} [d_{G_{+R}H}^\theta(y_1, z_1) + d_{G_{+R}H}^\theta(y_2, z_2)] \\ &= \sum A + \sum B + \sum C. \end{aligned} \quad (6)$$

Using equation 4, we directly have

$$\begin{aligned} \sum A &= 2^\theta(n_2^2e_1^2 - n_2e_1), \\ \sum B &= \sum B_1 + \sum B_2 + \sum B_3 + \sum B_4 + \sum B_5, \\ \sum B_1 &= \sum_{y \in V(G)} \sum_{z_1, z_2 \notin E(H)} [d_{G_{+R}H}^\theta(y, z_1) + d_{G_{+R}H}^\theta(y, z_2)] \\ &= \sum_{y \in V(G)} \sum_{z_1, z_2 \notin E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [2d_G^{\theta-i}(y)d_H^i(z_1) + 2d_G^{\theta-i}(y)d_H^i(z_2)] \\ &= 2 \sum_{y \in V(G)} \sum_{z_1, z_2 \notin E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} d_G^{\theta-i}(y)[d_H^i(z_1) + d_H^i(z_2)] = 2 \sum_{i=0}^{\theta} \binom{\theta}{i} M_1^{\theta-i}(G)\overline{M}_1^{i+1}(H), \\ \sum B_2 &= \sum_{y_1, y_2 \in E(G)} \sum_{z_1, z_2 \in E(H)} [d_{G_{+R}H}^\theta(y_1, z_1) + d_{G_{+R}H}^\theta(y_2, z_2)] \\ &= \sum_{y_1, y_2 \in E(G)} \sum_{z_1, z_2 \in E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [2d_G^{\theta-i}(y_1)d_H^i(z_1) + 2d_G^{\theta-i}(y_2)d_H^i(z_2)] = 2 \sum_{i=0}^{\theta} \binom{\theta}{i} M_1^{\gamma-i}(G)M_1^{i+1}(H), \\ \sum B_3 &= \sum_{y_1, y_2 \notin E(G)} \sum_{z_1, z_2 \in E(H)} [d_{G_{+R}H}^\theta(y_1, z_1) + d_{G_{+R}H}^\theta(y_2, z_2)] \\ &= \sum_{y_1, y_2 \notin E(G)} \sum_{z_1, z_2 \in E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [2d_G^{\theta-i}(y_1)d_H^i(z_1) + 2d_G^{\theta-i}(y_2)d_H^i(z_2)] = 2 \sum_{i=0}^{\theta} \binom{\theta}{i} \overline{M}_1^{\gamma-i}(G)M_1^{i+1}(H), \\ \sum B_4 &= \sum_{y_1, y_2 \in E(G)} \sum_{z_1, z_2 \notin E(H)} [d_{G_{+R}H}^\theta(y_1, z_1) + d_{G_{+R}H}^\theta(y_2, z_2)] \\ &= \sum_{y_1, y_2 \in E(G)} \sum_{z_1, z_2 \notin E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [2d_G^{\theta-i}(y_1)d_H^i(z_1) + 2d_G^{\theta-i}(y_2)d_H^i(z_2)] = 2 \sum_{i=0}^{\theta} \binom{\theta}{i} M_1^{\gamma-i}(G)\overline{M}_1^{i+1}(H), \end{aligned}$$

$$\begin{aligned}
 \sum B_5 &= \sum_{y_1, y_2 \notin E(G)} \sum_{z_1, z_2 \notin E(H)} [d_{G_{+R}H}^\theta(y_1, z_1) + d_{G_{+R}H}^\theta(y_2, z_2)] \\
 &= \sum_{y_1, y_2 \notin E(G)} \sum_{z_1, z_2 \notin E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [2d_G^{\theta-i}(y_1)d_H^i(z_1) + 2d_G^{\theta-i}(y_2)d_H^i(z_2)] = 2 \sum_{i=0}^{\theta} \binom{\theta}{i} \overline{M}_1^{\gamma-i}(G) \overline{M}_1^{i+1}(H), \\
 \sum B &= 2 \sum_{i=0}^{\theta} \binom{\theta}{i} \left[ \overline{M}_1^{\theta-i}(G) \overline{M}_1^{i+1}(H) + (M_1^{\gamma-i}(G) + \overline{M}_1^{\gamma-i}(G)) + \overline{M}_1^{i+1}(H) (M_1^{\gamma-i}(G) + \overline{M}_1^{\gamma-i}(G)) \right], \\
 \sum C &= \sum C_1 + \sum C_2 + \sum C_3 + \sum C_4 + \sum C_5, \\
 \sum C_1 &= \sum_{y_1, y_2 \notin E(R(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(R(G)-G)} \sum_{z \in H} [d_{G_{+R}H}^\theta(y_1, z) + d_{G_{+R}H}^\theta(y_2, z)] \\
 &= \sum_{y_1, y_2 \notin E(R(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(R(G)-G)} \sum_{z \in H} \sum_{i=0}^{\theta} \binom{\theta}{i} [2d_G^{\theta-i}(y_1)d_H^i(z) + 2^\theta] \\
 &= 2 \sum_{i=0}^{\theta} \binom{\theta}{i} \overline{M}_1^{\theta-i}(G) M_1^i(H) + 2^\theta n_2 e_1 (n_1 - 2), \\
 \sum C_2 &= \sum_{y_1, y_2 \in E(R(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(R(G)-G)} \sum_{z_1, z_2 \in E(H)} [d_{G_{+R}H}^\theta(y_1, z_1) + d_{G_{+R}H}^\theta(y_2, z_2)] \\
 &= \sum_{y_1, y_2 \in E(R(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(R(G)-G)} \sum_{z_1, z_2 \in E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [2d_G^{\theta-i}(y_1)d_H^i(z_1) + 2^\theta] \\
 \sum C_3 &= \sum_{y_1, y_2 \in E(R(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(R(G)-G)} \sum_{z_1, z_2 \notin E(H)} [d_{G_{+R}H}^\theta(y_1, z_1) + d_{G_{+R}H}^\theta(y_2, z_2)] \\
 &= \sum_{y_1, y_2 \in E(R(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(R(G)-G)} \sum_{z_1, z_2 \notin E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [2d_G^{\theta-i}(y_1)d_H^i(z_1) + 2^\theta] \\
 \sum C_4 &= \sum_{y_1, y_2 \notin E(R(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(R(G)-G)} \sum_{z_1, z_2 \in E(H)} [d_{G_{+R}H}^\theta(y_1, z_1) + d_{G_{+R}H}^\theta(y_2, z_2)] \\
 &= \sum_{y_1, y_2 \notin E(R(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(R(G)-G)} \sum_{z_1, z_2 \in E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [2d_G^{\theta-i}(y_1)d_H^i(z_1) + 2^\theta] \\
 \sum C_5 &= \sum_{y_1, y_2 \notin E(R(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(R(G)-G)} \sum_{z_1, z_2 \notin E(H)} [d_{G_{+R}H}^\theta(y_1, z_1) + d_{G_{+R}H}^\theta(y_2, z_2)] \\
 &= \sum_{y_1, y_2 \notin E(R(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(R(G)-G)} \sum_{z_1, z_2 \notin E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [2d_G^{\theta-i}(y_1)d_H^i(z_1) + 2^\theta] \\
 \sum C &= 2 \sum_{i=0}^{\theta} \binom{\theta}{i} \left[ \overline{M}_1^{\theta-i}(G) M_1^i(H) + M_1^{\gamma-i}(G) (M_1^i(H) + \overline{M}_1^i(H)) + \overline{M}_1^{\gamma-i}(G) (M_1^i(H) + \overline{M}_1^i(H)) \right] \\
 &\quad + 2^\theta e_1 [2n_1 (e_2 + \overline{e}_2) + n_2 (n_1 - 2)]. \tag{7}
 \end{aligned}$$

We arrived at desired result by putting the values in equation (6).  $\square$

**Theorem 3.** Let  $G_{+Q}H$  be  $Q$ -sum graph; then, its first general Zagreb co-index is given as

$$\begin{aligned}
\overline{M}_1^\gamma(G_{+Q}H) &= +4^\theta n_2 e_1 (n_1 - 2) + 4e_1 (n_1 + 2) (e_2 + \overline{e}_2) \\
&\cdot \sum_{i=0}^{\theta} \binom{\theta}{i} \left[ M_1^{\theta-i}(G) \overline{M}_1^{i+1}(H) + M_1^{\gamma-i}(G) M_1^i(H) + M_1^{i+1}(H) (M_1^{\gamma-i}(G) + \overline{M}_1^{\gamma-i}(G)) \right. \\
&+ \overline{M}_1^{i+1}(H) (M_1^{\gamma-i}(G) + \overline{M}_1^{\gamma-i}(G)) + M_1^i(H) (\overline{M}_1^{\theta-i}(G) + M_1^{\gamma-i}(G)) \\
&\left. + M_1^{\gamma-i}(G) \overline{M}_1^i(H) + \overline{M}_1^{\gamma-i}(G) (M_1^i(H) + \overline{M}_1^i(H)) \right] + \alpha_1,
\end{aligned} \tag{8}$$

where  $\gamma = \theta - 1$ .

*Proof.* Using equation (2), we have

$$\begin{aligned}
\overline{M}^\gamma(G_{+Q}H) &= \sum_{(y_1, y_2) (z_1, z_2) \notin E(G_{+Q}H)} [d^\theta(y_1, z_1) + d^\theta(y_2, z_2)], \\
\overline{M}_1^\gamma(G_{+Q}H) &= \sum_{z_1, z_2 \in H} \left[ \sum_{y_1, y_2 \in (V(Q(G)-G))} [d_{G_{+S}H}^\theta(y_1, z_1) + d_{G_{+QH}}^\theta(y_2, z_2)] + \sum_{y_1, y_2 \in V_G} [d_{G_{+QH}}^\theta(y_1, z_1) + d_{G_{+QH}}^\theta(y_2, z_2)] \right. \\
&+ \left. \sum_{y_1, y_2 \in V(S(G)) y_1 \in V(G) y_2 \in V(Q(G)-G)} [d_{G_{+QH}}^\theta(y_1, z_1) + d_{G_{+QH}}^\theta(y_2, z_2)] \right] \\
&= \sum A + \sum B + \sum C, \\
\sum A &= \sum_{y_1, y_2 \in V(Q(G)-G)} \sum_{z_1, z_2 \in V_{G_2}} [d_{G_{+QH}}^\theta(y_1, z_1) + d_{G_{+QH}}^\theta(y_2, z_2)] = \alpha_1.
\end{aligned} \tag{9}$$

Using equation 4, we directly have

$$\begin{aligned}
\sum B &= \sum_{i=0}^{\theta} \binom{\theta}{i} \left[ M_1^{\theta-i}(G) \overline{M}_1^{i+1}(H) + M_1^{\gamma-i}(G) M_1^i(H) + M_1^{i+1}(H) (M_1^{\gamma-i}(G) + \overline{M}_1^{\gamma-i}(G)) \right. \\
&\left. + \overline{M}_1^{i+1}(H) (M_1^{\gamma-i}(G) + \overline{M}_1^{\gamma-i}(G)) \right], \\
\sum C &= \sum C_1 + \sum C_2 + \sum C_3 + \sum C_4 + \sum C_5, \\
\sum C_1 &= \sum_{y_1 y_2 \notin E(Q(G)) y_1 \in V(G) y_2 \in V(Q(G)-G)} \sum_{z \in H} [d_{G_{+QH}}^\theta(y_1, z) + d_{G_{+QH}}^\theta(y_2, z)] \\
&= \sum_{y_1 y_2 \notin E(Q(G)) y_1 \in V(G) y_2 \in V(Q(G)-G)} \sum_{z \in H} \sum_{i=0}^{\theta} \binom{\theta}{i} [d_G^{\theta-i}(y_1) d_H^i(z) + 4^\theta] \\
&= \sum_{i=0}^{\theta} \binom{\theta}{i} \overline{M}_1^{\theta-i}(G) M_1^i(H) + 4^\theta n_2 e_1 (n_1 - 2), \\
\sum C_2 &= \sum_{y_1 y_2 \in E(Q(G)) y_1 \in V(G) y_2 \in V(Q(G)-G)} \sum_{z_1, z_2 \in E(H)} [d_{G_{+QH}}^\theta(y_1, z_1) + d_{G_{+QH}}^\theta(y_2, z_2)] \\
&= \sum_{y_1 y_2 \in E(Q(G)) y_1 \in V(G) y_2 \in V(Q(G)-G)} \sum_{z_1, z_2 \in E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [d_G^{\theta-i}(y_1) d_H^i(z_1) + 2^\theta] = \sum_{i=0}^{\theta} \binom{\theta}{i} M_1^{\gamma-i}(G) M_1^i(H) + 4^{\theta+2} e_1 e_2,
\end{aligned}$$



$$\begin{aligned}
 \sum C_3 &= \sum_{y_1, y_2 \in E(Q(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(Q(G)-G)} \sum_{z_1, z_2 \notin E(H)} \left[ d_{G+QH}^\theta(y_1, z_1) + d_{G+QH}^\theta(y_2, z_2) \right] \\
 &= \sum_{y_1, y_2 \in E(Q(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(Q(G)-G)} \sum_{z_1, z_2 \notin E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} \left[ d_G^{\theta-i}(y_1) d_H^i(z_1) + 2^\theta \right] = \sum_{i=0}^{\theta} \binom{\theta}{i} M_1^{\gamma-i}(G) \overline{M}_1^i(H) + 4^{\theta+2} e_1 \overline{e}_2, \\
 \sum C_4 &= \sum_{y_1, y_2 \notin E(Q(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(Q(G)-G)} \sum_{z_1, z_2 \in E(H)} \left[ d_{G+QH}^\theta(y_1, z_1) + d_{G+QH}^\theta(y_2, z_2) \right] \\
 &= \sum_{y_1, y_2 \notin E(Q(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(Q(G)-G)} \sum_{z_1, z_2 \in E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} \left[ d_G^{\theta-i}(y_1) d_H^i(z_1) + 4^\theta \right] = \sum_{i=0}^{\theta} \binom{\theta}{i} \overline{M}_1^{\gamma-i}(G) M_1^i(H) + 4^{\theta+1} e_1 e_2 (n_1 - 2), \\
 \sum C_5 &= \sum_{y_1, y_2 \notin E(Q(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(Q(G)-G)} \sum_{z_1, z_2 \notin E(H)} \left[ d_{G+QH}^\theta(y_1, z_1) + d_{G+QH}^\theta(y_2, z_2) \right] \\
 &= \sum_{y_1, y_2 \notin E(Q(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(Q(G)-G)} \sum_{z_1, z_2 \notin E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} \left[ d_G^{\theta-i}(y_1) d_H^i(z_1) + 4^\theta \right] = \sum_{i=0}^{\theta} \binom{\theta}{i} \overline{M}_1^{\gamma-i}(G) \overline{M}_1^i(H) + 4^{\theta+1} e_1 \overline{e}_2 (n_1 - 2), \\
 \sum C &= \sum_{i=0}^{\theta} \binom{\theta}{i} \left[ M_1^i(H) \left( \overline{M}_1^{\theta-i}(G) + M_1^{\gamma-i}(G) \right) + M_1^{\gamma-i}(G) \overline{M}_1^i(H) + \overline{M}_1^{\gamma-i}(G) \left( M_1^i(H) + \overline{M}_1^i(H) \right) \right] \\
 &\quad + 4^\theta [n_2 e_1 (n_1 - 2) + 4e_1 (n_1 + 2) (e_2 + \overline{e}_2)].
 \end{aligned}$$

(10)

We arrived at desired result by putting the values in equation (9).  $\square$

**Theorem 4.** Let  $G_{+T}H$  be  $T$ -sum graph; then, its first general Zagreb co-index is given as

$$\begin{aligned}
 \overline{M}_1^\gamma(G_{+T}H) &= 4^\theta (n_2^2 e_1^2 - n_2 e_1) + 2 \\
 &\quad \sum_{i=0}^{\theta} \binom{\theta}{i} \left[ M_1^{\theta-i}(G) \overline{M}_1^{i+1}(H) + (M_1^{\gamma-i}(G) + \overline{M}_1^{\gamma-i}(G)) \right. \\
 &\quad + \overline{M}_1^{i+1}(H) (M_1^{\gamma-i}(G) + \overline{M}_1^{\gamma-i}(G)) \overline{M}_1^{\theta-i}(G) M_1^i(H) + M_1^{\gamma-i}(G) (M_1^i(H) + \overline{M}_1^i(H)) \\
 &\quad \left. + \overline{M}_1^{\gamma-i}(G) (M_1^i(H) + \overline{M}_1^i(H)) \right] + 4^\theta [n_2 e_1 (n_1 - 2) + 4e_1 (n_1 + 2) (e_2 + \overline{e}_2)],
 \end{aligned}$$

(11)

where  $\gamma = \theta - 1$ .

*Proof.* Using equation (2), we have

$$\begin{aligned}
 \overline{M}_1^\gamma(G_{+T}H) &= \sum_{(y_1, y_2) \in E(G_{+T}H)} \left[ d^\theta(y_1, z_1) + d^\theta(y_2, z_2) \right], \\
 \overline{M}_1^\gamma(G_{+T}H) &= \sum_{z_1, z_2 \in H} \left[ \sum_{y_1, y_2 \in (V(T(G))-V(G))} \left[ d_{G+SH}^\theta(y_1, z_1) + d_{G+TH}^\theta(y_2, z_2) \right] + \sum_{y_1, y_2 \in V_G} \left[ d_{G+TH}^\theta(y_1, z_1) + d_{G+TH}^\theta(y_2, z_2) \right] \right. \\
 &\quad \left. + \sum_{y_1, y_2 \in V(S(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(T(G)-G)} \left[ d_{G+TH}^\theta(y_1, z_1) + d_{G+TH}^\theta(y_2, z_2) \right] \right] \\
 &= \sum A + \sum B + \sum C.
 \end{aligned}$$

(12)

Using equation 9, we directly have

$$\sum A = 4^\theta (n_2^2 e_1^2 - n_2 e_1). \quad (13)$$

The value  $\sum A$  and  $\sum B$  are by equations (7) and (9) as follows:

$$\begin{aligned} \sum C &= \sum C_1 + \sum C_2 + \sum C_3 + \sum C_4 + \sum C_5, \\ \sum C_1 &= \sum_{y_1 y_2 \notin E(T(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(T(G)-G)} \sum_{z \in H} [d_{G_{+T}H}^\theta(y_1, z) + d_{G_{+T}H}^\theta(y_2, z)] \\ &= \sum_{y_1 y_2 \notin E(T(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(T(G)-G)} \sum_{z \in H} \sum_{i=0}^{\theta} \binom{\theta}{i} [2d_G^{\theta-i}(y_1) d_H^i(z) + 4^\theta] \\ &= 2 \sum_{i=0}^{\theta} \binom{\theta}{i} \overline{M}_1^{\theta-i}(G) M_1^i(H) + 4^\theta n_2 e_1 (n_1 - 2), \\ \sum C_2 &= \sum_{y_1 y_2 \in E(T(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(T(G)-G)} \sum_{z_1 z_2 \in E(H)} [d_{G_{+T}H}^\theta(y_1, z_1) + d_{G_{+T}H}^\theta(y_2, z_2)] \\ &= \sum_{y_1 y_2 \in E(T(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(T(G)-G)} \sum_{z_1 z_2 \in E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [2d_G^{\theta-i}(y_1) d_H^i(z_1) + 4^\theta], \\ \sum C_3 &= \sum_{y_1 y_2 \in E(T(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(T(G)-G)} \sum_{z_1 z_2 \notin E(H)} [d_{G_{+T}H}^\theta(y_1, z_1) + d_{G_{+T}H}^\theta(y_2, z_2)] \\ &= \sum_{y_1 y_2 \in E(T(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(T(G)-G)} \sum_{z_1 z_2 \notin E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [2d_G^{\theta-i}(y_1) d_H^i(z_1) + 4^\theta] \\ &= 2 \sum_{i=0}^{\theta} \binom{\theta}{i} M_1^{\gamma-i}(G) \overline{M}_1^i(H) + 4^{\theta+2} e_1 \overline{e}_2, \\ \sum C_4 &= \sum_{y_1 y_2 \notin E(T(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(T(G)-G)} \sum_{z_1 z_2 \in E(H)} [d_{G_{+T}H}^\theta(y_1, z_1) + d_{G_{+T}H}^\theta(y_2, z_2)] \\ &= \sum_{y_1 y_2 \notin E(T(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(T(G)-G)} \sum_{z_1 z_2 \in E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [2d_G^{\theta-i}(y_1) d_H^i(z_1) + 4^\theta] \\ &= 2 \sum_{i=0}^{\theta} \binom{\theta}{i} \overline{M}_1^{\gamma-i}(G) M_1^i(H) + 4^{\theta+1} e_1 e_2 (n_1 - 2), \\ \sum C_5 &= \sum_{y_1 y_2 \notin E(T(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(T(G)-G)} \sum_{z_1 z_2 \notin E(H)} [d_{G_{+T}H}^\theta(y_1, z_1) + d_{G_{+T}H}^\theta(y_2, z_2)] \\ &= \sum_{y_1 y_2 \notin E(T(G))} \sum_{y_1 \in V(G)} \sum_{y_2 \in V(T(G)-G)} \sum_{z_1 z_2 \notin E(H)} \sum_{i=0}^{\theta} \binom{\theta}{i} [2d_G^{\theta-i}(y_1) d_H^i(z_1) + 4^\theta] \\ &= 2 \sum_{i=0}^{\theta} \binom{\theta}{i} \overline{M}_1^{\gamma-i}(G) \overline{M}_1^i(H) + 4^{\theta+1} e_1 \overline{e}_2 (n_1 - 2). \end{aligned} \quad (14)$$

By substituting the values of  $\sum A$ ,  $\sum b$ , and  $\sum C$  in equation (12), we obtained required proof.  $\square$

#### 4. Conclusion

The study of the basic or factor graphs is an interesting problem in the theory of graphs where the original graphs becomes complex. In this study, we have computed FGZ co-

index of graphs under operations such as  $\overline{M}_1(G_{+S}H)$ ,  $\overline{M}_1(G_{+R}H)$ ,  $\overline{M}_1(G_{+Q}H)$ , and  $\overline{M}_1(G_{+T}H)$  in the terms of indices and co-indices of their basic or factor graphs.

#### Data Availability

The data used to support the findings of the study are included within article. However, for more details of the data can be obtained from the corresponding author.

## Conflicts of Interest

The authors declare no conflicts of interest.

## Authors' Contributions

Muhammad Javaid and Uzma Ahmad contributed to the discussion of the problem, validation of results, final reading, and supervision; Muhammad Ibraheem contributed to the source of the problem, the collection of material, analyzing and computing the results, and initial drafting of the study; Ebenezer Bonyah and Shaohui Wang contributed to the discussion of the problem, the methodology, and the proofreading of the final draft.

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