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Research Article

Regularized Least Squares Recursive Algorithm with Forgetting Factor for Identifying Parameters in the Grinding Process

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This paper investigates a parameter identification problem in the grinding process. Due to the data saturated phenomenon and the ill-posed of parameter identification inverse problem, this paper presents a regularized least squares recursive algorithm with a forgetting factor (RLSRAFF), the basic idea of which is to combine the forgetting factor with regularization parameters. Moreover, based on RLSRAFF, this paper verifies the recursive calculation of criterion function, analyzes the effect of calculation error from the gain matrix and proves the convergence of the proposed algorithm. Finally, effectiveness of RLSRAFF is verified by simulation experiments and grinding data. Compared with other algorithms, RLSRAFF can give a more convergence rate to the real data and reduce the error from the true value.

1. Introduction

Parameters identification is one of the most important areas in system modeling and signal processing [1], and the related identification methods are attracted by many scholars. As one of most important parameters identification methods, the least square method (LSM) has been applied in various fields. In [2], to identify a ship's linear sway-yaw manoeuvring coefficients and drag-area parameters in current and wind, Wayne and Gash developed a simple least squares technique. In [3], the loss in localization accuracy induced by time difference of arrival noises and velocity errors is reduced by the constrained total least squares method. In [4], to deal with the problem of overenhancement results, an image enhancement scheme based on weighted least squares are proposed. In [5], due to the complex biochemical characteristics of the wastewater treatment process, an adaptive dynamic nonlinear partial least squares model is proposed to improve the prediction performance and stability of effluent quality indexes. In [6], Ramezani applied the collocated discrete least squares meshless method to improve the node moving technique. In order to enhance the performance of LSM, some improved algorithms are developed. Reference [7] presents an iteratively reweighted LSM to improve the antioutlier performance of the least squares support vector machine. Reference [8] combines the partial least square with the attention mechanism in a neural network named the attention-PLS. Zhang, et al. [9] proposed the Lagrange energy-least squares similitude method to deduce output scaling laws. Furthermore, some scholars study the recursive least squares (RLS) method with the forgetting factor. In [10], Paleologu et al. pointed out that the performance of the recursive least squares algorithm is governed by the forgetting factor and proposed a variable forgetting factor RLS (VFF-RLS) algorithm for system identification. Reference [11] based on the framework of recursive least squares-temporal difference proposed a new reinforcement learning method by using the forgetting factor. Sun et al. [12] presented an adaptive forgetting factor RLS method for online identification of the second order resistor-capacitance equivalent circuit model parameters. Meanwhile, the regularization method is used to improve

the LSM. Wang et al. [13] introduced the least squares regularization method to solve the ill-posed problem of the multiplicative error model. Zhou et al. [14] investigated the method of the anti-ill-conditioned population-weighted median based on the least squares regularization method. Jin et al. [15] used the kurtosis regularization algorithm for circuit joint optimization in neural network training to increase the information entropy of neural network weight data. Bai et al. [16] proposed a generic model for least squares nonnegative matrix factorizations with Tikhonov regularization. However, in the process of parameter identification, the data saturated phenomenon and the ill-posed problem often occur simultaneously. Therefore, this paper presents a regularized least squares recursive algorithm with forgetting factor (RLSRAFF).

At the same time, grinding process [17–19], as one of the most important procedures in the mineral processing, extracts the valuable minerals from the discernible gangue after physical grinding and classification. The role of this process can dissociate different useful minerals from each other and avail to the subsequent sorting process. The grinding and classification process has complex characteristics, such as large inertia, time-varying parameters, and nonlinearity. Recently, to realize the automatic production, some control technology strategies [20–24] for the grinding process have been attracted by majority of scientific researchers. As the core of the control system, the accuracy of the mathematical model of the grinding process plays a vital role. Reference [25] identified the parameters of the prediction model of steel ball wear law in the grinding process, [26] used a nonlinear parameter identification method based on the improved differential evolution algorithm to solve the parameter of the nonlinear model in the bauxite grinding classification process. Furthermore, Chen [27] investigated the least square recursive algorithm with forgetting factor to systematically identify the unknown parameters of the grinding process model.

Due to characteristics of complexity and nonlinearity, parameter identification in the mathematical model of the grinding process can lead to the ill-posed. The above literature does not consider the effectiveness of ill-posed on the identification results. Regularization [28] is proved to be an efficient approach for the inverse problem. Therefore, this paper develops RLSRAFF for identifying the parameters in the mathematical model of the grinding process. The main contributions can be summed as follows:

(i) The ill-posed problem is considered for identifying the parameters in the mathematical model of the grinding process.

(ii) RLSRAFF, which combines the forgetting factor with the regularization parameter, is presented.

(iii) The recursive calculation of criterion function, the effect of calculation error from the gain matrix, and the convergence of the proposed algorithm are analyzed.

The rest of the paper is organized as follows. In Section 2, the grinding process is described and then the parameter identification model is introduced. In Section 3, RLSRAFF is proposed and the calculation error of the gain matrix and convergence of the algorithm are proved. In Section 4, two experiments are carried out to verify the proposed method. In Section 5, the conclusion is given.

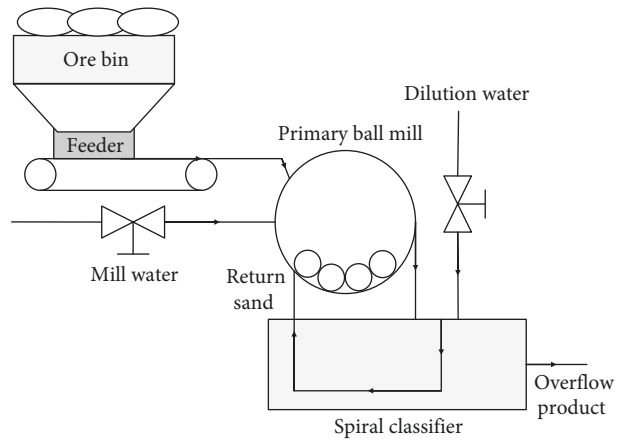

Figure 1: The grinding process of the primary ball mill.

2. Grinding Process and Parameter Identification Model

2.1. Grinding Process. The grinding process (see Figure 1) can be briefly described as follows. The fresh ores are firstly sent into the ball mill by the conveyor belt and then crushed ceaselessly by the steel balls to produce the pulp within a certain concentration. Meanwhile, a certain amount of water is added into the mill to adjust the concentration of ore pulp within limits. After grinding, the mixed ore pulp is continuously discharged from mill into the spiral classifier for classification, and a certain amount of water is added into the spiral classifier to adjust the concentration. The substandard ore pulp returns to the first stage of ball mill for regrinding, and the standard ore, extracting from the classifier, enters into the pump sump for the next process [27].

2.2. Mathematical Model of the Grinding Process Parameter Identification. Mathematical model of the grinding process mainly includes the ball mill model and spiral classifier model. The ball mill model [29] has been detailed in a variety of studies. It describes the relationship between the ore feeding quantity, the water feeding quantity, the return sand quantity, and the mill pulp concentration. The spiral classifier model gives the relationship between the classifier overflow concentration, the pulp fineness, the ore discharge quantity, and the return sand quantity.

However, these models only give internal mechanism of each part in the grinding process, and it has not reflected the multi-variable coupling and time delay between the primary ball mill and the classifier.

Thus, the mathematical model [27] between the ball mill and the classifier can be described as follows:

$$\begin{bmatrix} C_c \\ Q_D \end{bmatrix} = \begin{bmatrix} \dfrac{\beta_1}{\alpha_1 s+1} e^{-\tau_1 s} & \dfrac{\beta_2}{\alpha_2 s+1} e^{-\tau_2 s} \\ \dfrac{\beta_3}{(\alpha_3 s+1)(\alpha_5 s+1)} e^{-\tau_3 s} & \dfrac{\beta_4}{\alpha_4 s+1} e^{-\tau_4 s} \end{bmatrix} \begin{bmatrix} U_3 \\ U_1 \end{bmatrix}, \quad (1)$$

where C_c is the actual value of the classifier overflow concentration, (%); Q_D is the actual value of the return sand quantity, (m^3/h); U_1 is the actual value of the ore feeding quantity, (t/h); U_3 is the actual value of the classifier adding water quantity, (m^3/h).

By analyzing the model (1), we know that the U_1 and U_3 are the input variables, and C_c and Q_D are the output variables. The model identifies between the ball mill and the classifier is described as follows:

$$\begin{bmatrix} C_c \\ Q_D \end{bmatrix} = \begin{bmatrix} G'_{11}(S) & G'_{12}(S) \\ G'_{21}(S) & G'_{22}(S) \end{bmatrix} \begin{bmatrix} U_3 \\ U_1 \end{bmatrix}, \quad (2)$$

where $G'_{11}(S), G'_{12}(S), G'_{21}(S), G'_{22}(S)$ are consisted by the inertia link and the delay link, and $G'_{11}(S) = (\beta_1/(\alpha_1 S+1))e^{-\tau_1 s}, G'_{12}(S) = (\beta_2/(\alpha_2 S+1))e^{-\tau_2 s}, G'_{21}(S) = (\beta_3/(\alpha_3 S+1)(\alpha_5 S+1))e^{-\tau_3 s}, G'_{22}(S) = (\beta_4/(\alpha_4 S+1)(\alpha_4 S+1))e^{-\tau_4 s}$. Different identification methods are used to identify the parameters in these two links, respectively. The inertia link is identified by the least square algorithm, and the delay link is identified by the cross-correlation function algorithm [27]. The major study in this paper focuses on identification of the parameters for the inertia link $(a_1, b_1, c_1, d_1; a_2, a_3, b_2, b_3, c_2, d_2)$ to deal with the data saturated phenomenon and the ill-posed problem. For the inertia link, we have the following equations:

$$\begin{aligned} C_c &= G_{11}(S)U_3 + G_{12}(S)U_1 \\ &= \frac{\beta_1}{\alpha_1 S+1}U_3 + \frac{\beta_2}{\alpha_2 S+1}U_1, \end{aligned} \quad (3)$$

$$\begin{aligned} Q_D &= G_{21}(S)U_3 + G_{22}(S)U_1 \\ &= \frac{\beta_3}{(\alpha_3 S+1)(\alpha_5 S+1)}U_3 + \frac{\beta_4}{\alpha_4 S+1}U_1, \end{aligned} \quad (4)$$

where $G_{11}(S) = \beta_1/(\alpha_1 S+1)$, $G_{12}(S) = \beta_2/(\alpha_2 S+1), G_{21}(S) = \beta_3/(\alpha_3 S+1)(\alpha_5 S+1), G_{22}(S) = \beta_4/(\alpha_4 S+1)$. According to equations (3) and (4), we have

$$C_c(z) = \frac{b_1 z^{-1}}{1+a_1 z^{-1}}U_3(z) + \frac{c_1 z^{-1}}{1+d_1 z^{-1}}U_1(z), \quad (5)$$

$$Q_D(z) = \frac{b_2 z^{-1} + b_3 z^{-1}}{1+a_2 z^{-1} + a_3 z^{-1}}U_3(z) + \frac{c_2 z^{-1}}{1+d_2 z^{-1}}U_1(z), \quad (6)$$

Thus, the input and output observation data are used to estimate the unknown parameters $(a_1, b_1, c_1, d_1; a_2, a_3, b_2, b_3, c_2, d_2)$ in equations (5) and (6).

Remark 1. For the delay link, cross-correlation function [27] can be used to identify the delay time $(\tau_1, \tau_2, \tau_3, \tau_4)$, because the main work of this paper is to identify the unknown parameters, the details of identifying the delay time are not given.

3. Regularized Least Squares Recursive Algorithm with Forgetting Factor and Property Analysis

3.1. Regularized Least Squares Recursive Algorithm with Forgetting Factor. In order to identify the parameters in model (5), $U_3 z$ and $U_1(z)$ are defined as the input variable, $C_c(z)$ is defined as the output variable. The difference equation is used to discrete the linear system as

$$\begin{aligned} &2C_c(k) + a_1 C_c(k-1) + d_1 C_c(k-1) \\ &\quad = b_1 U_3(k-1) + c_1 U_1(k-1) + \varepsilon(k), \end{aligned} \quad (7)$$

where $k = 2, 3, \ldots, 1+N$, $\varepsilon(k)$is uncorrelated random variable that subjects to $N(0,1)$ distribution, $U_3(k-1)$ and $U_1(k-1)$ are the actual input signal of recorded data, $C_c(k)$ is the actual output signal of recorded data, and $C_c(k-1)$ is the function value of the output signal $C_c(k)$ at the previous sampling period. So we have the following equation from the following equation:

$$\begin{aligned} 2C_c(k) = &-a_1 C_c(k-1) - d_1 C_c(k-1) \\ &+ b_1 U_3(k-1) + c_1 U_1(k-1) + \varepsilon(k). \end{aligned} \quad (8)$$

It can be seen from (8) that the above parameters can be estimated by N sets of the observation equations:

$$\begin{cases} C_c(1+1) = -0.5a_1 C_c(1) - 0.5d_1 C_c(1) + 0.5b_1 U_3(1) + 0.5c_1 U_1(1) + 0.5\varepsilon(1+1) \\ C_c(1+2) = -0.5a_1 C_c(2) - 0.5d_1 C_c(2) + 0.5b_1 U_3(2) + 0.5c_1 U_1(2) + 0.5\varepsilon(1+2) \\ \qquad\qquad \vdots \\ C_c(1+N) = -0.5a_1 C_c(N) - 0.5d_1 C_c(N) + 0.5b_1 U_3(N) + 0.5c_1 U_1(N) + 0.5\varepsilon(1+N). \end{cases} \quad (9)$$

Equation (9) is defined as an observation equation. We define $C_c(N) = [C_c(1+1), C_c(1+2), \ldots, C_c(1+N)]^T$, $\theta(N) = [0.5a_1, 0.5d_1, 0.5b_1, 0.5c_1]^T$ and $\varepsilon(N) = [\varepsilon(1+1), \varepsilon(1+2), \ldots, \varepsilon(1+N)]^T$. Equation (9) can be written into the following form:

$$C_C(N) = \Phi(N)\theta(N) + \varepsilon(N), \tag{10}$$

We define

$$\Phi(N) = \left[\phi^T(1), \phi^T(2), \ldots, \phi^T(N)\right]^T, \tag{11}$$

$$\phi^T(i) = \left[-C_c(i), -C_c(i), U_3(i), U_1(i)\right], \tag{12}$$

where $i = 1, 2, \ldots, N$. So the observation equation (9) can be expressed as

$$C_c(1+i) = \phi^T(i)\theta(N) + \varepsilon(1+i). \tag{13}$$

The least square method is to minimize the sum of residuals by the observation equation (9). Thus, we assume the sum of residuals squares as

$$F = \sum_{i=1}^{N} \varepsilon^2(n+i) = \varepsilon^T(N)\varepsilon(N), \tag{14}$$

we use equation (10) into equation (14), take the derivative of θ and set this derivative as zero, we have

$$-2\Phi\left(C_C - \Phi\theta\right) = 0, \tag{15}$$

so we can obtain

$$\theta = \left(\Phi^T\Phi\right)^{-1}\Phi^T C_C, \tag{16}$$

due to the ill-posed of item $(\Phi^T\Phi)^{-1}$, the regularization term is introduced as

$$\theta = \left(\Phi^{\mathrm{T}}\Phi + \lambda I\right)^{-1}\Phi^T C_C, \tag{17}$$

where λ is the regularization parameter.

Because the phenomenon of data saturation, which makes the parameters to be identified and not improve with the increase of new sampling values, occurs in the least square method, this paper presents the regularized least squares recursive algorithm with forgetting factor (RLSRAFF). RLSRAFF combines the regularization parameter with forgetting factor, which is shown as follows:

$$\begin{cases} \theta(N+1) = \theta(N) + K(N)\left[C_c(N+2) - \phi^T\theta(N)\right], \\ K(N) = P(N)\phi\left[\mu + \phi^T P(N)\phi\right]^{-1}, \\ P(N+1) = \dfrac{1}{\mu}\left[I - P(N)\phi\left[\mu + \phi^T P(N)\phi\right]^{-1}\phi^T\right]P(N), \end{cases} \tag{18}$$

where $P(N) = (\phi^T\phi + \phi I)^{-1}$. N is the number of data group, μ is the forgetting factor $(0 < \mu \le 1)$. If μ is chosen as a smaller one, the forgetting rate of old data is larger. Therefore, the forgetting factor μ is adjusted according to the process characteristics.

Remark 2. The innovation of this paper mainly focuses on the combination of regularization parameter λ and the forgetting factor μ. The regularization parameter λ, added in the item$P(N) = (\phi^T\phi + \lambda I)^{-1}$, can eliminate the ill-posed caused by $(\phi^T\phi)^{-1}$ and reduce error.

3.2. Property Analysis of RLSRAFF. The property of recursive least square includes the following points: the recursive calculation of criterion function, the effect of calculation error from the gain matrix, and the convergence of the proposed algorithm. In this section, the property of RLSRAFF is analyzed.

Theorem 1. *We assume that $J(N+1)$ and $J(N)$ are described as follows:*

$$J(N) = {C_C}^T(N)C_C(N), \tag{19}$$

$$J(N+1) = {C_C}^T(N+1)C_C(N+1). \tag{20}$$

On the basis of RLSRAFF, the recurrence equation of the criterion function can be given as follows:

$$J(N+1) = \mu\left[J(N) + \left[\mu + \phi^T P(N)\phi\right]^{-1}{\widetilde{C}_c}^2(N+2)\right], \tag{21}$$

where $\widetilde{C}_c(N+2) = C_c(N+2) - \phi^T\theta(N)$, $J(N)$, and $J(N+1)$ are the criterion function of parameter estimation at N and $(N+1)$, respectively.

Proof. By analyzing Section 3.1, we have

$$\Phi(N+1) = \begin{pmatrix} \rho\Phi(N) \\ \phi^T(N+1) \end{pmatrix}, \tag{22}$$

$$C_C(N+1) = \begin{pmatrix} \rho C_C(N) \\ C_c(N+2) \end{pmatrix}, \tag{23}$$

where $\rho(0 < \rho \le 1)$ is a factor, and we have

$$\begin{aligned} C_{C1}(N+1) &= C_C(N+1) - \Phi(N+1)\theta(N+1) \\ \% \quad &= \begin{pmatrix} \rho\widetilde{C}_c(N) \\ \widetilde{C}_c(N+2) \end{pmatrix} \\ &\quad - \begin{pmatrix} \rho\Phi(N) \\ \phi^T(N+1) \end{pmatrix}K(N)\widetilde{C}_c(N+2). \end{aligned} \tag{24}$$

So we have

$$\begin{aligned} J(N+1) &= {C_{C1}}^T(N+1)C_{C1}(N+1) \\ &= \rho^2 J(N) - 2\rho^2 K^T(N)\Phi^T(N)C_{C1}(N)\widetilde{C}_c(N+2) \\ &\quad + \rho^2 K^T(N)\Phi^T(N)\Phi(N)K(N){\widetilde{C}_c}^2(N+2) \\ &\quad + \left[I - \Phi^T K(N)\right]{\widetilde{C}_c}^2(N+2). \end{aligned} \tag{25}$$

Using equation (19), the third and fourth terms of equation (26) can be transformed into

Table 1: The parameter identification results of different λ.

The value of λ	Identification results							
	a'_1	Absolute error	a'_2	Absolute error	b'_1	Absolute error	b'_2	Absolute error
$\lambda = 0.01$	−1.5434	0.0434	0.7694	0.0694	1.0543	0.0543	0.5164	0.0164
$\lambda = 0.1$	−1.5059	0.0059	0.7076	0.0076	1.1059	0.1059	0.4532	0.0468
$\lambda = 0.5$	−1.4527	0.0473	0.6951	0.0049	1.0817	0.0817	0.5151	0.0151
$\lambda = 1$	−1.5300	0.0300	0.7447	0.0447	1.0146	0.0146	0.7409	0.2409
$\lambda = 2$	−1.4797	0.0203	0.6642	0.0358	0.9826	0.0174	0.3803	0.0297
$\lambda = 3$	−1.4631	0.0369	0.6848	0.0152	0.9666	0.0364	0.5916	0.0916
$\lambda = 4$	−1.5227	0.0227	0.7230	0.0230	0.9214	0.0786	0.7239	0.2239
$\lambda = 5$	−1.5557	0.0557	0.7010	0.0010	0.7703	0.2297	0.4051	0.0949
$\lambda = 6$	−1.5805	0.0305	0.7668	0.0668	0.8559	0.1441	0.5573	0.0573
$\lambda = 7$	−1.6059	0.1059	0.7983	0.0983	0.9148	0.0852	0.4076	0.0924
$\lambda = 8$	−1.4907	0.0093	0.7546	0.0546	1.1587	0.1587	0.6139	0.1139
$\lambda = 9$	−1.4976	0.0022	0.6987	0.0013	1.0150	0.0150	0.5167	0.0167
$\lambda = 10$	−1.4407	0.0083	0.6825	0.0175	1.1735	0.0249	0.6839	0.1839

$$\begin{aligned}&\rho^2 K^T(N)\Phi^T(N)\Phi(N)K(N)\widetilde{C}_c{}^2(N+2)\\&= \rho^2\phi^T P(N)\phi\left[\mu+\phi^T P(N)\phi\right]^{-2}\widetilde{C}_c{}^2(N+2),\end{aligned} \tag{26}$$

$$\begin{aligned}&\left[I-\phi^T K(N)\right]^2\widetilde{C}_c{}^2(N+2)\\&\quad= \mu^2\left[\mu+\phi^T P(N)\phi\right]^{-2}\widetilde{C}_c{}^2(N+2).\end{aligned} \tag{27}$$

Substituting equation (26) into equations (27) and (28), and using $\phi^T(N)C_{C1}(N+1)=0$, we have

$$\begin{aligned}J(N+1) &= \mu J(N)+\mu\left[\mu+\phi^T P(N)\phi\right]^{-1}\widetilde{C}_c{}^2(N+2)\\&= \mu\left[J(N)+\left[\mu+\phi^T P(N)\phi\right]^{-1}\widetilde{C}_c{}^2(N+2)\right].\end{aligned} \tag{28}$$

Theorem 1 has been proved. □

Theorem 2. *In the process of recursive calculation, if the gain matrix $K(N)$ contains calculated error $\Delta K(N)$, which is affected by the rounding off and other factors, it can take the calculated error $\Delta P(N)$ into the matrix $P(N)$. So we can obtain the following equation:*

$$\Delta P(N+1) = -\Delta K(N)\phi^T P(N), \tag{29}$$

where $\Delta P(N)$ can transmit the calculation error to the gain matrix again by equation (19). If this cycle continues, the final identified result will be affected.

Proof. According to equation (19), equation $P(N+1)$ can be described as follows:

$$\begin{aligned}P(N+1) =& \left[I-K(N)\phi^T\right]P(N)\\&-P(N)\phi K^T(N)+P(N)\phi K^T(N)\\=& \left[I-K(N)\phi^T\right]P(N)-P(N)\phi K^T(N)\\&+K(N)\left[\mu+\phi^T P(N)\phi\right]K^T(N),\end{aligned} \tag{30}$$

so we have

Table 2: Parameter identification results.

Algorithm	Identification results			
	a'_1	a'_2	b'_1	b'_2
LSM [30]	−1.4713	0.6629	0.8065	0.5918
RLSM [30]	−1.4942	0.6674	0.8737	0.4274
LSRAFF [31]	1.4911	0.6883	0.9671	0.5446
RLSRAFF	−1.4976	0.6927	1.0450	0.5167

$$\begin{aligned}P(N+1) =& \left[I-K(N)\phi^T\right]P(N)\left[I-K(N)\phi^T\right]^T\\&+\mu K(N)K^T(N).\end{aligned} \tag{31}$$

By equation (27), we know that $\Delta P(N+1)$ is only related to the quadratic form of $\Delta K(N)$, which can effectively reduce the transmission of the calculation error and ensure the identification accuracy. However, if the number of parameters is less than 10, it is not necessary to avoid increasing the amount of calculation.

Meanwhile, we bring $K(N)=K(N)+\Delta K(N)$ in to equation (27) and subtract from equation (27), we can obtain equation (25). □

Theorem 2 has been proved.

Theorem 3. *If $\varepsilon(k)$ is known to be an uncorrelated random variable with zero mean, the parameter estimation $\theta(N)$ given in (19) is uniformly convergent, and we have*

$$\lim_{N\longrightarrow\infty}\theta(N)=\theta_0, \tag{32}$$

where θ_0 is true value of model parameters.

Proof. We define $\theta_1(N)=\theta_0-\theta(N)$, and suppose

$$\lim_{N\longrightarrow\infty}\theta_1(N)=\theta'_1, \tag{33}$$

Then according to equation (12), we have

$$C_C = \phi^T\theta_0+\varepsilon. \tag{34}$$

Based on equation (19), we have

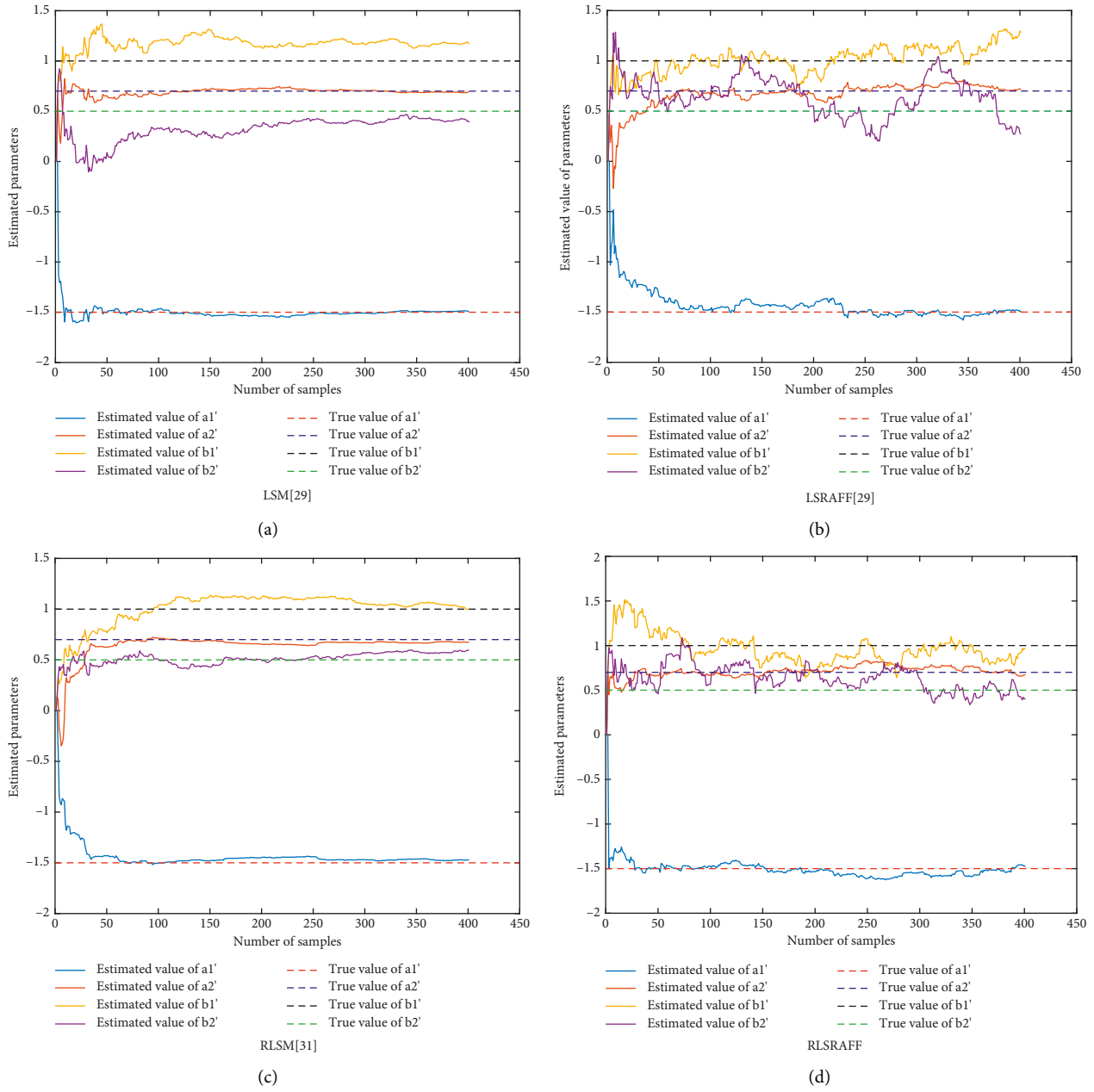

Figure 2: Parameter estimation variation process.

$$\begin{aligned}\theta_1(N+1) &= \theta_1(N) - K(N)\left[C_c(N+2) - \phi^T\theta(N)\right] \\ &= \theta_1(N) - K(N)\left[\phi^T\theta_0 + \varepsilon - \phi^T\theta(N)\right] \\ &= \left[I - K(N)\phi^T\right]\theta_1(N) - K(N)\varepsilon \\ &= \mu P(N+1)P^{-1}(N)\theta_1(N) - K(N)\varepsilon.\end{aligned} \tag{35}$$

We define

$$\begin{aligned}A(N+1) &= \mu P(N+1)P^{-1}(N) \\ &= \mu\left[\mu P^{-1}(N) + \phi\phi^T\right]^{-1}P^{-1}(N).\end{aligned} \tag{36}$$

Using (32), $\theta_1(N+1)$ can be obtained as

$$\theta_1(N+1) = A(N+1)\theta_1(N) - K(N)\varepsilon. \tag{37}$$

Further, we define the eigenvalue of matrix $A(N)$ is η, and then the following equation holds

$$A(N+1)x = \eta x, \tag{38}$$

where x is nonzero eigenvector. Substituting (32) into (34), we have

$$\mu(1-\eta)P^{-1}(N)x = \eta\phi\phi^T x. \tag{39}$$

Then, we get

$$\mu(1-\eta)x^T P^{-1}(N)x = \eta x^T\phi\phi^T x, \tag{40}$$

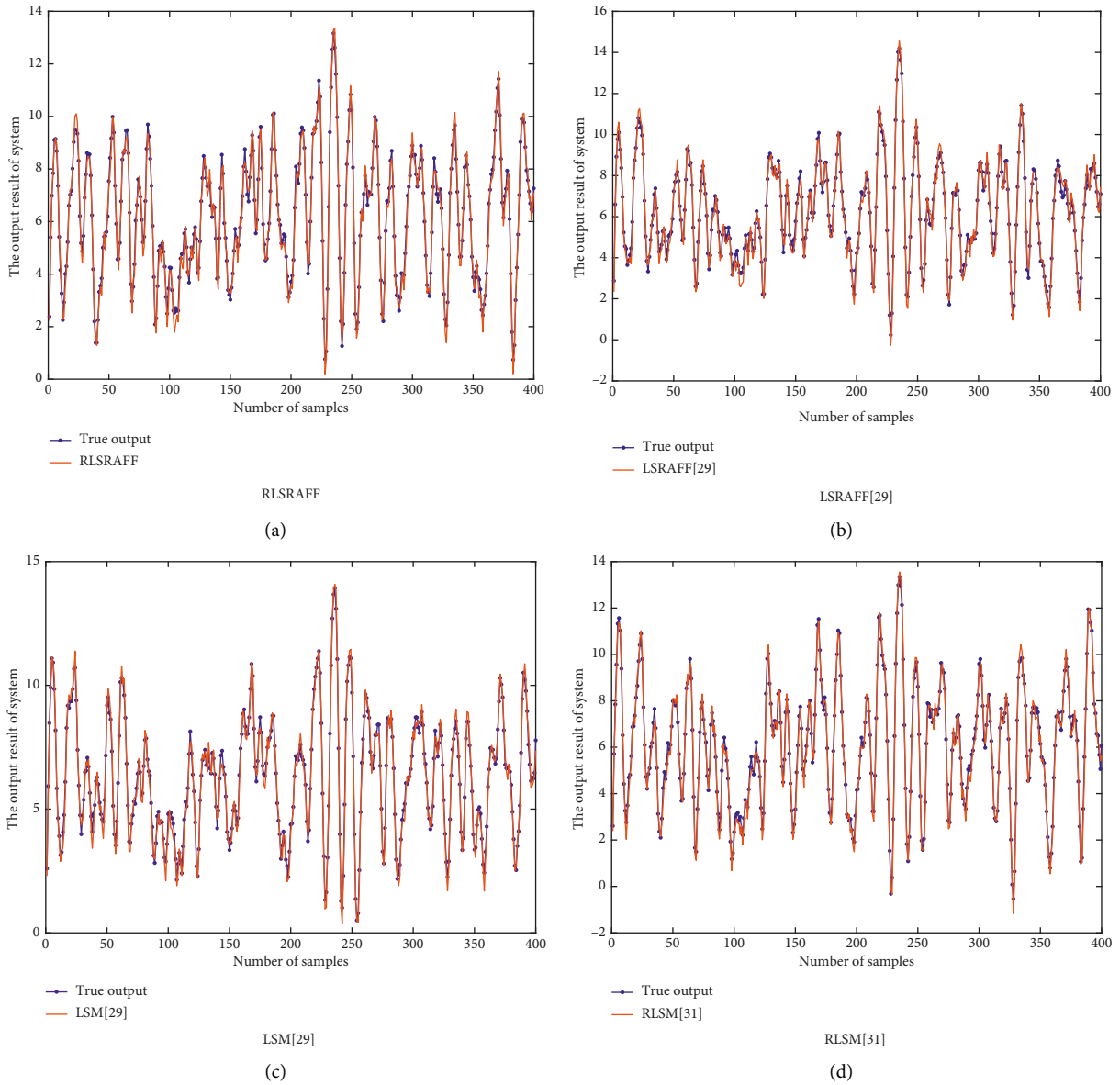

Figure 3: The output results of the system.

According to $\mu > 0, P^{-1}(N)$ and $\phi\phi T$ are all positive definite matrix. Thus, for all nonzero vector $x, \mu(1-\eta)$ and η are the same number on both sides of (36), we have

$$\frac{\mu(1-\eta)}{\eta} = \frac{\mu}{\eta} - \mu > 0, \tag{41}$$

we can gain that the eigenvalue of matrix $A(N)$ satisfies $0 < \eta < 1$, so (33) is stable at the balance point $\theta\prime 1 = 0$, and then according to (29), we have

$$\lim_{N \longrightarrow \infty} \theta_1(N) = 0. \tag{42}$$

Because $\theta_1(N) = \theta_0 - \theta(N)$, (28) is obtained. □

Theorem 3 has been proved.

4. Simulation Experiment

In this section, the proposed RLSRAFF is compared with other methods by two experiments. Firstly, the algorithm is compared with LSRAFF [30], the least squares method (LSM) [30], and the regularized least squares method (RLSM) [31] in Section 4.1 and Section 4.2. Then, the paper tests the performance of RLSRAFF in the parameter identification of the grinding process in Section 4.3.

4.1. Comparison of the Algorithm Performance. In this simulation, we choose the following equation to identify the model parameter as

$$\begin{aligned} z(k+2) = &-a_1' z(k+1) + a_2' z(k) \\ &+ b_1' u(k+1) + b_2' u(k) + \varepsilon(k), \end{aligned} \tag{43}$$

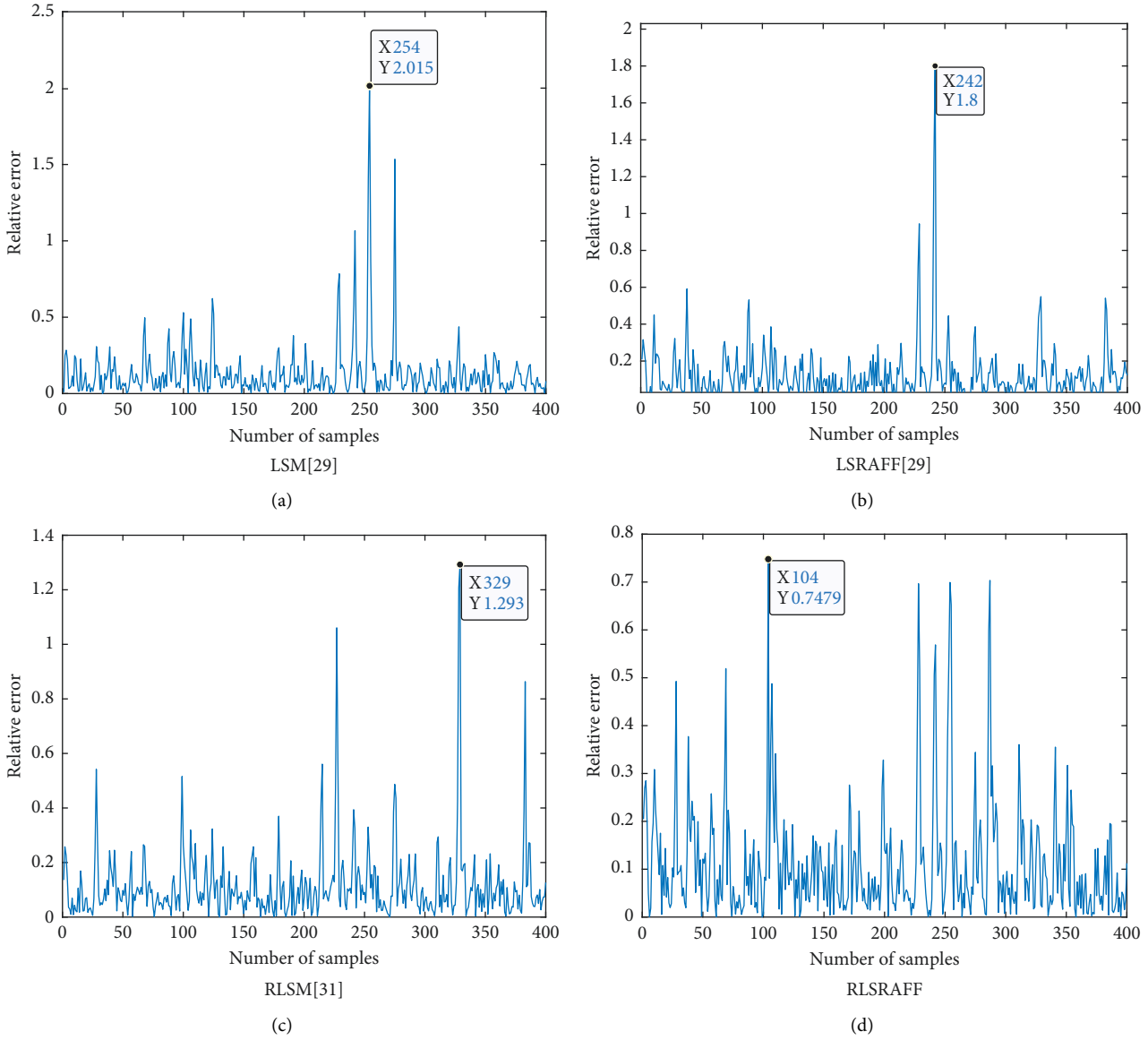

Figure 4: The relative error between the estimated value and the true value of system output.

where $a1 = -1.5, a2 = 0.7, b1 = 1.0, b2 = 0.5$; $\varepsilon(k)$ is a random variable that subjects to $N(0, 1)$ distribution; $u(k)$ is the system input, which is generated by fourth order M sequences (the amplitude is 1); $z(k)$ is the system output, and the length of observation data $L = 400$. Meanwhile, the forgetting factor is chosen as $\mu = 0.98$ in this simulation.

In this section, we choose 13 regularization parameters λ of the proposed RLSRAFF method to find the better identification results. The parameter identification results of different parameters λ are listed in Table 1. We can see from this table that the best one is given when $\lambda = 9$. Next, the proposed RLSRAFF, LSM [30], RLSM [30], and LSRAFF [31] methods are used to identify the identification results, which are given in Table 2. From this table, we can find that the proposed RLSRAFF method has better performance than others.

The parameter estimation process is shown in Figure 2. By observing the parameter estimation process from Figure 2, we can find that the curve (LSM and RLSM) changes very little in the later stage. However, the curve of RLSRAFF and LSRAFF [30] fluctuate all the time, and the error between the RLSRAFF curve and asymptotic property is smaller than the LSRAFF curve.

The output results of the system of these four algorithms are given in Figure 3, and the relative errors between the estimated value and the true value of system output are shown in Figure 4. From these figures, it can be seen that estimated values of system output obtained by the proposed RLSRAFF match with the true value very well. The average and maximum relative errors of system output of these four algorithms are described in Table 3, and the average and maximum relative error of RLSRAFF is 5.93% and 74.49%, respectively, which are smaller than others.

Table 3: The average and maximum relative error between the estimated value and the true value of system output.

Algorithm	Average relative error (%)	Maximum relative error (%)
LSM [30]	14.72	201.5
LSRAFF [30]	11.86	180.0
RLSM [31]	9.14	129.3
RLSRAFF	5.93	74.79

4.2. Statistical Results and Analysis. In this section, the performance of the proposed RLSRAFF method is tested by the statistical way. In this experiment, we select 10 groups of different parameters (a_1', a_2', b_1', b_2'), the true values of which are given in Table 3. The proposed RLSRAFF, LSM [30], RLSM [31], and LSRAFF [30] methods are used to identify these parameters, the results of which are shown in Table 4. From the identification results of Table 4, we can see that, the proposed RLSRAFF method gives a better result.

By analyzing the RLSRAFF algorithm, we can see that forgetting factors and regularization parameters are the key elements for this method. When forgetting factors are the same, the identification results of the two algorithms (RLSRAFF and LSRAFF [30]) with regularization parameters are better than those without regularization parameters. Furthermore, the forgetting factor of the proposed RLSRAFF method can eliminate the data saturated phenomenon.

Remark 3. According to equation (19), we can see that the proposed RLSRAFF method is equal to LSRAFF [30] when $\lambda = 0$. By analyzing the results of statistical results, it can be seen that the regularization parameters not only can solve ill-posed problems but also improve the identification results by adjusting the appropriate regularization parameters λ.

4.3. Application of Parameter Identification in the Grinding Process. In this section, the data in the grinding process are given to identify the parameters in the model by the proposed RLSRAFF. We set the sampling time is 10 seconds and collect 70 groups of data. The sampling data are shown in Table 5 [27]. In this table, the time is from 11 : 45 : 14 to 11 : 56 : 54; the process data of the ore feeding quantity is set from 25 t/h to 233 t/h; the process data of the classifier adding water quantity is set from 5.5 m^3/h to 35 m^3/h; the process data of the sand return quantity is set from 22 m^3/h to 105 m^3/h; the process data of the classifier overflow concentration is set from 42% to 92%, the forgetting factor is 0.3, and 0.5*randn is added in the sampling data [27].

The observation data of input variables and output variables are selected for identifying the parameters in the grinding process model and RLSRAFF is used. The average relative error between the calculated value and the actual value with different regularization parameters λ are used to test the performance of RLSRAFF, and the results are given in Table 6. The best result is given when $\lambda = 8$.

Table 4: Identification results of 10 experiments.

Parameters	a_1'	a_2'	b_1'	b_2'
Truth value	−1.5	1	1.5	1
RLSRAFF	−1.5009	0.9989	1.4985	0.9958
RLSM [31]	−1.4965	0.9984	1.4874	1.0582
LSM [30]	−1.5039	1.0044	1.5852	0.8855
LSRAFF [30]	−1.4980	0.9938	1.4812	0.9872
Truth value	−1.0	0.8	3.0	4.0
RLSRAFF	-0.9963	0.7996	2.9921	3.9940
RLSM [31]	−1.0191	0.8023	2.9365	3.8294
LSM [30]	−1.1040	0.8143	3.1093	3.7783
LSRAFF [30]	−1.0048	0.8054	3.0474	4.0382
Truth value	−0.9	0.3	2.0	3.0
RLSRAFF	−0.9037	0.3017	2.0170	2.9730
RLSM [31]	−0.9058	0.3175	2.0299	2.9587
LSM [30]	−0.9306	0.3233	1.9454	3.1175
LSRAFF [30]	−0.9257	0.3096	1.9708	3.0338
Truth value	−0.5	0.1	4.0	10.0
RLSRAFF	−0.4989	0.1008	3.9981	9.9925
RLSM [31]	−0.5128	0.0959	3.9381	9.9606
LSM [30]	−0.4790	0.0945	3.8541	9.9446
LSRAFF [30]	−0.4885	0.0948	4.1090	10.0459
Truth value	−0.1	0.4	10.0	7.0
RLSRAFF	−0.1021	0.4020	9.9249	6.9556
RLSM [31]	−0.0857	0.3694	9.8860	6.9263
LSM [30]	−0.0798	0.4077	10.0263	7.0824
LSRAFF [30]	−0.1077	0.3979	9.8964	6.8515
Truth value	−0.8	1.0	−2.0	6.0
RLSRAFF	−0.8015	0.0010	−2.0104	5.9948
RLSM [31]	−0.8013	0.9965	−1.8112	5.9769
LSM [30]	−0.8022	1.0106	−1.9778	6.1568
LSRAFF [30]	−0.8020	1.0003	−1.9851	5.9752
Truth value	−0.7	0.4	2.0	−3.0
RLSRAFF	−0.7011	0.4016	2.0014	−2.9984
RLSM [31]	−0.7157	0.3897	1.9152	−2.9785
LSM [30]	−0.6705	0.3668	1.8951	−3.0987
LSRAFF [30]	−0.7381	0.3899	2.1158	−2.9857
Truth value	−0.6	−0.2	−4.0	−3.0
RLSRAFF	−0.6060	−0.1988	−4.0408	−3.0135
RLSM [31]	−0.6247	−0.1962	−3.9870	−2.9425
LSM [30]	-0.6309	-0.1755	-4.0537	−2.8846
LSRAFF [30]	−0.5882	−0.2156	−3.9819	−2.9502
Truth value	0.3	0.4	8.0	5.5
RLSRAFF	0.3004	0.4006	7.9987	5.4976
RLSM [31]	0.2921	0.3982	7.9762	5.5475
LSM [30]	0.2834	0.3877	7.8914	5.3927
LSRAFF [30]	0.2895	0.3989	8.0239	5.4837
Truth value	0.3	−0.5	−3.0	1.0
RLSRAFF	0.2935	−0.4956	−3.0148	0.9929
RLSM [31]	0.3283	−0.4890	−2.9772	0.9802
LSM [30]	0.2553	−0.5161	−2.8933	1.0424
LSRAFF [30]	0.2621	−0.5128	−3.0151	1.0201

The experiment compares RLSRAFF and LSRAFF [27] (We select the obtained parameters $\theta^T = [-0.303\ -0.409\ 0.236\ 0.136]$ as the parameter identification results of LSRAFF in [27].), and the output results are given in Figure 5, which compares the output contrasting curves obtained by RLSRAFF and LSRAFF. It is observed from this figure that RLSRAFF can achieve a better performance than LSRAFF.

Table 5: Sampling data [27].

Sequence	Time	Ore feeding quantity (t/h)	Classifier adding water quantity (m^3/h)	Sand return quantity (m^3/h)	Classifier overflow concentration (m^3/h)
1	11 : 45 : 14	24.884	5.457	22.114	42.05
2	11 : 45 : 24	27.162	5.496	30.506	58.306
3	11 : 45 : 34	34.782	5.762	31.118	62.482
...	...	...	...	...	...
70	11 : 56 : 54	233.091	35.059	105.131	92.101

Table 6: The average relative error between the calculated value and the actual value.

Regularization parameters	Average relative error (%)	Regularization parameters	Average relative error (%)
$\lambda = 1$	3.87	$\lambda = 7$	3.85
$\lambda = 2$	3.94	$\lambda = 8$	3.83
$\lambda = 3$	4.02	$\lambda = 9$	3.88
$\lambda = 4$	4.12	$\lambda = 10$	3.89
$\lambda = 5$	4.00	$\lambda = 11$	3.94
$\lambda = 6$	3.93	$\lambda = 12$	4.03

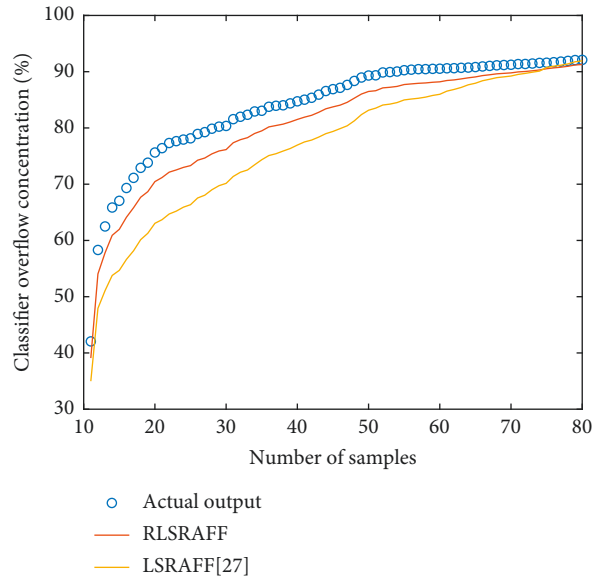

Figure 5: Output of the system.

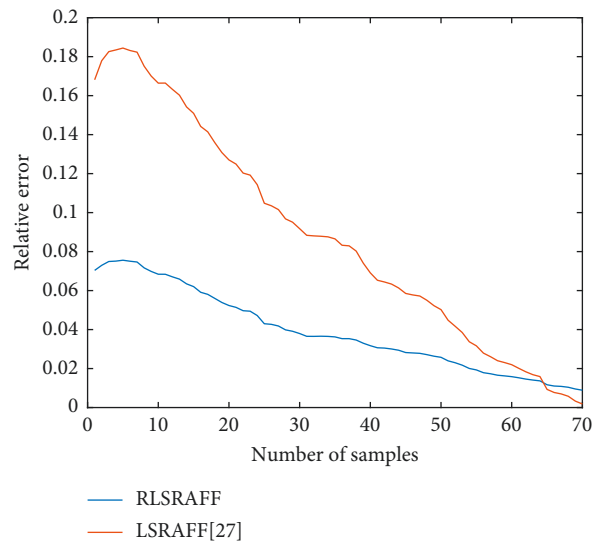

Figure 6: The relative error of the RLSRAFF method and the LSRAFF method [27].

Table 7: A List of symbols.

Symbols	Definition
$\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5; \beta_1, \beta_2, \beta_3, \beta_4$	Coefficient of inertia link
$\tau_1, \tau_2, \tau_3, \tau_4$	Coefficient of delay link
$G'_{11}(S), G'_{12}(S), G'_{21}(S), G'_{22}(S)$	Transfer function with delay link and inertia link
$G_{11}(S), G_{12}(S), G_{21}(S), G_{22}(S)$	Transfer function with inertia link
U_1	The actual value of ore feeding quantity (t/h)
U_3	The actual value of classifier adding water quantity (m^3/h)
Q_D	The actual value of the return sand quantity (m^3/h)
C_c	The actual value of classifier overflow concentration (%)
k	Observed times $(k = 2, 3, \ldots, 1 + N)$
i	$i = 1, 2, \ldots, N$
$U_3(k)$, $U_1(k)$U1K	The actual input signal of recorded data
$C_c(k)$	The actual output signal of recorded data
$a_1, b_1, c_1, d_1; a_2, a_3, b_2, b_3, c_2, d_2$a	Parameters to be identified
$\varepsilon(k)$	Uncorrelated random variable that subjects to $N(0,1)$ distribution
$N(0,1)$	Normal distribution
$\theta(N)$	$\theta(N) = [0.5a_1, 0.5d_1, 0.5b_1, 0.5c_1]^T$
$C_c(N)$	$C_c(N) = [C_c(1+1), C_c(1+2), \ldots, C_c(1+N)]^T$
$\Phi(N)$	$\Phi(N) = [\Phi^T(1), \Phi^T(2), \ldots, \Phi^T(N)]^T$
$\phi^T(i)$	$\phi^T(i) = [-C_c(i), -C_c(i), U_3(i), U_1(i)]^T$
F	The sum of residuals squares
N	Number of data group
μ	Forgetting factor $(0 < \mu \leq 1))$
ρ	A factor $(\rho^2 = \mu)$
λ	Regularization parameter
$K(N)$	Gain matrix
$P(N)$	$P(N) = (\phi^T\phi + \lambda I)^{-1}$
$J(N)$	Criterion function of parameter estimation at N
θ_0	True value of model parameters
η	Eigenvalue of matrix $A(N+1))$
x	Nonzero eigenvector
$u(k)$	System input of equation (43)
$z(k)$	System output of equation (43)
a'_1, a'_2, b'_1, b'_2	Parameters to be identified of equation (43)
L	Length of observation data

Further, the relative errors of the RLSRAFF method and the LSRAFF method are given in Figure 6, and the relative error of RLSRAFF is smaller than LSRAFF. Meanwhile, the average relative error of RLSRAFF is 3.83%, and the maximum relative error is 8.02%, but for LSRAFF is 8.79% and 18.32%, respectively. Therefore, RLSRAFF gives a better performance.

5. Conclusion

In the actual industrial production process, a large amount of data is often obtained by online real-time measurement, and the measured data often contains some error. Thus, the data saturated phenomenon and the ill-posed problem often occur simultaneously. The original LSM seldom considers the above effects to the identification of parameters. Therefore, this paper investigates the identification of parameters in the grinding process and considers the data saturated phenomenon and the ill-posed problem.

In order to solve the above problems, this paper presents a RLSRAFF algorithm, which combines the forgetting factor with regularization parameters. Furthermore, to analyze the performance of the RLSRAFF algorithm, this paper verifies the recursive calculation of criterion function and describes the effect of calculation error from the gain matrix and proves the convergence of the proposed algorithm. Finally, effectiveness of RLSRAFF is verified by the simulation experiments and grinding data. Comparing with other algorithms, the results show that RLSRAFF gives a better result and eliminates the ill-posed problem by the simulation and statistical results.

RLSRAFF is a universal method, the aim of which is to solve the data saturated phenomenon and the ill-posed problem. Both problems often occurs in the parameter identification process, such as the lithium ion batteries [32] and the wastewater treatment process [5]. Therefore, the RLSRAFF method and its extension method also can be applied in other fields. Furthermore, for LSM-based identification methods, noises can cause biased identification results. Some noise-compensated methods [33–35] can be used to reduce these biases. In the future research work, the noise-compensated methods can be considered to improve the RLSRAFF method and be applied in the grinding process (see Table 7).

Data Availability

The first experiment data used to support the findings of this study are included within the article. Previously reported

(Research on Modeling and Control Method in Grinding and Classification Process) data were used to support this study and are available at DOI or other persistent identifier. These prior studies (and datasets) are cited at relevant places within the text as references [26].

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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References

[1] J. Li, Y. Zheng, and Z. Lin, "Recursive identification of time-varying systems: self-tuning and matrix RLS algorithms," *Systems & Control Letters*, vol. 66, pp. 104–110, 2014.

[2] R. N. Wayne and R. Gash, "Least squares identification of linear sway-yaw manoeuvering coefficients and drag-area parameters of ships, proceedings of the Institution of Mechanical Engineers," *Journal of Engineering for the Maritime Environment*, vol. 235, 2021.

[3] Z. Zhou, Y. Rui, and X. Cai, "Constrained total least squares method using TDOA measurements for jointly estimating acoustic emission source and wave velocity," *Measurement*, vol. 182, Article ID 109758, 2021.

[4] Z. Huang, Z. Zhu, and Q. An, "Global–local image enhancement with contrast improvement based on weighted least squares," *Optik-International Journal for Light and Electron Optics*, vol. 2, Article ID 167433, 2021.

[5] C. Yang, Y. Zhang, and M. Huang, "Adaptive dynamic prediction of effluent quality in wastewater treatment processes using partial least squares embedded with relevance vector machine," *Journal of Cleaner Production*, vol. 314, 2021.

[6] L. M. Ramezani and F. A. Rahmani, "An improved node moving technique for adaptive analysis using collocated discrete least squares meshless method," *Engineering Analysis with Boundary Elements*, vol. 130, pp. 322–331, 2021.

[7] H. Xing and Z. He, "Adaptive loss function based least squares one-class support vector machine," *Pattern Recognition Letters*, vol. 156, pp. 174–182, 2022.

[8] Y. Xiong, W. Yang, and H. Liao, "Soft variable selection combining partial least squares and attention mechanism for multivariable calibration," *Chemometrics and Intelligent Laboratory Systems*, vol. 223, Article ID 104532, 2022.

[9] W. Zhang, Z. Luo, and S. Guo, "An improved least square similitude method based on Lagrange energy for estimating scaling laws and eliminating coupling effects," *Thin-Walled Structures*, vol. 174, Article ID 109018, 2022.

[10] C. Paleologu, J. Benesty, and S. Ciochina, "A robust variable forgetting factor recursive least squares algorithm for system identification," *IEEE Signal Processing Letters*, vol. 15, pp. 597–600, 2008.

[11] B. Simone, Z. Zhang, and D. Liu, "Eligibility traces and forgetting factor in recursive least-squares- based temporal difference," *International Journal of Adaptive Control and Signal Processing*, vol. 36, pp. 334–353, 2021.

[12] X. Sun, J. Ji, and B. Ren, "Adaptive forgetting factor recursive least square algorithm for online identification of equivalent circuit model parameters of a lithium-ion battery," *Energies*, vol. 12, p. 2242, 2019.

[13] L. Wang, T. Chen, and C. Zou, "Weighted least squares regularization iterative method and accuracy evaluation of ill conditioned multiplicative error model," *Journal of Surveying and mapping*, vol. 50, pp. 589–599, 2021.

[14] S. Zou, X. Wu, and F. Wang, "Regularized robust method for ill conditioned weighted total least squares model," *Geodesy and geodynamics*, vol. 41, pp. 1106–1110, 2021.

[15] J. Jin, H. Tang, and S. Gao, "A method of optimizing the performance of memory computing integrated chip combined with kurtosis regularization," *Journal of Functional Materials and Devices*, vol. 27, pp. 514–518, 2021.

[16] Y. Teng, S. Qi, and F. Han, "A framework for least squares nonnegative matrix factorizations with Tikhonov regularization," *Neurocomputing*, vol. 387, pp. 78–90, 2020.

[17] P. Zhou, T. Chai, and J. Sun, "Intelligence-based supervisory control for optimal operation of a DCS-controlled grinding system," *IEEE Transactions on Control Systems Technology*, vol. 21, pp. 162–175, 2013.

[18] Y. Chi, H. Yue, and J. Ding, "Design and implementation of the simulation experiment for grinding process," *Control Engineering China*, vol. 15, pp. 598–601, 2008.

[19] L. A. Houseman, J. H. Schubert, and J. R. Hart, "Plant Star 2000:a plant-wide control platform for minerals processing," *Minerals Engineering*, vol. 14, pp. 593–600, 2001.

[20] G. Chu, "Explore the current situation of automatic control strategy of grinding and classification system," *World nonferrous metals*, vol. 24, pp. 12–14, 2018.

[21] Y. Wu, Q. Zhang, and S. Huang, "Research Progress on intelligent control of grinding and classification," *Nonferrous Metals*, vol. 1, pp. 88–94, 2021.

[22] Y. Du, D. Liu, and R. Sun, "Development trend of intelligent grinding and classification," *Nonferrous Metals*, vol. 01, pp. 95–100, 2021.

[23] Y. Liu and C. Han, "Research on optimization of grinding and classification process based on case-based reasoning," *China Metal Bulletin*, vol. 04, pp. 178–180, 2020.

[24] Y. Zhang, Z. Chen, and X. Cai, "RBF grinding classification optimization control process based on adaptive law," *Digital technology and application*, vol. 37, pp. 1-2, 2019.

[25] Y. Wang, Z. Liu, and X. Wang, "Prediction method of ball wear law of ball mill based on differential evolution," *Journal of East China University of Science and Technology*, vol. 40, pp. 349–356, 2014.

[26] Z. Liu, *Nonlinear Parameter Identification of Bauxite Grinding and Classification Process Based on Differential Evolution*, Central South University, Changsha, China, 2014.

[27] Y. Chen, *Research on Modeling and Control Method in Grinding and Classification Process*, Hebei University of technology, Tianjin, China, 2016.

[28] G. Huang, Y. Liu, and F. Yin, "Tikhonov regularization with MTRSVD method for solving large-scale discrete ill-posed problems," *Journal of Computational and Applied Mathematics*, vol. 405, Article ID 113969, 2022.

[29] R. P. King, "Modeling and simulation of mineral processing systems," *Butterworth Heinemann*, 2001.

[30] Z. Fang and D. Xiao, *Process Identification*, Tsinghua University Press, Beijing, China, 1988.

[31] F. Luo, *Restricted Preconditioned Conjugate Gradient Method for Solving Weighted Toeplitz Regularized Least Squares Problem*, Lanzhou University, Lanzhou, China, 2021.
[32] J. Shi, H. Guo, and D. Chen, "Parameter identification method for lithium-ion batteries based on recursive least square with sliding window difference forgetting factor," *Journal of Energy Storage*, vol. 44, p. 103485, 2021.
[33] Z. Wei, D. Zhao, and H. He, "A noise-tolerant model parameterization method for lithium-ion battery management system," *Applied Energy*, vol. 268, Article ID 114932, 2020.
[34] Z. Wei, S. Meng, and B. Xiong, "Enhanced online model identification and state of charge estimation for lithium-ion battery with a FBCRLS based observer," *Applied Energy*, vol. 181, pp. 332–341, 2016.
[35] J. Hu, H. He, and S. Member, "Disturbance-immune and aging-robust internal short circuit diagnostic for lithium-ion battery, IEEE transactions on industrial electronics," *Chimia*, vol. 69, Article ID 3063968, 2021.