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Research Article

A Novel Discrete DGM (2, 1, t^2) Model Based on Stochastic Oscillation Sequence and Its Application

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This article aims to improve the prediction accuracy of the grey model for stochastic oscillation sequence, to overcome the defect of the direct span from difference to differential and to better depict the growth trend of data series. Firstly, the accelerated translation smoothing transformation is constructed, which transforms the stochastic oscillation sequence into the monotonic increasing sequence that is suitable for modeling. Secondly, the quadratic time power term is introduced to establish a novel discrete DGM (2, 1, t^2) model, then derive a novel simulation prediction formula, and analyze properties of this novel model; it was concluded that its application scope is expanded. Finally, a novel discrete DGM (2, 1, t^2) model based on a stochastic oscillation sequence is established to forecast a group of stochastic oscillation sequences with different amplitudes and two different city traffic flow series and compare the prediction accuracy with other grey models; the results show that the feasibility of this model is verified and has good simulated prediction effect in the field of traffic flow.

1. Introduction

The grey prediction model is an important component of the grey system theory [1], and the DGM (2, 1) model has been widely used in various fields [2–6]. Moreover, many scholars have performed different aspects of research and improvement for the model: optimizing the grey derivative of the DGM (2, 1) model [7], the parameter optimization for the DGM (2, 1) model [8], and studying the predictive performance of the DGM (2, 1) model and its improved models [9, 10].

For the traditional grey model, system parameters are estimated by the differential equation; however, the time response is estimated by the whitening differential equation; this leap from difference directly to differential will produce large errors. With the deepening of research, some scholars have found that discrete models can overcome this defect, and applied the discrete ideas to various grey models [11–14].

Owing to a wide variety of factors, the sequences are mostly stochastic oscillation sequences in some practical applications, and the simulated prediction accuracy for those nonmonotonic sequences is not ideal. Therefore, in order to make up for this defect, scholars have researched both data and models, respectively.

1.1. Data Aspects. Scholars have performed data treatment, which improves the smoothness of the stochastic oscillation sequence and presented various data transformations: a weighting mean value generating transformation, a translation transformation, and a geometric mean transformation, which maintains the monotony, weakens the randomness, and improves the smoothness of the original sequence [15–17]; a smoothness operator can compress the amplitude of the stochastic oscillation sequence [18–20]; an empirical mode decomposition (EDM) algorithm is introduced to decompose the original stochastic oscillation sequences data information [21].

1.2. Model Aspects. Some scholars make up for this deficiency with other different modeling ideas: a new grey interval model is constructed by building envelope lines of the original oscillation sequence [22–24], the structure of the grey prediction model is improved by introducing periodic operator of triangular function [25], and an optimizing AF-MNGBM(1, 1) model is established by combining a self-adaptive artificial fish swarm algorithm (SAFSA) and the metabolic method, to overcome the randomness of the stochastic oscillation sequences [26].

The DGM (2, 1) model is a class of monotonic second-order linear dynamic models that will fit better to data with strongly changing trends. So, in this article, the DGM (2,1) model is used to simulate the stochastic oscillation sequence. Firstly, this article applies an accelerated translation smoothing transformation to the original stochastic oscillation sequence, turning it into a monotonic increasing sequence. Then, in view of the error problem of the transition from difference to the differential of the model and the advantage of introducing the time power in the model to characterize the increasing trend of its data series [27], a model's structure is an important factor affecting the performance of the model. The structure of the existing DGM (2, 1) grey prediction models is simple, which leads to poor simulation and prediction performance of the model [28]; the discrete DGM (2, 1, t^2) model is constructed. Furthermore, this article derives the simulation prediction formula of the model and analyzes the property of this model, and the scope of application is expanded for this novel model. Finally, this novel model is built based on a batch of stochastic oscillation sequences with different amplitudes and two city traffic flow, the prediction effect of this novel model and the other four grey models is compared, and the result indicated that this model has the feasibility and good simulated prediction effect.

2. Principle of the Stochastic Oscillation Sequence and the Data Transformation

2.1. Principle of the Stochastic Oscillation Sequence

Definition 1 (see [1]). Assume that a sequence $Y = \{y(1), y(2), \cdots, y(n)\}$ is given, then

(1) If for $\forall k \in \{2, 3, \cdots, n\}$, $y(k) - y(k-1) > 0$, then Y is called a monotonic increasing sequence.

(2) If for $\forall k \in \{2, 3, \cdots, n\}$, $y(k) - y(k-1) < 0$, then Y is called a monotonic decreasing sequence.

(3) If for $\exists k, k' \in \{2, 3, \cdots, n\}$, $y(k) - y(k-1) > 0$ and $y(k') - y(k'-1) < 0$, then Y is called a stochastic oscillation sequence. And$M = \max\{y(k) \mid k = 1, 2, \cdots, n\}$, $m = \min\{y(k) \mid k = 1, 2, \cdots, n\}$, then $M - m$ is called the amplitude of Y.

2.2. Accelerated Translation Smoothing Transformation

Definition 2 (see [15]). Assume that a sequence $Y = \{y(1), y(2), \cdots, y(n)\}$, T is the amplitude of$Y$, then

$$f_1(y(k)) = y(k) + (k-1)T, \tag{1}$$

is called accelerated translation transformation, where

$$\begin{aligned} T &= M - m, \\ M &= \max\{y(k) \mid k = 1, 2, \cdots, n\}, \\ m &= \min\{y(k) \mid k = 1, 2, \cdots, n\}. \end{aligned} \tag{2}$$

Definition 3 (see [18]). Let $X = \{y(1), y(2), \cdots, y(n)\}$, $y(k) > 0$, $k = 1, 2, \cdots, n-1$ be a sequence, and another sequence

$$YD = \{y(1)d, y(2)d, \cdots, y(n)d\}, \quad k = 1, 2, \cdots, n-1, \tag{3}$$

where

$$y(k)d = \frac{[y(k) + T] + [y(k+1) + T]}{4}, \tag{4}$$

where $T = M - m$ is the amplitude of$Y$, $M = \max\{y(k) \mid k = 1, 2, \cdots, n\}$, $m = \min\{y(k) | k = 1, 2, \cdots, n\}$, D is a sequence operator and called a (first-order) smoothness operator of$Y$, and $Y\ D$ is called a smoothness sequence of Y.

Definition 4. Assume that a sequence $Y = \{y(1), y(2), \cdots, y(n)\}$, $y(k) > 0k = 1, 2, \cdots, n-1$, and $T = M - m (T > 1)$, where $M = \max\{y(k) | k = 1, 2, \cdots, n\}$, $m = \min\{y(k) | k = 1, 2, \cdots, n\}$, then

$$f_2(y(k)) = \frac{[y(k) + (k-1)T] + [y(k+1) + kT]}{4}, \tag{5}$$

is called accelerated translation smoothing transformation.

And

$$\begin{aligned} f_2(y(k)) &= \frac{[y(k) + (k-1)T] + [y(k+1) + kT]}{4} \\ &= \frac{1}{4}[y(k) + y(k+1) + (2k-1)T], \end{aligned} \tag{6}$$

where $4 > 0$; the discrete grey model will not change the relative error of the simulation and will not change the simulation and prediction effect of the model [29]. The following calculation example analysis also proves this.

Theorem 1. *For any sequence $Y = \{y(1), y(2), \cdots, y(n)\}$, $y(k) > 0k = 1, 2, \cdots, n-1$, the sequence obtained after the accelerated translation smoothing transformation is monotonic increasing ones, namely,*

$$x(k) < x(k+1), \quad k = 1, 2, \cdots, n-1, \tag{7}$$

where $X = \{x(1), x(2), \cdots, x(n)\}$ is transformed sequence, and

$$\begin{aligned} x(k) &= f_2(y(k)) \\ &= \frac{[y(k) + (k-1)T] + [y(k+1) + kT]}{4}. \end{aligned} \tag{8}$$

Proof. Assume that a sequence $Y = \{y(1), y(2), \cdots, y(n)\}$, $y(k) > 0k = 1, 2, \cdots, n-1T = M - m$, where $M = \max\{y(k) | k = 1, 2, \cdots, n\}$, $m = \min\{y(k) | k = 1, 2, \cdots, n\}$; according to (5), we get

$$\begin{aligned} x(k)-x(k+1) &= f_2(y(k))-f_2(y(k+1)) \\ &= \frac{y(k)+y(k+1)+(2k-1)T}{4} \\ &\quad -\frac{y(k+1)+y(k+2)+(2k+1)T}{4} \\ &= \frac{y(k)-y(k+2)-2T}{4}, \end{aligned} \tag{9}$$

and $y(k)-y(k+2)\le T$, therefore $x(k)<x(k+1)$, $k=1,2,\cdots,n-1$. □

3. A New Discrete DGM (2, 1, t^2) Model Based on Stochastic Oscillation Sequence

3.1. The Construction of the Discrete DGM (2, 1, t^2) Model Based on Stochastic Oscillation Sequence. The system parameters are estimated by the difference equations, and the time response is estimated by the whitening differential equations in the original DGM (2, 1) model; crossing directly from the differential equation to the differential equation will cause errors. Moreover, there is the problem of a constant growth rate of the simulated values arises for the original DGM(2, 1) model [30], and according to the modeling mechanism in literature [31], the time power is introduced to establish a discrete DGM (2, 1, t^2) model to achieve the unification of the estimated system parameters and the time response formula. Moreover, modeled with a nonnegative smoothing sequence $X^{(0)}=\{x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n)\}$, a sequence is formed by an accelerated translation smoothing transformation for a stochastic oscillation sequence.

Definition 5. Let $X^{(0)}=\{x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n)\}$ be a transformed nonnegative smoothing sequence. Its 1-AGO (accumulating generation operator) sequence is $X^{(1)}=\{x^{(1)}(1),x^{(1)}(2),\cdots,x^{(1)}(n)\}$, where $x^{(1)}(k)=\sum_{i=1}^{k} x^{(0)}(i)$, $k=1,2,\cdots,n$, then

$$\frac{d^2x^{(1)}}{dt^2}+a\frac{dx^{(1)}}{dt}=bk^2+ck+d, \tag{10}$$

is called the whitening differential equation of the DGM(2, 1 t^2) model.

According to the discrete whitening differential equation by the second-order difference of the 1-AGO sequence $X^{(1)}$ of$X^{(0)}$ [32] and according to the modeling mechanism of literature [33], the derivative in (10) can be approximated by

$$\begin{aligned} \frac{dx^{(1)}}{dt}\Big|_{t=k} &\approx \lim_{\Delta t}\frac{\Delta x^{(1)}(t)}{\Delta t}\Big|_{t=k}=\frac{x^{(1)}(k)-x^{(1)}(k-1)}{(k)-(k-1)} \\ &= x^{(1)}(k)-x^{(1)}(k-1), \\ \frac{d^2x^{(1)}}{dt^2}\Big|_{t=k} &\approx \frac{dx^{(1)}(k)}{dt}-\frac{dx^{(1)}(k-1)}{dt}. \end{aligned} \tag{11}$$

Then, substituting them into (10), we get

$$x^{(1)}(k)=\beta_1x^{(1)}(k-1)+\beta_2x^{(1)}(k-2)+\beta_3k^2+\beta_4k+\beta_5, \tag{12}$$

which is called discrete DGM(2, 1, t^2) model where

$$\begin{aligned} \beta_1&=\frac{2+a}{1+a}, \\ \beta_2&=\frac{-1}{1+a}, \\ \beta_3&=\frac{b}{1+a}, \\ \beta_4&=\frac{c}{1+a}, \\ \beta_5&=\frac{d}{1+a}. \end{aligned} \tag{13}$$

Theorem 2. *Assume that a nonnegative sequence $X^{(0)}=\{x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n)\}$, then another sequence $X^{(1)}=\{x^{(1)}(1),x^{(1)}(2),\cdots,x^{(1)}(n)\}$ is the 1-AGO series of $X^{(0)}$, where $x^{(1)}(k)=\sum_{i=1}^{k}x^{(0)}(i)$, $k=1,2,\cdots,n$ and then the parameter column $\widehat{\beta}=(\beta_1,\beta_2,\beta_3,\beta_4,\beta_5)^T$, and*

$$\begin{aligned} B&=\begin{bmatrix} x^{(1)}(2) & x^{(1)}(1) & 3^2 & 3 & 1 \\ x^{(1)}(3) & x^{(1)}(2) & 4^2 & 4 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x^{(1)}(n-1) & x^{(1)}(n-2) & n^2 & n & 1 \end{bmatrix}, \\ Y&=\begin{bmatrix} x^{(1)}(3) \\ x^{(1)}(4) \\ \vdots \\ x^{(1)}(n) \end{bmatrix}, \end{aligned} \tag{14}$$

then from the least square estimation, parameters of the discrete DGM (2, 1 t^2) model are satisfied:

$$\widehat{\beta}=(\beta_1,\beta_2,\beta_3,\beta_4,\beta_5)^T=\left(B^TB\right)^{-1}B^TY. \tag{15}$$

Proof. According to (12) in Definition 5, we get

$$\begin{aligned} x^{(1)}(3)&=\beta_1x^{(1)}(2)+\beta_2x^{(1)}(1)+\beta_3 3^2+\beta_4 3+\beta_5 \\ x^{(1)}(4)&=\beta_1x^{(1)}(3)+\beta_2x^{(1)}(2)+\beta_3 4^2+\beta_4 4+\beta_5 \\ &\cdots \\ x^{(1)}(n)&=\beta_1x^{(1)}(n-1) \\ &=+\beta_2x^{(1)}(n-2)+\beta_3n^2+\beta_4n+\beta_5, \end{aligned} \tag{16}$$

then $Y=B\widehat{\beta}=B[\beta_1,\beta_2,\beta_3,\beta_4,\beta_5]^T$, error sequence $\varepsilon=Y-B\widehat{\beta}$ can be obtained, and then assuming

$$s = \varepsilon^T \varepsilon = (Y - B\widehat{\beta})^T (Y - B\widehat{\beta}) = \sum_{k=3}^{n} \left[x^{(1)}(k) - \beta_1 x^{(1)}(k-1) - \beta_2 x^{(1)}(k-2) - \beta_3 k^2 - \beta_4 k - \beta_5\right]^2, \quad (17)$$

we get

$$\begin{aligned}
\frac{\partial s}{\partial \beta_1} &= -2\sum_{k=3}^{n}\left[x^{(1)}(k) - \beta_1 x^{(1)}(k-1) - \beta_2 x^{(1)}(k-2) - \beta_3 k^2 - \beta_4 k - \beta_5\right] x^{(1)}(k-1) = 0,\\
\frac{\partial s}{\partial \beta_2} &= -2\sum_{k=3}^{n}\left[x^{(1)}(k) - \beta_1 x^{(1)}(k-1) - \beta_2 x^{(1)}(k-2) - \beta_3 k^2 - \beta_4 k - \beta_5\right] x^{(1)}(k-2) = 0,\\
\frac{\partial s}{\partial \beta_3} &= -2\sum_{k=3}^{n}\left[x^{(1)}(k) - \beta_1 x^{(1)}(k-1) - \beta_2 x^{(1)}(k-2) - \beta_3 k^2 - \beta_4 k - \beta_5\right] k^2 = 0,\\
\frac{\partial s}{\partial \beta_4} &= -2\sum_{k=3}^{n}\left[x^{(1)}(k) - \beta_1 x^{(1)}(k-1) - \beta_2 x^{(1)}(k-2) - \beta_3 k^2 - \beta_4 k - \beta_5\right] k = 0,\\
\frac{\partial s}{\partial \beta_5} &= -2\sum_{k=3}^{n}\left[x^{(1)}(k) - \beta_1 x^{(1)}(k-1) - \beta_2 x^{(1)}(k-2) - \beta_3 k^2 - \beta_4 k - \beta_5\right] = 0,
\end{aligned} \quad (18)$$

and by means of the above system of homogeneous linear equations, we can get the parameter column $\widehat{\beta} = (\beta_1, \beta_2, \beta_3, \beta_4, \beta_5)^T$; then, from $Y = B\widehat{\beta} = B[\beta_1, \beta_2, \beta_3, \beta_4, \beta_5]^T$, we can get $\widehat{\beta} = (B^T B)^{-1} B^T Y$. □

3.2. Simulation Prediction Formula in the General Term Form of the Discrete DGM (2, 1, t^2) Model. The discrete model in this paper is a recursive form of the simulation prediction formula, and it can be directly used for simulation prediction. However, such formulations are more complex and may bring relatively large cumulative errors for medium- and long-term predictions. In addition, the simulation prediction formula in the recursive form will directly mask the formal features of the sequence itself [32]. If medium- and long-term prediction is needed in the case, it is necessary to derive the simulation prediction formula in the form of model general term which overcomes the above defects and is completely equivalent to the discrete model.

Theorem 3. *Assume that a nonnegative sequence $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\}$, its 1-AGO is $X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\}$, where $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)$, $k = 1, 2, \cdots, n$, and the parameter column $\widehat{\beta} = (\beta_1, \beta_2, \beta_3, \beta_4, \beta_5)^T$ as shown above, if parametersβ_1, β_2 satisfy $\lambda^2 - \beta_1\lambda - \beta_2 = 0$ and ${\beta_1}^2 + 4\beta_2 \geq 0$, this equation has two real number roots λ_1 and λ_2. The simulation prediction formula for the general term form of the discrete DGM(2, 1 t^2) model is as follows:*

(1) When $\lambda_1, \lambda_2 \neq 1$, the simulation prediction formula for the general term form is

$$x^{(1)}(k) = \mu_1 k^2 + \mu_2 k + \mu_3 + \mu_4 \lambda_2^k + \mu_5 \lambda_2^k, \quad k = 3, 4, \cdots, n, \quad (19)$$

where

$$\begin{aligned}
\mu_1 &= \frac{\beta_3}{1 - \beta_1 - \beta_2},\\
\mu_2 &= \frac{-2\beta_3(\beta_1 + 2\beta_2) + \beta_4(1 - \beta_1 - \beta_2)}{(1 - \beta_1 - \beta_2)^2},\\
\mu_3 &= \frac{\beta_1(\mu_1 - \mu_2) + 2\beta_2(2\mu_1 - \mu_2) + \beta_5}{(1 - \beta_1 - \beta_2)^2},\\
\mu_4 &= \frac{\lambda_2\left(x^{(1)}(1) - \mu_1 - \mu_2 - \mu_3\right) - x^{(1)}(2) + 4\mu_1 + 2\mu_2 + \mu_3}{-\beta_2 - {\lambda_1}^2},\\
\mu_5 &= \frac{\lambda_1\left(x^{(1)}(1) - \mu_1 - \mu_2 - \mu_3\right) - x^{(1)}(2) + 4\mu_1 + 2\mu_2 + \mu_3}{-\beta_2 - {\lambda_2}^2}.
\end{aligned} \quad (20)$$

(2) When one of the values is 1 in λ_1, λ_2, suppose that $\lambda_1 = 1$, so according to the Vieta theorem $\lambda_1 + \lambda_2 = \beta_1$ and$\lambda_1\lambda_2 = -\beta_2$, we get $\lambda_2 = -\beta_2 = \beta_1 - 1$, and then the simulation prediction formula for the general term form is

$$\begin{aligned}
x^{(1)}(k) &= \mu_1 k^3 + \mu_2 k^2 + \mu_3 k + \mu_4\\
&\quad + \mu_5 (\beta_1 - 1)^k, \quad k = 3, 4, \cdots, n,
\end{aligned} \quad (21)$$

where

$$\begin{aligned}
\mu_1 &= \frac{\beta_3}{6-3\beta_1},\\
\mu_2 &= \frac{\beta_3(3\beta_1-4)}{(4-6\beta_1)(2-\beta_1)},\\
\mu_3 &= \frac{\beta_1(\mu_2-\mu_1)+4\beta_4(\mu_2-2\mu_1)+\beta_5}{\beta_1+2\beta_2},\\
\mu_4 &= \frac{x^{(1)}(2)-x^{(1)}(1)(\beta_1-1)-(-\beta_1+9)\mu_1-(-\beta_1+5)\mu_2-(-\beta_1+3)\mu_3}{2-\beta_1},\\
\mu_5 &= \frac{x^{(1)}(2)-x^{(1)}(1)-3\mu_1-\mu_2}{(1-\beta_1)(2-\beta_1)}.
\end{aligned} \tag{22}$$

(3) When $\lambda_1=\lambda_2=1$, according to the Vieta theorem, we get $\beta_1=2$ and $\beta_2=-1$; then, the simulation prediction formula for the general term form is

$$x^{(1)}(k)=\mu_1k^4+\mu_2k^3+\mu_3k^2+\mu_4k+\mu_5,\quad k=3,4,\cdots,n, \tag{23}$$

where

$$\begin{aligned}
\mu_1 &= \frac{\beta_3}{12},\\
\mu_2 &= \frac{\beta_3}{3},\\
\mu_3 &= \frac{\beta_1(\mu_1-\mu_2)+8\beta_2(2\mu_1-\mu_2)+\beta_5}{-\beta_1-4\beta_2},\\
\mu_4 &= x^{(1)}(1)-x^{(1)}(2)-15\mu_1-7\mu_2-3\mu_3,\\
\mu_5 &= 2x^{(1)}(1)-x^{(1)}(2)+14\mu_1+6\mu_2+2\mu_3.
\end{aligned} \tag{24}$$

Proof. According to Definition 5 mentioned above and the discrete DGM(2, 1 t^2) model as (12), suppose that the concomitant homogeneous recursive equations are$x^{(1)}(t)=\beta_1x^{(1)}(t-1)+\beta_2x^{(1)}(t-2)$, then $F(t)=\beta_3t^2+\beta_4t+\beta_5$ is the free term. And parameters β_1 and β_1 satisfy $(\beta_1)_2+4\beta_2\geq 0$; thus, the characteristic equation $\lambda_2-\beta_1\lambda-\beta_1=0$ has two real number roots λ_1 and λ_2, and according to the Vieta theorem, the two roots satisfy $\lambda_1+\lambda_2=\beta_1$ and $\lambda_1\lambda_2=\beta_2$.

(1) When λ_1, $\lambda_2\neq 1$, then special solution form of the model equation is

$$x^*(t)=\mu_1t^2+\mu_2t+\mu_3. \tag{25}$$

(i) Taking the above $x^*(t)$ into the original model equation, we get

$$\mu_1t^2+\mu_2t+\mu_3=\beta_1\left(\mu_1(t-1)^2+\mu_2(t-1)+\mu_3\right)+\beta_2\left(\mu_1(t-2)^2+\mu_2(t-2)+\mu_3\right)+\beta_3t^2+\beta_4t+\beta_5. \tag{26}$$

So,

$$\begin{aligned}
\mu_1 &= \frac{\beta_3}{1-\beta_1-\beta_2},\\
\mu_2 &= \frac{-2\beta_3(\beta_1+2\beta_2)+\beta_4(1-\beta_1-\beta_2)}{(1-\beta_1-\beta_2)^2},\\
\mu_3 &= \frac{\beta_1(\mu_1-\mu_2)+2\beta_2(2\mu_1-\mu_2)+\beta_5}{(1-\beta_1-\beta_2)^2}.
\end{aligned} \tag{27}$$

(ii) Two real number roots of the characteristic equation $\lambda^2-\beta_1\lambda-\beta_2=0$ are as follows:

$$\lambda_1=\frac{\beta_1-\sqrt{\beta_1^{\,2}+4\beta_2}}{2}. \tag{28}$$

Then, the solution of the model equation may be expressed as follows:

$$x^{(1)}(t)=\mu_1t^2+\mu_2t+\mu_3+\mu_4\lambda_1^t+\mu_5\lambda_2^t. \tag{29}$$

(iii) Taking the initial value $x^{(1)}(1)$ and $x^{(1)}(2)$ into the above$x^{(1)}(t)$, we get

$$\begin{aligned}
x^{(1)}(1) &= \mu_4\lambda_1+\mu_5\lambda_2+\mu_1+\mu_2+\mu_3,\\
x^{(1)}(2) &= \mu_4\lambda_1^{\,2}+\mu_5\lambda_2^{\,2}+4\mu_1+2\mu_2+\mu_3.
\end{aligned} \tag{30}$$

We get

$$\mu_4 = \frac{\lambda_2\left(x^{(1)}(1) - \mu_1 - \mu_2 - \mu_3\right) - x^{(1)}(2) + 4\mu_1 + 2\mu_2 + \mu_3}{-\beta_2 - \lambda_1^{\,2}},$$

$$\mu_5 = \frac{\lambda_1\left(x^{(1)}(1) - \mu_1 - \mu_2 - \mu_3\right) - x^{(1)}(2) + 4\mu_1 + 2\mu_2 + \mu_3}{-\beta_2 - \lambda_2^{\,2}}. \tag{31}$$

So, the simulation prediction formula for the general term form of the discrete DGM(2,1 t^2) model can be represented as follows:

$$x^{(1)}(k) = \mu_1 k^2 + \mu_2 k + \mu_3 + \mu_4 \lambda_1^k + \mu_5 \lambda_2^k, \quad k = 3, 4, \cdots, n. \tag{32}$$

(3) When one of these values is 1 in λ_1, λ_2, suppose that $\lambda_1 = 1$, according to the Vieta theorem $\lambda_1 + \lambda_2 = \beta_1$ and$\lambda_1 \lambda_2 = -\beta_2$, we get $\lambda_2 = -\beta_2 = \beta_1 - 1$ and then special solution form of the model equation is

$$x^*(t) = \left(\mu_1 t^2 + \mu_2 t + \mu_3\right) t. \tag{33}$$

According to the calculation process (i) of the abovementioned (1), then getμ_1, μ_2, and μ_3.

The solution of the model equation may be expressed as follows:

$$x^{(1)}(t) = \left(\mu_1 t^2 + \mu_2 t + \mu_3\right) t + \mu_4 + \mu_5 \left(\beta_1 - 1\right)^t. \tag{34}$$

According to the calculation process (ii) and (iii) of the abovementioned (1), we get μ_4 and μ_5, so the simulation prediction formula for the general term form of the discrete DGM(2, 1 t^2) model can be represented as follows:

$$\begin{aligned} x^{(1)}(k) &= \mu_1 k^3 + \mu_2 k^2 + \mu_3 k + \mu_4 \\ &\quad + \mu_5 \left(\beta_1 - 1\right)^k, \quad k = 3, 4, \cdots, n. \end{aligned} \tag{35}$$

(4) When $\lambda_1 = \lambda_2 = 1$, according to the Vieta theorem, get $\beta_1 = 2$ and $\beta_2 = -1$, then the special solution form of the model equation is

$$x^*(t) = \left(\mu_1 t^2 + \mu_2 t + \mu_3\right) t^2. \tag{36}$$

According to the calculation process (i) of the abovementioned (1), then we get μ_1, μ_2, and μ_3.

The solution of the model equation may be expressed as follows:

$$x^{(1)}(t) = \left(\mu_1 t^2 + \mu_2 t + \mu_3\right) t^2 + \mu_4 t + \mu_5. \tag{37}$$

According to the calculation process (ii) and (iii) of the abovementioned (1), get μ_4 and μ_5, so the simulation prediction formula for the general term form of the discrete DGM(2, 1 t^2) model can be represented as

$$\begin{aligned} x^{(1)}(k) &= \mu_1 k^4 + \mu_2 k^3 + \mu_3 k^2 \\ &\quad + \mu_4 k + \mu_5, \quad k = 3, 4, \cdots, n. \end{aligned} \tag{38}$$

Note: as long as $a \neq -1$, it must exist $\beta_1^{\,2} + 4\beta_2 \geq 0$; then, for this situation, $\beta_1^{\,2} + 4\beta_2 < 0$ is left in another article for discussion. □

Inference 1. If the sequence $x^{(1)}(k) = \alpha_1 k^2 + \alpha_2 k + \alpha_3$ is that of quadratic functional type, then the simulation prediction formula of the general term form of this the discrete DGM (2, 1 t^2) model has whiteness coincidence for a sequence that $x^{(0)}(k) = \xi_1 k + \xi_2$, $k = 1, 2, \cdots, n$, namely, $\widehat{x}^{(0)}(k) = x^{(0)}(k)$.

Proof. Let $x^{(0)}(k) = \xi_1 k + \xi_2$, $k = 1, 2, \cdots, n$, be the original sequence, then its 1-AGO sequence is $x^{(1)}(k) = \alpha_1 k^2 + \alpha_2 k + \alpha_3$, $k = 3, 4, \cdots, n$, where $\alpha_1 = \xi_1/2$, $\alpha_2 = \xi_1/2 + \xi_2$, and $\alpha_3 = 0$; according to the process of (1) in Theorem 3, we can get both roots of the characteristic equation as 0, so the simulation prediction formula for the general term form is $x^{(1)}(k) = \alpha_1 k^2 + \alpha_2 k + \alpha_3$, $k = 3, 4, \cdots, n$. Finally, the inverse accumulating restored value is

$$\widehat{x}^{(0)}(k) = \widehat{x}^{(1)}(k) - \widehat{x}^{(1)}(k-1) = \xi_1 k + \xi_2 = x^{(0)}(k). \tag{39}$$

□

Inference 2. If $x^{(0)}(k) = \alpha_1 \lambda_1^{\,k} + \alpha_2 \lambda_2^{\,k} + \alpha_3 k^2 + \alpha_4 k + \alpha_5$, $k = 1, 2, \cdots, n\lambda_1$, $\lambda_2 \neq 0, 1$ is the original sequence, then the simulation prediction formula of the general term form of this discrete DGM (2, 1, t^2) model has whiteness coincidence for sequences that strictly obey this functional form.

Proof. Let $x^{(0)}(k) = \alpha_1 \lambda_1^{\,k} + \alpha_2 \lambda_2^{\,k} + \alpha_3 k^2 + \alpha_4 k + \alpha_5$, $k = 1, 2, \cdots, n$, λ_1, $\lambda_2 \neq 0, 1$, be the original sequence, then its 1-AGO sequence still satisfies $x^{(1)}(k) = \alpha_1' \lambda_1^{\,k} + \alpha_2' \lambda_2^{\,k} + \alpha_3' k^2 + \alpha_4' k + \alpha_5'$, $k = 1, 2, \cdots, n$. According to the process of (1) in Theorem 3, it is easy to get the simulation value obtained from the simulation prediction formula in the form of the discrete DGM (2, 1, t^2) model which highly coincides with the actual value. Therefore, there is high simulation prediction accuracy for sequences approximating this form. □

Inference 3. If $x^{(0)}(k) = \alpha_1 \lambda_1^{\,k} + \alpha_2 \lambda_2^{\,k} + \alpha_3 k^2 + \alpha_4 k + \alpha_5$, $k = 1, 2, \cdots, n$, $\beta \neq 0, 1$ is the original sequence, then the simulation prediction formula of the general term form of this the discrete DGM (2, 1, t^2) model has whiteness coincidence for sequences that strictly obey this functional form.

The proof process is the same as in (2) in Theorem 3 and inference 2.

Inference 4. If the sequence $x^{(0)}(k) = \alpha_1 \lambda_1^{\,k} + \alpha_2 \lambda_2^{\,k} + \alpha_3 k^2 + \alpha_4 k + \alpha_5$, $k = 1,2, \cdots, n$ is the original sequence, then the simulation prediction formula of the general term form of this the discrete DGM (2, 1 t^2) model has whiteness coincidence for sequences that strictly obey this functional form.

The proof process is the same as in (3) in Theorem 3 and inference 2.

3.3. The Property of the Discrete DGM (2, 1, t^2) Model

Property 1. discrete DGM (2, 1, t^2) model is suitable for sequence modeling with approximate homogeneous exponential laws, namely, $x^{(0)}(t) = me^{\theta t}$.

Property 2. The discrete DGM (2, 1, t^2) model is suitable for sequence modeling with approximate in-homogeneous exponential laws, namely, $x^{(0)}(t) = me^{\theta t} + c$.

According to properties 1 and 2 mentioned above, it can be shown that the discrete model is equally applicable to high-growth sequences.

Property 3. When $a = -1$, then the discrete DGM (2, 1, t^2) model is $x^{(1)}(k) = x^{(1)}(k-1) + \eta_1 k^2 + \eta_2 k + \eta_3$; hence, the simulation prediction formula for the general term form is

$$x^{(1)}(k) = x^{(1)}(1) + \eta_1\left(\frac{(k+1)(k+2)(2k+3)}{6} - 1\right) + \eta_2\left(\frac{k(1+k)}{2} - 1\right) + (k-1)\eta_3. \tag{40}$$

Property 4. If $a = -2$, then $\beta_1 = 0$ and $\beta_2 = 1$, and the discrete DGM (2, 1, t^2) model is $x^{(1)}(k) = x^{(1)}(k-2) + \beta_3 k^2 + \beta_4$; according to Theorem 3, one of the two real number roots λ_1 andλ_2 in the concomitant homogeneous recursive equations these values is 1; hence, this is a special circumstance for (2) in Theorem 3.

Property 5. If $c = 0$, then $\beta_4 = 0$, and the discrete DGM (2, 1, t^2) model is $x^{(1)}(k) = \beta_1 x^{(1)}(k-1) + \beta_2 x^{(1)}(k-2) + \beta_3 k^2 + \beta_5$; according to Theorem 3, we can get the following.

When $\lambda_1, \lambda_2 \neq 1$, the simulation prediction formula for the general term form is

$$x^{(1)}(k) = \mu_1 k^2 + \mu_2 k + \mu_3 + \mu_4 \lambda_1^k + \mu_5 \lambda_2^k, \quad k = 3, 4, \cdots, n, \tag{41}$$

where

$$\begin{aligned}
\mu_1 &= \frac{\beta_3}{1-\beta_1-\beta_2},\\
\mu_2 &= \frac{2\beta_3(\beta_1+2\beta_2)}{(1-\beta_1-\beta_2)^2},\\
\mu_3 &= \frac{\beta_3(\beta_1+4\beta_2)(1-\beta_1-\beta_2) + 2\beta_3(\beta_1+2\beta_2)^2 + \beta_5(1-\beta_1-\beta_2)^2}{(1-\beta_1-\beta_2)^3},\\
\mu_4 &= \frac{\lambda_2\left(x^{(1)}(1) - \mu_1 - \mu_2 - \mu_3\right) - x^{(1)}(2) + 4\mu_1 + 2\mu_2 + \mu_3}{-\beta_2 - \lambda_1{}^2},\\
\mu_5 &= \frac{\lambda_1\left(x^{(1)}(1) - \mu_1 - \mu_2 - \mu_3\right) - x^{(1)}(2) + 4\mu_1 + 2\mu_2 + \mu_3}{-\beta_2 - \lambda_2{}^2}.
\end{aligned} \tag{42}$$

When one of the values is 1 in λ_1, λ_2, then the simulation prediction formula for the general term form is

$$x^{(1)}(k) = \mu_1 k^3 + \mu_2 k^2 + \mu_3 k + \mu_4 + \mu_5(\beta_1 - 1)^k, \quad k = 3, 4, \cdots, n, \tag{43}$$

where

$$\begin{aligned}
\mu_1 &= \frac{\beta_3}{6-3\beta_1},\\
\mu_2 &= \frac{\beta_3(3\beta_1-4)}{(4-6\beta_1)(2-\beta_1)},\\
\mu_3 &= \frac{-\beta_3(-7\beta_1+8)(2-3\beta_1) - \beta_3(3\beta_1-4)^2 + 2\beta_5(2-3\beta_1)(2-\beta_1)}{2(2-3\beta_1)(2-\beta_1)^2},
\end{aligned}$$

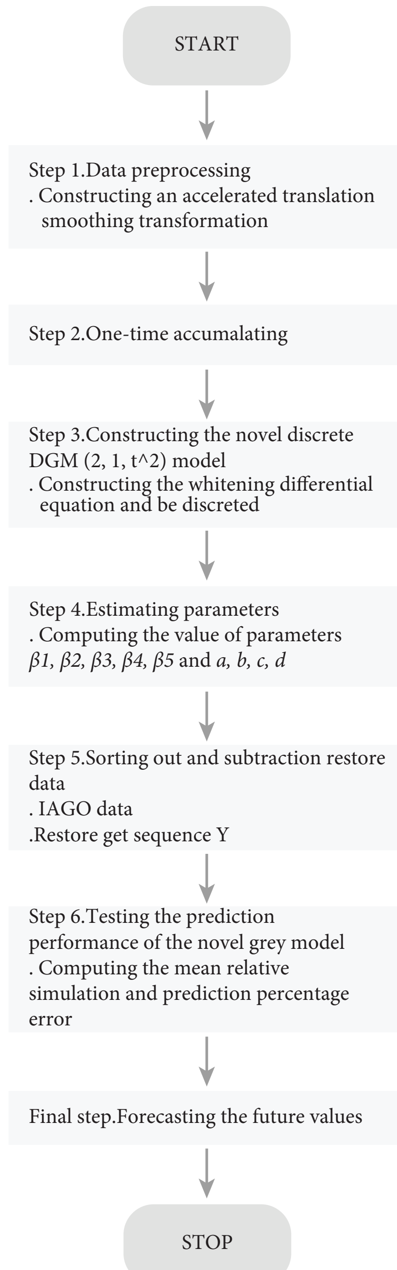

Figure 1: Flow chart of the algorithmic steps.

Table 1: Comparison of modeling accuracy of four models based on four stochastic oscillation sequences with different amplitudes.

Sequence data (2005–2010)	Model	Simulation value	Mean relative error (%)	Prediction value (2011)	Mean prediction error (%)
X_1	1	0.874, 1.874, 1.262, 2.009, 1.400, 2.152	11.558	1.547	19.232
	2	0.874, 1.874, 1.390, 1.679, 1.770, 1.800	0.000	1.914	0.001
	3	0.874, 1.874, 1.802, 1.382, 1.650, 1.648	10.416	2.429	26.817
	4	0.874, 1.874, 1.390, 1.679, 1.770, 1.800	0.000	1.915	0.000

Table 1: Continued.

Sequence data (2005–2010)	Model	Simulation value	Mean relative error (%)	Prediction value (2011)	Mean prediction error (%)
X_2	1	0.874, 1.278, 2.257, 1.395, 2.376, 1.515	27.319	2.498	30.471
	2	0.874, 1.278, 1.390, 2.874, 1.770, 1.800	0.000	1.914	0.001
	3	0.874, 1.278, 3.688, 0.015, 3.109, 0.289	70.738	4.451	132.423
	4	0.874, 1.278, 1.390, 2.874, 1.770, 1.800	0.000	1.915	0.000
X_3	1	0.874, 1.278, 4.033, 0.741, 3.521, 0.523	40.806	3.057	59.644
	2	0.874, 1.278, 3.874, 1.679, 1.770, 1.800	0.000	1.914	0.001
	3	0.874, 1.278, 5.446, −0.919, 4.168, −1.055	81.582	5.451	184.621
	4	0.874, 1.278, 3.874, 1.679, 1.770, 1.800	0.000	1.915	0.000
X_4	1	0.874, 1.278, 2.323, 1.731, 2.794, 2.221	22.771	3.302	72.439
	2	0.874, 1.278, 1.390, 1.679, 4.874, 1.800	0.000	1.916	0.001
	3	0.874, 1.278, 5.788, −1.364, 4.652, −0.572	105.662	7.914	313.310
	4	0.874, 1.278, 1.390, 1.679, 4.874, 1.800	0.000	1.915	0.000

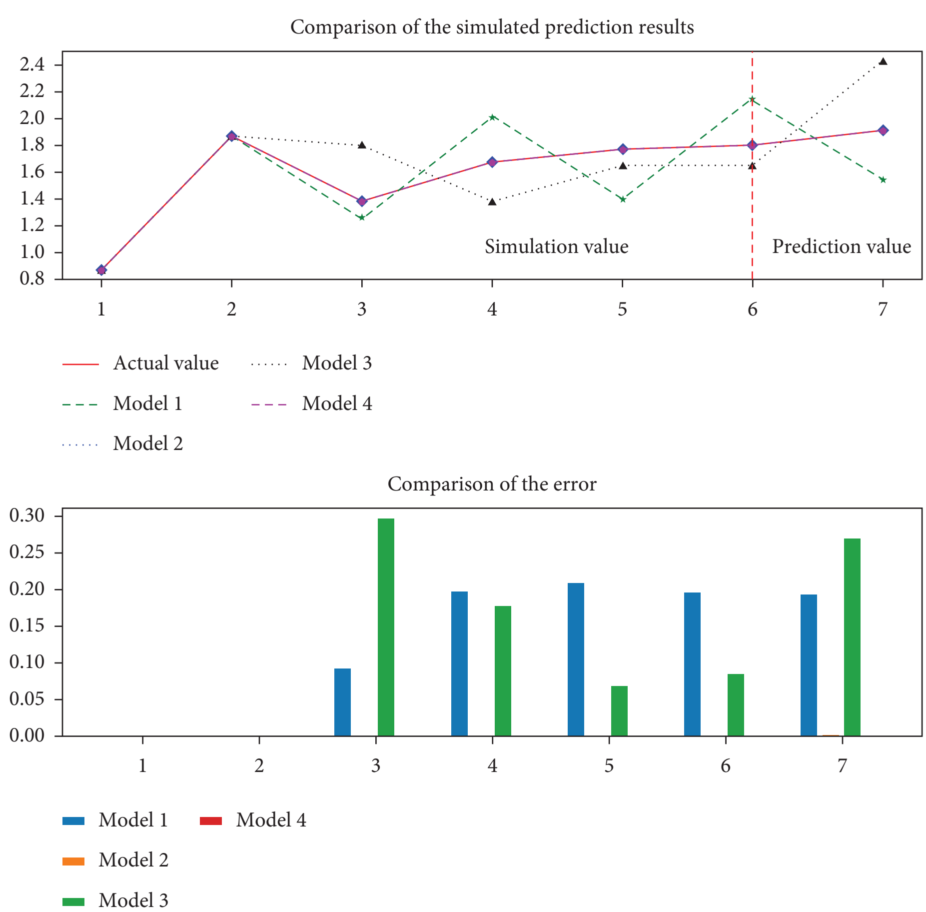

Figure 2: Schematic diagram of the predicted results for stochastic oscillation sequence X_1.

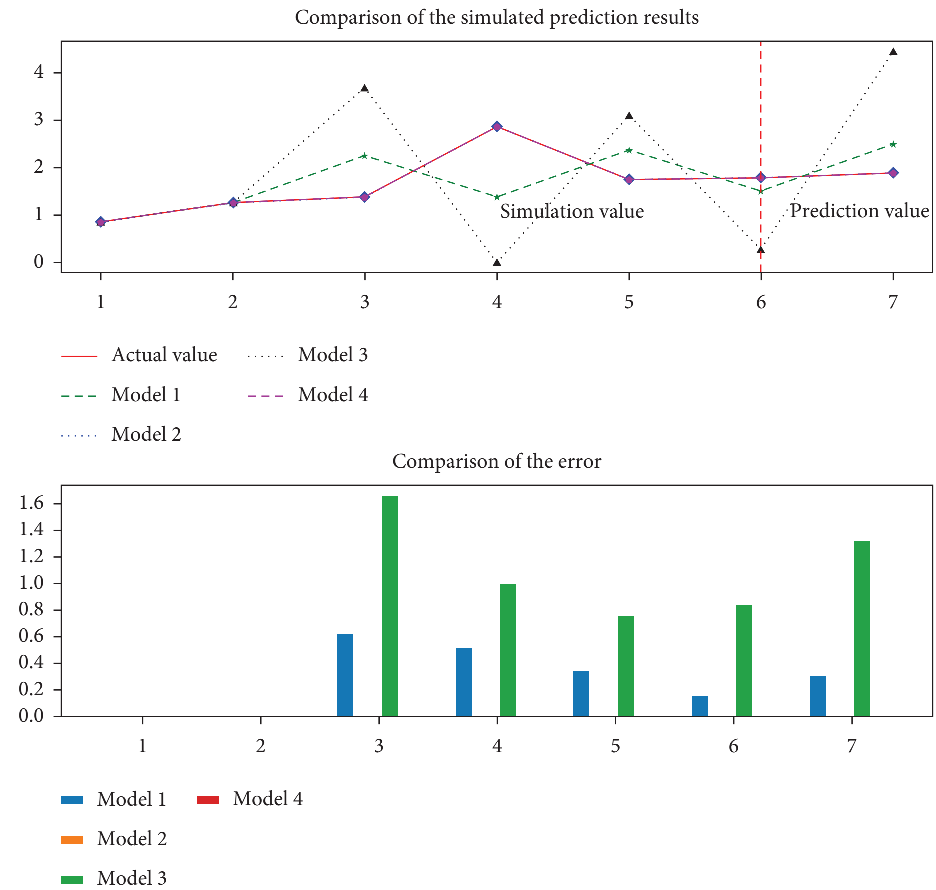

Figure 3: Schematic diagram of the predicted results for stochastic oscillation sequence X_2.

$$\begin{aligned}\mu_4 &= \frac{x^{(1)}(2) - x^{(1)}(1)(\beta_1 - 1) - (-\beta_1 + 9)\mu_1 - (-\beta_1 + 5)\mu_2 - (-\beta_1 + 3)\mu_3}{2 - \beta_1},\\ \mu_5 &= \frac{x^{(1)}(2) - x^{(1)}(1) - 3\mu_1 - \mu_2}{(1 - \beta_1)(2 - \beta_1)}.\end{aligned} \tag{44}$$

When $\lambda_1 = \lambda_2 = 1$, then the simulation prediction formula for the general term form is

$$x^{(1)}(k) = \mu_1 k^4 + \mu_2 k^3 + \mu_3 k^2 + \mu_4 k + \mu_5, \quad k = 3, 4, \cdots, n, \tag{45}$$

where

$$\begin{aligned}\mu_1 &= \frac{\beta_3}{12},\\ \mu_2 &= \frac{\beta_3}{3},\\ \mu_3 &= \frac{5\beta_3 + 6\beta_5}{12},\\ \mu_4 &= x^{(1)}(1) - x^{(1)}(2) - 15\mu_1 - 7\mu_2 - 3\mu_3,\\ \mu_5 &= 2x^{(1)}(1) - x^{(1)}(2) + 14\mu_1 + 6\mu_2 + 2\mu_3.\end{aligned} \tag{46}$$

Property 6. When $a = -1$ and $c = 0$, then the discrete DGM $(2, 1, t^2)$ model is $x^{(1)}(k) = x^{(1)}(k-1) + \eta_1 (k+1)^2 + \eta_2$; hence, the simulation prediction formula for the general term form is

$$x^{(1)}(k) = x^{(1)}(1) + \eta_1 \left(\frac{(k+1)(k+2)(2k+3)}{6} - 5 \right) + (k-1)\eta_2. \tag{47}$$

4. Algorithmic Steps for the Novel Discrete DGM (2, 1 t^2) Model Based on Stochastic Oscillation Sequence

Assume that $Y = \{y(1), y(2), \cdots, y(n)\}$ is the original stochastic oscillation sequence, where $y(k) > 0$, $k = 1, 2, \cdots, n$, the algorithmic procedure is shown in Figure 1.

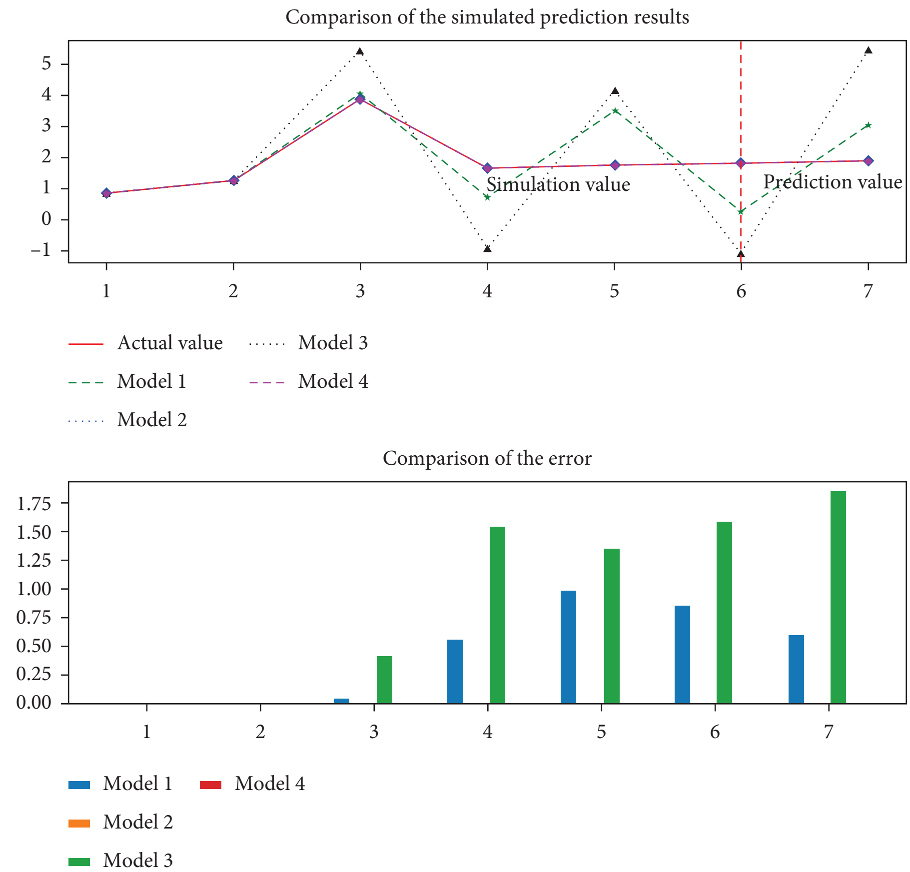

Figure 4: Schematic diagram of the predicted results for stochastic oscillation sequence X_3.

5. The Instance Analysis

5.1. Example Analysis. According to literature [8], assume that the data sequence of the annual revenue in 2005–2011 years is X where

$$X = \{0.874, 1.278, 1.390, 1.679, 1.770, 1.800, 1.915\}. \tag{48}$$

Based on the sequence X, there are four stochastic oscillation sequences X_k, $k = 1, 2, 3, 4$, with amplitudes of 1, 2, 3, and 4 where

$$\begin{aligned} X_1 &= \{0.874, 1.874, 1.390, 1.679, 1.770, 1.800, 1.915\},\\ X_2 &= \{0.874, 1.278, 1.390, 2.874, 1.770, 1.800, 1.915\},\\ X_3 &= \{0.874, 1.278, 3.874, 1.679, 1.770, 1.800, 1.915\},\\ X_4 &= \{0.874, 1.278, 1.390, 1.679, 4.874, 1.800, 1.915\}. \end{aligned} \tag{49}$$

Based on the four stochastic oscillation sequences described in this paper and modeling them based on the first six data, the last one data is predicted, the idea of controlling for different combinations of parameters is predicted according to literature [34], the prediction model of stochastic oscillation sequence based on amplitude compression (model 1) is established, and thus the following four models are established separately: the grey model of stochastic oscillation sequence based on amplitude compression (model 1), the discrete DGM (2, 1, t_2) model based on the first-order smoothing operator (model 2), the DGM (1,1) model based on the accelerated translation smoothing transformation (model 3), and the discrete DGM (2, 1, t_2) model based on stochastic oscillation sequence (model 4). According to the idea of special situation case analysis in literature [35], this paper takes the special cases as presented here ($c = 0$), respectively. Both model 1 and model 2 are based on the same first-order smoothing operator data transformation, and model 3 and model 4 are modeled based on the same accelerated translation smoothing transformation. Model 1 and model 3 are DGM (1, 1) models based on the first-order smoothing operator and the accelerated translation smoothing transformation, respectively. Model 2 and model 4 are the discrete DGM (2, 1, t^2) models in this paper based on the above two data transformations. The simulated prediction results of four models are shown in Table 1. Schematic diagrams are shown in Figures 2–5.

Table 1 and Figures 2–5 show that the discrete DGM (2, 1, t^2) model presented here is even more obviously effective based on the same data transformation, and the data transformation modeling presented in this paper with the discrete DGM (2, 1, t^2) model simulated predictions accuracy is better. Although the simulated prediction accuracy of model 3 is not as good as model 1, as a whole, the simulation effect of the discrete DGM (2, 1, t^2) model based on the stochastic oscillation sequence is also more ideal for the other three types of models.

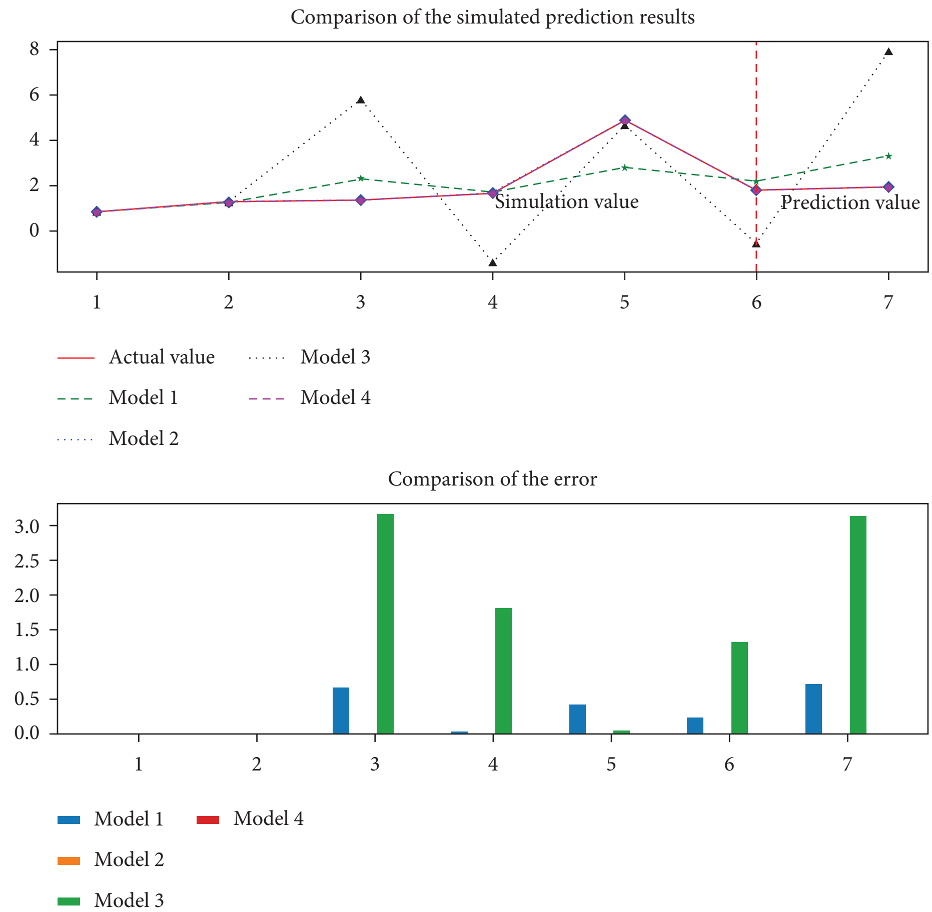

Figure 5: Schematic diagram of the predicted results for stochastic oscillation sequence X_4.

Table 2: Traffic flow data veh/h.

Time	Real value
8.12	138
8.13	293
8.14	266
8.15	205
8.16	257
8.17	270
8.18	182
8.19	136
8.20	182
8.21	227
8.22	235
8.23	230
8.24	231
8.25	183

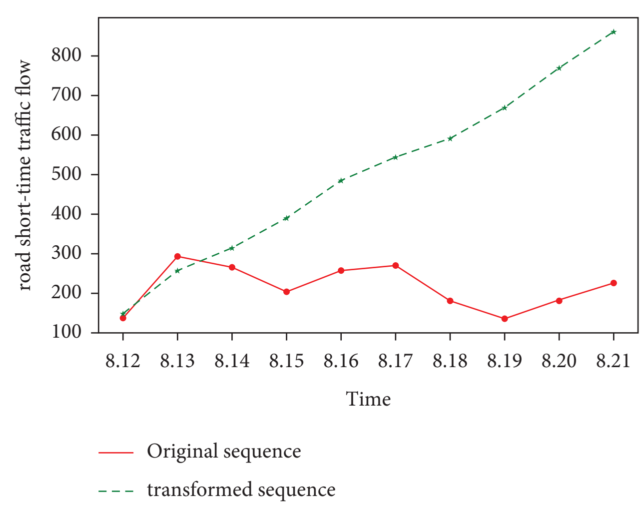

Figure 6: Schematic diagram of the original stochastic oscillation sequence and transformed sequence.

5.2. Applied Analysis. In recent years, with the development of the global economy, the expansion of urban scale, and the population explosion, there is the rise of traffic demand resulting in traffic congestion; this problem has become one of the most important problems faced by large- and medium-sized cities in the world. For various reasons, most of the sequence of traffic flow data is a stochastic oscillation sequence.

5.2.1. Applied Analysis 1. According to literature [36], we can get the short-time traffic flow data from 7:00–7:30 am of the east straight road section in Nan tong city, Jiangsu Province from August 12 to August 25, 2018. The data are shown in Table 2. Using this data sequence as the original

Table 3: Comparison of the simulated predictions of the four models on short-time traffic flow.

Real value (8.12–8.21)	Model 1 Simulation value	Model 1 Simulation error (%)	Model 2 Simulation value	Model 2 Simulation error (%)	Model 3 Simulation value	Model 3 Simulation error (%)	Model 4 Simulation value	Model 4 Simulation error (%)
138.00	138.00	0.00	138.00	0.00	138.00	0.00	138.00	0.00
293.00	293.00	0.00	293.00	0.00	293.00	0.00	293.00	0.00
266.00	226.59	14.81	266.00	0.00	434.36	63.29	266.00	0.00
205.00	233.01	13.66	216.22	5.47	152.57	25.57	216.21	5.47
257.00	273.94	6.59	228.65	11.30	319.06	24.15	228.65	11.03
270.00	202.36	25.05	282.05	4.46	66.05	75.54	282.05	4.46
182.00	170.27	6.44	205.36	12.84	265.49	45.88	205.36	12.83
136.00	239.65	76.21	100.75	25.91	50.19	63.09	100.76	25.91
182.00	267.49	46.97	207.69	14.12	292.83	60.89	207.69	14.11
227.00	203.77	10.22	237.03	4.42	126.96	44.07	237.03	4.41
Mean relative simulation error (%)	20.00		7.83		40.25		7.82	
Real value (8.22–8.25)	Prediction value	Prediction error (%)	Prediction value	Prediction error (%)	Prediction value	Prediction error (%)	Prediction value	Prediction error (%)
235.00	141.50	39.78	180.51	23.19	426.19	81.36	180.50	23.18
230.00	116.65	49.28	63.30	72.48	325.13	41.36	17.06	92.58
231.00	105.22	54.45	-40.26	117.43	698.53	202.39	60.54	73.79
183.00	88.19	51.80	274.84	50.19	682.39	272.89	236.28	29.11

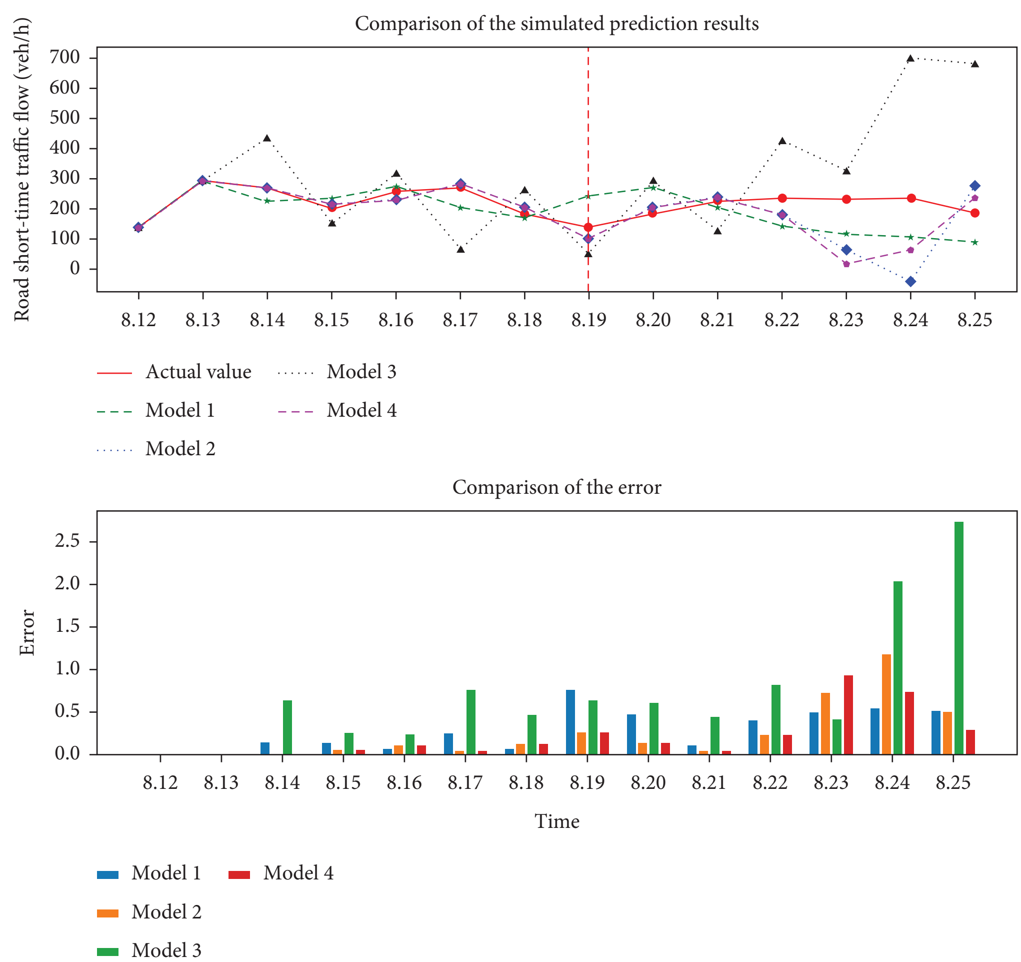

Figure 7: Schematic diagram of the predicted value of the four models on short-time traffic flow.

Table 4: One day 6 : 00–18 : 00 New York City road traffic flow veh/h.

Time point	Data
6 : 00	37908
7 : 00	71249
8 : 00	76975
9 : 00	68285
10 : 00	63315
11 : 00	56786
12 : 00	53642
13 : 00	53179
14 : 00	54194
15 : 00	58615
16 : 00	64593
17 : 00	70076
18 : 00	74693

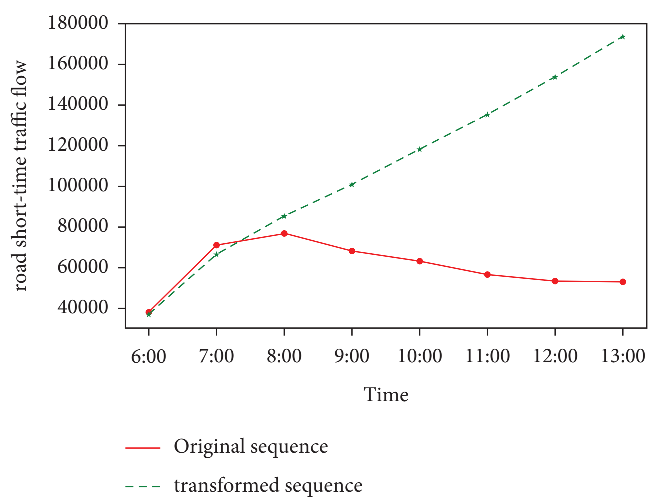

Figure 8: Schematic diagram of the original stochastic oscillation sequence and transformed sequence.

Table 5: Comparison of the simulated predictions of the four models on city traffic flow.

Real value (6 : 00–13 : 00)	Model 1		Model 2		Model 3		Model 4	
	Simulation value	Simulation error (%)	Simulation value	Simulation error (%)	Simulation value	Simulation error (%)	Simulation value	Simulation error (%)
37908.00	37908.00	0.00	37908.00	0.00	37908.00	0.00	37908.00	0.00
71249.00	71249.00	0.00	71249.00	0.00	71249.00	0.00	71249.00	0.00
76975.00	77695.74	0.94	76975.00	0.00	106884.38	38.86	76975.00	0.00
68285.00	62471.98	8.51	68309.18	0.03	38615.54	41.98	68309.17	0.03
63315.00	69257.98	9.39	63364.37	0.08	82572.46	30.42	63364.36	0.07
56786.00	54360.35	4.27	56140.90	1.14	23777.92	58.13	56140.86	1.13
53642.00	61459.89	14.57	55451.32	3.37	76543.44	42.69	55451.32	3.37
53179.00	46863.66	11.88	50353.73	5.31	29101.87	45.28	50353.72	5.31
Mean relative simulation error (%)	6.19		1.24		32.17		1.23	
Real value (14 : 00–18 : 00)	Prediction value	Prediction error (%)	Prediction value	Prediction error (%)	Prediction value	Prediction error (%)	Prediction value	Prediction error (%)
54194.00	54252.94	0.11	57429.00	6.00	95007.87	75.31	57428.95	5.96
58615.00	39935.29	31.87	57023.30	2.72	62775.77	7.09	57148.19	2.50
64593.00	47592.36	26.31	72244.74	11.84	146285.98	126.47	72151.90	11.70
70076.00	33532.16	52.14	81025.43	15.63	134429.88	91.83	81152.04	15.802
74693.00	41436.72	44.52	107448.11	43.85	241524.31	223.35	107361.86	43.73

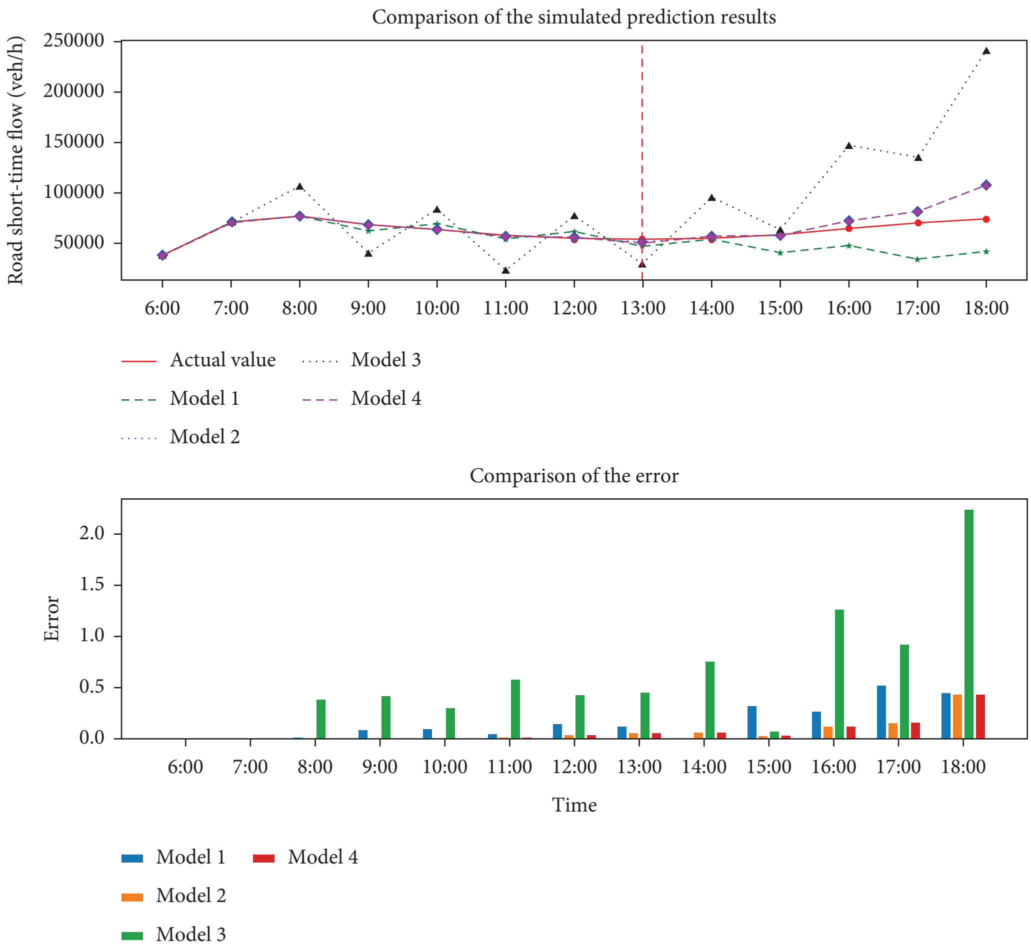

Figure 9: Schematic diagram of the predicted value of the four models on city traffic flow.

stochastic oscillation sequence and modeling it based on the first ten data, the last four data are predicted; the four models in the above example analysis were established separately. Moreover, Figure 6 shows that the transformed sequence is smoother than that of the original stochastic oscillation sequence. The simulated predictions result of four models is shown in Table 3. Schematic diagrams of the predicted value of the four models are shown in Figure 7.

According to the data of Table 3 and Figure 7, overall, the simulation of the discrete DGM (2, 1, t^2) model based on the stochastic oscillation sequence is much smaller than the average relative error of the other three types of models; that is, the simulation effect will be more ideal. In conclusion, according to the simulation and prediction analysis of road short-time traffic flow in Nan tong city, Jiangsu Province, this new model is feasible and effective in practice.

5.2.2. Applied Analysis 2. According to literature [37], the data of road traffic flow in New York City from 6 : 00 to 18 : 00 (New York time, with an interval of one hour) are shown in Table 4. Using this data sequence as the original sequence and modeling it based on the first eight data, the last five data are predicted; the four models in the above example analysis were established separately. Moreover, Figure 8 shows that the transformed sequence is smoother than that of the original stochastic oscillation sequence. The simulated predictions results of four models are shown in Table 5. Schematic diagrams of the predicted value of the four models are shown in Figure 9.

According to the data of Table 5 and Figure 9, in general, the simulation predictions of the discrete DGM (2, 1, t^2) model based on the stochastic oscillation sequence would be better than the other three types of models; that is, according to the simulation and prediction analysis of road traffic flow in New York City, this model is feasible and effective in practice.

6. Conclusions

In this article, we first transform the stochastic oscillation sequence into new ones suitable for modeling by establishing an accelerated translation smoothing transformation. Then, in order to avoid errors of the mismatch between the grey differential equation and whitening differential equation and the constant parameters, a quadratic time power term $bk^2 + ck + d$ is introduced, establishing a discrete DGM (2, 1, t^2) model. Therefore, it not only overcomes the defect of the span from difference directly to the differential but also achieves the purpose of depicting the growth trend of its data sequence.

In this article, there is the simulation prediction formula of the recurrence form and the pass-term form, which can be selected in the practical application modeling. Moreover, according to the analysis of the discrete DGM (2, 1, t^2) model properties, it is known that the scope of the new model of both the simulation prediction formula and the general term form is extended. Moreover, according to examples, verification concluded that models are based on the same data transformation, and the simulation and short-term prediction effect of the discrete DGM (2, 1, t^2) model are more significant. Moreover, in the establishment of the same model based on the two different data transformations, the simulation and short-term prediction accuracy proposed in this paper are generally better. In brief, the novel discrete DGM (2, 1, t^2) model based on stochastic oscillatory sequence achieves better short-term prediction effects in example analysis and in the field of traffic flow.

Data Availability

Data are available from corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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