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Research Article

Suppressing Chaos for a Fractional-Order Chaotic Chemical Reaction Model via PD$^{\zeta}$ Controller

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In this work, based on the earlier publications, we build a new fractional-order chemical reaction model. Computer simulations manifest that the fractional-order chemical reaction model presents chaotic behavior under a certain parameter condition. To eliminate the chaotic dynamical property, a suitable fractional-order PD$^{\zeta}$ controller with time delay is designed. Regarding the time delay as a bifurcation parameter, we set up a novel delay-independent stability and bifurcation criterion guaranteeing the stability and the creation of Hopf bifurcation of the controlled fractional-order chemical reaction model. The influence of time delay on the stability and Hopf bifurcation of the controlled fractional-order chemical reaction model is revealed. At last, numerical simulations are performed to sustain the rationality of the designed PD$^{\zeta}$ controller. The obtained conclusions of this work are completely novel and have immense application prospects in the chaos control of chemical reaction systems. Furthermore, the research idea can also be utilized to suppress the chaos of a lot of fractional-order chaotic models.

1. Introduction

Chaos exists widely in many areas such as climate, physics, chemistry, engineering, economy and finance, complex networks, and population systems [1–6]. The chaotic phenomenon depends on the initial condition of the original system. The chaotic behavior occurring in the nonlinear dynamical systems owns the very complex property and unpredictability. In many cases, chaotic behavior is not what we want in our practical life. Thus, a natural problem arises: how to suppress the chaotic behavior of the original system has been an important theme in many disciplines. During the past several decades, suppressing chaos has received great attention from many scholars. For example, Chen [7] controlled the chaos via a simple adaptive feedback control technique. Du et al. [8] applied the phase space compression method to control the chaos of an economic model. In 1990, Yorke [9] utilized Ott, Grebogi, and Yorke (OGY) control approach to control the chaos of a chaotic model. Paula and Savi [10] designed an extended time-delayed feedback control approach to suppress the chaos of a nonlinear pendulum. For more detailed literature on this aspect, one can refer [11–14].

In 1996, Geysermans and Baras [15] proposed a homogeneous chaotic Wilamowski–Rossler model. The balance equations of this model have a well-defined microscopic counterpart and all the reaction follows the following "elementary" steps:

$$\begin{cases} \mathscr{A}_1 + \mathscr{X} \xrightarrow{\kappa_1} 2\mathscr{X}, \mathscr{A}_1 + \mathscr{X} \xleftarrow{\kappa_{-1}} 2\mathscr{X}, \\ \mathscr{X} + \mathscr{Y} \xrightarrow{\kappa_2} 2\mathscr{Y}, \\ \mathscr{A}_5 + \mathscr{Y} \xrightarrow{\kappa_3} \mathscr{A}_2, \\ \mathscr{X} + \mathscr{Z} \xrightarrow{\kappa_4} \mathscr{A}_3, \\ \mathscr{A}_4 + \mathscr{Z} \xrightarrow{\kappa_5} 2\mathscr{Z}, \mathscr{A}_4 + \mathscr{Z} \xleftarrow{\kappa_{-5}} 2\mathscr{Z}. \end{cases} \tag{1}$$

System (1) includes two autocatalytic steps involving constituents $\mathscr{X}$ and $\mathscr{Z}$, coupled via three other steps involving the three constituents $\mathscr{X}$, $\mathscr{Z}$, and $\mathscr{Y}$. The initial $(\mathscr{A}_1, \mathscr{A}_4, \mathscr{A}_5)$ and final $(\mathscr{A}_2, \mathscr{A}_3)$ product concentrations remain fixed. The distance from thermodynamic equilibrium is controlled by the values of $\mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_3, \mathscr{A}_4, \mathscr{A}_5$. $\kappa_{\pm i}\,(i = 1, 2, 3, 4, 5)$ stands for the rate constant. In model (1), there are 15 free parameters. To reduce the number of free parameters, Geysermans and Baras [15] selected the rate coefficients $\kappa_{-2} = 0, \kappa_{-3} = 0$, and $\kappa_{-4} = 0$. Note that the last two relations imply that $\mathscr{A}_2$ and $\mathscr{A}_3$ are continuously removed from the reactor [15, 16].

Assuming that there exists an ideal mixture and a well-stirred reactor, then the macroscopic rate equations of model (1) can be expressed as follows:

$$\begin{cases} \dfrac{du_1(t)}{dt} = \beta_1 u_1(t) - \kappa_{-1}u_1^2(t) - u_1(t)u_2(t) - u_1(t)u_3(t), \\ \dfrac{du_2(t)}{dt} = u_1(t)u_2(t) - \beta_5 u_2(t), \\ \dfrac{du_3(t)}{dt} = \beta_4 u_3(t) - u_1(t)u_3(t) - k_{-5}u_3^2(t), \end{cases} \tag{2}$$

where $u_1(t), u_2(t)$, and $u_3(t)$ stand for the mole fractions of $\mathscr{X}, \mathscr{Y}$, and $\mathscr{Z}$ at the time t. The rate constants κ_1, κ_3, and κ_5 are incorporated in the parameters β_1, β_5, and β_4 (e.g., $\beta_1 = \kappa_1[\mathscr{A}_1], \ldots, o$) and $\beta_1 > 0, \beta_4 > 0, \beta_5 > 0, \kappa_{-1} > 0, \kappa_{-5} > 0$ stand for the constants. In detail, one can refer [15, 16]. In 2015, Xu and Wu [17] dealt with the bifurcation control of chaos for model (2) via three-time delay feedback controllers. Namely, they considered the following controller chemical model:

$$\begin{cases} \dfrac{du_1(t)}{dt} = \beta_1 u_1(t) - \kappa_{-1}u_1^2(t) - u_1(t)u_2(t) - u_1(t)u_3(t) + \mu_1[u_1(t) - u_1(t-\theta)], \\ \dfrac{du_2(t)}{dt} = u_1(t)u_2(t) - \beta_5 u_2(t) + \mu_2[u_2(t) - u_2(t-\theta)], \\ \dfrac{du_3(t)}{dt} = \beta_4 u_3(t) - u_1(t)u_3(t) - k_{-5}u_3^2(t) + \mu_3[u_3(t) - u_3(t-\theta)], \end{cases} \tag{3}$$

where $\mu_i (i = 1, 2, 3)$ stands for a real constant and θ is a delay.

It is worth mentioning that all the literature above (see [15–17]) are only concerned with the integer-order chemical models and they are not concerned with the fractional-order chemical models. Recent studies have shown that fractional-order differential equation is deemed as a more effective tool to portray natural phenomena than the classical integer-order ones since it has great advantages in memory trait and hereditary property of numerous materials and development processes [18–20]. Recently, fractional-order dynamical systems have displayed great application in lots of areas such as network systems, intelligent control, physical science, biological engineering, chemistry, finance, and so on [21–26]. Rich achievements on fractional-order dynamical models have been obtained. For example, Ke [27] dealt with the Mittag–Leffler stability and asymptotic ω-periodicity for a class of fractional-order inertial delayed neural networks. Du and Lu [28] focused on the finite-time stability for fractional-order fuzzy delayed cellular neural networks. Xiao et al. [29] probed into the bifurcation control problem for fractional-order small-world networks; Huang et al. [30] studied the bifurcation problem of fractional-order delayed neural networks. In detail, we refer the readers to [31–34].

Inspired by the exploration above and on the basis of system (2), in order to describe the continuous change process of the mole fractions of $\mathscr{X}, \mathscr{Y}$, and $\mathscr{Z}$ and characterize the memory trait and hereditary property of the variables $\mathscr{X}, \mathscr{Y}$, and $\mathscr{Z}$, we modify system (2) as the following fractional-order form:

$$\begin{cases} \dfrac{d^\zeta u_1(t)}{dt^\zeta} = \beta_1 u_1(t) - \kappa_{-1}u_1^2(t) - u_1(t)u_2(t) - u_1(t)u_3(t), \\ \dfrac{d^\zeta u_2(t)}{dt^\zeta} = u_1(t)u_2(t) - \beta_5 u_2(t), \\ \dfrac{d^\zeta u_3(t)}{dt^\zeta} = \beta_4 u_3(t) - u_1(t)u_3(t) - k_{-5}u_3^2(t), \end{cases} \tag{4}$$

where $\zeta \in (0, 1]$. The research indicates that when $\zeta = 0.97, \beta_1 = 30, \kappa_{-1} = 0.55, \beta_5 = 9.5, \beta_4 = 16.5, \kappa_{-5} = 0.5$, then there is a chaotic phenomenon in system (3). The software simulation figures are presented in Figure 1.

In this current work, we are to deal with the chaos control of system (4) by virtue of a fractional-order PD$^\zeta$ controller. The key contributions of this study are as follows:

(i) On the basis of the earlier studies, a new fractional-order chaotic chemical reaction model is set up

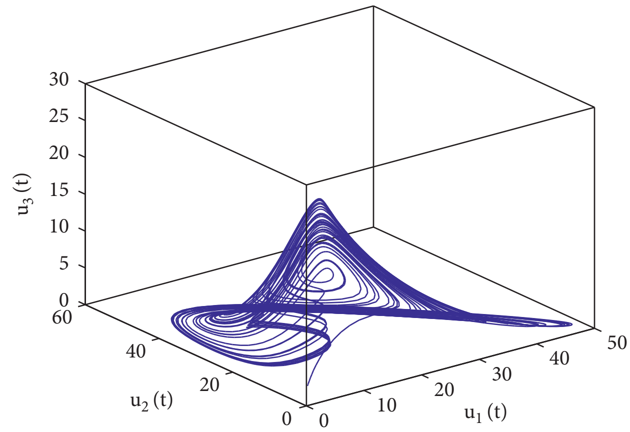

Figure 1: Software simulation figures of system (4) with $\zeta = 0.97, \beta_1 = 30, \kappa_{-1} = 0.55, \beta_5 = 9.5, \beta_4 = 16.5, \kappa_{-5} = 0.5$.

(ii) The chaotic phenomenon of system (4) is suppressed by means of an appropriate fractional-order PD$^\zeta$ controller

(iii) The study approach can be utilized to suppress the chaos of lots of fractional-order dynamical models in many subjects

This manuscript can be arranged as follows: some prerequisite theory on the fractional-order differential equation is prepared in Section 2; in Section 3, we prove the existence and uniqueness of the solution of system (4); in Section 4, the chaos of system (4) is suppressed via fractional-order PD$^\zeta$ controller and a delay-independent sufficient condition that ensures the stability and the creation of Hopf bifurcation of the fractional-order controlled chaotic chemical reaction model is built; in Section 5, software simulation results are presented to sustain the established conclusions; and Section 6 completes this article.

2. Preliminary Knowledge

In this part, we present some indispensable theories on a fractional-order differential equation.

Definition 1 (see [35]). The fractional type integral of the order ζ of the function $u(\xi)$ is given by

$$\mathcal{I}^\zeta u(\xi) = \frac{1}{\Gamma(\zeta)} \int_{\xi_0}^{\xi} (\xi - \nu)^{\zeta-1} u(\nu) \mathrm{d}\nu, \tag{5}$$

where $\xi > \xi_0, \zeta > 0$ and $\Gamma(\nu) = \int_0^\infty s^{\nu-1} e^{-s} \mathrm{d}s$.

Definition 2 (see [35]). The Caputo fractional order derivative of the order ζ of the function $u(\nu) \in ([\nu_0, \infty), R)$ is defined as follows:

$$\mathcal{D}^\zeta u(\nu) = \frac{1}{\Gamma(\kappa - \zeta)} \int_{\nu_0}^{\nu} \frac{u^{(\kappa)}(s)}{(\nu - s)^{\zeta-\kappa+1}} \mathrm{d}s, \tag{6}$$

where $\nu \geq \nu_0$ and κ represents a positive integer ($\zeta \in [\kappa - 1, \kappa)$). In particular, if $\zeta \in (0, 1)$, then

$$\mathcal{D}^\zeta u(\nu) = \frac{1}{\Gamma(1 - \zeta)} \int_{\nu_0}^{\nu} \frac{u'(s)}{(\nu - s)^{\zeta}} \mathrm{d}s. \tag{7}$$

Lemma 1 (see [36]). *Consider the fractional-order system $\mathcal{D}^\zeta w = \mathcal{F} w, w(0) = w_0$ where $\zeta \in (0, 1), w \in R^l, \mathcal{F} \in R^{l\times l}$. Assuming that $\chi_i (i = 1, 2, \ldots, l)$ is the root of the characteristic equation of $\mathcal{D}^\zeta w = \mathcal{F} w$, then the equilibrium point of the system $\mathcal{D}^\zeta w = \mathcal{F} w$ is locally asymptotically stable if $|arg(\chi_i)| > (\zeta\pi/2) (i = 1, 2, \ldots, l)$ and the equilibrium point of the system $\mathcal{D}^\zeta w = \mathcal{F} w$ is stable if $|arg(\chi_i)| > (\zeta\pi/2) (i = 1, 2, \ldots, l)$ and all critical eigenvalues that satisfy $|arg(\chi_i)| = (\zeta\pi/2) (i = 1, 2, \ldots, l)$ own geometric multiplicity one.*

3. Existence and Uniqueness of the Solution of System (4)

In this section, we will prove the existence and uniqueness of the solution of system (4).

Theorem 1. *Let $\Lambda = \{(u_1, u_2, u_3) \in R^3\colon \max\{|u_1|, |u_2|, |u_3|\} \leq A\}$, where $A > 0$ is a constant. $\forall (u_{10}, u_{20}, u_{30}) \in \Lambda$, system (4) with the initial value (u_{10}, u_{20}, u_{30}) has a unique solution $U = (u_1, u_2, u_3) \in \Lambda$.*

Proof. Define the following mapping:

$$f(U) = \left(f_1(U), f_2(U), f_3(U)\right), \tag{8}$$

where

$$\begin{cases} f_1(U) = \beta_1 u_1(t) - \kappa_{-1} u_1^2(t) - u_1(t) u_2(t) - u_1(t) u_3(t), \\ f_2(U) = u_1(t) u_2(t) - \beta_5 u_2(t), \\ f_3(U) = \beta_4 u_3(t) - u_1(t) u_3(t) - k_{-5} u_3^2(t). \end{cases} \tag{9}$$

$\forall U, \overline{U} \in \Lambda$, one obtains

$$\begin{aligned}
&\|f(U)-f(\widetilde{U})\| \\
&= |\beta_1 u_1(t) - \kappa_{-1} u_1^2(t) - u_1(t)u_2(t) - u_1(t)u_3(t) \\
&\quad - \left[\beta_1 \overline{u}_1(t) - \kappa_{-1}\overline{u}_1^2(t) - \overline{u}_1(t)\overline{u}_2(t) - \overline{u}_1(t)\overline{u}_3(t)\right]| \\
&\quad + \left|u_1(t)u_2(t) - \beta_5 u_2(t) - \left[\overline{u}_1(t)\overline{u}_2(t) - \beta_5\overline{u}_2(t)\right]\right| \\
&\quad + |\beta_4 u_3(t) - u_1(t)u_3(t) - k_{-5}u_3^2(t) \\
&\quad - \left[\beta_4\overline{u}_3(t) - \overline{u}_1(t)\overline{u}_3(t) - k_{-5}\overline{u}_3^2(t)\right]| \\
&\le \left(\beta_1 + 2\kappa_{-1}A + A\right)\left|u_1(t)-\overline{u}_1(t)\right| + A\left|u_2(t)-\overline{u}_2(t)\right| + A\left|u_3(t)-\overline{u}_3(t)\right| + A\left|u_1(t)-\overline{u}_1(t)\right| \\
&\quad + \left(A+\beta_5\right)\left|u_2(t)-\overline{u}_2(t)\right| + A\left|u_1(t)-\overline{u}_1(t)\right| \\
&\quad + \left(\beta_4 + A + 2\kappa_{-5}A\right)\left|u_3(t)-\overline{u}_3(t)\right| \\
&= A_1\left|u_1(t)-\overline{u}_1(t)\right| + A_2\left|u_2(t)-\overline{u}_2(t)\right| + A_3\left|u_3(t)-\overline{u}_3(t)\right| \\
&\le A_0\|U-\overline{U}\|,
\end{aligned} \tag{10}$$

where

$$\begin{cases} A_1 = \beta_1 + 2\kappa_{-1}A + 3A, \\ A_2 = 2A + \beta_5, \\ A_3 = \beta_4 + 2A + 2\kappa_{-5}A, \end{cases} \tag{11}$$

$$A_0 = \max\{A_1, A_2, A_3\}. \tag{12}$$

Then $f(U)$ satisfies Lipschitz condition with respect to U (see [39, 40]). According to Banach fixed point theorem, we know that Theorem 1 is true. □

4. Suppressing Chaos via Fractional-Order PD$^\zeta$ Controller

In this part, we are to apply a suitable controller to eliminate the chaotic phenomenon of system (4). By virtue of the idea of Tang et al. [37], the fractional-order PD$^\zeta$ controller can be designed as follows:

$$\psi(t) = \varrho_p u_1(t-\theta) + \varrho_d \frac{d^\zeta u_1(t)}{dt^\zeta}, \tag{13}$$

where ϱ_p and $\varrho_d \neq 1$ present the proportional control parameter and the derivative control parameter, respectively, and θ denotes a delay. Adding (13) to the first equation of system (4), we get

$$\begin{cases} \dfrac{d^\zeta u_1(t)}{dt^\zeta} = \beta_1 u_1(t) - \kappa_{-1}u_1^2(t) - u_1(t)u_2(t) - u_1(t)u_3(t) + \psi(t), \\[2ex] \dfrac{d^\zeta u_2(t)}{dt^\zeta} = u_1(t)u_2(t) - \beta_5 u_2(t), \\[2ex] \dfrac{d^\zeta u_3(t)}{dt^\zeta} = \beta_4 u_3(t) - u_1(t)u_3(t) - k_{-5}u_3^2(t). \end{cases} \tag{14}$$

That is

$$
\begin{cases}
\dfrac{d^{\zeta}u_1(t)}{dt^{\zeta}} = \beta_1 u_1(t) - \kappa_{-1}u_1^2(t) - u_1(t)u_2(t) - u_1(t)u_3(t) \\
+ \varrho_p u_1(t-\theta) + \varrho_d \dfrac{d^{\zeta}u_1(t)}{dt^{\zeta}}, \\
\dfrac{d^{\zeta}u_2(t)}{dt^{\zeta}} = u_1(t)u_2(t) - \beta_5 u_2(t), \\
\dfrac{d^{\zeta}u_3(t)}{dt^{\zeta}} = \beta_4 u_3(t) - u_1(t)u_3(t) - k_{-5}u_3^2(t).
\end{cases}
\tag{15}
$$

System (15) can be rewritten as the following form:

$$
\begin{cases}
\dfrac{d^{\zeta}u_1(t)}{dt^{\zeta}} = \dfrac{\beta_1}{1-\varrho_d}u_1(t) - \dfrac{\kappa_{-1}}{1-\varrho_d}u_1^2(t) - \dfrac{1}{1-\varrho_d}u_1(t)u_2(t) \\
-\dfrac{1}{1-\varrho_d}u_1(t)u_3(t) + \dfrac{\varrho_p}{1-\varrho_d}u_1(t-\theta), \\
\dfrac{d^{\zeta}u_2(t)}{dt^{\zeta}} = u_1(t)u_2(t) - \beta_5 u_2(t), \\
\dfrac{d^{\zeta}u_3(t)}{dt^{\zeta}} = \beta_4 u_3(t) - u_1(t)u_3(t) - k_{-5}u_3^2(t).
\end{cases}
\tag{16}
$$

It is not difficult to obtain that if the following condition

$$
(\mathcal{Q}_1)\ \beta_4 > \beta_5,\ (\beta_1 - \kappa_{-1})\kappa_{-5} > \kappa_4 - \kappa_5, \tag{17}
$$

holds, then system (16) owns the following unique positive equilibrium $\mathcal{U}(u_1^*, u_2^*, u_3^*)$, where

$$
\begin{cases}
u_1^* = \beta_5, \\
u_2^* = \dfrac{(\beta_1 - \kappa_{-1}\beta_5)\kappa_{-5} - \beta_4 + \beta_5}{\kappa_{-5}}, \\
u_3^* = \dfrac{\beta_4 - \beta_5}{\kappa_{-5}}.
\end{cases}
\tag{18}
$$

The linear system of (16) around the positive equilibrium $\mathcal{U}(u_1^*, u_2^*, u_3^*)$ takes the following form:

$$
\begin{cases}
\dfrac{d^{\zeta}u_1(t)}{dt^{\zeta}} = a_1 u_1(t) + a_2 u_2(t) + a_2 u_3(t) + a_3 u_1(t-\theta), \\
\dfrac{d^{\zeta}u_2(t)}{dt^{\zeta}} = a_4 u_1(t) + a_5 u_2(t), \\
\dfrac{d^{\zeta}u_3(t)}{dt^{\zeta}} = a_6 u_1(t) + a_7 u_3(t),
\end{cases}
\tag{19}
$$

where

$$
\begin{cases}
a_1 = \dfrac{\beta_1 - 2\kappa_{-1}u_1^* - u_2^* - u_3^*}{1-\varrho_d}, \\
a_2 = -\dfrac{u_1^*}{1-\varrho_d}, \\
a_3 = \dfrac{\varrho_p}{1-\varrho_d}, \\
a_4 = u_2^*, \\
a_5 = u_1^* - \beta_5, \\
a_6 = -u_3^*, \\
a_7 = \beta_4 - u_1^* - 2\kappa_5 u_3^*.
\end{cases}
\tag{20}
$$

The characteristic equation of system (19) takes the form

$$
\det \begin{bmatrix} s^{\zeta} - a_1 - a_3 e^{-s\theta} & -a_2 & -a_2 \\ -a_4 & s^{\zeta} - a_5 & 0 \\ -a_6 & 0 & s^{\zeta} - a_7 \end{bmatrix} = 0. \tag{21}
$$

Then,

$$
s^{3\zeta} + b_1 s^{2\zeta} + b_2 s^{\zeta} + b_3 + \left(c_1 s^{2\zeta} + c_2 s^{\zeta} + c_3\right)e^{-s\theta} = 0, \tag{22}
$$

where

$$\begin{cases} b_1 = -(a_1 + a_5 + a_7), \\ b_2 = a_5a_7 + a_1a_5 + a_1a_7 - a_2a_6 - a_2a_4, \\ b_3 = a_2a_5a_6 + a_2a_4a_7 - a_1a_5a_7, \\ c_1 = -a_3, \\ c_2 = a_3(a_5 + a_7), \\ c_3 = -a_3a_5a_7. \end{cases} \tag{23}$$

When $\theta = 0$, then (22) becomes

$$\lambda^3 + (b_1 + c_1)\lambda^2 + (b_2 + c_2)\lambda + b_3 + c_3 = 0. \tag{24}$$

Assuming that

$$(\mathcal{Q}_2)\begin{cases} b_1 + c_1 > 0, \\ (b_1 + c_1)(b_2 + c_2) > b_3 + c_3, \\ (b_3 + c_3)[(b_1 + c_1)(b_2 + c_2) - (b_3 + c_3)] > 0, \end{cases} \tag{25}$$

is true, then the three roots $\lambda_1, \lambda_2, \lambda_3$ of (24) satisfy $|\arg(\lambda_1)| > (\zeta\pi/2)$, $|\arg(\lambda_2)| > (\zeta\pi/2)$, and $|\arg(\lambda_3)| > (\zeta\pi/2)$. By virtue of Lemma 1, we can conclude that the positive equilibrium point $\mathcal{U}(u_1^*, u_2^*, u_3^*)$ of system (14) is locally asymptotically stable when $\theta = 0$.

Assume that $s = i\rho = \rho(\cos(\zeta\pi/2) + i\sin(\pi/2))$ is the root of equation (22). It follows from (22) that

$$\begin{aligned} &\rho^{3\zeta}\left(\cos\frac{3\zeta\pi}{2} + i\sin\frac{3\zeta\pi}{2}\right) + b_1\rho^{2\zeta}(\cos\zeta\pi + i\sin\zeta\pi) \\ &\quad + b_2\rho^{\zeta}\left(\cos\frac{\zeta\pi}{2} + i\sin\frac{\zeta\pi}{2}\right) + b_3 \\ &\quad + \left[c_1\rho^{2\zeta}(\cos\zeta\pi + i\sin\zeta\pi) + c_2\rho^{\zeta}\left(\cos\frac{\zeta\pi}{2} + i\sin\frac{\zeta\pi}{2}\right) + c_3\right] \\ &\quad \times(\cos\rho\theta - i\sin\rho\theta) = 0. \end{aligned} \tag{26}$$

Then,

$$\begin{cases} \mathcal{G}_1\cos\rho\theta + \mathcal{G}_2\sin\rho\theta = \mathcal{H}_1, \\ \mathcal{G}_2\cos\rho\theta - \mathcal{G}_1\sin\rho\theta = \mathcal{H}_2, \end{cases} \tag{27}$$

where

$$\begin{cases} \mathcal{G}_1 = d_1\rho^{2\zeta} + d_2\rho^{\zeta} + d_3, \\ \mathcal{G}_2 = d_4\rho^{2\zeta} + d_5\rho^{\zeta}, \\ \mathcal{H}_1 = e_1\rho^{3\zeta} + e_2\rho^{2\zeta} + e_3\rho^{\zeta} + e_4, \\ \mathcal{H}_2 = e_5\rho^{3\zeta} + e_6\rho^{2\zeta} + e_7\rho^{\zeta}, \end{cases} \tag{28}$$

where

$$\begin{cases} d_1 = c_1\cos\zeta\pi, \\ d_2 = c_2\cos\dfrac{\zeta\pi}{2}, \\ d_3 = c_3, \\ d_4 = c_1\sin\zeta\pi, \\ d_5 = c_2\sin\dfrac{\zeta\pi}{2}, \\ e_1 = -\cos\dfrac{\zeta\pi}{2}, \\ e_2 = -b_1\cos\zeta\pi, \\ e_3 = -b_2\cos\dfrac{\zeta\pi}{2}, \\ e_4 = -b_3, \\ e_5 = -\sin\dfrac{\zeta\pi}{2}, \\ e_6 = -b_1\sin\zeta\pi, \\ e_7 = -b_2\sin\dfrac{\zeta\pi}{2}. \end{cases} \tag{29}$$

It follows from (27) that

$$\cos\rho\theta = \frac{\mathcal{H}_1\mathcal{G}_1 + \mathcal{H}_2\mathcal{G}_2}{\mathcal{G}_1^2 + \mathcal{G}_2^2}, \tag{30}$$

$$\mathcal{G}_1^2 + \mathcal{G}_2^2 = \mathcal{H}_1^2 + \mathcal{H}_2^2. \tag{31}$$

By virtue of (28) and (31), one gets

$$\begin{aligned} &\left(d_1\rho^{2\zeta} + d_2\rho^{\zeta} + d_3\right)^2 + \left(d_4\rho^{2\zeta} + d_5\rho^{\zeta}\right)^2 = \\ &\left(e_1\rho^{3\zeta} + e_2\rho^{2\zeta} + e_3\rho^{\zeta} + e_4\right)^2 + \left(e_5\rho^{3\zeta} + e_6\rho^{2\zeta} + e_7\rho^{\zeta}\right)^2, \end{aligned} \tag{32}$$

which leads to

$$\epsilon_1\rho^{6\zeta} + \epsilon_2\rho^{5\zeta} + \epsilon_3\rho^{4\zeta} + \epsilon_4\rho^{3\zeta} + \epsilon_5\rho^{2\zeta} + \epsilon_6\rho^{\zeta} + \epsilon_7 = 0, \tag{33}$$

where

$$\begin{cases} \epsilon_1 = e_1^2 + e_5^2, \\ \epsilon_2 = 2(e_1e_2 + e_5e_6), \\ \epsilon_3 = e_2^2 + e_6^2 - d_1^2 - d_4^2 + 2(e_1e_3 + e_5e_7), \\ \epsilon_4 = 2(e_1e_4 + e_2e_3 + e_6e_7 - d_1d_2 - d_4d_5), \\ \epsilon_5 = e_3^2 + e_7^2 - d_2^2 - d_5^2 + 2(e_2e_4 - d_1d_3), \\ \epsilon_6 = 2(e_3e_4 - d_2d_3), \\ \epsilon_7 = e_4^2 - d_3^2. \end{cases} \tag{34}$$

Set

$$\Theta(\rho) = \epsilon_1\rho^{6\zeta} + \epsilon_2\rho^{5\zeta} + \epsilon_3\rho^{4\zeta} + \epsilon_4\rho^{3\zeta} + \epsilon_5\rho^{2\zeta} + \epsilon_6\rho^{\zeta} + \epsilon_7. \tag{35}$$

Assuming that

$$(\mathcal{Q}_3)\ |e_4| < |d_3| \tag{36}$$

is true, since $\lim_{\rho\longrightarrow\infty}\Theta(\rho) = +\infty$, then equation (33) owns at least one real positive root. So equation (22) has at least one pair of pure roots. Making use of Sun et al. [38], one can easily establish the conclusion as follows.

Lemma 2. *(a) Supposing that $\epsilon_k > 0\,(k = 1, 2, 3, 4, 5, 6)$, equation (22) owns no root with zero real parts for $\theta \geq 0$. (b) Supposing that $(\mathcal{Q}_3)$ holds and $\epsilon_k > 0\,(k = 1, 2, 3, 4, 5)$, then equation (22) owns a pair of purely imaginary roots $\pm i\rho_0$ if $\theta = \theta_0^{(l)}\,(l = 1, 2, \ldots,)$ where*

$$\theta_0^{(l)} = \frac{1}{\rho_0}\left[\arccos\left(\frac{\mathcal{H}_1\mathcal{G}_1 + \mathcal{H}_2\mathcal{G}_2}{\mathcal{G}_1^2 + \mathcal{G}_2^2}\right) + 2l\pi\right], \tag{37}$$

where $l = 0, 1, \ldots$, and $\rho_0 > 0$ represents the unique zero of $\Theta(\rho)$.

Denote $\theta_0 = \theta_0^{(0)}$. Now the following hypothesis is given:

$$(\mathcal{Q}_4)\ \mathcal{C}_{1R}\mathcal{C}_{2R} + \mathcal{C}_{1I}\mathcal{C}_{2I} > 0, \tag{38}$$

where

$$\begin{cases} \mathcal{C}_{1R} = 3\zeta\rho_0^{3\zeta-1}\cos\dfrac{(3\zeta-1)\pi}{2} + 2\zeta c_1\rho_0^{2\zeta-1}\cos\dfrac{(2\zeta-1)\pi}{2} \\ \quad + \zeta c_2\rho_0^{\zeta-1}\cos\dfrac{(\zeta-1)\pi}{2} + \left[2\zeta c_1\rho_0^{2\zeta-1}\cos\dfrac{(2\zeta-1)\pi}{2} + \zeta c_2\rho_0^{\zeta-1}\cos\dfrac{(\zeta-1)\pi}{2}\right]\cos\rho_0\theta_0 + \sin\rho_0\theta_0 \\ \quad \times\left[2\zeta c_1\rho_0^{2\zeta-1}\sin\dfrac{(2\zeta-1)\pi}{2} + \zeta c_2\rho_0^{\zeta-1}\sin\dfrac{(\zeta-1)\pi}{2}\right], \\ \mathcal{C}_{1I} = 3\zeta\rho_0^{3\zeta-1}\sin\dfrac{(3\zeta-1)\pi}{2} + 2\zeta c_1\rho_0^{2\zeta-1}\sin\dfrac{(2\zeta-1)\pi}{2} \\ \quad + \zeta c_2\rho_0^{\zeta-1}\sin\dfrac{(\zeta-1)\pi}{2} - \left[2\zeta c_1\rho_0^{2\zeta-1}\cos\dfrac{(2\zeta-1)\pi}{2} + \zeta c_2\rho_0^{\zeta-1}\cos\dfrac{(\zeta-1)\pi}{2}\right]\sin\rho_0\theta_0 + \cos\rho_0\theta_0 \\ \quad \times\left[2\zeta c_1\rho_0^{2\zeta-1}\sin\dfrac{(2\zeta-1)\pi}{2} + \zeta c_2\rho_0^{\zeta-1}\sin\dfrac{(\zeta-1)\pi}{2}\right], \\ \mathcal{C}_{2R} = \left(c_1\rho_0^{2\zeta}\cos\zeta\pi + c_2\rho_0^{\zeta}\cos\dfrac{\zeta\pi}{2} + c_3\right)\rho_0\sin\rho_0\theta_0 \\ \quad - \left(c_1\rho_0^{2\zeta}\sin\zeta\pi + c_2\rho_0^{\zeta}\sin\dfrac{\zeta\pi}{2} + c_3\right)\rho_0\cos\rho_0\theta_0, \\ \mathcal{C}_{2I} = \left(c_1\rho_0^{2\zeta}\cos\zeta\pi + c_2\rho_0^{\zeta}\cos\dfrac{\zeta\pi}{2} + c_3\right)\rho_0\cos\rho_0\theta_0 \\ \quad + \left(c_1\rho_0^{2\zeta}\sin\zeta\pi + c_2\rho_0^{\zeta}\sin\dfrac{\zeta\pi}{2} + c_3\right)\rho_0\sin\rho_0\theta_0. \end{cases} \tag{39}$$

Lemma 3. *Let $s(\theta) = \phi_1(\theta) + i\phi_2(\theta)$ be the root of (22) at $\theta = \theta_0$ satisfying $\phi_1(\theta_0) = 0, \phi_2(\theta_0) = \rho_0$, then $Re(ds/d\theta)|_{\theta=\theta_0,\rho=\rho_0} > 0$.*

Proof. Making use of (22), we get

$$\begin{aligned}&\left(3\zeta s^{3\zeta-1} + 2\zeta b_1 s^{2\zeta-1} + \zeta b_2 s^{\zeta-1}\right)\frac{\mathrm{d}s}{\mathrm{d}\theta}\\ &+\left(2\zeta c_1 s^{2\zeta-1} + \zeta c_2 s^{\zeta-1}\right)e^{-s\theta}\frac{\mathrm{d}s}{\mathrm{d}\theta}\\ &- e^{-s\theta}\left(\frac{\mathrm{d}s}{\mathrm{d}\theta}\theta + s\right)\left(c_1 s^{2\zeta} + c_2 s^{\zeta} + c_3\right) = 0,\end{aligned} \tag{40}$$

which leads to

$$\begin{aligned}&\left[3\zeta s^{3\zeta-1} + 2\zeta b_1 s^{2\zeta-1} + \zeta b_2 s^{\zeta-1} + \left(2\zeta c_1 s^{2\zeta-1} + \zeta c_2 s^{\zeta-1}\right)e^{-s\theta} - \theta e^{-s\theta}\left(c_1 s^{2\zeta} + c_2 s^{\zeta} + c_3\right)\right]\frac{\mathrm{d}s}{\mathrm{d}\theta}\\ &= s e^{-s\theta}\left(c_1 s^{2\zeta} + c_2 s^{\zeta} + c_3\right).\end{aligned} \tag{41}$$

Then,

$$\left(\frac{\mathrm{d}s}{\mathrm{d}\theta}\right)^{-1} = \frac{\mathscr{C}_1(s)}{\mathscr{C}_2(s)} - \frac{\theta}{s}, \tag{42}$$

where

$$\begin{cases}\mathscr{C}_1(s) = 3\zeta s^{3\zeta-1} + 2\zeta b_1 s^{2\zeta-1} + \zeta b_2 s^{\zeta-1}\\ +\left(2\zeta c_1 s^{2\zeta-1} + \zeta c_2 s^{\zeta-1}\right)e^{-s\theta},\\ \mathscr{C}_2(s) = s e^{-s\theta}\left(c_1 s^{2\zeta} + c_2 s^{\zeta} + c_3\right).\end{cases} \tag{43}$$

Then,

$$\begin{aligned}\mathrm{Re}\left[\left(\frac{\mathrm{d}s}{\mathrm{d}\theta}\right)^{-1}\right]_{\theta=\theta_0,\rho=\rho_0} &= \mathrm{Re}\left[\frac{\mathscr{C}_1(s)}{\mathscr{C}_2(s)}\right]_{\theta=\theta_0,\rho=\rho_0}\\ &= \frac{\mathscr{C}_{1R}\mathscr{C}_{2R} + \mathscr{C}_{1I}\mathscr{C}_{2I}}{\mathscr{C}_{2R}^2 + \mathscr{C}_{2I}^2}.\end{aligned} \tag{44}$$

In view of $(\mathcal{Q}_4)$, we have

$$\mathrm{Re}\left[\left(\frac{\mathrm{d}s}{\mathrm{d}\theta}\right)^{-1}\right]_{\theta=\theta_0,\rho=\rho_0} > 0, \tag{45}$$

which completes the proof.

Making use of Lemma 1, we can easily obtain the following conclusion. □

Theorem 2. *Supposing that $(\mathcal{Q}_1)$–$(\mathcal{Q}_4)$ hold, then the positive equilibrium point $\mathscr{U}(u_1^*, u_2^*, u_3^*)$ of system (16) is locally asymptotically stable if the time delay θ lies in the interval $[0, \theta_0)$ and the Hopf bifurcation phenomenon of system (16) will arise near the positive equilibrium point $\mathscr{U}(u_1^*, u_2^*, u_3^*)$ if $\theta = \theta_0$.*

Remark 1. Xu and Wu [17] dealt with the chaos control for an integer-order chaotic chemical reaction model by time-delay feedback control technique. This manuscript deals with the chaos control issue for a fractional-order chaotic chemical reaction model via a fractional-order PD^{ζ} controller. The model and the research approach is very different from those in [17]. From this viewpoint, we think that the obtained results and the research method of this manuscript supplement the work of [17] and promote the development of the chaos control theory of fractional-order differential equation to some degree.

Remark 2. In this paper, we use the fractional-order PD^{ζ} controller to control the chaos of the fractional-order chaotic chemical reaction model (4). Compared with the time delay feedback controller, the fractional-order PD^{ζ} controller has more adjustable parameters and then can control the chaos of model (4) neatly.

5. Example

Consider the following controlled fractional-order chaotic chemical reaction model:

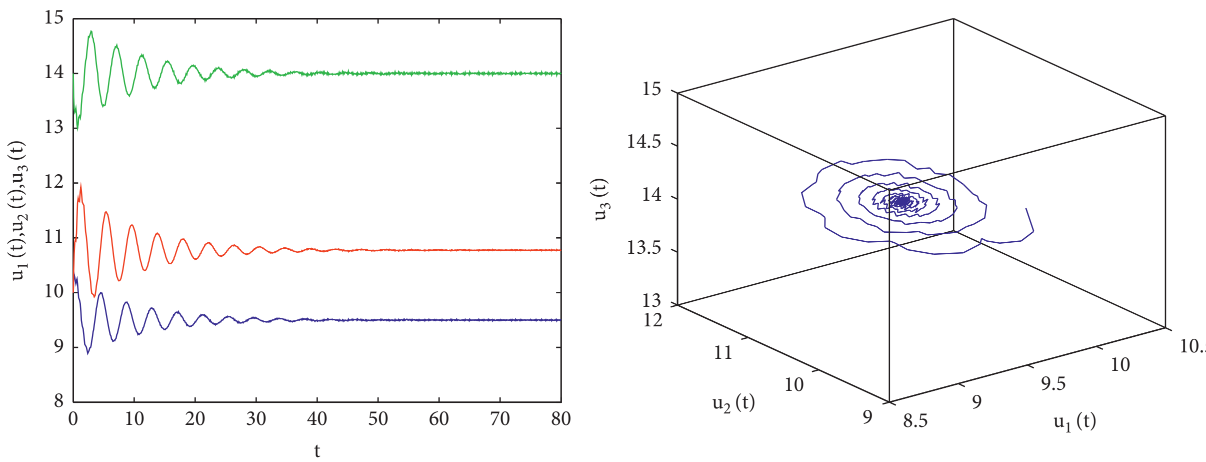

Figure 2: Computer simulation figures of the controlled fractional-order chaotic chemical reaction model (46) with $\theta = 0.20 < \theta_0 = 0.25$. The blue line represents $u_1(t)$, the red line represents $u_2(t)$, and the green line represents $u_3(t)$.

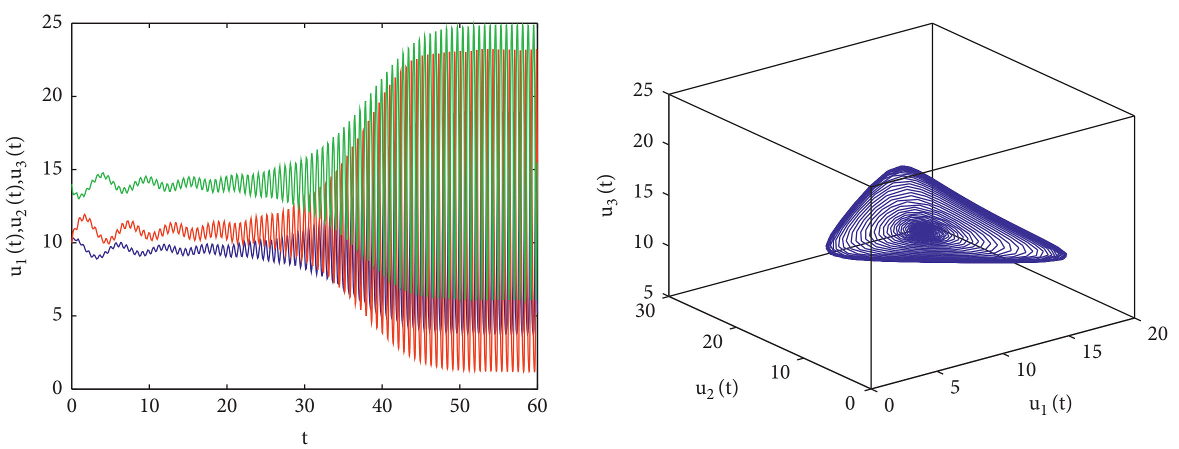

Figure 3: Computer simulation figures of the controlled fractional-order chaotic chemical reaction model (46) with $\theta = 0.28 > \theta_0 = 0.25$. The blue line represents $u_1(t)$, the red line represents $u_2(t)$, and the green line represents $u_3(t)$.

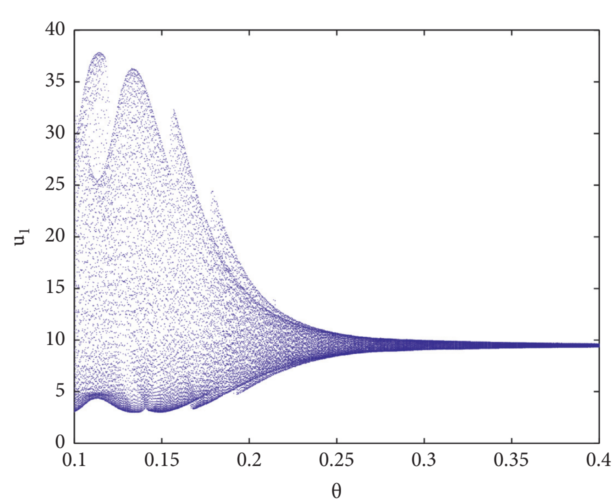

Figure 4: Bifurcation plot of the controlled fractional-order chaotic chemical reaction model (46): θ-u_1.

$$\begin{cases} \dfrac{d^{\zeta}u_1(t)}{dt^{\zeta}} = \beta_1 u_1(t) - \kappa_{-1}u_1^2(t) - u_1(t)u_2(t) - u_1(t)u_3(t) \\ \\ +\varrho_p u_1(t-\theta) + \varrho_d \dfrac{d^{\zeta}u_1(t)}{dt^{\zeta}}, \\ \\ \dfrac{d^{\zeta}u_2(t)}{dt^{\zeta}} = u_1(t)u_2(t) - \beta_5 u_2(t), \\ \\ \dfrac{d^{\zeta}u_3(t)}{dt^{\zeta}} = \beta_4 u_3(t) - u_1(t)u_3(t) - k_{-5}u_3^2(t), \end{cases} \tag{46}$$

where ζ=0.97,β_1=30,κ_{-1}=0.55,β_5=9.5,β_4=16.5,κ_{-5}= 0.5. Let ϱ_p=0.5,ϱ_d=0.9. It is not difficult to obtain the unique positive equilibrium point of system (46) is $\mathscr{U}(9.5,10.775,14)$. By direct computation via MATLAB software, we can easily get ρ_0=4.0239 and θ_0=0.25. The three assumptions $(\mathscr{Q}_1)$–$(\mathscr{Q}_4)$ of Theorem 2 are easily verified to be right. So we can conclude that the positive equilibrium point $\mathscr{U}(9.5, 10.775,14)$ of system (46) is locally asymptotically stable if the time delay θ lies in the interval [0,0.25) and the Hopf bifurcation phenomenon for system (46) will arise near the positive equilibrium point $\mathscr{U}(9.5,10.775,14)$ if θ=0.25. In this paper, we use the predictor-correctors approach [39, 41, 42] to discretize system (46) and by virtue of the MATLAB software to carry out numerical simulations. In order to display these results, we select two sets of different delay parameters. Firstly, we choose θ=0.20<θ_0=0.25, and the software simulation plots are presented in Figure 2, which implies that $u_1 \longrightarrow 9.5, u_2 \longrightarrow 10.775, u_3 \longrightarrow 14$ as the time t tends to infinity. From the chemical point of view, the mole fraction of the constituent $\mathscr{X}$ will be close to 9.5, the mole fraction of the constituent $\mathscr{Y}$ will be close to 10.775, and the mole fraction of the constituent $\mathscr{Z}$ will be close to 14. Secondly, we choose θ=0.28>θ_0=0.25, and the software simulation plots are presented in Figure 3, which implies that a Hopf bifurcation periodic solution of system (46) will arise near the positive equilibrium point $\mathscr{U}(9.5,10.775,14)$ as the time t tends to infinity. From the chemical point of view, the mole fraction of the constituent $\mathscr{X}$, the mole fraction of the constituent $\mathscr{Y}$, and the mole fraction of the constituent $\mathscr{Z}$ will remain periodically oscillatory situations near the values 9.5, 10.775, 14, respectively. Furthermore, we give the bifurcation plot, which can be seen in Figure 4, to indicate that the bifurcation value of system (46) is 0.25.

6. Conclusions

Suppressing the chaotic behavior of nonlinear dynamical systems has been a significant and classic issue in many disciplines. For a long time, the suppression of chaos has attracted much attention from many scholars in mathematics, physics, chemistry, engineering, and numerous other areas. In the present manuscript, based on the earlier publications, we set up a novel fractional-order chaotic chemical reaction model. Taking advantage of an appropriate fractional-order PD$^{\zeta}$ controller, we can effectively eliminate the chaotic phenomenon of the involved fractional-order chaotic chemical reaction model. A delay-independent sufficient condition to guarantee the stability and the creation of Hopf bifurcation of the fractional-order controlled chaotic chemical reaction model is built. The exploration manifests that the delay occurring in fractional-order PD$^{\zeta}$ controller is the key factor in suppressing the chaotic phenomenon of the fractional-order chaotic chemical reaction model. The derived conclusions of this manuscript are entirely new and the exploration approach of this manuscript can also be utilized to inquire into numerous chaos control problem of lots of fractional-order chaotic dynamical systems.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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