Research Article

Orthogonally Fixed Points and (m, m) -Hom-Derivation Equations

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In this paper, we introduce the concept of m -Hom- m -derivation (briefly (m, m) -Hom-derivation) equations in orthogonally Banach algebras. We use the orthogonally fixed point to investigate the hyperstability of (m, m) -Hom-derivation equations.

1. Introduction and Preliminaries

The stability problem of functional equations originated from a question of Ulam [1] concerning the stability of group homomorphisms. Hyers [2] answered to the question of Ulam between Banach spaces. In 1978, Rassias [3] generalized for additive mappings. See [4–9] for more information.

In addition, Hyers–Ulam stability of functional equations on the normed space, Banach space, and Banach algebras was given. Later, other research studies on the Hyers–Ulam stability of fractional-order systems were presented. Obloza, in 1997, was the first author who introduced the Hyers–Ulam stability for linear differential equations [10, 11]. In recent years, Hyers–Ulam stability theory of fractional differential equations has attracted great attention, for example, with the extension of the theory of fractional calculus (integral and derivative of arbitrary order) [12–16]; some researchers tried to extend the above concept for the fractional differential equations [17–20].

Recently, Eshaghi Gordji et al. [21] introduced the notion of the orthogonal set which contains the notion of the orthogonality in normed space. By using the concept of orthogonal sets, Bahraini et al. [22] proved the generalization of the Diaz–Margolis [23] fixed point theorem on these sets. The study on orthogonal sets has been done by several authors (for example, see [24–26]).

Definition 1 (see [21]). Let $A \neq \emptyset$ and $\perp \subseteq A \times A$ be a binary relation. If $\perp$ satisfies

$$\exists a_0 (\forall b; b \perp a_0) \text{ or } (\forall b; a_0 \perp b), \quad (1)$$

it is called an orthogonal set (or O -set), and we denote this O -set by $(A, \perp)$.

Let $(A, \perp)$ be an O -set and (A, d) be a generalized metric space; then, $(A, \perp, d)$ is called an orthogonally generalized metric space.

Definition 2 (see [21]). Let $(X, \perp, d)$ be an orthogonally metric space.

- (i) A sequence $\{a_n\}_{n \in \mathbb{N}}$ is called an orthogonal sequence (briefly O -sequence) if

$$(\forall n; a_n \perp a_{n+1}) \text{ or } (\forall n; a_{n+1} \perp a_n). \quad (2)$$

- (ii) A mapping g from X into X is $\perp$ -continuous in $a \in X$ if for all O -sequences $\{a_n\}_{n \in \mathbb{N}}$ in X with $a_n \rightarrow a$, $g(a_n) \rightarrow g(a)$. Clearly, every continuous map is $\perp$ -continuous at any $a \in X$.
- (iii) $(X, \perp, d)$ is orthogonally complete (briefly O -complete) if every Cauchy O -sequence is convergent to a point in X .
- (iv) A mapping g from X into X is $\perp$ -preserving if for all $a, b \in X$ with $a \perp b$, $g(a) \perp g(b)$.

- (v) A mapping g from X into X is called an orthogonality contraction with Lipschitz constant $0 < \lambda < 1$ if

$$d(ga, gb) \leq \lambda d(a, b) \text{ where } a \perp b. \quad (3)$$

Example 3. Let $X = [0, 1]$, and let the metric on X be Euclidean metric. For all $a, b \in X$, $a \perp b$ is defined by $ab \leq \max\{(a/s), (b/s)\}$. Let $g: X \rightarrow X$ be a mapping defined by

$$g(a) = \begin{cases} \frac{a}{s}, & a \leq \frac{1}{s}, \\ 0, & a > \frac{1}{s}, \end{cases} \quad (4)$$

where $s = 2, 3, \dots$. Since $a \perp b$ and $ab \leq \max\{(a/s), (b/s)\}$, we have the following cases:

- (1) If $a = 0$, $b \leq (1/s) \Rightarrow g(a) = 0, g(b) = (b/s)$
- (2) If $a = 0$, $b \leq (1/s) \Rightarrow g(a) = g(b) = 0$
- (3) If $b \leq (1/s)$, $a \leq (1/s) \Rightarrow g(b) = (b/s), f(a) = (a/s)$
- (4) If $b \leq (1/s)$, $a \leq (1/s) \Rightarrow g(b) = (b/s), g(a) = 0$

Hence, it is clear that X is not complete, but it is easy to see that O -complete, g is not continuous and contraction on X but it is easy to see that $\perp$ -continuous and $\perp$ - λ -contraction for $\lambda = (1/s)$, and in the end, g is $\perp$ -preserving on X .

Theorem 1 (see [22]). Suppose that $(X, d, \perp)$ is an O -complete generalized metric space. Let $J: X \rightarrow X$ be a $\perp$ -preserving, $\perp$ -continuous, and $\perp$ - λ -contraction. Let $a_0 \in X$ satisfy for all $b \in X$, $a_0 \perp b$ or for all $b \in X$, $b \perp a_0$, and consider the “ O -sequence of successive approximations with initial element a_0 :” $a_0, J(a_0), J^2(a_0), \dots, J^n(a_0), \dots$. Then, either

$$d(J^n(a_0), J^{n+1}(a_0)) = \infty, \quad \forall n \geq 0, \quad (5)$$

or there exists a positive integer n_0 such that

$$d(J^n(a_0), J^{n+1}(a_0)) < \infty, \quad (6)$$

for all $n > n_0$. If the second alternative holds, then

- (1) The O -sequence of $\{J^n(a_0)\}$ is convergent to a fixed point a^* of J .

- (2) a^* is the unique fixed point of J in $X^* = \{b \in X: d(J^n(a_0), b) < \infty\}$.

- (3) If $b \in X$, then

$$d(b, a^*) \leq \frac{1}{1-\lambda} d(b, J(b)). \quad (7)$$

Consider the generalized additive, quadratic, cubic, and quartic functional equation

$$f(sa + b) + f(sa - b) = s^{m-2} [f(a + b) + f(a - b)] + 2(s^2 - 1) \cdot \left[s^{m-2} f(a) + \frac{(m-2)(1 - (m-2)^2)}{6} f(b) \right], \quad (8)$$

where $1 \leq m \leq 4$ is an integer number and $s \neq 0, \pm 1$ is a real number.

Definition 4. Let $\mathfrak{A}$ be an orthogonally Banach algebra, and $H: \mathfrak{A} \rightarrow \mathfrak{A}$ satisfying (8) and

$$H(ab) = H(a)H(b), \quad \forall a, b \in \mathfrak{A} \text{ with } a \perp b, \quad (9)$$

for $m = 1, 2, 3, 4$, is called an additive homomorphism, quadratic homomorphism, cubic homomorphism, and quartic homomorphism, respectively, or, briefly, m -homomorphisms.

In 2018, Park et al. [27] defined Hom-derivation, and in the following, we defined (m, m) -Hom-derivations (for $m = 1, 2, 3, 4$) on orthogonally Banach algebra.

Definition 5. Let $H: \mathfrak{A} \rightarrow \mathfrak{A}$ be m -homomorphisms, $m \in \{1, 2, 3, 4\}$. An m -mapping $D: \mathfrak{A} \rightarrow \mathfrak{A}$ is called (m, m) -Hom-derivations if D satisfies

$$D(ab) = D(a)H(b) + H(a)D(b), \quad \forall a, b \in \mathfrak{A} \text{ with } a \perp b. \quad (10)$$

In the present paper, we investigate the stability of orthogonally (m, m) -Hom-derivations for functional equation (8) on orthogonally Banach algebra by using the orthogonally fixed point.

2. Main Results

Throughout the section, let $\mathfrak{A}$ be an orthogonally Banach algebra. Let f, h be mappings between $\mathfrak{A}$, for all m positive integers less than 5 and $k \neq 0, \pm 1$, for all $a, b, u, v \in \mathfrak{A}$ with $a \perp b$, and $u \perp v$; for simplicity, denote

$$\Phi_m f(a, b) = f(ka + b) + f(ka - b)$$

$$-k^{m-2}[f(a+b) + f(a-b)] - 2(k^2 - 1)\left[k^{m-2}f(a) + \frac{(m-2)(1-(m-2)^2)}{6}f(b)\right], \quad (11)$$

$$\Psi_m f(uv) = f(u)h(v) + h(u)f(v).$$

Suppose that φ_m and ψ_m are mappings from $\mathfrak{A}^2$ into $[0, \infty)$ such that

$$\varphi_m\left(\frac{a}{s}, \frac{b}{s}\right) \leq \frac{L}{|s|^m} \varphi_m(a, b), \quad (12)$$

$$\psi_m\left(\frac{u}{s}, \frac{v}{s}\right) \leq \frac{L}{s^{2m}} \psi_m(u, v), \quad (13)$$

for all $a, b, u, v \in \mathfrak{A}$ with $a \perp b$ and $u \perp v$ and for some constant $0 < L < 1$. If we take $a = b = u = v = 0$, $\varphi_m(0, 0) = 0$ and $\psi_m(0, 0) = 0$.

It follows (12) and (13):

$$\lim_{n \rightarrow \infty} |s|^{mn} \varphi_m\left(\frac{a}{s^n}, \frac{b}{s^n}\right) = 0, \quad (14)$$

$$\lim_{n \rightarrow \infty} s^{2mn} \psi_m\left(\frac{a}{s^n}, \frac{b}{s^n}\right) = 0, \quad (15)$$

for all $a, b, u, v \in \mathfrak{A}$ with $a \perp b$ and $u \perp v$ and $s \neq 0, \pm 1$.

Theorem 2. Let $f, \zeta: \mathfrak{A} \rightarrow \mathfrak{A}$ be two mappings satisfying

$$\|\Phi_m f(a, b)\| \leq \varphi_m(a, b), \quad (16)$$

$$\|\Phi_m \zeta(a, b)\| \leq \varphi_m(a, b), \quad (17)$$

$$\|\zeta(uv) - \zeta(u)\zeta(v)\| \leq \psi_m(u, v), \quad (18)$$

$$\|f(uv) - \Psi_m f(uv)\| \leq \psi_m(u, v), \quad (19)$$

where $\varphi_m, \psi_m(a, b)$ fulfills (12) and (13). Then, there exist unique orthogonally m -homomorphisms $H_m: \mathfrak{A} \rightarrow \mathfrak{A}$ and unique orthogonally (m, m) -Hom-derivations $D_m: \mathfrak{A} \rightarrow \mathfrak{A}$ such that

$$\|\zeta(a) - H_m(a)\| \leq \frac{1}{2|s|^m(L^{-1} - 1)} \varphi_m(a, 0), \quad (20)$$

$$\|f(a) - D_m(a)\| \leq \frac{1}{2|s|^m(L^{-1} - 1)} \varphi_m(a, 0), \quad (21)$$

for all $a \in \mathfrak{A}$ and $m = 1, 2, 3, 4$.

Proof. Let Δ be the set of all mappings $k: \mathfrak{A} \rightarrow \mathfrak{A}$ such that $k(a) \perp s^m k((1/s)a)$ or $s^m k((1/s)a) \perp k(a)$, for all $a \in \mathfrak{A}$. Define d_φ on Δ by

$$d_\varphi(k, t) = \inf\{\alpha \in \mathbb{R}^+: \|k(a) - t(a)\| \leq \alpha \varphi_m(a, 0) \quad \forall a \in \mathfrak{A}\}, \quad (22)$$

and suppose that, for all $k, t \in \Delta$,

$$k \perp t \iff k(a) \perp t(a) \text{ or } t(a) \perp k(a), \quad (23)$$

for all $a \in \mathfrak{A}$.

Clearly, $(\mathfrak{A}, \Delta, \perp)$ is an O -complete generalized metric space. Define $\Gamma: \Delta \rightarrow \Delta$ by $\Gamma k(a) = s^m k((1/s)a)$, $a \in \mathfrak{A}$ and $m = 1, 2, 3, 4$. Then,

$$\begin{aligned} \|\Gamma k(a) - \Gamma t(a)\| &= \left\| s^m k\left(\frac{1}{k}a\right) - s^m t\left(\frac{1}{k}a\right) \right\| \leq s^m \alpha \varphi_m\left(\frac{a}{s}, 0\right) \\ &\leq L \alpha \varphi_m(a, 0). \end{aligned} \quad (24)$$

So, by the definition of d_φ on Δ , for $k, t \in \Delta$ with $k \perp t$ or $t \perp k$ and $d_\varphi(k, t) < \alpha$, we have $\|\Gamma k - \Gamma t\| \leq \alpha L$. This shows that $d_\varphi(\Gamma k, \Gamma t) \leq L d_\varphi(k, t)$, i.e., Γ is $\perp$ -contraction. The function Γ is $\perp$ -continuous. In fact, if $\{k_n\}$ is an O -sequence in Δ which converges to $k \in \Delta$, then for given $\varepsilon > 0$, there exist $\alpha > 0$ with $\alpha < \varepsilon$ and $n \in \mathbb{N}$ such that

$$\|k_n(a) - k(a)\| \leq \alpha \varphi(a, 0). \quad (25)$$

Therefore, by the similar argument, for all $a \in \mathfrak{A}$, $m = 1, 2, 3, 4$, and $n \geq N$, we have

$$d_\varphi(\Gamma(k_n), \Gamma(k)) \leq L\alpha < L\varepsilon. \quad (26)$$

Clearly, Γ is $\perp$ -preserving.

We show that, for any $f \in \Delta$, we have

$$d_\varphi(\Gamma^{n+1}f, \Gamma^n f) \leq \infty. \quad (27)$$

In (16), put $b = 0$. By induction, we have

$$\left\| f(a) - s^{mn} f\left(\frac{a}{s^n}\right) \right\| \leq \frac{L^{n-1}}{|s|^{m(n-1)}} \varphi_m(a, 0). \quad (28)$$

Then, for $L \in (0, 1)$,

$$\begin{aligned} \|\Gamma^{n+1}f - \Gamma^n f\| &= s^{nm} \left\| s f\left(\frac{a}{s^{n+1}}\right) - f\left(\frac{a}{s^n}\right) \right\| \\ &\leq L^n \varphi_m(a, 0) \\ &\longrightarrow 0 \quad \text{as } n \longrightarrow +\infty, \end{aligned} \quad (29)$$

and then, all conditions of Theorem 3 hold. Furthermore, similarly, mapping ζ holds.

So, the O -sequence $\{\Gamma^n f\}$ converges to the unique fixed point D_m in the set of $\{k \in \Delta: d_\varphi(\Gamma^n f, k) < \infty\}$, i.e.,

$$D_m(a) = \lim_{n \rightarrow \infty} \Gamma^n f = \lim_{n \rightarrow \infty} s^{mn} f\left(\frac{a}{s^n}\right). \quad (30)$$

Also, for $f \in \Delta$,

$$d_\varphi(f, D_m) \leq \frac{L}{1-L} d_\varphi(f, \Gamma f), \quad (31)$$

and by (29), $d(f, D_m) \leq L\varphi_m(a, 0)$. Therefore, D_m satisfies in (21), i.e.,

$$\|f(a) - D_m(a)\| \leq \frac{L}{2|s|^m(1-L)} \varphi_m(a, 0). \quad (32)$$

Similarly,

$$H_m(a) = \lim_{n \rightarrow \infty} \Lambda^n \zeta = \lim_{n \rightarrow \infty} s^{mn} \zeta\left(\frac{a}{s^n}\right), \quad (33)$$

and H satisfies in (20), i.e.,

$$\|\zeta(a) - H_m(a)\| \leq \frac{L}{2|s|^m(1-L)} \varphi_m(a, 0). \quad (34)$$

We claim that

- (1) H_m is the unique desired orthogonally m -homomorphism which satisfies in (20)
- (2) D_m is the unique desired orthogonally (m, m) -Hom-derivation which satisfies in (21)

First of all, D_m and H are two m -mappings. In fact, for all $a, b \in \mathfrak{A}$ with $a \perp b$ and $m = 1, 2, 3, 4$ and using (12), we have

$$\begin{aligned} \|\Phi_m D(a, b)\| &= \lim_{n \rightarrow \infty} s^{mn} \left\| \Phi_m f\left(\frac{a}{s^n}, \frac{b}{s^n}\right) \right\| \\ &\leq \lim_{n \rightarrow \infty} s^{2mn} \Phi_m \left(\frac{a}{s^n}, \frac{b}{s^n}\right) \\ &= 0. \end{aligned} \quad (35)$$

So, $\Phi_m D(a, b) = 0$. Similarly, $\Phi_m H(a, b) = 0$. By the result in [28], D_m and H_m are orthogonally m -mappings. $\square$

Proof. of Claim (1). By using (14) and (18), we have

$$\begin{aligned} \|H_m(uv) - H_m(u)H_m(v)\| &= \lim_{n \rightarrow \infty} s^{2mn} \left\| \zeta\left(\frac{uv}{s^{2n}}\right) - \zeta\left(\frac{u}{s^n}\right)\zeta\left(\frac{v}{s^n}\right) \right\| \\ &\leq \lim_{n \rightarrow \infty} s^{2mn} \psi_m\left(\frac{u}{s^n}, \frac{v}{s^n}\right) \\ &= 0, \end{aligned} \quad (36)$$

for all $u, v \in \mathfrak{A}$ with $u \perp v$ and $m = 1, 2, 3, 4$.

Therefore, H_m is an orthogonally m -homomorphism. $\square$

Proof. of Claim (2). By using (14) and (19), we have

$$\begin{aligned} \|D_m(uv) - \Psi_m D(uv)\| &= \lim_{n \rightarrow \infty} s^{2mn} \left\| f\left(\frac{uv}{s^{2n}}\right) - f\left(\frac{u}{s^n}\right)\zeta\left(\frac{v}{s^n}\right)^m - \zeta\left(\frac{u}{s^n}\right)^m f\left(\frac{v}{s^n}\right) \right\| \\ &\leq \lim_{n \rightarrow \infty} s^{2mn} \psi_m\left(\frac{u}{s^n}, \frac{v}{s^n}\right) \\ &= 0, \end{aligned} \quad (37)$$

for all $u, v \in \mathfrak{A}$ with $u \perp v$ and $m = 1, 2, 3, 4$. So, $D_m(uv) = \Psi_m(uv)$ for all $u, v \in \mathfrak{A}$ with $u \perp v$ and $m = 1, 2, 3, 4$. Therefore, D_m is an orthogonally (m, m) -Hom-derivation. $\square$

Corollary 1. Let $\theta, p \in \mathbb{R}^+$ with $p > m$, and $f, \zeta: \mathfrak{A} \rightarrow \mathfrak{A}$ are mappings such that

$$\|\Phi_m f(a, b)\| \leq \theta(\|a\|^p + \|b\|^p), \quad (38)$$

$$\|\Phi_m \zeta(a, b)\| \leq \theta(\|a\|^p + \|b\|^p), \quad (39)$$

$$\|\zeta(uv) - \zeta(u)\zeta(v)\| \leq \theta(\|u\|^p + \|v\|^p), \quad (40)$$

$$\|f(uv) - \Psi_m f(uv)\| \leq \theta(\|u\|^p + \|v\|^p), \quad (41)$$

for all $a, b, u, v \in \mathfrak{A}$ with $a \perp b$ and $u \perp v$; then, there exist unique orthogonally m -homomorphisms $H_m: \mathfrak{A} \longrightarrow \mathfrak{A}$ and unique orthogonally (m, m) -Hom-derivations $D_m: \mathfrak{A} \longrightarrow \mathfrak{A}$ such that

$$\|\zeta(a) - H_m(a)\| \leq \frac{\theta}{2(|s|^p - |s|^m)} \|a\|^p, \quad (42)$$

$$\|f(a) - D_m(a)\| \leq \frac{\theta}{2(|s|^p - |s|^m)} \|a\|^p. \quad (43)$$

Proof. By taking

$$\begin{aligned} \varphi_m(a, b) &:= \theta(\|a\|^p + \|b\|^p), \\ \psi_m(u, v) &:= \theta(\|u\|^p + \|v\|^p), \\ L &= |s|^{m-p}, \end{aligned} \quad (44)$$

and using Theorem 2, we get the desired results. $\square$

In the next theorem, we prove the hyperstability of orthogonally (m, m) -Hom-derivations for functional equation (8).

Theorem 3. Suppose there exist functions φ_m, ψ_m such that

$$\lim_{n \rightarrow \infty} |s|^{mn} \varphi_m\left(0, \frac{b}{s^n}\right) = 0, \quad (45)$$

$$\lim_{n \rightarrow \infty} s^{2mn} \psi_m\left(\frac{u}{s^n}, \frac{v}{s^n}\right) = 0, \quad (46)$$

for all $a, b, u, v \in \mathfrak{A}$ with $a \perp b$ and $u \perp v$. Moreover, assume that $f, \zeta: \mathfrak{A} \longrightarrow \mathfrak{A}$ are mappings such that

$$\|\Phi_m f(a, b)\| \leq \varphi_m(0, b), \quad (47)$$

$$\|\Phi_m \zeta(a, b)\| \leq \varphi_m(0, b), \quad (48)$$

$$\|\zeta(uv) - \zeta(u)\zeta(v)\| \leq \psi_m(u, v), \quad (49)$$

$$\|f(uv) - \Psi_m f(uv)\| \leq \psi_m(u, v), \quad (50)$$

for all $a, b, u, v \in A$ with $a \perp b$ and $u \perp v$; then, f is an orthogonally (m, m) -Hom-derivation.

Proof. Letting $b = 0$ in (47), we get $f(sa) = s^m f(a)$, and by using induction on $n \in \mathbb{N}$, we get

$$f(a) = s^{mn} f\left(\frac{a}{s^n}\right). \quad (51)$$

Similarly, we have

$$\zeta(a) = s^{mn} \zeta\left(\frac{a}{s^n}\right). \quad (52)$$

Hence, we have

$$\|\Phi_m f(a, b)\| = |s|^{mn} \left\| \Phi_m f\left(\frac{a}{s^n}, \frac{b}{s^n}\right) \right\| \leq |s|^{mn} \varphi_m\left(0, \frac{b}{s^n}\right), \quad (53)$$

for all $a, b \in \mathfrak{A}$ with $a \perp b$ and $n \in \mathbb{N}$. So, by $n \longrightarrow \infty$ in (53) and using (45), we have

$$\Phi_m f(a, b) = 0. \quad (54)$$

It follows from (49) and (52) that

$$\begin{aligned} \|\zeta(uv) - \zeta(u)\zeta(v)\| &= s^{2mn} \left\| \zeta\left(\frac{uv}{s^{2n}}\right) - \zeta\left(\frac{u}{s^n}\right)\zeta\left(\frac{v}{s^n}\right) \right\| \\ &\leq s^{2mn} \psi_m\left(\frac{u}{s^n}, \frac{v}{s^n}\right), \end{aligned} \quad (55)$$

for all $u, v \in \mathfrak{A}$ with $u \perp v$ and $n \in \mathbb{N}$. Hence, by $n \longrightarrow \infty$ in (55) and using (46), we have $\zeta(uv) = \zeta(u)\zeta(v)$ for all $u, v \in \mathfrak{A}$ with $u \perp v$, and it follows from (50) and (51) that

$$\begin{aligned} \|f(uv) - \Psi_m f(uv)\| &= s^{2mn} \left\| f\left(\frac{uv}{s^{2n}}\right) - f\left(\frac{u}{s^n}\right)\zeta\left(\frac{v}{s^n}\right)^m - \zeta\left(\frac{u}{s^n}\right)^m f\left(\frac{v}{s^n}\right) \right\| \\ &\leq s^{2mn} \psi_m\left(\frac{u}{s^n}, \frac{v}{s^n}\right), \end{aligned} \quad (56)$$

for all $u, v \in \mathfrak{A}$ with $u \perp v$ and $n \in \mathbb{N}$. Hence, by $n \longrightarrow \infty$ in (56) and using (46), we have $f(uv) = \Psi_m f(uv)$. Therefore, f is an orthogonally (m, m) -Hom-derivation. $\square$

Corollary 2. Let $\theta, p \in \mathbb{R}^+$ with $p > m$, and $f, \zeta: \mathfrak{A} \longrightarrow \mathfrak{A}$ are mappings such that

$$\begin{aligned} \|\Phi_m f(a, b)\| &\leq \theta \|b\|^p, \\ \|\Phi_m g(a, b)\| &\leq \theta \|b\|^p, \\ \|\zeta(uv) - \zeta(u)\zeta(v)\| &\leq \theta(\|u\|^p + \|v\|^p), \\ \|f(uv) - \Psi_m f(uv)\| &\leq \theta(\|u\|^p + \|v\|^p), \end{aligned} \quad (57)$$

for all $a, b, u, v \in \mathfrak{A}$ with $a \perp b$ and $u \perp v$; then, f is an (m, m) -Hom-derivation.

3. Conclusions

In this paper, the author introduced orthogonally (m, m) -Hom-derivation equations. Using an orthogonally fixed point theorem, it is proved that orthogonally (m, m) -Hom-derivation equations for orthogonally m -mappings on orthogonally Banach algebras can be stable and hyperstable.

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares that there are no conflicts of interest.

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