Hindawi
Journal of Mathematics
Volume 2023, Article ID 9806481, 1 page
https://doi.org/10.1155/2023/9806481

Retraction

Retracted: A New Four-Parameter Inverse Weibull Model: Statistical Properties and Applications

Journal of Mathematics

Received 10 October 2023; Accepted 10 October 2023; Published 11 October 2023

This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope
(2) Discrepancies in the description of the research reported
(3) Discrepancies between the availability of data and the research described
(4) Inappropriate citations
(5) Incoherent, meaningless and/or irrelevant content included in the article
(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

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[1] N. Yahia and N. Alsadat, "A New Four-Parameter Inverse Weibull Model: Statistical Properties and Applications," *Journal of Mathematics*, vol. 2022, Article ID 5936783, 6 pages, 2022.

Hindawi
Journal of Mathematics
Volume 2022, Article ID 5936783, 6 pages
https://doi.org/10.1155/2022/5936783

Research Article

A New Four-Parameter Inverse Weibull Model: Statistical Properties and Applications

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Received 15 March 2022; Revised 15 April 2022; Accepted 5 May 2022; Published 13 June 2022

Academic Editor: Naeem Jan

A four-parameter Type II Topp Leone generalized inverse Weibull (TIITLGINW) model is suggested. The reliability study of the new model is provided. Quantiles, moments, moment generating function, and probability weighted moment are some of the mathematical properties being researched. The maximum likelihood (ML) estimate is employed for TIITLGINW parameters. A simulation study is conducted to estimate the model parameters of the TIITLGINW model. A single real-world collection of data is used to analyze TIITLGINW's significance and accessibility.

1. Introduction

Because of its failure rate, the inverse Weibull (INW) model has a broader applicability in the field of dependability and biological investigations. Keller and Kanath [1] proposed the INW model to investigate the form of the density function (pdf) and hazard rate function (FUN) (hrf). The INW model fits numerous datasets through terms of the time required for an insulated fluid to decompose, the topic of which led to the action of continuous tension. Nelson and Jiang et al. [2, 3] presented Weibull (W) and INW mixing models [4]. Models with two INW models were examined [5]. The flexibility of the INW model was investigated [6]. Bayesian and maximum likelihood estimates of the INW parameters with progressive type-II censoring were examined.

The probability density function (pdf) and cumulative FUN (cdf) of the generalized INW (GINW) model are given by [7]

$$f(z;\mu,\eta,\gamma)=\frac{\gamma\eta\mu^{\eta}}{z^{\eta+1}}e^{-\gamma(\mu/z)^{\eta}},\quad z,\mu,\eta,\ \gamma>0, \tag{1}$$

and

$$F(z;\mu,\eta,\gamma)=e^{-\gamma(\mu/z)^{\eta}},\quad z,\mu,\eta,\ \gamma>0. \tag{2}$$

The INW model has recently been introduced in statistical theory literature [8]. A modified INW model was suggested, while Shahbaz et al. [9] proposed the Kumaraswamy INW model. Hanook et al. [10] developed the beta INW model, whereas Khan et al. [11] investigated features of the transmuted INW model [12]. The Topp Leone (TL) INW model was presented [13]. The beta generalized INW geometric model was investigated. Alkarni et al. [14] proposed the extended INW model. Even power weighted generalized INW distribution was studied by Mutlk and Al-Dubaicy [15], Algarni et al. [16] proposed classical and Bayesian estimation of the INW distribution under progressive type-I censoring scheme, Al-Moisheer et al. [17] discussed the odd inverse power generalized Weibull generated family of distributions, and Ahmadini et al. [18] studied estimation of the constant stress partially accelerated life test for INW distribution with type-I censoring.

Ahmadini et al. [18] investigated the Type II TL class (TIITL) class of models. In addition, Elgarhy et al. [19] developed a three-parameter TIITLINW model. The TIITL class cdf is supplied via

$$F(z;\ \mu,\ \omega)=1-\left[1-[G(x;\omega)]^{2}\right]^{\mu}. \tag{3}$$

The equivalent pdf to (3) is produced via

$$f(z;\,\mu,\omega) = 2\mu g(z;\omega)G(z;\omega) \cdot \left[1-[G(z;\omega)]^2\right]^{\mu-1}, \quad z\in R,\ \mu>0. \tag{4}$$

The main goal of this study is as follows:

To introduce a new four-parameter model which is called Type II Topp Leone generalized inverse Weibull model

The suggested model is very flexible and contains many submodels

The suggested model has closed form of quantile

The pdf of the suggested model can be unimodal and right skewness. Also, the hazard rate function can be increasing and J-shaped.

The following is how this document is structured. The Section 2 defines the new model (which is a broad model). The Section 3 investigates the linear formulation of the TIITLGINW model's pdf. Section 4 investigates statistical characteristics. In the Section 5, the ML estimation approach is used to generate the estimates of the TIITLGINW parameters. In Section 6, a simulation study is carried out to determine the model parameters of the TIITLGINW model. Section 7 employs the study of a single real-world data collection. Section 8 has concluding observations.

2. The New Model

Inside this section, we develop the TIITLGINW model, a novel lifespan model. The cdf of the TIITLGINW model with set of parameters $\varphi = (\mu,\eta,\rho,\gamma)$ is computed by inserting (2) into (3) as follows:

$$G(z;\,\varphi) = 1-\left[1-e^{-2\gamma(\mu/z)^{\eta}}\right]^{\rho}. \tag{5}$$

Inserting (1) and (2) into (4) yields the matching pdf to (5):

$$g(z;\,\varphi) = 2\rho\gamma\eta\mu^{\eta}z^{-\eta-1}e^{-2\gamma(\mu/z)^{\eta}} \cdot \left[1-e^{-2\gamma(\mu/z)^{\eta}}\right]^{\rho-1}, \quad z,\rho,\ \mu,\eta,\gamma>0, \tag{6}$$

where μ and γ are the scale parameters and ρ, η are the two shape parameters.

The TIITLGINW model is a highly adaptable model that contains several additional models. The submodels of the TIITLGINW model are given in Table 1.

Figure 1 shows different TIITLGINW pdf graphs for appropriate parameter combinations.

X's survival FUN (sf), hrf, inverted hrf, and cumulative hrf are described as follows:

$$R(z;\,\varphi) = \left[1-e^{-2\gamma(\mu/z)^{\eta}}\right]^{\rho}, \quad z,\ \rho,\ \mu,\eta,\gamma>0, \tag{7}$$

$$h(z;\,\varphi) = \frac{2\rho\gamma\eta\mu^{\eta}z^{-\eta-1}e^{-2\gamma(\mu/z)^{\eta}}}{1-e^{-2\gamma(\mu/z)^{\eta}}}, \tag{8}$$

$$\tau(z;\,\varphi) = \frac{2\rho\gamma\eta\mu^{\eta}z^{-\eta-1}e^{-2\gamma(\mu/z)^{\eta}}\left[1-e^{-2\gamma(\mu/z)^{\eta}}\right]^{\rho-1}}{1-\left[1-e^{-2\gamma(\mu/z)^{\eta}}\right]^{\rho}}, \tag{9}$$

$$H(z;\,\varphi) = -\ln\left[1-e^{-2\gamma(\mu/z)^{\eta}}\right]^{\rho}. \tag{10}$$

Figure 2 shows different TIITLGINW hrf graphs for appropriate parameter combinations.

3. Useful Expansion

Inside this part, we propose two useful pdf and cdf expansions for the TIITLGINW model. Now, examine the binomial series:

$$(1-a)^{n} = \sum_{k=0}^{\infty}(-1)^{k}\binom{a}{k}a^{k}, \quad 0<a<1. \tag{11}$$

As a result of using (11), the accompanying phrase within (6) can be indicated:

$$\left(1-e^{-2\gamma\left(\mu/z_i\right)^{\eta}}\right)^{\rho-1} = \sum_{i=0}^{\infty}(-1)^{i}\binom{\rho-1}{i}e^{-2\gamma\left(\mu/z_i\right)^{\eta}}. \tag{12}$$

After several simplifications, we arrive at

$$f(z) = \eta\mu^{\eta}\sum_{i=0}^{\infty}W_{i}z^{-\eta-1}e^{-2(i+1)\gamma\left(\mu/z_i\right)^{\eta}}, \tag{13}$$

where $w_i = 2(-1)^{i}\binom{\rho-1}{i}\rho\gamma$.

Also, the expansion of cdf can be expressed as follows:

$$[F(z)]^{h} = \left[1-\left(1-e^{-2\gamma\left(\mu/z_i\right)^{\eta}}\right)^{\rho}\right]^{h}. \tag{14}$$

Then,

$$[F(z)]^{h} = \sum_{k=0}^{\infty}W_{k}e^{-2k\gamma\left(\mu/z_i\right)^{\eta}}, \tag{15}$$

where $w_k = \sum_{j=0}^{h}(-1)^{j+k}\binom{h}{j}\binom{\rho j}{k}$.

4. Fundamental Mathematical Features

Numerous statistical features of the TIITLGINW model are obtained in this section.

4.1. Quantile Function. The quantile FUN of Z, denoted by Z_u, is determined via

$$Zu = \mu\left(\frac{-1}{\gamma}ln\sqrt{1-(1-u)^{1/\rho}}\right)^{(-1/\eta)}. \tag{16}$$

Table 1: Submodels of the TIITLGINW model.

	Model	ρ	μ	η	γ	cdf	Author
1	TIITLGINR	—	—	—	1	$F(z;\ \rho,\ \mu,\gamma) = 1 - [1 - e^{-2\gamma(\mu/z)^2}]^{\rho}$	New
2	TIITLGINE	—	—	—	2	$F(z;\ \rho,\ \mu,\gamma) = 1 - [1 - e^{-(2\gamma\mu/z)}]^{\rho}$	New
3	TIITLINW	—	1	—	—	$F(z;\ \rho,\ \mu) = 1 - [1 - e^{-2(\mu/z)^2}]^{\rho}$	New
4	TIITLINR	—	1	—	1	$F(z;\ \rho,\ \mu) = 1 - [1 - e^{-2(\mu/z)^2}]^{\rho}$	New
5	TIITLINE	—	1	—	2	$F(z;\ \rho,\ \mu) = 1 - [1 - e^{-(2\mu/z)}]^{\rho}$	New

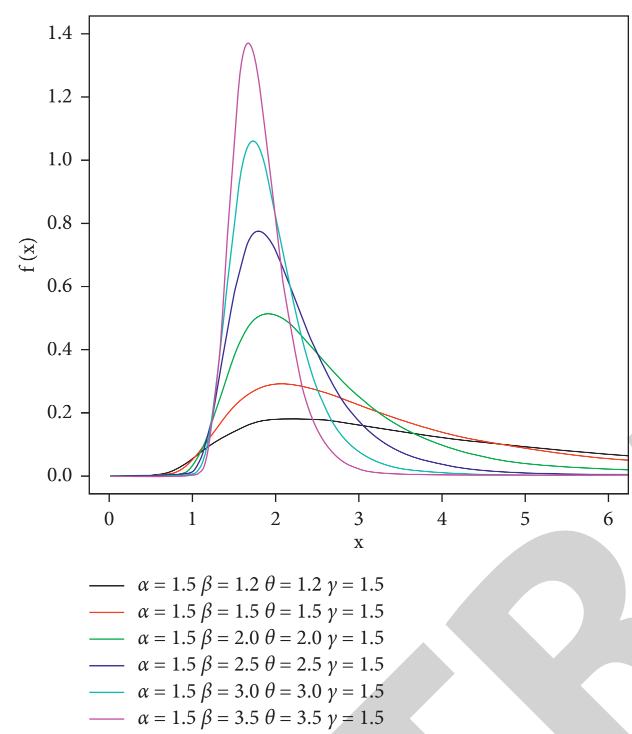

Figure 1: TIITLGINW pdf graphs for appropriate parameter combinations.

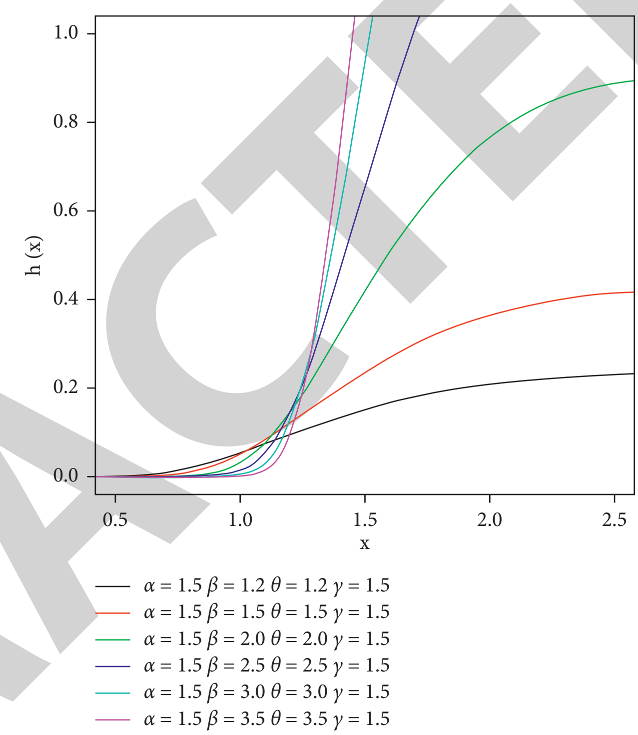

Figure 2: TIITLGINW hrf graphs for appropriate parameter combinations.

4.2. Different Types of Moments. The r$^{\text{th}}$ moment (Mom) of Z may be determined utilizing relation.

$$\mu_r' = E(Z^r) = \int_{-\infty}^{\infty} z^r g(z;\varphi)dz. \tag{17}$$

Simply replacing (13) within (17) results in

$$\mu_r' = \sum_{i=0}^{\infty} W_i \int_0^{\infty} z^{r-\eta-1} e^{-2(i+1)\gamma(\mu/z_i)^{\eta}} dz. \tag{18}$$

Suppose $y = (\mu/z_i)^{\eta}$; after that,

$$\mu_r' = \mu^r \sum_{i=0}^{\infty} W \int_i^{\infty} y e^{-2(i+1)\gamma y} dy. \tag{19}$$

Then, μ_r' becomes

$$\mu_r' = \mu^r \sum_{i=0}^{\infty} W_i \frac{\Gamma(1-(r/\eta))}{(2(i+1)\gamma)^{(1-(r/\eta))}}, \quad \frac{r}{\eta} < 1. \tag{20}$$

The TIITLGINW model's Mom generating FUN is provided by

$$M_Z(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} E(Z^r) = \sum_{r,i=0}^{\infty} \frac{t^r}{r!} \frac{w_i \Gamma(1-(r/\beta))}{[2(i+1)\gamma]^{(1-(r/\beta))}}, \quad r/\beta < 1. \tag{21}$$

The probability weighted Mom (PrWMs) may be computed as follows:

$$\tau_{r,s} = E[Z^r G(z)^s] = \int_{-\infty}^{\infty} x^r g(z)(G(z))^s dz. \tag{22}$$

Trying to insert (13) and (6) within (22), we get

$$\tau_{r,s} = \sum_{i,k=0}^{\infty} W_i W_k \int_0^{\infty} z^{r-\eta-1} e^{-2(i+k+1)\gamma(\mu/z_i)^{\eta}} dz. \tag{23}$$

As a result, the PrWM of the TIITLGINW model looks like

$$\tau_{r,s} = \mu^r \sum_{i,k=0}^{\infty} W_i W_k \frac{\Gamma(1-(r/\eta))}{(2(i+k+1)\gamma)^{(1-(r/\eta))}}, \quad \frac{r}{\eta} < 1. \tag{24}$$

Table 2: Numerous numerical values of MLEs and C1 of the TIITLGINW model.

n	Par	MLE	C1	MLE	C1
		Set 1: (0.5, 0.5, 0.5, 0.5)		Set 2: (0.5, 0.5, 0.5, 0.5)	
30	μ	0.4870	0.0071	0.4991	0.0042
	η	0.4711	0.0212	1.5740	0.2106
	ρ	0.4664	0.0163	0.5386	0.0424
	γ	0.3743	0.0329	0.8978	0.5204
50	μ	0.5217	0.0031	0.4917	0.0030
	η	0.4862	0.0102	1.5609	0.1372
	ρ	0.5095	0.0071	0.5054	0.0227
	γ	0.3871	0.0245	0.7790	0.3332
200	μ	0.5007	0.0011	0.5021	0.0016
	η	0.4889	0.0033	1.5440	0.0447
	ρ	0.5078	0.0027	0.5155	0.0117
	γ	0.4073	0.0127	0.7120	0.0856
		Set 3: (0.5, 0.5, 0.5, 0.3)		Set 4: (0.5, 1.5, 0.5, 0.3)	
30	μ	0.5410	0.0162	0.5118	0.0136
	η	0.5525	0.0512	1.8293	0.6032
	ρ	0.5455	0.0332	1.0048	2.5698
	γ	0.4392	0.1288	0.4504	0.2833
50	μ	0.5526	0.0097	0.5280	0.0082
	η	0.5478	0.0276	1.7470	0.2578
	ρ	0.5551	0.0173	0.6603	0.1249
	γ	0.3953	0.0391	0.3532	0.0251
200	μ	0.4849	0.0012	0.4809	0.0017
	η	0.4405	0.0061	1.6474	0.0560
	ρ	0.4713	0.0028	0.4649	0.0086
	γ	0.2932	0.0028	0.2548	0.0042
		Set 5: (0.3, 0.8, 0.3, 0.5)		Set 6: (0.3, 0.5, 0.3, 0.3)	
30	μ	0.2961	0.0030	0.3516	0.0106
	η	0.8937	0.2663	0.4791	0.0338
	ρ	0.3563	0.0362	0.4123	0.0572
	γ	0.5952	0.5313	0.1735	0.6286
50	μ	0.2888	0.0010	0.3049	0.0021
	η	0.6398	0.0470	0.3800	0.0218
	ρ	0.2708	0.0048	0.3028	0.0061
	γ	0.2629	0.0633	0.1901	0.4063
200	μ	0.3048	0.0005	0.3041	0.0003
	η	0.7341	0.0092	0.3934	0.0131
	ρ	0.3122	0.0017	0.3094	0.0014
	γ	0.3184	0.0354	0.4173	0.0356

5. ML Method of Approach

The ML estimates (MLEs) of the unknown parameters for such TIITLGINW model are produced using complete samples. Assume $Z_1, \ldots, Z_n$ be seen from the TIITL-GINW model with something like a certain number of parameters $\varphi = (\mu, \eta, \rho, \gamma)^T$. The total log-likelihood (LL) FUN for the vector of parameters φ may be phrased as

$$\ln L(\varphi) = n \ln 2\rho + n \ln \gamma + n \ln \eta + n\eta \ln \mu - (\eta+1)\sum_{i=1}^{n} \ln z_i - 2\gamma \sum_{i=1}^{n}\left(\frac{\mu}{z_i}\right)^{\eta} + (\rho-1)\sum_{i=1}^{n}\ln\left[1 - e^{-2\gamma(\mu/z_i)^{\eta}}\right]. \tag{25}$$

The scoring FUN $U(\varphi) = (U_\mu, U_\eta, U_\rho, U_\gamma)$ elements are specified by

$$U_\mu = \frac{n\eta}{\mu} - 2\gamma\eta\mu^{\eta-1}\sum_{i=1}^{n} z_i^{-\eta} + 2\gamma\eta\mu^{\eta-1}(\rho-1)\sum_{i=1}^{n}\frac{z_i^{-\eta} e^{-2\gamma(\mu/z_i)^{\eta}}}{1 - e^{-2\gamma(\mu/x_i)^{\eta}}}, \tag{26}$$

$$U_\eta = \frac{n}{\eta} - n\ln\mu - \sum_{i=1}^{n}\ln z_i - 2\gamma\sum_{i=1}^{n}\left(\frac{\mu}{z_i}\right)^{\eta}\ln\left(\frac{\mu}{z_i}\right) + (\rho-1)\sum_{i=1}^{n}\frac{2\gamma(\mu/z_i)^{\eta}\ln(\mu/z_i) e^{-2\gamma(\mu/z_i)^{\eta}}}{1 - e^{-2\gamma(\mu/z_i) z_i^{\eta}}}, \tag{27}$$

Table 3: MLEs, C2, C3, C4, and C5 for the dataset.

Model	MLE and C2				C3	C4	C5
TIITLGINW (μ, η, ρ, γ)	1.494 (195300)	3.724 (2.265)	1.146 (1.21)	0.664 (323200)	30.8	0.09702	0.99177
TIITLGIR (μ, ρ, γ)	1.112 (333800)	3.61 (1.334)	2.204 (1323000)		31.736	0.12642	0.90655
GINW (μ, β, γ)	1.868 (282500)	4.017 (0.697)	0.489 (296900)		30.817	0.10195	0.98545
GIR (μ, γ)	1.953 (244100)	0.724 (181000)			42.365	0.25659	0.14358
TIITLIE (μ, ρ)	3.086 (0.543)	20.766 (10.928)			34.209	0.15423	0.72826
IE (μ)	1.725 (0.386)				65.337	0.38725	0.00497

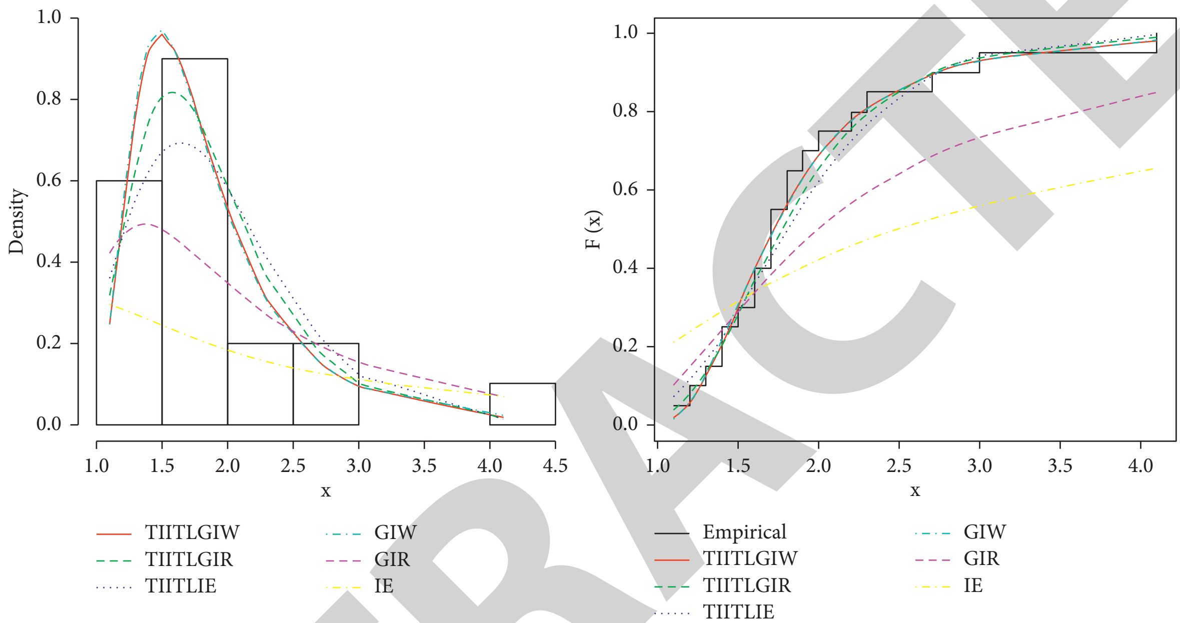

Figure 3: The empirical pdf and empirical cdf of the TIITLGINW model.

$$U_{\rho} = \frac{n}{\rho} + \sum_{i=1}^{n} \ln\left[1 - e^{-2\gamma (\mu/z_i)^{\eta}}\right], \tag{28}$$

$$U_{\gamma} = \frac{n}{\gamma} - 2\sum_{i=1}^{n}\left(\frac{\mu}{z_i}\right)^{\eta} + 2(\rho - 1)\sum_{i=1}^{n}\frac{(\mu/z_i)^{\eta} e^{-2\gamma (\mu/z_i)^{\eta}}}{1 - e^{-2\gamma (\mu/z_i)^{\eta}}}. \tag{29}$$

The MLEs of the φ parameters are then produced via assigning $U(\varphi) = 0$ and calculating them.

6. Numerical Outcomes

Comparing the theoretical performances of alternative estimators MLE for the TIITLGINW model is extremely challenging. Mathematica 9 software is used to do a numerical analysis. The experiments take into account different sample sizes of $n = 30$, 50, and 200, and furthermore, the various values of the φ parameters.

The study will indeed be repeated 5000 times in total. In each experiment, ML estimation techniques will be utilized to provide parameter estimates. As a consequence of these experiments, the MLEs and mean square errors (C1) for the various estimators will be reported.

7. Modelling

Throughout this section, we test the adaptability of the TIITLGINW model by applying it to a real-world data collection. The TIITLGINW model is compared to the TIITLGIR, TIITLIE, GINW, GIR, and IE models.

The used data are reported in [20], and it is 2.7, 4.1, 1.8, 1.5, 1.1, 1.4, 1.8, 1.6, 2.2, 1.7, 1.2, 1.4, 3, 1.3, 1.7, 1.9, 1.7, 2.3, 1.6, 2.

Tables 2 and 3 provide the ML estimates as well as the standard errors (C2) of the model parameters. Analytical metrics such as 2LL (C3), Kolmogorov–Smirnov (C4), and p value (C5) are included in the identical tables.

The numerical values of MLEs, C2, C3, C4, and C5 are given in Table 3.

The fits of the TIITLGINW model to the TIITLGIR, TIITLIE, GINW, GIR, and IE models are compared and given in Table 3. The statistics in these tables demonstrate that the

TIITLGINW model has the lowest C3 and C4 values and the highest C5 of any fitted model. As a result, it may be picked as the best model. Figure 3 shows the fitted pdf and estimated cdf plots for the TIITLGINW model.

8. Summary and Conclusion

The TIITLGINW model, a novel four-parameter model, is presented throughout this study. Simply put, TIITLGINW pdfs are a linear combination of GINW densities. We compute accurate formulations for certain of its statistical properties. We look into estimation using machine learning. The proposed model outperforms certain rival models in terms of fit when tested on real data. Also, in the future work, many authors can use this model to generalize it or study it as statistical inference using censored schemes.

Abbreviations

TIITLGINW: Type II Topp Leone generalized inverse Weibull
ML: Maximum likelihood
pdf: Density function
hrf: Hazard rate function
FUN: Function
cdf: Cumulative function
sf: Survival function
Mom: Moment
PrWMs: Probability weighted moment
MLEs: ML estimates.

Data Availability

The data used to support this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

Acknowledgments

This research project was supported by a grant from the "Research Center of the Female Scientific and Medical Colleges," Deanship of Scientific Research, King Saud University.

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