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Research Article

Computation of Topological Indices of Double and Strong Double Graphs of Circumcoronene Series of Benzenoid (H_m)

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Topological indices are very useful to assume certain physiochemical properties of the chemical compound. A molecular descriptor which changes the molecular structures into certain real numbers is said to be a topological index. In chemical graph theory, to create quantitative structure activity relationships in which properties of molecule may be linked with their chemical structures relies greatly on topological indices. The benzene molecule is a common chemical shape in chemistry, physics, and nanoscience. This molecule could be very beneficial to synthesize fragrant compounds. The circumcoronene collection of benzenoid H_m is one family that generates from benzene molecules. The purpose of this study is to calculate the topological indices of the double and strong double graphs of the circumcoronene series of benzenoids (H_m). In addition, we also present a numerical and graphical comparison of topological indices of the double and strong double graphs of the circumcoronene series of benzenoid (H_m).

1. Introduction and Preliminaries

For undetermined notations and terminologies, we refer the readers to read the book [1].

Let $G(V, E)$ be a simple, finite connected graph, where the set of vertices is $V(G)$ and the set of edges is $E(G)$. For every vertex $x \in V(G)$, the edge connecting x and z is denoted by xz. In graph G, the total number of edges that connects to each vertex is known as the degree of vertex. The number of connected vertices to a fixed vertex is known as neighborhood. The degree of a vertex is denoted by d_x, where $x \in V(G)$. Hand-shaking lemma is very productive for calculating the size of a graph G.

Lemma 1. *If a graph G is having size k, then*

$$\sum_{x \in V(G)} \deg(x) = 2k. \quad (1)$$

In chemical graph theory, topological indices show a significant role in assisting chemists for modeling the molecular structure of chemical compounds and studying their chemical and physical characteristics. In chemistry, discovery of the drugs commonly relies on the topological descriptors. Drugs are characterized as molecular graphs, where graphs considered are simple with no multiple edges and no cycle formation. These topological descriptors provide information of a chemical compound based on the arrangement of its atoms and their bonds. A wide range of topological indices have been studied, and some of the more frequent forms of topological indices include degree-based, distance-based topological indices, and counting-related polynomials. In the topological indices, very famous and the oldest index is the Wiener index $W(G)$.

The Wiener index [2] is defined as follows:

$$W(G) = \frac{1}{2}\sum_{(x,z)} d(x,z), \tag{2}$$

where $d(x,z)$ is the distance among vertices x and z of a graph G.

A graph G's geometric arithmetic index (GA) [3] is defined as follows:

$$\mathrm{GA}(G) = \sum_{xz\in E(G)} \frac{2\sqrt{d_x d_z}}{d_x + d_z}. \tag{3}$$

A graph G's atomic bond connectivity index (ABC) [4] is defined as follows:

$$\mathrm{ABC}(G) = \sum_{xz\in E(G)} \sqrt{\frac{d_x + d_z - 2}{d_x d_z}}. \tag{4}$$

A graph G's forgotten index (F) [5] is defined as follows:

$$F(G) = \sum_{xz\in E(G)} \left(d_x^2 + d_z^2\right). \tag{5}$$

A graph G's inverse sum indeg index (ISI) [6] is defined as follows:

$$\mathrm{ISI}(G) = \sum_{xz\in E(G)} \frac{1}{(1/d_x) + (1/d_z)}. \tag{6}$$

A graph G's general inverse sum indeg index $(\mathrm{ISI}_{(\alpha,\beta)})$ [7] is defined as follows:

$$\mathrm{ISI}_{(\alpha,\beta)}(G) = \sum_{xz\in E(G)} [d_x d_z]^{\alpha}[d_x + d_z]^{\beta}, \tag{7}$$

where α and β are the real numbers.

A graph G's first multiplicative-Zagreb (PM_1) and second multiplicative-Zagreb indices (PM_2) are defined [8] as follows:

$$\mathrm{PM}_1(G) = \prod_{xz\in E(G)} (d_x)^2, \tag{8}$$

$$\mathrm{PM}_2(G) = \prod_{xz\in E(G)} (d_x \cdot d_z). \tag{9}$$

It is also possible to write the first multiplicative-Zagreb index (PM_1) [9] for G as follows:

$$\mathrm{PM}_1(G) = \prod_{xz\in E(G)} (d_x + d_z). \tag{10}$$

Imran et al. [10–12] studied the edge Mostar index of nanostructures and chemical structures by using graph operations and also computed the eccentric connectivity polynomial of connected graphs and Mostar indices for melem chain nanostructures. For more details about topological indices, we refer the works of Xiong et al. [13], Hong et al. [14], Alaeiyan et al. [15], Ch et al. [16], and Sardar et al. [17].

Definition 1. The well-known family of the benzenoid molecular graph is circumcoronene series of benzenoid (H_m), where $(m \geq 1)$ [18]. This family of graph constructed exclusively from benzene C_6 on circumference. Certain main members of circumcoronene series of benzenoid are benzene (H_1), coronene (H_2), circumcoronene (H_3), and circumcircumcoronene (H_4) [19]. Generally, circumcoronene series of benzenoid (H_m) is shown in Figure 1.

Definition 2. In order to make a double graph $D(H_m)$ of a graph **G**, take two copies of the graph **G** and join the nodes in each copy with their neighbors in the other copy [20]. For example, the graph (H_1) and its double graph $D(H_1)$ are shown in Figure 2. In double graph of circumcoronene series of benzenoid, there are $12m^2$ vertices and $4(9m^2 - 3m)$ edges, respectively. In $D(H_m)$, we have $12m$ vertices of degree 4 and $12(m^2 - m)$ vertices of degree.

Definition 3. Consider the two copies of graph **G**, and by joining the closed neighborhoods of one graph's vertex to the vertex in an adjacent graph, one can obtain the strong double graph $\mathrm{SD}(\mathbf{G})$ of graph **G** [21]. For example, strong double graph of graph H_1 is shown in Figure 3.

This study is laid out as follows. We will examine some vertex-based topological indices of double and strong double graphs of circumcoronene series of benzenoid (H_m) in Sections 2 and 4, respectively. The comparison is given in Sections 3 and 5. In Section 6, we provide final remarks for the whole study.

2. Degree-Based Topological Indices of Double Graph of Circumcoronene Series of Benzenoid Graph (H_m)

This section contains a calculation of the degree-based indices of the double graph of circumcoronene series of benzenoid (H_m).

Theorem 1. *Let $D(H_m)$ be the double graph of circumcoronene series of benzenoid graph (H_m); then, the geometric arithmetic index of $D(H_m)$ is*

$$GA[D(H_m)] = \frac{(96m - 96)\sqrt{6}}{5} + 36m^2 - 60m + 48. \tag{11}$$

Proof. In the double graph of circumcoronene series of benzenoid, there are $12m^2$ vertices and $4(9m^2 - 3m)$ edges, respectively. There are $12m$ vertices in $D(H_m)$ of degree 4 and $12(m^2 - m)$ of degree 6.

We separate the edges of $D(H_m)$ into the edges of the type$E[d_x, d_z]$, where xz is an edge. In $D(H_m)$, we get edge of types $E_{(4,4)}$ and $E_{(4,6)}$ and $E_{(6,6)}$. A list of their edges is given in Table 1.

By using Table 1 and equation (1), the result that we obtain is

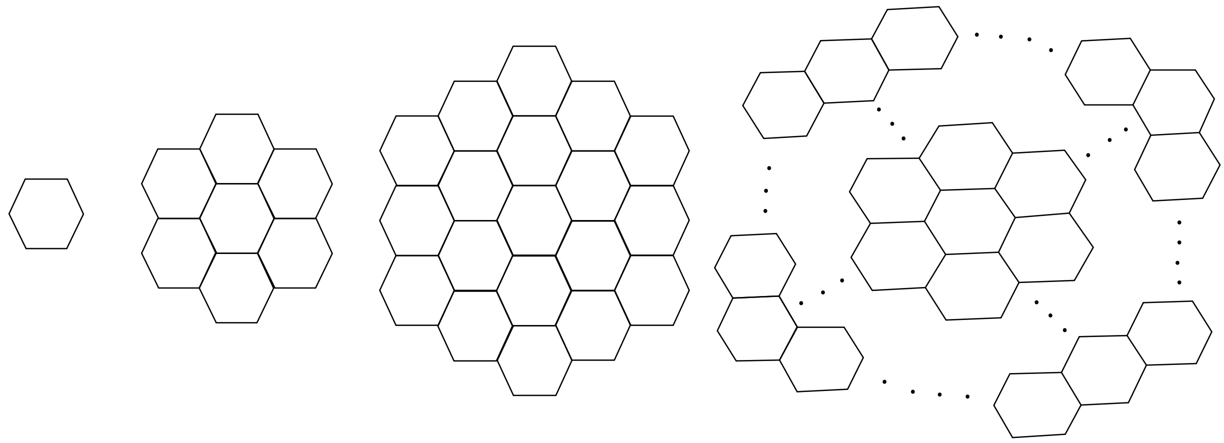

Figure 1: Circumcoronene series of benzenoid (H_1, H_2, H_3, and H_m).

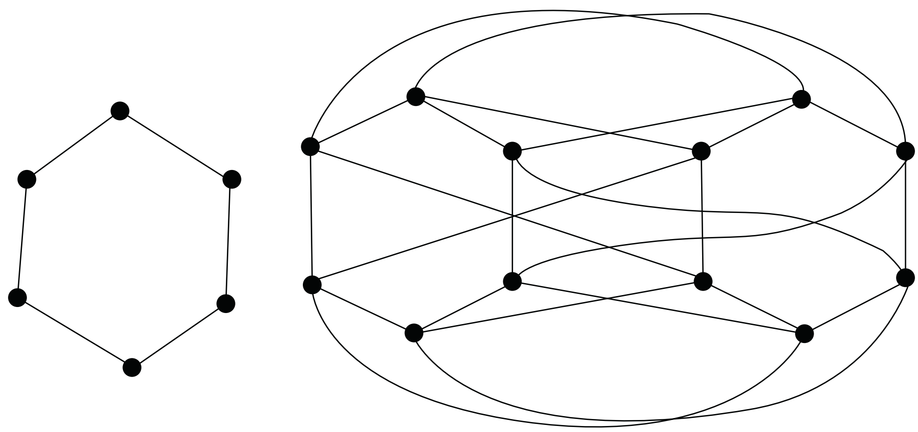

Figure 2: Circumcoronene series of benzenoid (H_1) and its double graph ($D(H_1)$).

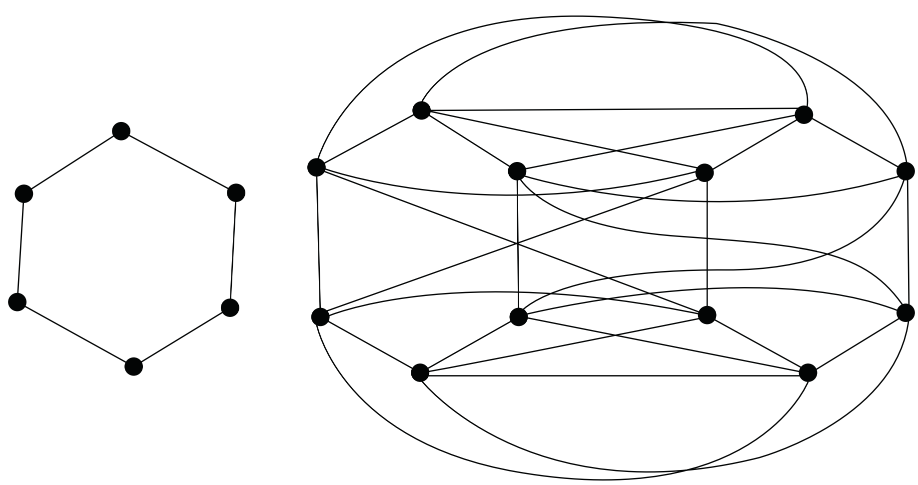

Figure 3: Circumcoronene series of benzenoid (H_1) and its strong double graph ($\mathrm{SD}(H_1)$).

Table 1: Separation of edges.

$E[d_x, d_z]$	$E_{(4,4)}$	$E_{(4,6)}$	$E_{(6,6)}$
Number of edges	24	$48(m-1)$	$36m^2 - 60m + 24$

$$\mathrm{GA}[G] = \sum_{xz\in E(G)} \frac{2\sqrt{d_x d_z}}{d_x + d_z}.$$

$$\begin{aligned}
\mathrm{GA}[D(H_m)] &= |E_{(4,4)}| \sum_{xz\in E[D(H_m)]} \frac{2\sqrt{d_x d_z}}{d_x + d_z} + |E_{(4,6)}| \sum_{xz\in E[D(H_m)]} \frac{2\sqrt{d_x d_z}}{d_x + d_z} + |E_{(6,6)}| \sum_{xz\in E[D(H_m)]} \frac{2\sqrt{d_x d_z}}{d_x + d_z}.\\
\mathrm{GA}[D(H_m)] &= 24\left[\frac{2\sqrt{16}}{8}\right] + 48(m-1)\left[\frac{2\sqrt{24}}{10}\right] + \left(36m^2 - 60m + 24\right)\left[\frac{2\sqrt{36}}{12}\right].\\
\mathrm{GA}[D(H_m)] &= 24 + 48(m-1)\left[\frac{\sqrt{24}}{5}\right] + 36m^2 - 60m + 24.\\
\mathrm{GA}[D(H_m)] &= \frac{(96m - 96)\sqrt{6}}{5} + 36m^2 - 60m + 48.
\end{aligned} \tag{12}$$

□

Theorem 2. *Let $D(H_m)$ be the double graph of circum-coronene series of the benzenoid graph (H_m); then, the ABC index of $D(H_m)$ is*

$$\begin{aligned}\mathrm{ABC}[D(H_m)] = &\left(6\sqrt{3} + \left(6m^2 - 10m + 4\right)\sqrt{5}\right)\sqrt{2}\\ &+ 16\sqrt{3}(m-1).\end{aligned} \tag{13}$$

Proof. By using Table 1 and equation (4), the result that we obtain is

$$\begin{aligned}
\mathrm{ABC}[D(H_m)] &= |E_{(4,4)}| \sum_{xz\in E[D(H_m)]} \sqrt{\frac{d_x + d_z - 2}{d_x d_z}} + |E_{(4,6)}| \sum_{xz\in E[D(H_m)]} \sqrt{\frac{d_x + d_z - 2}{d_x d_z}} + |E_{(6,6)}| \sum_{xz\in E[D(H_m)]} \sqrt{\frac{d_x + d_z - 2}{d_x d_z}}.\\
&= 24\sqrt{\frac{4+4-2}{(4)(4)}} + 48(m-1)\sqrt{\frac{4+6-2}{(4)(6)}} + \left(36m^2 - 60m + 24\right)\sqrt{\frac{6+6-2}{(6)(6)}}.\\
&= 6\sqrt{6} + 48(m-1)\sqrt{\frac{1}{3}} + \left(36m^2 - 60m + 24\right)\sqrt{\frac{5}{18}}.\\
\mathrm{ABC}[D(H_m)] &= \left(6\sqrt{3} + \left(6m^2 - 10m + 4\right)\sqrt{5}\right)\sqrt{2} + 16\sqrt{3}(m-1).
\end{aligned} \tag{14}$$

□

Theorem 3. *Let $D[H_m]$ be the double graph of circum-coronene series of benzenoid graph (H_m); then, the forgotten index of $D(H_m)$ is*

$$F[D(H_m)] = 2592m^2 - 1824m. \tag{15}$$

Proof. By using Table 1 and equation (5), the result that we obtain is

$$\begin{aligned}
F[D(H_m)] &= |E_{(4,4)}| \sum_{xz\in E[D(H_m)]} \left(d_x^2 + d_z^2\right) + |E_{(4,6)}| \sum_{xz\in E[D(H_m)]} \left(d_x^2 + d_z^2\right) + |E_{(6,6)}| \sum_{xz\in E[D(H_m)]} \left(d_x^2 + d_z^2\right).\\
&= 24\left(4^2 + 4^2\right) + 48(m-1)\left(4^2 + (6)^2\right) + \left(36m^2 - 60m + 24\right)\left(6^2 + (6)^2\right)\\
&= 768 + 2496(m-1) + \left(36m^2 - 60m + 24\right)(72).\\
F[D(H_m)] &= 2592m^2 - 1824m.
\end{aligned} \tag{16}$$

□

Theorem 4. *Let $D[H_m]$ be the double graph of circumcoronene series of the benzenoid graph (H_m); then, the inverse sum indeg index of $D(H_m)$ is*

$$\mathrm{ISI}[D(H_m)] = 108m^2 - \frac{324}{5}m + \frac{24}{5}. \tag{17}$$

Proof. By using Table 1 and equation (6), the result that we obtain is

$$\begin{aligned}
\mathrm{ISI}[D(H_m)] &= |E_{(4,4)}| \sum_{xz\in E[D(H_m)]} \frac{(d_x d_z)}{(d_x + d_z)} + |E_{(4,6)}| \sum_{xz\in E[D(H_m)]} \frac{(d_x d_z)}{(d_x + d_z)} + |E_{(6,6)}| \sum_{xz\in E[D(H_m)]} \frac{(d_x d_z)}{(d_x + d_z)}\\
&= 24\left[\frac{(4)(4)}{(4+4)}\right] + 48(m-1)\left[\frac{(4)(6)}{(4+6)}\right] + \left(36m^2 - 60m + 24\right)\left[\frac{(6)(6)}{(6+6)}\right]\\
&= 48 + 48(m-1)\left[\frac{12}{5}\right] + \left(36m^2 - 60m + 24\right)[3],\\
\mathrm{ISI}[D(H_m)] &= 108m^2 - \frac{324}{5}m + \frac{24}{5}.
\end{aligned} \tag{18}$$

□

Theorem 5. *Let $D[H_m]$ be the double graph of circumcoronene series of the benzenoid graph (H_m); then, the general inverse sum indeg index ($ISI_{(\alpha,\beta)}$) of $D(H_m)$ is*

$$\mathrm{ISI}_{(\alpha,\beta)}[D(H_m)] = 4p[16]^{\alpha}[8]^{\beta} + 8p[16p]^{\alpha}[4(1+p)]^{\beta}. \tag{19}$$

Proof. By using Table 1 and equation (7), the result that we obtain is

$$\begin{aligned}
\mathrm{ISI}_{(\alpha,\beta)}[D(H_m)] &= |E_{(4,4)}| \sum_{xz\in E[D(H_m)]} [d_x d_z]^{\alpha}[d_x + d_z]^{\beta}\\
&\quad + |E_{(4,6)}| \sum_{xz\in E[D(H_m)]} [d_x d_z]^{\alpha}[d_x + d_z]^{\beta} + |E_{(6,6)}| \sum_{xz\in E[D(H_m)]} [d_x d_z]^{\alpha}[d_x + d_z]^{\beta}\\
&= 24[(4)(4)]^{\alpha}[4+4]^{\beta} + 48(m-1)[(4)(6)]^{\alpha}[4+6]^{\beta} + \left(36m^2 - 60m + 24\right)[(6)(6)]^{\alpha}[6+6]^{\beta}\\
&= 24[16]^{\alpha}[8]^{\beta} + 48(m-1)[24]^{\alpha}[10]^{\beta} + \left(36m^2 - 60m + 24\right)[36]^{\alpha}[12]^{\beta},
\end{aligned} \tag{20}$$

where α and β are the real numbers. □

Theorem 6. *Let $D[H_M]$ be the double graph of circumcoronene series of the benzenoid graph (H_m); then, the first multiplicative-Zagreb index of $D(H_m)$ is*

$$\mathrm{PM}_1[D(H_m)] = (3m-2)\left[13271040(m-1)^2\right]. \tag{21}$$

Proof. By using Table 1 and equation (10), the result that we obtain is

$$\begin{aligned}
\mathrm{PM}_1[D(H_m)] &= |E_{(4,4)}| \prod_{xz\in E[D(H_m)]} (d_x + d_z) \times |E_{(4,6)}| \prod_{xz\in E[D(H_m)]} (d_x + d_z) \times |E_{(6,6)}| \prod_{xz\in E[D(H_m)]} (d_x + d_z),\\
PM_1[D(H_m)] &= 24(8) \times 48(m-1)(10) \times \left(36m^2 - 60m + 24\right)(12),\\
PM_1[D(H_m)] &= 192 \times (480m - 480) \times \left(432m^2 - 720m + 288\right),\\
PM_1[D(H_m)] &= (3m-2)\left[13271040(m-1)^2\right].
\end{aligned} \tag{22}$$

□

Theorem 7. *Let $D[H_m]$ be the double graph of circumcoronene series of the benzenoid graph (H_m); then, the second multiplicative-Zagreb index of $D(H_m)$ is*

$$\mathrm{PM}_2[D(H_m)] = \left(m - \frac{2}{3}\right)\left[573308928(m-1)^2\right]. \quad (23)$$

Proof. By using Table 1 and equation (9), the result that we obtain is

$$\begin{aligned}
\mathrm{PM}_2[D(H_m)] &= |E_{(4,4)}| \prod_{xz\in E[D(H_m)]} (d_x \cdot d_z) \times |E_{(4,6)}| \prod_{xz\in E[D(H_m)]} (d_x \cdot d_z) \times |E_{(6,6)}| \prod_{xz\in E[D(H_m)]} (d_x \cdot d_z),\\
\mathrm{PM}_2[D(H_m)] &= 24(16) \times 48(m-1)(24) \times \left(36m^2 - 60m + 24\right)(36),\\
\mathrm{PM}_2[D(H_m)] &= 442368(m-1) \times \left(1296m^2 - 2160m + 864\right),\\
\mathrm{PM}_2[D(H_m)] &= \left(m - \frac{2}{3}\right)\left[573308928(m-1)^2\right].
\end{aligned} \quad (24)$$

□

3. Comparison

In this section, we present a numerical and graphical comparison of topological indices that included the first multiplicative-Zagreb index (PM_1), general inverse sum indeg index ($\mathrm{ISI}_{(\alpha,\beta)}$), atom bond connectivity index (ABC), forgotten index (F), geometric arithmetic index (GA), second multiplicative-Zagreb index (PM_2), and inverse sum indeg index (ISI) for $m = 1, 2, 3, 4, \ldots, 10$ for the double graph of circumcoronene series of the benzenoid graph ($D(H_m)$), as given in Table 2 and Figure 4.

4. Degree-Based Topological Indices of Strong Double Graphs of Circumcoronene Series of Benzenoid Graph (H_m)

This section contains a calculation of the degree-based indices of the strong double graph of circumcoronene series of benzenoid (H_m). Figure 3 shows the strong double graph of (H_1).

Theorem 8. *Let $SD(H_M)$ be the double graph of circumcoronene series of the benzenoid graph (H_m); then, the geometric arithmetic index of $SD(H_m)$ is*

$$\mathrm{GA}[\mathrm{SD}(H_m)] = (8m-8)\sqrt{35} + 42m^2 - 60m + 48. \quad (25)$$

Proof. In the strong double graph of circumcoronene series of benzenoid, there are $12m^2$ vertices and $6(7m^2 - 2m)$ edges, respectively. There are $12m$ vertices in $\mathrm{SD}(H_m)$ of degree 5 and $12m(m^2 - 1)$ of degree 7.

We separate the edges of $\mathrm{SD}(H_m)$ into the edges of the type$E(d_x, d_z)$, where xz is an edge. In $\mathrm{SD}(H_m)$, we get edge of types $E_{(5,5)}$ and $E_{(5,7)}$ and $E_{(7,7)}$. A list of their edges is given in Table 3.

By using Table 3 and equation (1), the result that we obtain is

$$\begin{aligned}
\mathrm{GA}[G] &= \sum_{xz\in E(G)} \frac{2\sqrt{d_x d_z}}{d_x + d_z},\\
\mathrm{GA}[\mathrm{SD}(H_m)] &= |E_{(5,5)}| \sum_{xz\in E[\mathrm{SD}(H_m)]} \frac{2\sqrt{d_x d_z}}{d_x + d_z} + |E_{(5,7)}| \sum_{xz\in E[\mathrm{SD}(H_m)]} \frac{2\sqrt{d_x d_z}}{d_x + d_z} + |E_{(7,7)}| \sum_{xz\in E[\mathrm{SD}(H_m)]} \frac{2\sqrt{d_x d_z}}{d_x + d_z},\\
\mathrm{GA}[\mathrm{SD}(H_m)] &= (6m+24)\frac{2\sqrt{25}}{10} + 48(m-1)\frac{2\sqrt{35}}{12} + \left(42m^2 - 66m + 24\right)\frac{2\sqrt{49}}{14},\\
\mathrm{GA}[\mathrm{SD}(H_m)] &= (6m+24) + 48(m-1)\left[\frac{\sqrt{35}}{6}\right] + 42m^2 - 66m + 24,\\
\mathrm{GA}[\mathrm{SD}(H_m)] &= (8m-8)\sqrt{35} + 42m^2 - 60m + 48.
\end{aligned} \quad (26)$$

□

Table 2: Computation of topological indices of double graph of circumcoronene series of benzenoid $(D(H_m))$.

m	$GA(D(H_m))$	$ABC(D(H_m))$	$F(D(H_m))$	$ISI(D(H_m))$	$PM_1(D(H_m))$	$PM_2(D(H_m))$
1	24	14.697	768	48	0	0
2	119.03	67.710	6720	307.20	5.3084×10^7	7.6441×10^8
3	286.06	158.67	17856	782.40	3.7159×10^8	5.3509×10^9
4	525.09	287.58	34176	1473.6	1.1944×10^9	1.7199×10^{10}
5	836.12	454.42	55680	2380.8	2.7604×10^9	3.9749×10^{10}
6	1219.2	659.24	82360	3504	5.3084×10^9	7.6441×10^{10}
7	1674.2	901.98	1.1424×10^5	4843.2	9.0774×10^9	1.3071×10^{11}
8	2201.2	1182.7	1.5129×10^5	6398.4	1.4306×10^{10}	2.0601×10^{11}
9	2800.2	1501.3	1.9353×10^5	8169.6	2.1234×10^{10}	3.0576×10^{11}
10	3471.3	1857.9	2.4096×10^5	10156.8	3.0099×10^{10}	4.3342×10^{11}

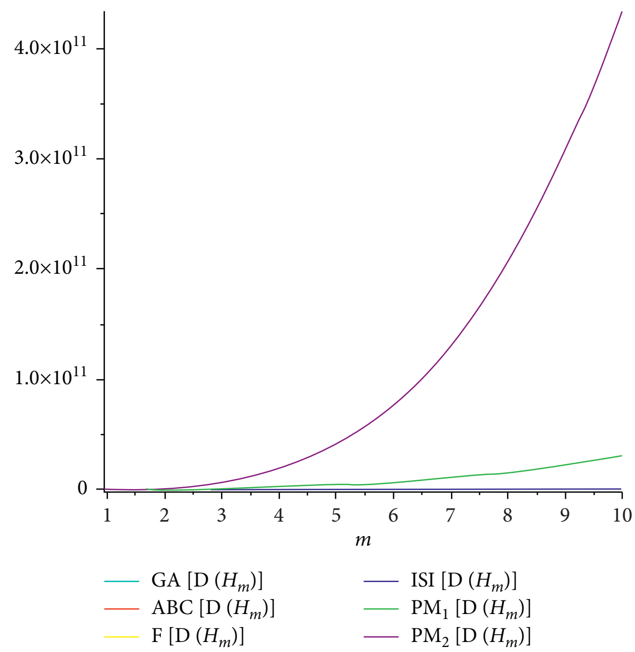

Figure 4: Graphical representation of topological indices of double graph of circumcoronene series of benzenoid $(\mathbf{H_m})$.

Table 3: Separation of edges.

$E(d_x, d_z)$	$E_{(5,5)}$	$E_{(5,7)}$	$E_{(7,7)}$
Number of edges	$6m + 24$	$48(m - 1)$	$42m^2 - 66m + 24$

Theorem 9. *Let $SD(H_m)$ be the strong double graph of circumcoronene series of the benzenoid graph (H_m); then, the ABC index of $SD(H_m)$ is*

$$ABC[SD(H_m)] = \frac{((240m - 240)\sqrt{7} + 84m + 336)\sqrt{2}}{35} + \left(12m - \frac{48}{7}\right)\sqrt{3}(m - 1). \tag{27}$$

Proof. By using Table 3 and equation (4), the result that we obtain is

$$\begin{aligned}
\mathrm{ABC}[\mathrm{SD}(H_m)] &= |E_{(5,5)}| \sum_{xz\in E[\mathrm{SD}(H_m)]} \sqrt{\frac{d_x+d_z-2}{d_xd_z}} + |E_{(5,7)}| \sum_{xz\in E[\mathrm{SD}(H_m)]} \sqrt{\frac{d_x+d_z-2}{d_xd_z}} + |E_{(7,7)}| \sum_{xz\in E[\mathrm{SD}(H_m)]} \sqrt{\frac{d_x+d_z-2}{d_xd_z}}\\
&= (6m+24)\sqrt{\frac{5+5-2}{(5)(5)}} + 48(m-1)\sqrt{\frac{5+7-2}{(5)(7)}} + \left(42m^2-66m+24\right)\sqrt{\frac{7+7-2}{(7)(7)}}\\
&= (6m+24)\frac{\sqrt{8}}{5} + 48(m-1)\sqrt{\frac{2}{7}} + \left(42m^2-66m+24\right)\frac{\sqrt{12}}{7},\\
\mathrm{ABC}[\mathrm{SD}(H_m)] &= \frac{((240m-240)\sqrt{7}+84m+336)\sqrt{2}}{35} + \left(12m-\frac{48}{7}\right)\sqrt{3}(m-1).
\end{aligned} \tag{28}$$

□

Theorem 10. *Let $SD[H_m]$ be the strong double graph of circumcoronene series of the benzenoid graph (H_m); then, the forgotten index of $SD(H_m)$ is*

$$F[\mathrm{SD}(H_m)] = 4116m^2 - 2616m. \tag{29}$$

Proof. By using Table 3 and equation (5), the result that we obtain is

$$\begin{aligned}
F[\mathrm{SD}(H_m)] &= |E_{(5,5)}| \sum_{xz\in E[\mathrm{SD}(H_m)]} \left(d_x^2+d_z^2\right) + |E_{(5,7)}| \sum_{xz\in E[\mathrm{SD}(H_m)]} \left(d_x^2+d_z^2\right) + |E_{(7,7)}| \sum_{xz\in E[\mathrm{SD}(H_m)]} \left(d_x^2+d_z^2\right)\\
&= (6m+24)\left(5^2+5^2\right) + 48(m-1)\left(5^2+7^2\right) + \left(42m^2-66m+24\right)\left(7^2+7^2\right)\\
&= (300m+1200) + 3552(m-1) + \left(42m^2-66m+24\right)(98),\\
F[\mathrm{SD}(H_m)] &= 4116m^2 - 2616m.
\end{aligned} \tag{30}$$

□

Theorem 11. *Let $SD[H_m]$ be the strong double graph of circumcoronene series of the benzenoid graph (H_m); then, the inverse sum indeg index of $SD(H_m)$ is*

$$\mathrm{ISI}[\mathrm{SD}(H_m)] = 147m^2 - 76m + 4. \tag{31}$$

Proof. By using Table 3 and equation (6), the result that we obtain is

$$\begin{aligned}
\mathrm{ISI}[\mathrm{SD}(H_m)] &= |E_{(5,5)}| \sum_{xz\in E[\mathrm{SD}(H_m)]} \frac{(d_xd_z)}{(d_x+d_z)} + |E_{(5,7)}| \sum_{xz\in E[\mathrm{SD}(H_m)]} \frac{(d_xd_z)}{(d_x+d_z)}\\
&\quad + |E_{(7,7)}| \sum_{xz\in E[\mathrm{SD}(H_m)]} \frac{(d_xd_z)}{(d_x+d_z)}\\
&= (6m+24)\left[\frac{(5)(5)}{(5+5)}\right] + 48(m-1)\left[\frac{(5)(7)}{(5+7)}\right] + \left(42m^2-66m+24\right)\left[\frac{(7)(7)}{(7+7)}\right]\\
&= (15m+60) + 140(m-1) + \left(42m^2-66m+24\right)\left[\frac{49}{14}\right],\\
\mathrm{ISI}[\mathrm{SD}(H_m)] &= 147m^2 - 76m + 4.
\end{aligned} \tag{32}$$

□

Theorem 12. *Let $SD(H_m)$ be the strong double graph of circumcoronene series of the benzenoid graph (H_m); then, the general inverse sum indeg index ($ISI_{(\alpha,\beta)}$) of $SD(H_m)$ is*

$$\mathrm{ISI}_{(\alpha,\beta)}[\mathrm{SD}(H_m)] = (6m+24)[25]^{\alpha}[10]^{\beta} + 48(m-1)[35]^{\alpha}[12]^{\beta} + \left(42m^2 - 66m + 24\right)[49]^{\alpha}[14]^{\beta}. \tag{33}$$

Proof. By using Table 3 and equation (7), the result that we obtain is

$$\begin{aligned}\mathrm{ISI}_{(\alpha,\beta)}[\mathrm{SD}(H_m)] &= \left|E_{(5,5)}\right| \sum_{xz\in E[\mathrm{SD}(H_m)]} [d_x d_z]^{\alpha}[d_x + d_z]^{\beta} + \left|E_{(5,7)}\right| \sum_{xz\in E[\mathrm{SD}(H_m)]} [d_x d_z]^{\alpha}[d_x + d_z]^{\beta} + \left|E_{(7,7)}\right| \\ &\quad \cdot \sum_{xz\in E[\mathrm{SD}(H_m)]} [d_x d_z]^{\alpha}[d_x + d_z]^{\beta} \\ &= (6m+24)[(5)(5)]^{\alpha}[5+5]^{\beta} + 48(m-1)[(5)(7)]^{\alpha}[5+7]^{\beta} + \left(42m^2 - 66m + 24\right)[(7)(7)]^{\alpha}[7+7]^{\beta} \\ &= (6m+24)[25]^{\alpha}[10]^{\beta} + 48(m-1)[35]^{\alpha}[12]^{\beta} + \left(42m^2 - 66m + 24\right)[49]^{\alpha}[14]^{\beta},\end{aligned} \tag{34}$$

where α and β are the real numbers. □

Theorem 13. *Let $SD[H_m]$ be the strong double graph of circumcoronene series of the benzenoid graph (H_m); then, the first multiplicative-Zagreb index of $SD(H_m)$ is*

$$\mathrm{PM}_1[\mathrm{SD}(H_m)] = 20321280\left(m - \frac{4}{7}\right)(m+4)(m-1)^2. \tag{35}$$

Proof. By using Table 3 and equation (10), the result that we obtain is

$$\begin{aligned}\mathrm{PM}_1[\mathrm{SD}(H_m)] &= \left|E_{(5,5)}\right| \sum_{xz\in E[\mathrm{SD}(H_m)]} (d_x + d_z) \times \left|E_{(5,7)}\right| \sum_{xz\in E[\mathrm{SD}(H_m)]} (d_x + d_z) \times \left|E_{(7,7)}\right| \sum_{xz\in E[\mathrm{SD}(H_m)]} (d_x + d_z), \\ \mathrm{PM}_1[\mathrm{SD}(H_m)] &= (6m+24)(10) \times 48(m-1)(12) \times \left(42m^2 - 66m + 24\right)(14), \\ \mathrm{PM}_1[\mathrm{SD}(H_m)] &= (60m+240) \times (576m - 576) \times \left(588m^2 - 924m + 336\right), \\ \mathrm{PM}_1[\mathrm{SD}(H_m)] &= 20321280\left(m - \frac{4}{7}\right)(m+4)(m-1)^2.\end{aligned} \tag{36}$$

□

Theorem 14. *Let $SD[H_m]$ be the strong double graph of circumcoronene series of the benzenoid graph (H_m); then, the second multiplicative-Zagreb index of $SD(H_m)$ is*

$$\mathrm{PM}_2[\mathrm{SD}(H_m)] = 518616000\left(m - \frac{4}{7}\right)(m+4)(m-1)^2. \tag{37}$$

Table 4: Computation of topological indices of strong double graph of circumcoronene series of benzenoid $(SD(H_m))$.

m	$GA(SD(H_m))$	$ABC(SD(H_m))$	$F(SD(H_m))$	$ISI(SD(H_m))$	$PM_1(SD(H_m))$	$PM_2(SD(H_m))$
1	30	16.970	1500	75	0	0
2	143.33	75.715	11232	440	1.7418×10^{8}	4.4453×10^{9}
3	340.66	176.03	29196	1099	1.3818×10^{9}	3.5266×10^{10}
4	621.99	317.91	55392	2052	5.0165×10^{9}	1.2802×10^{11}
5	987.32	501.37	89820	3299	1.2959×10^{10}	3.3073×10^{11}
6	1436.6	726.39	132480	4840	2.7579×10^{10}	7.0384×10^{11}
7	9070.0	993.00	183372	6675	5.1732×10^{10}	1.3202×10^{12}
8	2587.3	1301.1	242496	8804	8.8763×10^{10}	2.2653×10^{12}
9	3288.6	1650.9	309852	11227	1.4250×10^{11}	3.6368×10^{12}
10	4074.0	2042.2	385440	13944	2.1728×10^{11}	5.5450×10^{12}

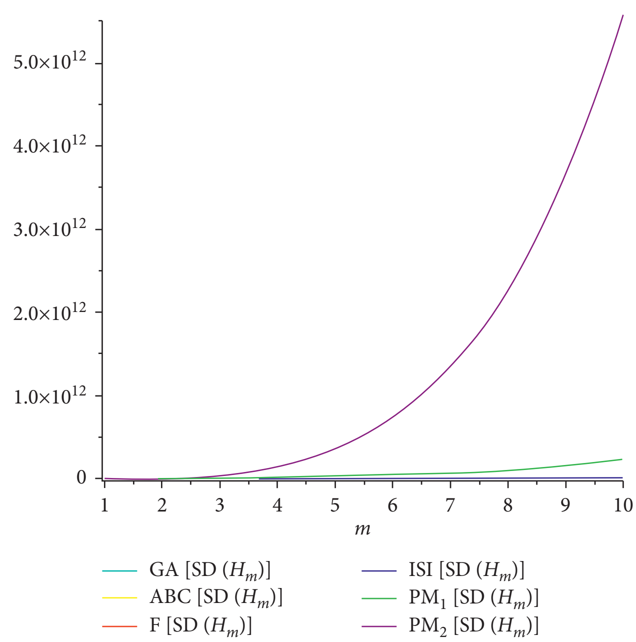

Figure 5: Graphical representation of topological indices of the strong double graph of circumcoronene series of benzenoid (H_m).

Proof. By using Table 3 and equation (9), the result that we obtain is

$$
\begin{aligned}
PM_2[SD(H_m)] &= |E_{(5,5)}| \prod_{xz\in E[SD(H_m)]} (d_x\cdot d_z)\times|E_{(5,7)}| \prod_{xz\in E[SD(H_m)]} (d_x\cdot d_z)\times|E_{(7,7)}| \prod_{xz\in E[SD(H_m)]} (d_x\cdot d_z),\\
PM_2[SD(H_m)] &= (6m+24)(25)\times 48(m-1)(35)\times\left(42m^2-66m+24\right)(49),\\
PM_2[SD(H_m)] &= 252000(m+3)(m-1)\times\left(2058m^2-3234m+1176\right),\\
PM_2[SD(H_m)] &= 518616000\left(m-\frac{4}{7}\right)(m+4)(m-1)^2.
\end{aligned}
\tag{38}
$$

□

5. Comparison

In this section, we present a numerical and graphical comparison of topological indices that included the first multiplicative-Zagreb index (PM_1), general inverse sum indeg index ($ISI_{(\alpha,\beta)}$), atom bond connectivity index (ABC), forgotten index (F), geometric arithmetic index (GA), second multiplicative-Zagreb index (PM_2), and inverse sum indeg index (ISI) for $m = 1, 2, 3, 4, \ldots, 10$ for the strong double graph of circumcoronene series of the benzenoid graph ($SD(H_m)$), as given in Table 4 and Figure 5.

6. Conclusion

We have computed the closed formulae of topological indices such as the first multiplicative-Zagreb index (PM_1), general inverse sum indeg index ($ISI_{(\alpha,\beta)}$), atom bond connectivity index (ABC), forgotten index (F), geometric arithmetic index (GA), second multiplicative-Zagreb index (PM_2), and inverse sum indeg index (ISI) of double and strong double graphs of circumcoronene series of benzenoid H_m ($m \geq 1$). Chemical compounds can be studied by these indices in order to understand their diverse properties. The geometric structure and comparison of obtained results are shown graphically and numerically. Those results are convenient for further study as they do not include any polynomial.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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