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Research Article

Two New Approaches (RAMS-RATMI) in Multi-Criteria Decision-Making Tactics

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When a decision must be made, a tool called multi-criteria decision-making (MCDM) is used to assess and select alternatives among numerous criteria. For a wide variety of complex problems, MCDM methods have demonstrated usefulness in finding the optimal solutions. Despite the abundance of MCDM methods available today, there has been slow progress in developing new methodologies in MCDM in the past decade. In this context, this paper presents new MCDM tools which ranks alternatives based on median similarity (RAMS) between optimal alternatives and other alternatives. RAMS is an extension to the most recently developed technique that used perimeter similarity (RAPS). This paper also introduces a further tool that combines the RAMS method with the multiple criteria ranking by alternative trace (MCRAT) methodology using a majority index and the concept of the VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method. This tool is ranking the alternatives based on the trace to median index (RATMI). An illustration of the use of RAMS and RATMI is given through a case study of ranking different materials for the selection of break booster valve body in a vehicle. The validity of the new two techniques was tested against seven well-known MCDM techniques (ARAS, SAW, TOPSIS, COPRAS, VIKOR, WASPAS, and MOORA) using fifteen real problems data taken from the literature. The RATMI technique was more promising than RAPS and RAMS for 87% and 93% of the fifteen difficulties, respectively, according to the results of the correlation coefficient tests between the developed techniques and the selected seven techniques.

1. Introduction

Different aspects denote the progression of the multi-criteria decision-making (MCDM) technique from old to new methods. MCDM is a tool that can be used to support decision-making in different spheres of business and science [1]. The use of MCDM facilitates the selection process and ensures that the decision will be based on reliable solutions as it generates progressive solutions to a variety of decisions. In order for the approach to be applied, the inclusion of various alternatives and selection from predefined criteria is essential [2]. The approach is used in the presence of conflicting criteria that require either maximizing or minimizing values. Hence, the approach is helpful in finding the optimal solution and the most appropriate settlement among the various alternatives available [3].

There is a large amount of literature in relation to the scientific background of MCDM methods, including descriptions of their distinctions, classifications, and applications, and there is even more written every day, especially in relation to their use within a professional setting. The development of MCDM has been examined from a historical perspective, and its techniques have attracted high adaption magnitude. MCDM was firstly introduced and become evident in the early eighteenth century, and the decision-making systems have been advancing to take the modern shape in the early 1970s by Howard Raiffa [4]. Despite the fact that the MCDM concept has roots in the eighteenth century, and possibly even before, however, the most widely MCDM methods that are now used are ÉLimination et Choix Traduisant la REalité (ELECTRE), the analytic hierarchy process (AHP), the technique for order of preference by

similarity to ideal solution (TOPSIS), and preference ranking organization method for enrichment evaluations (PROMETHEE) [5]. Numerous observed case studies indicate that selecting the appropriate MCDM method for a given problem is a recurring issue because of the large number of MCDM methods available. Methods can be selected according to how they fit a specific situation; there are no good or bad methods [5, 6].

A significant outlook of validating the development of MCDM techniques is the description and analysis of their peculiarities and potential applications. The main characteristic of MCDM is dealing with uncertainty when it comes to providing the most effective and optimal solution founded on the rational decision [7]. From an exploration study that determines the objectives characteristics relative to MCDM approaches, its findings reveal the methods to assist in exhibiting patterns or tendencies of the dual verification mechanism [8]. This assertion manifests that the potential of MCDM techniques is evident in terms of demonstrating capabilities in evaluating as well as comparing different results. The most notable categories encompass selection between alternatives, alternative rating, alternatives classification, and identifying alternatives [9].

In the past decade, there has been slow progress in developing new methodologies in MCDM [5]. One aspect to help explain the slow progress is the degree of stakeholder involvement. The development of new MCDM methods by scholars does not seem to be the most prevalent research direction at present, and a large number of existing MCDM methods available may be a factor contributing to the lack of interest among researchers. Within this scope, one of the new methods that have been developed in the last decade and have been the most popular and used are weighted aggregated sum product assessment (WASPAS) and (total area based on orthogonal vectors) TOV, as well as two very recent methods, ranking the alternatives by perimeter similarity (RAPS) and multiple criteria ranking by alternative trace (MCRAT).

Competitiveness in MCRAT and RAPS as recent additional MCDM processes ascertain reasons for their adoption as well as their advantages. A paper confirms that these novel approaches have demonstrated their effectiveness in decision-making owing to their optimal design [5]. The MCRAT and RAPS are an extension of solving a problem using categories of ranking and problem choice. The deviation in their resolving of an issue using a decision-making model could be obvious in the criteria considered to reach an optimal solution.

A major advantage of the two recently added methods RAPS and MCRAT is their simplicity, logic, justification, generality, and validity. However, as RAPS and MCRAT are considered modern methods, both methods were only tested and validated in a mining engineering setting only. For this reason, this paper aims at developing and expanding the two methods, RAPS and MCRAT, to include their uses in all settings rather than the mining setting only. So, this paper answers the question "How can the two RAPS and MCRAT approaches be modified for use in a variety of situations, such as education, banking sector, construction industry, housing, business, …etc.?"

The rest of this paper is organized as follows: Section 2 provides additional background relating to the history of MCDM. Section 3 describes the new proposed methodologies. An illustrative numerical example of the proposed methods appears in Section 4. In tabular and graphical forms, Section 5 compares the proposed techniques to other seven alternative techniques as well as the original one RAPS using 15 different problems. The conclusion comprises in Section 6.

2. Literature Review

A literature analysis assists in validating different MCDM methods and concepts behind their innovation. The development history of some of the most commonly MCDM techniques utilized is listed based on their development history from oldest to newest. Firstly, ELECTRE, the technique was first introduced in 1965 by a research team affiliated to the European consultancy firm [10]. The intention of this initiation was clear as a decision-making approach to formulating solutions characterized by multi-criteria problems. Its expansion is evident displayed in the development from ELECTRE I to ELECTRE II. According to Akram et al. [11], ELECTRE I is a model introduced with the consideration of incorporating a set of different concepts to enhance the sets of Pythagorean fuzzy concordance and discordance from the perspective of outranking when exposed to various alternatives. The application of ELECTRE I has extended from staff selection and intuitionistic fuzzy environment [11]. The justification for this application is flexibility while making a decision involving comparative analysis. For the other extensions, the literature provides the ELECTRE II, ELECTRE III, ELECTRE IV, ELECTRE IS, and ELECTRE TRI as the other models [12]. These details establish this approach to decision-making framework to respond to situations, especially situations that are relative to a complex algorithm. The limitation cited is insufficient performance on a single criterion, which may disregard some of the alternatives [5]. Based on this limitation, it was certain that other models with multi-criteria consideration were to be initiated.

In MCDM, AHP is an old but popular technique that was developed in the 1980s by L. Saaty [13]. This framework quantifies the criteria and alternative possibilities for a necessary decision and links them to its overall purpose. AHP generates weights from pairwise comparison metrics based on mathematically defined structures [14]. The process of evaluating alternatives involves evaluating relative values, judging relative importance, grouping judgments, and analyzing inconsistencies between judgments [15]. Following the calculation of the criteria's weights, alternatives' priorities, and sensitivity analysis results, decision-makers select optimal alternatives [16].

TOPSIS is represented from the viewpoint of its procedures and validation. Hwang and Yoon are credited to the proposal of this method and its use in 1981, although Yoon is accredited for its extension in 1987 [17]. The concept that is mostly associated with this approach is provision of most viable solution amongst a set of alternatives. Specificity is noted in the form of delivering the positive ideal solution,

which is a hypothetical alternative with maximum benefit coupled with maximization of cost criteria [17]. This description implies that this method has strengths and drawbacks. Its advantages include clear logic through the demarcation of best and worst possible alternatives and the representation of performance evaluations with a minimum of two dimensions. These aspects of the model do not eliminate its weaknesses. These disadvantages encompass the failure to account correlation of attributes and deviation from ideal solution, which could be interpreted as a high probability to alter final outcomes [5].

A multianalysis decision framework that accounted for qualitative and quantitative data was fundamental in extending solving of complex problems. Preference ranking organization method for enrichment evaluation (PROMETHEE) was developed in 1986 [5]. Functionality of the method is in its exceptionality in guiding a multi-criteria analysis designating characteristics of simplicity and clarity not forgetting stability as well as value in outranking [18]. The idea of maximal utilization of data cannot be forgotten in affirming efficiency in the course of its operation. The results associated with the PROMETHEE are its possibility to report great outcomes as a necessary standard portraying the improvement it has made accounting additional set of alternatives [19]. The usefulness of PROMETHEE in different fields attributed to its efficiency could have proved other novelties in the subsequent 21st century.

During the twenty-first century, a series of multi-criteria models have been developed and their concepts expanded as a demonstration of their ineffectiveness or building prototypes with additional features. Of great mention in this category is the multiobjective optimization on the basis of ration analysis (MOORA) and the complex proportional assessment (COPRAS). MOORA was developed in 2006 as a robust method and consisting of independent attributed, and MOORA is a relatively simple process but with a complex calculation procedure [5]. Another vital element of this technique is its wider application, especially in the production environment, and other processes assumed to be having conflicting objectives [20]. These applications contrast the practicality of the COPRAS method despite their successful use in solving problems comprising multiobjective criteria and needing optimization. The peculiar concept of COPRAS is its utilization to rank safe regions and its preference for deterministic data [21]. COPRAS is crucial given the period of its invention. COPRAS was introduced in 2007 as a suitable method for evaluating single alternative, although this use denotes its limitation of lower stability in the context of data variation and its sensitivity to data variations eve at the slightest change [5].

In the literature, there are numerous numbers of MCDM methods. For instance, the MCDM method simple additive weighting (SAW) aims to evaluate the effectiveness of various solutions [22]. Decision-makers are crucial to the implementation of SAW since they must select the preferred weights for each criterion.

MCDM includes, also, stepwise weight assessment ratio analysis (SWARA). The SWARA method gives decision-makers the chance to select the optimum course of action based on many circumstances. The criteria needs are ranked in order of significance when employing the SWARA approach. The given criteria will be ranked by experts according to their importance [23]. For example, the most important criterion will be listed first, and the least important criterion will be included last. The SWARA technique mostly relies on experts.

Rezaei [24] introduced the MCDM technique known as the best worst method (BWM). The BWM approach has been applied by several researchers in a wide range of industries and fields [25]. It can be used to evaluate alternatives in light of the criteria and examine the applicability of the criteria that are applied when coming up with a solution to reach the main goal(s) of the problem. In comparison to other MCDM techniques, the BWM uses fewer paired comparisons and fewer data points, and it is distinguished by its reference pairwise comparison.

Another MCDM technique is called VlseKriterijuska Optimizacija I Komoromisno Resenje (VIKOR). The VIKOR method's name, which is Serbian, can be roughly translated as "multi-criteria optimization and compromise solution" [26]. The development of this methodology was a response to Yu's [27] first public appeal for the creation of tools for reaching a compromise solution. By outlining in detail how near each alternative is to the "ideal" hypothetical solution, the strategy implies weighing and selecting alternatives based on competing criteria.

Deng [28] developed the grey relational analysis (GRA) method for issues that needed to be resolved in a situation with a lot of uncertainty. GRA demonstrated its effectiveness in systems with insufficient information when compared to other strategies [29]. Similar to VIKOR, GRA evaluates both positive and negative ideal solutions, comparing them to various alternatives based on their "degree of grey connection" [30]. The major benefits of GRA are their reliance on actual data and their ease of use in computations [29]. The method offers a versatile procedure that could be combined with other MCDM methods.

To sum up, MCDM techniques are employed to solve complex real-world problems because of its ability to examine various alternatives and choose the best option. For instance, Kalita et al. [31] presented a comprehensive literature review on the applications of MCDM techniques for parametric optimization of nontraditional machining (NTM) processes. Kalita et al. [32] applied six popular MCDM techniques to identify the most appropriate combination of milling parameters, leading to a compromise solution with a higher material removal rate and a lower average surface roughness. Using the combined compromise solution (CoCoSo) method and the MCDM tool, Panchagnula et al. [33] investigated the ideal combination of drilling parameters by employing MCDM tool. To assess the benchmarking process of active queue management (AQM) methods of internet network congestion control, Albahri et al. [34] presented an extension of the MCDM approach called fuzzy decision by opinion score (FDOSM). An integrated MCDM tool was created by Krishnan et al. [35] for benchmarking and assessing smart e-tourism data management solutions. The integrated tool used VIKOR approach and interval type 2 trapezoidal-fuzzy weighted with zero inconsistency (IT2TR-FWZIC). The sensitivity analysis of ranking the management

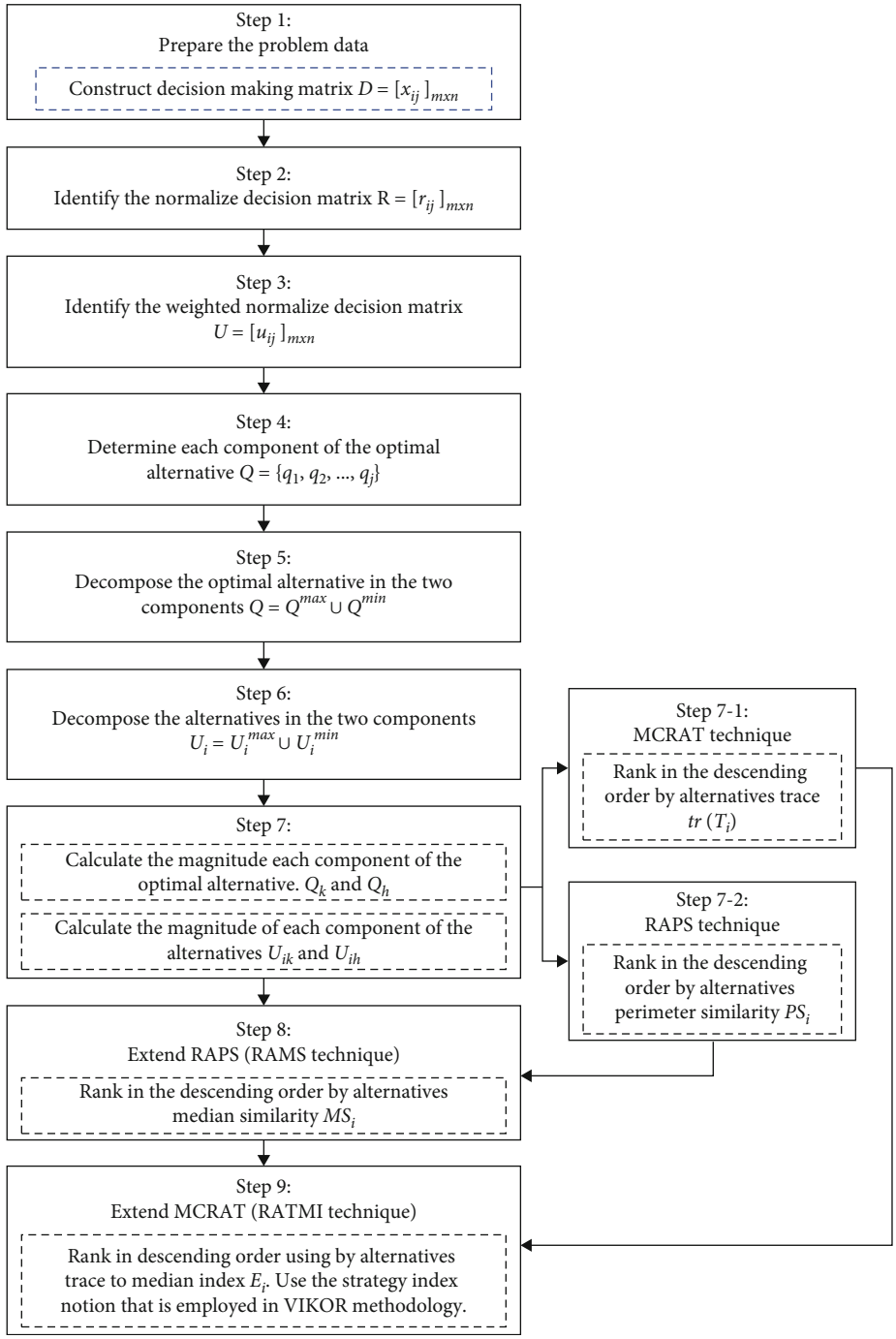

Figure 1: Steps of RAMS and RATMI methodologies.

of e-tourism data was evaluated using 12 intelligent criteria and 31 scenarios of modifying the weight of the criterion. The researchers were interested in more current studies of how to use MCDM approaches to attack COVID-19. Comprehensive review of the integration of MCDM applications for coronavirus disease 2019 was presented by Alsalem et al. [36]. They divided the examined studies into development- and evaluation-based categories. The bulk of studies in the assessment category were medical in nature, whereas studies in the development category were more concerned with developing fresh approaches to dealing with COVID-19-related decision-making problems that were either patient- or service-based. They also discussed the shortcomings of the recent studies and their recommendations for improvements in future research. In this context, Albahri et al. [37] have extended two MCDM methods the fuzzy-weighted zero-inconsistency (FWZIC) method and fuzzy decision by opinion score method (FDOSM) under the fuzzy environment. The intriguing case study of the COVID-19 vaccination dose distribution was used to test the proposed

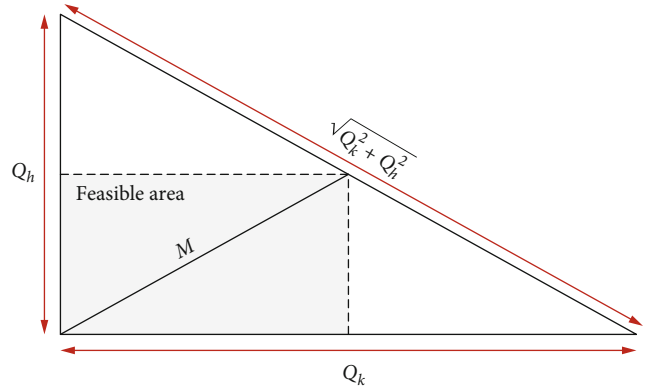

Figure 2: Geometric interpretation of the median $M = (\sqrt{Q_k^2 + Q_h^2})/2$.

extension of these two approaches. Albahri et al. [38] provide another extended two MCDM methods for a case study of sign language. The two methods, Pythagorean mm-polar fuzzy-weighted zero-inconsistency (Pm-PFWZIC) and Pythagorean mm-polar fuzzy decision by opinion score (Pm-PFDOSM), are designed to weigh the assessment of sign language criteria and to determine alternate rankings in a progressive manner.

3. The Proposed Methodologies

This paper proposes an extension to the two most recent MCDM techniques, MCRAT and RAPS [5]. The first proposed technique is associated with the RAPS methodology. Instead of ranking the alternatives based on the perimeter similarity that represents the ratio between the perimeter of each alternative and the optimal alternative as is the case with RAPS, the newly proposed method uses the median similarity. Thus, the proposed extension to RAPS will be named RAMS. The letter "M" refers to the word median instead of letter "P" that referred to word perimeter. RAMS scrutinizes the search space preciously towards the best rankings. Following that, the properties of both RAMS and MCRAT approaches will then be combined using the strategy index notion that is employed in VIKOR methodology. As a result, the second newly proposed technique uses the trace to median index to rank the alternatives and will be named RATMI. Figure 1 illustrates the RAMS and RATMI steps, which are further detailed as follows:

Step 1: Preparation of problem data

Construct the problem data in the form of decision-making matrix X_{ij}:

$$D = \left[x_{ij}\right]_{mxn} = \begin{bmatrix} A/C & C_1 & C_2 & \cdots & C_n \\ A_1 & x_{11} & x_{12} & \cdots & x_{1n} \\ A_2 & x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}, \quad (1)$$

Table 1: Input decision-making matrix.

Alternative/criteria	C_1 Max	C_2 Max	C_3 Min	C_4 Min
A_1	80	80	1.37	1.0
A_2	185	222	1.66	1.5
A_3	36	53	0.90	1.6
A_4	110	150	1.10	2.3
A_5	62	132	1.20	2.0
A_6	128	143	1.43	2.8
A_7	84	63	1.13	1.5
A_8	180	205	1.37	2.1
A_9	46	85	1.06	1.3
A_{10}	70	96	1.29	1.8
A_{11}	28	44	0.96	1.1
A_{12}	52	121	1.17	1.6
A_{13}	54	71	1.41	1.1
A_{14}	103	78	1.62	1.6
A_{15}	59	86	1.05	1.3
A_{16}	110	135	1.35	1.8
Weight	0.5405	0.1802	0.0688	0.2015

where $A = [A_1, A_2, \cdots, A_m]$ is a given set of alternatives and m is the total number of alternatives, $C = [C_1, C_2, \cdots, C_n]$ is a given set of criteria and n is the total number of criteria, and $[x_{ij}]_{mxn}$ is the assessment of alternative A_i with respect to a set of criteria.

Some of the criteria should be maximized, while some should be minimized.

Step 2: Normalization of problem data

The problem data is multidimensional since each criterion is described by its dimension. Making judgments in this situation is incredibly challenging. To avoid such complications, the multidimensional decision space must be transformed into a nondimensional decision space. For the max criteria, determine the normalization in the following way:

Table 2: Normalized decision-making matrix.

Alternative/criteria	C_1 Max	C_2 Max	C_3 Min	C_4 Min
A_1	0.4324	0.3604	0.6569	1.0000
A_2	1.0000	1.0000	0.5422	0.6667
A_3	0.1946	0.2387	1.0000	0.6250
A_4	0.5946	0.6757	0.8182	0.4348
A_5	0.3351	0.5946	0.7500	0.5000
A_6	0.6919	0.6441	0.6294	0.3571
A_7	0.4541	0.2838	0.7965	0.6667
A_8	0.9730	0.9234	0.6569	0.4762
A_9	0.2486	0.3829	0.8491	0.7692
A_{10}	0.3784	0.4324	0.6977	0.5556
A_{11}	0.1514	0.1982	0.9375	0.9091
A_{12}	0.2811	0.5450	0.7692	0.6250
A_{13}	0.2919	0.3198	0.6383	0.9091
A_{14}	0.5568	0.3514	0.5556	0.6250
A_{15}	0.3189	0.3874	0.8571	0.7692
A_{16}	0.5946	0.6081	0.6667	0.5556

Table 3: Weighted normalized decision-making matrix.

Alternative/criteria	C_1 Max	C_2 Max	C_3 Min	C_4 Min
A_1	0.2337	0.0649	0.0452	0.2015
A_2	0.5405	0.1802	0.0373	0.1343
A_3	0.1052	0.0430	0.0688	0.1259
A_4	0.3214	0.1218	0.0563	0.0876
A_5	0.1811	0.1071	0.0516	0.1008
A_6	0.3740	0.1161	0.0433	0.0720
A_7	0.2454	0.0511	0.0548	0.1343
A_8	0.5259	0.1664	0.0452	0.0960
A_9	0.1344	0.0690	0.0584	0.1550
A_{10}	0.2045	0.0779	0.0480	0.1119
A_{11}	0.0818	0.0357	0.0645	0.1832
A_{12}	0.1519	0.0982	0.0529	0.1259
A_{13}	0.1578	0.0576	0.0439	0.1832
A_{14}	0.3009	0.0633	0.0382	0.1259
A_{15}	0.1724	0.0698	0.0590	0.1550
A_{16}	0.3214	0.1096	0.0459	0.1119

Table 4: Decomposition of the optimal alternative.

Optimal alternative/criteria	C_1 Max q_1	C_2 Max q_2	C_3 Min q_3	C_4 Min q_4
$Q^{\max}$	0.2921	0.0325	—	—
$Q^{\min}$	—	—	0.0047	0.0406

Table 5: Decomposition of alternatives.

Alternative/criteria	C_1 Max u_1	C_2 Max u_2	$C3$ Min u_3	C_4 Min u_4
$A_1U^{\max}$	0.0546	0.0042	—	—
$A_1U^{\min}$	—	—	0.0020	0.0406
$A_2U^{\max}$	0.2921	0.0325	—	—
$A_2U^{\min}$	—	—	0.0014	0.0180
$A_3U^{\max}$	0.0111	0.0019	—	—
$A_3U^{\min}$	—	—	0.0047	0.0159
$A_4U^{\max}$	0.1033	0.0148	—	—
$A_4U^{\min}$	—	—	0.0032	0.0077
$A_5U^{\max}$	0.0328	0.0115	—	—
$A_5U^{\min}$	—	—	0.0027	0.0102
$A_6U^{\max}$	0.1399	0.0135	—	—
$A_6U^{\min}$	—	—	0.0019	0.0052
$A_7U^{\max}$	0.0602	0.0026	—	—
$A_7U^{\min}$	—	—	0.0030	0.0180
$A_8U^{\max}$	0.2766	0.0277	—	—
$A_8U^{\min}$	—	—	0.0020	0.0092
$A_9U^{\max}$	0.0181	0.0048	—	—
$A_9U^{\min}$	—	—	0.0034	0.0240
$A_{10}U^{\max}$	0.0418	0.0061	—	—
$A_{10}U^{\min}$	—	—	0.0023	0.0125
$A_{11}U^{\max}$	0.0067	0.0013	—	—
$A_{11}U^{\min}$	—	—	0.0042	0.0336
$A_{12}U^{\max}$	0.0231	0.0096	—	—
$A_{12}U^{\min}$	—	—	0.0028	0.0159
$A_{13}U^{\max}$	0.0249	0.0033	—	—
$A_{13}U^{\min}$	—	—	0.0019	0.0336
$A_{14}U^{\max}$	0.0906	0.0040	—	—
$A_{14}U^{\min}$	—	—	0.0015	0.0159
$A_{15}U^{\max}$	0.0297	0.0049	—	—
$A_{15}U^{\min}$	—	—	0.0035	0.0240
$A_{16}U^{\max}$	0.1033	0.0120	—	—
$A_{16}U^{\min}$	—	—	0.0021	0.0125

$$r_{\mathrm{ij}} = \frac{x_{\mathrm{ij}}}{\max_i\left(x_{\mathrm{ij}}\right)}, \forall i \in [1, 2, \cdots, m] \wedge j \in S_{\max}, \tag{2}$$

while for the min criteria

$$r_{\mathrm{ij}} = \frac{\min_i\left(x_{\mathrm{ij}}\right)}{x_{\mathrm{ij}}}, \forall i \in [1, 2, \cdots, m] \wedge j \in S_{\min}, \tag{3}$$

where: $S_{\max}$ is a set of criteria that should be maximized and $S_{\min}$ is a set of criteria that should be minimized.

As a result, the normalized decision matrix will have the following form:

$$R = \left[r_{\mathrm{ij}}\right]_{\mathrm{mxn}} = \begin{bmatrix} A/C & C_1 & C_2 & \cdots & C_n \\ A_1 & r_{11} & r_{12} & \cdots & r_{1n} \\ A_2 & r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & r_{m1} & \mathrm{r}_{m2} & \cdots & r_{\mathrm{mn}} \end{bmatrix}. \tag{4}$$

Step 3: Weighted normalization

Do the weighted normalization as follows for each normalized assessment r_{ij}:

$$u_{\mathrm{ij}} = w_j r_{\mathrm{ij}}, \forall i \in [1, 2, \cdots, m],\ \forall j \in [1, 2, \cdots, n], \tag{5}$$

where w_j is a weight of criterion j that can be determined either from a group of experts or from using one of the MCDM tools such as the AHP technique. The sum of the weights must equal one: $\sum_{j=1}^{n} w_j = 1$.

Then, the weighted normalization matrix can be formed as follows:

$$U = \left[u_{\mathrm{ij}}\right]_{\mathrm{mxn}} = \begin{bmatrix} A/C & C_1 & C_2 & \cdots & C_n \\ A_1 & u_{11} & u_{12} & \cdots & u_{1n} \\ A_2 & u_{21} & u_{22} & \cdots & u_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & u_{m1} & u_{m2} & \cdots & u_{\mathrm{mn}} \end{bmatrix}. \quad (6)$$

Step 4: Determination of optimal alternative

Determine each component of the optimal alternative as follows:

$$q_j = \max\left(u_{\mathrm{ij}} | 1 \le j \le n\right), \forall i \in [1, 2, \cdots, m]. \quad (7)$$

The optimal alternative is represented by the following set:

$$Q = \left\{q_1, q_2, \cdots, q_j\right\}, j = 1, 2, \cdots, n. \quad (8)$$

Step 5: Decomposition of the optimal alternative

Decompose the optimal alternative in the two sets or two components.

$$Q = Q^{\max} \cup Q^{\min}, \quad (9)$$

$$Q = \{q_1, q_2, \cdots, q_k\} \cup \{q_1, q_2, \cdots, q_h\}\ ;\ k + h = j, \quad (10)$$

where k represents the total number of criteria that should be maximized and h represents the total number of criteria that should be minimized.

Step 6: Decomposition of the alternative

Similarly, to step 5, decompose each alternative.

$$U_i = U_i^{\max} \cup U_i^{\min}, \forall i \in [1, 2, \cdots, m], \quad (11)$$

$$U_i = \{u_{i1}, u_{i2}, \cdots, u_{\mathrm{ik}}\} \cup \{u_{i1}, u_{i2}, \cdots, u_{\mathrm{ih}}\}; \forall i = [1, 2, \cdots, m]. \quad (12)$$

Step 7: Magnitude of component

For each component of the optimal alternative, calculate the magnitude defined by

$$Q_k = \sqrt{q_1^2 + q_2^2 + \cdots q_k^2}, \quad (13)$$

$$Q_h = \sqrt{q_1^2 + q_2^2 + \cdots q_h^2}. \quad (14)$$

Table 6: Magnitude of optimal alternative and alternatives.

Alternative/Q	Max Qk 0.5697 U_{ik}	Min Qh 0.2129 U_{ih}
A_1	0.2426	0.2065
A_2	0.5697	0.1394
A_3	0.1136	0.1435
A_4	0.3437	0.1041
A_5	0.2105	0.1132
A_6	0.3916	0.0840
A_7	0.2507	0.1451
A_8	0.5516	0.1061
A_9	0.1511	0.1656
A_{10}	0.2189	0.1218
A_{11}	0.0893	0.1942
A_{12}	0.1809	0.1366
A_{13}	0.1680	0.1884
A_{14}	0.3075	0.1316
A_{15}	0.1860	0.1658
A_{16}	0.3395	0.1210

Table 7: Alternatives ranked according to MCRAT method.

Alternative	Trace	Value	Rank
A_1	$\mathrm{tr}(T_1)$	0.1822	7
A_2	$\mathrm{tr}(T_2)$	0.3543	1
A_3	$\mathrm{tr}(T_3)$	0.0953	15
A_4	$\mathrm{tr}(T_4)$	0.2180	5
A_5	$\mathrm{tr}(T_5)$	0.1440	10
A_6	$\mathrm{tr}(T_6)$	0.2410	3
A_7	$\mathrm{tr}(T_7)$	0.1737	8
A_8	$\mathrm{tr}(T_8)$	0.3369	2
A_9	$\mathrm{tr}(T_9)$	0.1213	14
A_{10}	$\mathrm{tr}(T_{10})$	0.1506	9
A_{11}	$\mathrm{tr}(T_{11})$	0.0922	16
A_{12}	$\mathrm{tr}(T_{12})$	0.1322	13
A_{13}	$\mathrm{tr}(T_{13})$	0.1358	12
A_{14}	$\mathrm{tr}(T_{14})$	0.2032	6
A_{15}	$\mathrm{tr}(T_{15})$	0.1413	11
A_{16}	$\mathrm{tr}(T_{16})$	0.2192	4

The same approach is applied for each alternative.

$$U_{ik} = \sqrt{u_{i1}^2 + u_{i2}^2 + \cdots u_{ik}^2},\ \forall i = [1, 2, \cdots, m], \quad (15)$$

$$U_{ih} = \sqrt{u_{i1}^2 + u_{i2}^2 + \cdots u_{ih}^2},\ \forall i = [1, 2, \cdots, m]. \quad (16)$$

From this point, the following two methods were developed to create the rank of alternatives:

Table 8: Alternatives ranked according to RAPS method.

	Max Q_k 0.5697 U_{ik}	Min Q_h 0.2129 U_{ih}	Perimeter P 1.3909 P_i	Perimeter Similarity $PS_i = P/P_i$	Rank
A_1	0.2426	0.2065	0.7677	0.5519	7
A_2	0.5697	0.1394	1.2957	0.9316	1
A_3	0.1136	0.1435	0.4402	0.3165	16
A_4	0.3437	0.1041	0.8069	0.5801	5
A_5	0.2105	0.1132	0.5626	0.4045	12
A_6	0.3916	0.0840	0.8760	0.6298	3
A_7	0.2507	0.1451	0.6854	0.4928	8
A_8	0.5516	0.1061	1.2193	0.8767	2
A_9	0.1511	0.1656	0.5409	0.3889	14
A_{10}	0.2189	0.1218	0.5911	0.4250	11
A_{11}	0.0893	0.1942	0.4972	0.3575	15
A_{12}	0.1809	0.1366	0.5442	0.3913	13
A_{13}	0.1680	0.1884	0.6087	0.4376	9
A_{14}	0.3075	0.1316	0.7736	0.5562	6
A_{15}	0.1860	0.1658	0.6010	0.4321	10
A_{16}	0.3395	0.1210	0.8210	0.5902	4

Table 9: Alternatives ranked according to RAMS method.

	Max Q_k 0.5697 U_{ik}	Min Q_h 0.2129 U_{ih}	Median M 0.3041 M_i	Perimeter Similarity $MS_i = M_i/M$	Rank
A_1	0.2426	0.2065	0.1593	0.5238	7
A_2	0.5697	0.1394	0.2933	0.9644	1
A_3	0.1136	0.1435	0.0915	0.3010	16
A_4	0.3437	0.1041	0.1795	0.5904	5
A_5	0.2105	0.1132	0.1195	0.3929	12
A_6	0.3916	0.0840	0.2002	0.6584	3
A_7	0.2507	0.1451	0.1448	0.4762	8
A_8	0.5516	0.1061	0.2808	0.9235	2
A_9	0.1511	0.1656	0.1121	0.3686	14
A_{10}	0.2189	0.1218	0.1252	0.4118	10
A_{11}	0.0893	0.1942	0.1069	0.3514	15
A_{12}	0.1809	0.1366	0.1133	0.3727	13
A_{13}	0.1680	0.1884	0.1262	0.4149	9
A_{14}	0.3075	0.1316	0.1672	0.5499	6
A_{15}	0.1860	0.1658	0.1246	0.4097	11
A_{16}	0.3395	0.1210	0.1802	0.5926	4

Step 7.1: Ranking by alternatives trace (MCRAT)

Create the matrix F composed of optimal alternative components:

$$F = \begin{bmatrix} Q_k & 0 \\ 0 & Q_h \end{bmatrix}. \tag{17}$$

Create the matrix G_j composed of alternative components:

$$G_j = \begin{bmatrix} U_{ik} & 0 \\ 0 & U_{ih} \end{bmatrix}, \forall i = [1, 2, \cdots, m]. \tag{18}$$

Create the matrix T_i as follows:

Table 10: Alternatives ranked according to RATMI method.

	MCRAT $tr^*=0.0922$ $tr^-=0.3543$ tr_i	RAMS $MS^*=0.3010$ $MS^-=0.9644$ MS_i	Majority Index E_i	Rank
A_1	0.1822	0.5238	0.3396	7
A_2	0.3543	0.9644	1.0000	1
A_3	0.0953	0.3010	0.0059	16
A_4	0.2180	0.5904	0.4581	5
A_5	0.1440	0.3929	0.1681	12
A_6	0.2410	0.6584	0.5532	3
A_7	0.1737	0.4762	0.2876	8
A_8	0.3369	0.9235	0.9359	2
A_9	0.1213	0.3686	0.1066	14
A_{10}	0.1506	0.4118	0.1950	9
A_{11}	0.0922	0.3514	0.0380	15
A_{12}	0.1322	0.3727	0.1303	13
A_{13}	0.1358	0.4149	0.1691	11
A_{14}	0.2032	0.5499	0.3995	6
A_{15}	0.1413	0.4097	0.1755	10
A_{16}	0.2192	0.5926	0.4621	4

Table 11: Ranking materials of break booster valve body by different MCDM methods.

	MCRAT	RAPS	ARAS	SAW	TOPSIS	COPRAS	VIKOR	WASPAS	MOORA
A_1	7	7	7	6	7	7	7	6	7
A_2	1	1	1	1	1	1	1	1	1
A_3	15	16	16	16	16	16	15	15	15
A_4	5	5	4	5	5	5	5	5	5
A_5	10	12	10	12	10	11	11	11	11
A_6	3	3	3	3	3	3	3	3	4
A_7	8	8	8	8	8	8	8	8	8
A_8	2	2	2	2	2	2	2	2	2
A_9	14	14	14	14	14	14	14	14	14
A_{10}	9	11	9	11	9	9	9	9	9
A_{11}	16	15	15	15	15	15	16	16	16
A_{12}	13	13	13	13	13	12	12	13	12
A_{13}	12	9	12	10	12	13	13	12	13
A_{14}	6	6	6	7	6	6	6	7	6
A_{15}	11	10	11	9	11	10	10	10	10
A_{16}	4	4	5	4	4	4	4	4	3

$$T_i = F \times G_j = \begin{bmatrix} t_{11;i} & 0 \\ 0 & t_{22;i} \end{bmatrix}, \forall i = [1, 2, \cdots, m]. \tag{19}$$

Then, the trace of the matrix T_i is as follows:

$$tr(T_i) = t_{11;i} + t_{22;i}, \forall i = [1, 2, \cdots, m]. \tag{20}$$

Alternatives are now ranked according to the descending order of $tr(T_i)$.

Step 7.2: Ranking by alternatives perimeter similarity (RAPS)

Perimeter of the optimal alternative is expressed as the perimeter of the right angle. Components Q_k and Q_h represent the base and perpendicular side of this triangle, respectively.

	RAMS	RATMI	MCRAT	RAPS	ARAS	SAW	TOPSIS	COPRAS	VIKOR	WASPAS	MOORA
RAMS	1	0.991	0.975	0.997	0.976	0.988	0.979	0.971	0.968	0.976	0.965
RATMI	0.991	1	0.988	0.988	0.988	0.988	0.991	0.991	0.988	0.991	0.985
MCRAT	0.975	0.988	1	0.971	0.994	0.971	0.997	0.991	0.994	0.994	0.991
RAPS	0.997	0.988	0.971	1	0.971	0.994	0.974	0.968	0.965	0.974	0.962
ARAS	0.976	0.988	0.994	0.971	1	0.971	0.997	0.991	0.988	0.988	0.982
SAW	0.988	0.988	0.971	0.994	0.971	1	0.974	0.974	0.971	0.982	0.968
TOPSIS	0.979	0.991	0.997	0.974	0.997	0.974	1	0.994	0.991	0.991	0.988
COPRAS	0.971	0.991	0.991	0.968	0.991	0.974	0.994	1	0.997	0.991	0.994
VIKOR	0.968	0.988	0.994	0.965	0.988	0.971	0.991	0.997	1	0.994	0.997
WASPAS	0.976	0.991	0.994	0.974	0.988	0.982	0.991	0.991	0.994	1	0.991
MOORA	0.965	0.985	0.991	0.962	0.982	0.968	0.988	0.994	0.997	0.991	1
Color Code	Greater than or equal to 0.997				Other colors are greater than 0.965 and less than 0.997				Less than or equal to 0.965		

Figure 3: The density of the correlation between the compared techniques is displayed on a heatmap. Greater significance of the correlation factor is shown by denser colour than by lighter colour. That is shown by the table's colour code.

$$P = Q_k + Q_h + \sqrt{Q_k^2 + Q_h^2}. \tag{21}$$

Perimeter of each alternative is calculated the same way

$$P_i = U_{\text{ik}} + U_{\text{ih}} + \sqrt{U_{ik}^2 + U_{ih}^2}. \tag{22}$$

Perimeter similarity represents the ratio between the perimeter of each alternative and the optimal alternative:

$$\text{PS}_i = \frac{P_i}{P}, \forall i = [1, 2, \cdots, m]. \tag{23}$$

Alternatives are now ranked according to the descending order of PS_i.

Step 8: Ranking by alternatives median similarity (RAMS)

The median of the optimal alternative is expressed as the median of the right angle used for the RAPS technique, as portrayed in Figure 2.

$$M = \frac{\left(\sqrt{Q_k^2 + Q_h^2}\right)}{2}. \tag{24}$$

Median of each alternative is calculated the same way.

$$M_i = \frac{\left(\sqrt{U_{\text{ik}}^2 + U_{\text{ih}}^2}\right)}{2} \tag{25}$$

Median similarity represents the ratio between the perimeter of each alternative and the optimal alternative:

$$\text{MS}_i = \frac{M_i}{M}, \forall i = [1, 2, \cdots, m]. \tag{26}$$

Alternatives are now ranked according to the descending order of MS_i.

Step 9: Ranking the alternatives using the trace to median index (RATMI)

If v is the weight of MCRAT's strategy and the $(1 - v)$ is the weight of RAMS's strategy, then, the majority index E_i between the two strategies is as follows:

$$E_i = v\frac{(tr_i - tr^*)}{(tr^- - tr^*)} + (1 - v)\frac{(MS_i - MS^*)}{(MS^- - MS^*)}, \tag{27}$$

where

$$\begin{aligned} \text{tr}_i &= \text{tr}(T_i), \forall i = [1, 2, \cdots, m], \\ \text{tr}^* &= \min\ (\text{tr}_i, \forall i = [1, 2, \cdots, m]), \\ \text{tr}^- &= \max\ (\text{tr}_i, \forall i = [1, 2, \cdots, m]), \\ \text{MS}^* &= \min\ (\text{MS}_i, \forall i = [1, 2, \cdots, m]), \\ \text{MS}^- &= \max\ (\text{MS}_i, \forall i = [1, 2, \cdots, m]), \end{aligned} \tag{28}$$

where v is a value from 0 to 1. Here, $v = 0.5$

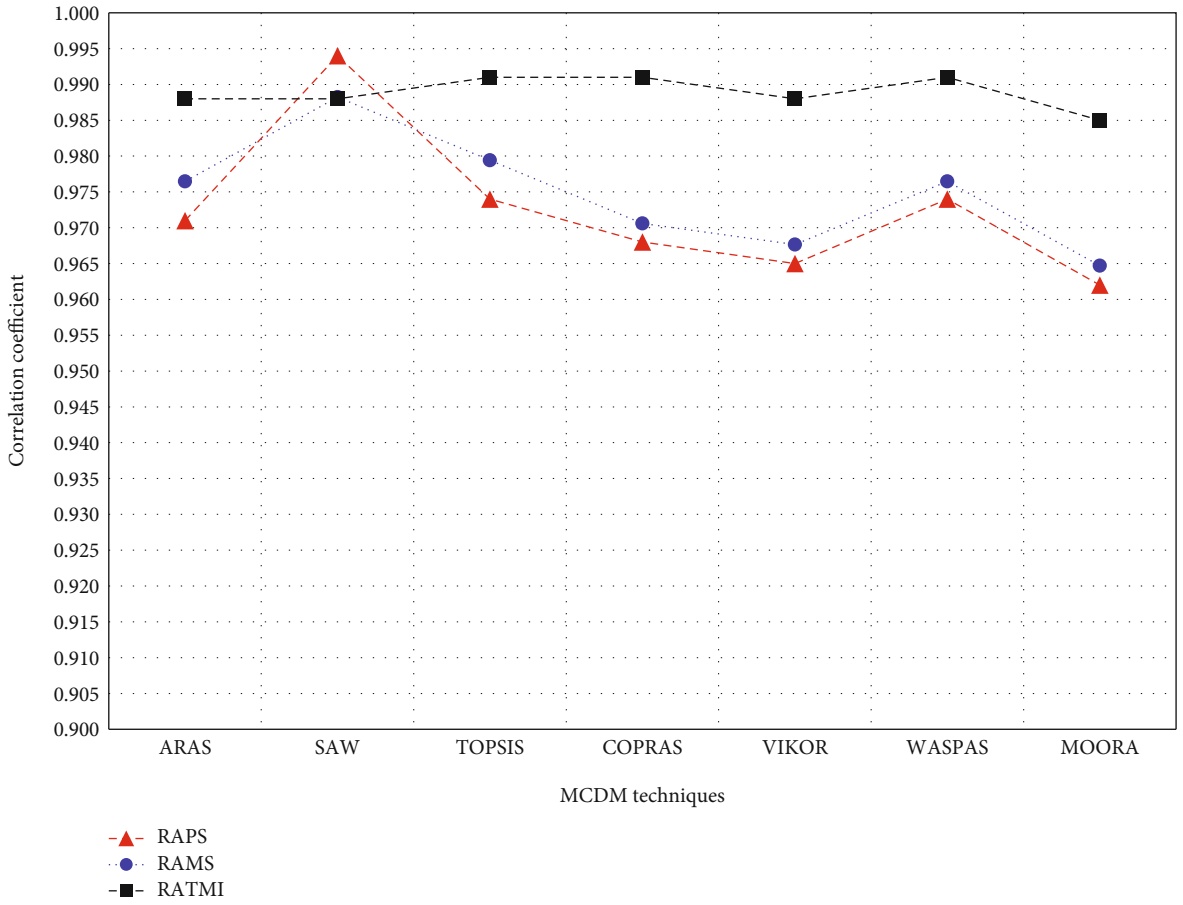

Figure 4: Relative comparison between the two RAMS and RATMI technique and RAPS technique.

4. Illustrative Numerical Example

This section applied the two proposed RAMS and RATMI methods using the data-driven by Moradian et al. [39] for material selection of break booster valve body in a vehicle. The criteria of selecting the materials were C1 (tensile strength), C2 (deflection temperature of the material), C3 (material's density), and C4 (cost of the product). Table 1 shows the input decision matrix, while Tables 2 and 3 show the normalized and weighted normalized input data based on steps 1, 2, and 3, respectively, and Equations (2)–(6).

Step 4 determined the optimal alternative by applying Equations (7) and (8). Followed this step, steps 5 and 6 defined the decomposition of the optimal alternative and the decomposition of each of the alternatives by using Equations (9)–(12). The decomposition results are shown in Tables 4 and 5.

Step 6 calculates the magnitude of the optimal alternative and other alternatives using Equations (13)–(16). Values obtained within this step are shown in Table 6. The steps 7.1 and 7.2 ranked the alternatives by applying MCRAT and RAPS techniques using the Equations (17)–(23). Tables 7 and 8 show the ranking by the trace of the matrix (MCRAT) and perimeter similarity (RAPS) methods.

The two new methods RAMS and RATMI were illustrated in steps 8 and 9, respectively. From step 8, the alternatives are ranked based on the median similarity between the optimal alternatives and other alternatives by applying Equations (24)–(26). This is followed by step 9, which focuses on the majority index between MCRAT and RAMS methods by using Equation (27) with $v = 0.5$. The results of these two steps are shown in Tables 9 and 10.

5. Testing the Validity of the RAMS and RATMI Methods

For the same numerical example, Table 11 demonstrates the ranking by a variety of other MCDM methods. Figure 3 illustrates the correlation coefficient between the RAMS and RATMI methods and other methods in heat map format. From the figure, it can be concluded that the best correlation of the RAMS method is 99.7% with the RAPS method and over 96.5% with other methods. The best correlation of the RATMI method is 99.1% with each of the TOPSIS, COPRAS, and WASPAS methods, while it is over 98.5% with other methods. The correlation between RAMS and RAPS is 99.7%, and the correlation between RATMI and both MCRAT and RAMS are 98.8% and 99.1%, respectively. Figure 4 shows

Table 12: A few details of the chosen 15 problems.

Prob. No.	Ref. No.	Problem field	Objective of the study	MCDM tool(s) used	Results obtained
1	[40]	Climate change	Evaluate the indoor climate of the premises in the workplace. Six criteria that should be considered when evaluating the indoor climate of a building: air turnover inside the premises, air humidity, air temperature, illumination intensity, air flow rate, and dew point. 14 rooms were considered as alternatives in the study.	ARAS	Room 9 had the ideal environment, while room 5 had the unfavorable microclimate. The microclimate in room 9 only made up 77% of an optimally balanced microclimate, and the ratio in the worst room was just 55%.
2	[41]	Banking sector	Based on 6 chosen criteria (liquidity, capital adequacy, efficiency, and profitability) and 13 adopted subcriteria, it was required to rank the top 5 commercial banks in Serbia.	SAW, ARAS, COPRAS, GRA, CP, MOORA, VIKOR, and TOPSIS	Different techniques of MCDM result in various bank rankings. Because of this, selecting an MCDM method may be a challenging decision.
3	[5]	Mining engineering	Mining of raw materials typically involves drilling and blasting operations. Choosing the most appropriate blasting pattern parameters as well as ensuring that the working environment and production conditions are respected are key factors in optimizing blasting design. The ranking of 12 alternatives in this study was based on five criteria (powder factor, fragmentation, fly rock, air shock, and cost).	MCRAT and RAPS	According to the geometry and parameters described in this study, the option with a 76 mm diameter hole, 16.3 m hole length, three-meter stemming, a three-meter load, and a 0.76 m sub drill was found to be the most suitable for dacite blasting.
4	[42]	Higher education	According to 8 criteria (course content, time/day of the course, lecturer of the course, lecturer's experience, course adequacy, course's being theoretical/applied, personal interest in the course, and Relation of the course with the desired field of study), doctoral students in the business department of the Graduate School of Social Sciences at Dumlupinar University are required to choose their 11 courses as alternatives.	AHP and TOPSIS	The relation of the course with the desired field of study was the most essential consideration for decision-makers when choosing courses for doctoral education, and the time/day of the course was the least relevant aspect.
5	[43]	Housing sustainability	The objective was to test an assessment method of economic, social and environmental criteria that affect a household's quality of life in order to determine the affordability of sustainable housing. In Liverpool, England, 10 alternative areas were taken into consideration as well as an investigation into the comparative performance of various MCDM tools for 20 evaluation criteria.	VPM, WSM, AHP, TOPSIS, and VOPRAS	COPRAS was exhibited the highest potential in sustainable housing affordability decision analysis, but in a case of higher level uncertainty in criteria importance, TOPSIS, WSM and WPM were considered for their better tolerance to the higher level of the criterion weight change.
6	[44]	*Sidi* education	*Sidi* education is one of the most crucial lessons for Christians in the Christian society. The Sanskrit word "*sidi*," which means complete or perfect, is where the word originates. The graduates from Learning *Sidi* (17 students as alternatives) must meet a number of criteria, including attendance, test scores, leadership, and attitudes.	VIKOR	A list of individuals who completed the *Sidi* catechism learning procedure was the study's result. Students (A8 and A15) were found to not have passed the course based on the VIKOR value, while the other SIDI participants were found to have graduated.

Table 12: Continued.

Prob. No.	Ref. No.	Problem field	Objective of the study	MCDM tool(s) used	Results obtained
7, 8	[45]	Construction industry	The first necessary step in applying a MADM approach is normalization. The choice of normalization techniques directly affects the results. The Logarithmic Normalizing (LN) approach is one of the most recent normalization techniques to be introduced. The investigation of two traditional approaches of MCDM based on the LN method was the subject of this research study based on actual cases in construction company.	VIKOR and TOPSIS	An evaluation of the normalization methods was presented, along with their sensitivity to both methodologies and results. As a result, despite some small differences between these methods, those differences may have significant effects on ultimately selecting among feasible alternatives and determining the quality of decision-making. Both VIKOR and TOPSIS appeared to be positively affected by LN.
9, 10, 11, 12, 13	[46]	Banking sector	The study's objective was to assess and rank the banking sectors of seven developing nations using the TOPSIS technique for the years 2009 to 2013. Ten financial measures from the IMF's Financial Soundness Indicators were investigated and evaluated in terms of these nations' sectors: Argentina, Brazil, Indonesia, Poland, Russia, South Africa, and Turkey. The nations were taken into consideration as alternatives, and the financial ratios were taken into consideration as evaluation criteria.	TOPSIS	Based on the five primary financial criteria—capital adequacy, assets quality, liquidity, profitability, and sensitivity to market risk—and ten frequently used financial ratios chosen as sub-criteria for the performance evaluation of the sector, the results revealed that among the study's participating nations, the banking sectors of Turkey and South Africa performed the best and Argentina's did the worst.
14	[47]	Computer science	This study concerned with Web service selection. AHP was used to evaluate the weights of 8 service criteria (response time, security, reliability, and cost) while VIKOR was used to identify and rank 5 candidate services.	AHP and VIKOR	The results showed that the hybrid approach (AHP-VIKOR) can select the best and most candidate selection
15	[48]	Sustainable communities	This study presented a new multicriteria decision analysis (MCDA) method called data variability assessment technique for order of preference by similarity to ideal solution (the DARIA-TOPSIS method). In 26 European countries, this methodology was used to evaluate sustainable cities and communities. This study took into account 10 evaluation criteria in which three economic, four environmental, and three social.	DARIA-TOPSIS	From a methodical point of view, the proposed method (DARIA-TOPSIS) extended the classical MCDA paradigm of evaluating the set of alternatives to the dynamic approach. The study showed numerous benefits of this approach, including its ability to handle time-based overall evaluation of alternatives as well as detailed efficiencies and ranks of alternatives, which give the user more analytical power.

Table 13: Comparative results between the two new techniques RAMS and RATMI and the original technique RAPS with the other seven MCDM techniques.

Method	Problem #1		$n = 6$, $m = 14$			Ref. [40]	
	ARAS	SAW	TOPSIS	COPRAS	VIKOR	WASPAS	MOORA
RATMI	0.956	0.996	0.930	0.952	0.969	0.969	0.950
RAMS	0.952	1.000	0.934	0.956	0.956	0.965	0.950
RAPS	0.947	0.881	0.956	0.960	0.798	0.947	0.950
	Problem #2		$n = 13$, $m = 5$			Ref. [41]	
RATMI	0.800	0.900	1.000	0.800	0.900	0.800	0.800
RAMS	0.600	0.800	0.900	0.600	0.700	0.600	0.600
RAPS	0.800	0.900	1.000	0.800	0.900	0.800	0.800
	Problem #3		$n = 5$, $m = 12$			Ref. [5]	
RATMI	0.993	0.993	1.000	0.972	0.972	0.993	0.972
RAMS	0.993	0.993	1.000	0.972	0.972	0.993	0.972
RAPS	0.993	0.993	1.000	0.972	0.972	0.993	0.972
	Problem #4		$n = 8$, $m = 11$			Ref. [42]	
Method	ARAS	SAW	TOPSIS	COPRAS	VIKOR	WASPAS	MOORA
RATMI	0.918	0.945	0.991	—	0.991	0.918	0.918
RAMS	0.918	0.945	0.991	—	0.991	0.918	0.918
RAPS	0.918	0.945	0.991	—	0.991	0.918	0.918
	Problem #5		$n = 20$, $m = 10$			Ref. [43]	
RATMI	0.576	0.988	0.442	0.612	0.564	0.721	0.709
RAMS	0.576	0.988	0.442	0.612	0.564	0.721	0.709
RAPS	0.576	0.988	0.442	0.612	0.564	0.721	0.709
	Problem #6		$n = 4$, $m = 17$			Ref. [44]	
RATMI	0.914	0.907	0.919	—	0.564	0.895	0.914
RAMS	0.914	0.907	0.919	—	0.564	0.895	0.914
RAPS	0.914	0.907	0.919	—	0.564	0.895	0.972
	Problem #7		$n = 5$, $m = 4$			Ref. [45]	
RATMI	0.800	1.000	0.400	—	1.000	1.000	1.000
RAMS	0.800	1.000	0.400	—	1.000	1.000	1.000
RAPS	0.800	1.000	0.400	—	1.000	1.000	1.000
	Problem #8		$n = 6$, $m = 8$			Ref. [45]	
RATMI	1.000	0.976	0.952	0.976	0.976	0.976	0.976
RAMS	1.000	0.976	0.952	0.976	0.976	0.976	0.976
RAPS	1.000	0.976	0.952	0.976	0.976	0.976	0.976
	Problem #9		$n = 10$, $m = 7$			Ref. [46]	
RATMI	0.679	0.964	0.357	0.429	0.786	0.607	0.571
RAMS	0.679	0.964	0.357	0.429	0.786	0.607	0.571
RAPS	0.679	0.964	0.357	0.429	0.786	0.607	0.571
	Problem #10		$n = 10$, $m = 7$			Ref. [46]	
RATMI	0.964	0.964	0.607	0.607	0.429	0.964	0.500
RAMS	0.929	0.929	0.536	0.643	0.464	0.929	0.571
RAPS	0.929	0.929	0.536	0.643	0.464	0.929	0.571
	Problem #11		$n = 10$, $m = 7$			Ref. [46]	
RATMI	0.286	-0.321	-0.179	-0.143	-0.250	-0.321	-0.143
RAMS	0.286	-0.321	-0.179	-0.143	-0.250	-0.321	-0.143
RAPS	0.286	-0.321	-0.179	-0.143	-0.250	-0.321	-0.143

Table 13: Continued.

	Problem #12		$n = 10$, $m = 7$			Ref. [46]	
RATMI	0.107	-0.500	0.536	0.500	0.536	-0.500	0.429
RAMS	0.107	-0.500	0.536	0.500	0.536	-0.500	0.429
RAPS	0.107	-.0500	0.536	0.500	0.536	-0.500	0.429
	Problem #13		$n = 10$, $m = 7$			Ref. [46]	
RATMI	-0.179	-0.179	0.214	0.143	0.036	-0.179	0.143
RAMS	-0.179	-0.179	0.214	0.143	0.036	-0.179	0.143
RAPS	-0.179	-0.179	0.214	0.143	0.036	-0.179	0.143
	Problem #14		$n = 4$, $m = 5$			Ref. [47]	
RATMI	1.000	0.800	0.900	0.900	0.900	0.900	0.900
RAMS	0.900	0.900	0.800	0.800	0.800	0.800	0.800
RAPS	1.000	0.800	0.900	0.900	0.900	0.900	0.900
	Problem #15		$n = 10$, $m = 26$			Ref. [48]	
Method	ARAS	SAW	TOPSIS	COPRAS	VIKOR	WASPAS	MOORA
RATMI	0.913	0.954	0.675	0.716	0.685	0.884	0.732
RAMS	0.899	0.941	0.661	0.700	0.665	0.871	0.717
RAPS	0.921	0.951	0.670	0.707	0.681	0.878	0.726

Table 14: Comparative percentage between the three methods RAPS, RAMS, and RATMI.

Rule of comparison	Percentage of rule satisfaction out of 15 problems		
	RAMS vs RAPS	RATMI vs RAMS	RATMI vs RAPS
(a) Number of problems the correlation coefficient degrees between the first method and the other seven MCDM techniques are better than the second method	1 (6.7%)	4 (26.7%)	2 (13.3%)
(b) Number of problems the correlation coefficient degrees of the two methods and the other seven MCDM techniques are identical	10 (66.6%)	10 (66.6%)	11 (73.3%)
(c) Number of problems the correlation coefficient degrees of the second method and the other seven MCDM techniques are better than the first method	4 (26.7%)	1 (6.7%)	2 (13.3%)
(d) Combined (a) + (b)	11 (73%)	*14 (93%)*	(87%)

a comparison between the two new methods RAMS and RATMI with RAPS method. RATMI method has a strong correlation degree over the other methods, RAPS and RAMS.

Validity tests were conducted using data taken from 15 additional problems in the literature. Some details of these problems are demonstrated in Table 12. The number of criteria (n) ranged from 4 to 20, and the number of alternatives (m) ranged from 4 to 26. Table 13 compares the correlation coefficient degrees between the original RAPS methodology, the two new methods RAMS and RATMI, and seven other MCDM techniques. The following findings obtained from this comparison are as follows:

5.1. Comparison between RAMS and RAPS

(i) For problem 1, the correlection coefficient degrees between RAMS and the other seven MCDM techniques were better to RAPS correlation coefficient degrees by, on average, 0.039

(ii) For problems 3-5 and 7-13, RAMS and RAPS showed identical correlation coefficient degrees with the other seven techniques

(iii) For problems 2, 6, 14, and 15, the correlection coefficient degrees between RAMS and the other seven techniques were, on average, lower than RAPS correlection coefficient degrees by 0.171, 0.010, 0.071, and 0.015, respectively

(iv) To sum up, in 11 out of 15 problems (73%), the correlection coefficient degrees of RAMS with the other seven MCDM approaches were superior to or equal to the correlection coefficient degrees of RAPS

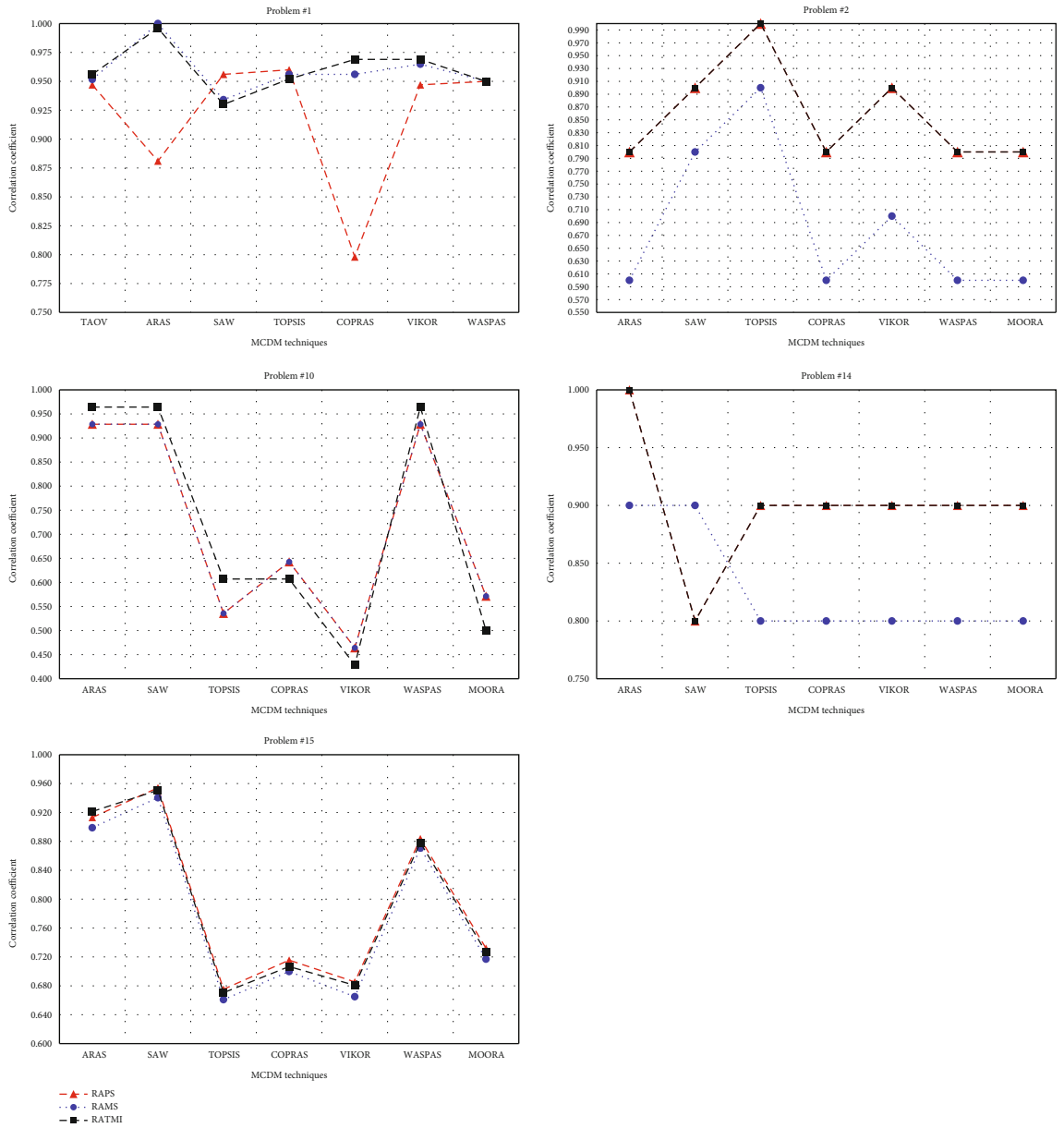

Figure 5: Relative comparison between RAPS, RAMS, and RATMI method for different problems.

5.2. Comparison between RATMI and RAMS

(i) For problems 2, 10, 14, and 15, the correlection coefficient degrees between RATMI and the other seven MCDM techniques were superior to RAMS correlection coefficient degrees by, on average, 0.171, 0.012, 0.067, and 0.013, respectively

(ii) For problems 3–9 and 11–13, the RATMI and RAMS exhibited identical correlection coefficient degrees with the other seven techniques

(iii) For problem 1, RAMS outperformed RATMI with an average correlection coefficient degree of 0.001

(iv) To sum up, in 14 out of 15 problems (93%), the correlection coefficient degrees of RATMI with the other seven approaches were superior to or on par with the correlection coefficient degrees of RAMS

5.3. Comparison between RATMI and RAPS

(i) For problems 1 and 10, the correlection coefficient degrees between RATMI and the other seven MCDM techniques were, on average, better than RAPS correlection coefficient degrees by 0.040 and 0.005, respectively

(ii) For problems 2-5, 7-9, and 11-14, the RATMI and RAPS displayed identical correlection coefficient degrees with the other seven MCDM approaches

(iii) For problems 6 and 15, the correlection coefficient degrees between RAPS and the other seven MCDM

techniques were, on average, better than RATMI correlection coefficient degrees by 0.010 and 0.004, respectively

(iv) To sum up, in 13 out of 15 problems (87%), the correlation coefficient degrees between RATMI and the other seven approaches were superior to or on par with the correlation coefficient degrees of RAPS

Table 14 summarizes the comparison results, reported in the previous points, between the two new techniques RAMS and RATMI and the original technique RAPS. Table 14 makes it obvious that the RATMI approach is a strong rival to RAPS and RAMS. Figure 5 shows comparative results for problems 1, 2, 10, 14, and 15 in a graphical form.

6. Conclusion

The practice of using multiple criteria decision-making (MCDM) as a supporting tool is common in many branches of science and business. In real-world scenarios, the goal of MCDM tools is to assist decision-makers in selecting or ranking alternatives based on assessing and contrasting criteria. Ranking alternatives by perimeter similarity (RAPS) and multiple criteria ranking by alternative trace (MCRAT) are two current MCDM methods that are tested and used in the mining engineering industry. In addition to this, the development of new MCDM techniques is progressing slowly. Therefore, the objectives of this paper were as follows: (1) add two new MCDM techniques called RAMS and RATMI, and (2) these techniques can be applied to a variety of contexts, including education, the financial industry, construction, housing, and business.

The RAMS technique ranks alternatives based on median similarity, as an extension of RAPS. Another method was proposed by integrating the RAMS technique with the MRCAT methodology developed by Urošević et al. [5], employing a majority index and the VIKOR method's premise. This process is known as RATMI or ranking the alternatives based on the trace to median index.

Both methodologies (RAMS and RATMI) were fully presented using a numerical example from a real-world situation of evaluating materials for the body of a break booster valve. In addition to 9 more MCDM techniques, these techniques were evaluated alongside the original RAPS and MCRAT procedures. It may be said that the best correlation between the RAMS technique and the RAPS method was 99.7%, and that it was better than 96.5% with other approaches. The best correlation of the RATMI method was 99.1% with each of the TOPSIS, COPRAS, and WASPAS methods, while it was over 98.5% with other methods. There was a 99.7% correlation between RAMS and RAPS, and 98.8% and 99.1% correlations between MCRAT and RAMS and RATMI, respectively.

The effectiveness of the proposed techniques RAMS and RATMI was compared to the other seven well-known MCDM tools, including additive ratio assessment (ARAS), simple additive weighting (SAW), technique for order of preference by similarity to ideal solution (TOPSIS), complex proportional assessment (CORPAS), VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR), weighted aggregates sum product assessment (WASPAS), and multi-objective optimization on the basis of ratio analysis)MOOR-A(as well as a more recent one RAPS using fifteen tested MCDM problems taken from the literature with a number of criteria ranging from 4 to 20 and number of alternatives ranging from 4 to 26. Results revealed that for 13.3 percent and 26.7 percent of the total 15 problems, respectively, RATMI is more efficient than RAPS and RAMS. As a general remark, the proposed technique RATMI had an equivalent or better correlation coefficient degree with the RAMS technique for 93% of the investigated problems. Future research could focus on the sensitivity analysis of RATMI within different problem sizes ($n \times m$). The techniques can be related, in future studies, by increasing the uncertainty of problem data.

Data Availability

The data supporting the findings of this study are available within the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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