

Research Article

Some Results in Neutrosophic Cubic Graphs with an Application in School's Management System

Zheng Kou ¹, Maryam Akhouni ², M. Ghassemi,³ A. A. Talebi,³ and G. Muhiuddin ⁴

¹Institute of Computing Science and Technology, Guangzhou University, Guangzhou 510006, China

²Clinical Research Development Unit of Rouhani Hospital, Babol University of Medical Sciences, Babol, Iran

³Department of Mathematics, University of Mazandaran, Babolsar, Iran

⁴Department of Mathematics, University of Tabuk, Tabuk 71491, Saudi Arabia

Correspondence should be addressed to Maryam Akhouni; maryam.akhouni@mubabol.ac.ir

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Neutrosophic cubic graph (NCG) belonging to FG family has good capabilities when facing problems that cannot be expressed by FGs. When an element membership is not clear, neutrality is a good option that can be well supported by a NCG. Hence, in this paper, some types of edge irregular neutrosophic cubic graphs (EI-NCGs) such as neighborly edge totally irregular (NETI), strongly edge irregular (SEI), and strongly edge totally irregular (SETI) are introduced. A comparative study between NEI-NCGs and NETI-NCGs is done. Finally, an application of neutrosophic cubic digraph to find the most effective person in a school has been presented.

1. Introduction

The fuzzy set theory was introduced by Zadeh [1]. It focuses on the membership degree of an object in a particular set. Kaufmann [2] represented FGs based on Zadeh's fuzzy relation [3, 4]. Rosenfeld [5] described the structure of FGs obtaining analogs of several graph theoretical concepts. Bhattacharya [6] gave some remarks on FGs. Several concepts on FGs were introduced by Mordeson et al. [7]. The existence of a single degree for a true membership could not resolve the ambiguity on uncertain issues, so the need for a degree of membership was felt. Afterward, to overcome the existing ambiguities, Atanassov [8] defined an extension of fuzzy set by introducing non-membership function and defined intuitionistic fuzzy set (IFS). But after a while, Atanassov and Gargov [9] developed IFS and presented interval-valued intuitionistic fuzzy set (IVIFS). Hongmei and Lianhua [10] defined interval-valued fuzzy graph and studied its properties. Zhang et al. [11] introduced bipolar fuzzy sets and relations. Smarandache [12–14] gave the idea of neutrosophic sets. Kandasamy [15] defined neutrosophic graphs. Akram et al. [16–19] studied new results in NGs. Jun et al. [20] introduced cubic set. For more details about cubic

sets and their applications in different research areas, we refer the readers to [21–23]. Rashid et al. [24] investigated cubic graphs. Jun et al. [25, 26] gave the idea of neutrosophic cubic set and defined different operations on it. Gulistan et al. [27, 28] presented complex bipolar fuzzy sets, NCGs, and some binary operations on it. Karunambigai et al. [29] discussed edge regular-IFG. Gani and Radha [30] studied the concept of regular fuzzy graphs and defined degree of a vertex in FGs. Gani et al. [31] investigated the concept of IFGs, NI-FGs, and HI-FGs in 2008. Nandhini [32] described the concept of SI-FG and studied its properties. Maheswari and Sekar defined the concepts of edge irregular-FGs and edge totally irregular-FGs [33]. Also, they analyzed some properties of NEI-FGs, NETI-FGs, SEI-FGs, and SETI-FGs [34, 35]. Rao et al. [36–38] studied dominating set, equitable dominating set, valid degree, isolated vertex, and some properties of VGs with novel application. Kou et al. [39] investigated g -eccentric node and vague detour g -boundary nodes in VGs. Shi et al. [40, 41] introduced total dominating set, perfect dominating set, and global dominating set in product vague graphs. Rashmanlou et al. [42] presented some properties of cubic graphs. Amanathulla et al. [43] studied on distance two surjective labeling of paths and

TABLE 1: Some basic notations.

Notation	Meaning
FG	Fuzzy graph
NCG	Neutrosophic cubic graph
CNCG	Connected neutrosophic cubic graph
I-FG	Irregular fuzzy graph
EI	Edge irregular
NI	Neighborly irregular
NEI	Neighborly edge irregular
NETI	Neighborly edge totally irregular
SI	Strongly irregular
SEI	Strongly edge irregular
SETI	Strongly edge totally irregular
HI	Highly irregular
HEI	Highly edge irregular
HETI	Highly edge totally irregular
TD	Total degree
TI	Total irregular
VD	Various degree
AE	Adjacent edge
CF	Constant function

interval graphs. Bhattacharya and Pal [44] gave the fuzzy covering problem of fuzzy graphs and its application. Borzooei et al. [45, 46] defined inverse fuzzy graphs and new results of domination in vague graphs. Kalaiarasi et al. [47] presented regular and irregular m-polar fuzzy graphs. Ramprasad et al. [48] investigated some properties of highly irregular, edge regular, and totally edge regular m-polar fuzzy graphs. Poulik and Ghorai [49] defined certain indices of graphs under bipolar fuzzy environment. Ullah et al. [50] introduced new results on bipolar-valued hesitant fuzzy sets. Jan et al. [51] presented some root level modifications in interval valued fuzzy graphs. Broumi et al. [52] introduced a novel system and method for telephone network planning based on neutrosophic graph. Muhiuddin et al. [53, 54] presented reinforcement number of a graph and new results in cubic graphs. Talebi et al. [55–57] presented some properties of irregularity and edge irregularity on intuitionistic fuzzy graphs and single valued neutrosophic graphs.

NCGs have many applications in psychology and medical sciences and can play a significant role in solving the vague and complex problems that exist around our lives. With the help of this fuzzy graph, the most effective person in an organization can be determined according to the amount of its performance in a specific period. Therefore, in this paper, some types of EI-NCGs such as neighborly edge totally irregular (NETI)-NCGs, strongly edge irregular (SEI)-NCGs, and strongly edge totally irregular (SETI)-NCGs are introduced. Also, we have given some interesting results about EI-NCGs, and several examples are investigated. Finally, an application of neutrosophic cubic digraph to find the most effective person in a school has been presented.

2. Preliminaries

Definition 1. A graph $G = (V, E)$ is a mathematical model consisting of a set of nodes V and a set of edges E , where each is an unordered pair of distinct nodes.

Definition 2 (see [5]). A FG $Z = (V, \nu, \xi)$ is a non-empty set V together with a pair of functions $\nu: V \rightarrow [0, 1]$ and $\xi: V \times V \rightarrow [0, 1]$ so that $\xi(xy) \leq \min\{\nu(x), \nu(y)\}$, $\forall x, y \in V$.

All the basic notations are shown in Table 1.

3. New Concepts of Edge Irregular-NCGs

Definition 3. Let $G^*: (V, E)$ be a graph. By NCG of G^* , we mean a pair $G: (M, N)$ where $M = (A, B) = ((\tilde{T}_A, T_B), (\tilde{I}_A, I_B), (\tilde{F}_A, F_B))$ is the NCS representation of V and $N = (C, D) = ((\tilde{T}_C, T_D), (\tilde{I}_C, I_D), (\tilde{F}_C, F_D))$ is the NCS representation of E so that

- (i) $(\tilde{T}_C(u, \nu) < rmin\{\tilde{T}_A(u), \tilde{T}_A(\nu)\}, T_D(u, \nu) \leq \max\{T_B(u), T_B(\nu)\})$.
- (ii) $(\tilde{I}_C(u, \nu) < rmin\{\tilde{I}_A(u), \tilde{I}_A(\nu)\}, I_D(u, \nu) \leq \max\{I_B(u), I_B(\nu)\})$.
- (iii) $(\tilde{F}_C(u, \nu) < rmax\{\tilde{F}_A(u), \tilde{F}_A(\nu)\}, F_D(u, \nu) \leq \min\{F_B(u), F_B(\nu)\})$.

Definition 4. Let $G: (M, N)$ be a NCG on $G^*: (V, E)$. Then, the degree of a node u is defined as $d_G(u) = ((d_{\tilde{T}_A}^-(u), td_{T_B}(u)), t(d_{\tilde{T}_A}^-(u), d_{I_B}(u)), n, q(d_{\tilde{F}_A}^-(u), d_{F_B}(u)))$ where

$$\begin{aligned} d_{\tilde{T}_A}^-(u) &= \sum_{v \neq u} \tilde{T}_C(uv), & d_{T_B}(u) &= \sum_{v \neq u} T_D(uv). \\ d_{\tilde{I}_A}^-(u) &= \sum_{v \neq u} \tilde{I}_C(uv), & d_{I_B}(u) &= \sum_{v \neq u} I_D(uv). \\ d_{\tilde{F}_A}^-(u) &= \sum_{v \neq u} \tilde{F}_C(uv) & \text{and } d_{F_B}(u) &= \sum_{v \neq u} F_D(uv). \end{aligned}$$

Definition 5. Let $G: (M, N)$ be a NCG on $G^*: (V, E)$. The TD of a node u is defined by $td_G(u) = ((td_{\tilde{T}_A}^-(u), td_{T_B}(u)), (td_{\tilde{T}_A}^-(u), td_{I_B}(u)), (td_{\tilde{F}_A}^-(u), td_{F_B}(u)))$ where

$$\begin{aligned} td_{\tilde{T}_A}^-(u) &= \sum_{v \neq u} \tilde{T}_C(uv) + \tilde{T}_A(u), & td_{T_B}(u) &= \sum_{v \neq u} T_D(uv) + T_B(u). \\ td_{\tilde{I}_A}^-(u) &= \sum_{v \neq u} \tilde{I}_C(uv) + \tilde{I}_A(u), & td_{I_B}(u) &= \sum_{v \neq u} I_D(uv) + I_B(u). \\ td_{\tilde{F}_A}^-(u) &= \sum_{v \neq u} \tilde{F}_C(uv) + \tilde{F}_A(u) & \text{and } td_{F_B}(u) &= \sum_{v \neq u} F_D(uv) + F_B(u). \end{aligned}$$

Definition 6. Let $G: (M, N)$ be a NCG on $G^*: (V, E)$. Then:

- (i) G is irregular, if there is a node that is neighbor to nodes with VDs.
- (ii) G is TI, if there is a node which is neighbor to nodes with various TDs.

Definition 7. Let $G: (M, N)$ be a CNCG. Then, G is called a

- (i) NI-NCG if each pair of neighbor nodes has VDs.
- (ii) NTI-NCG if each pair of neighbor nodes has various TDs.
- (iii) SI-NCG if each pair of nodes has VDs.
- (iv) STI-NCG if each pair of nodes has various TDs.

- (v) HI-NCG if each node in G is neighbor to the nodes having VDs.
 (vi) HTI-NCG if each node in G is neighbor to the nodes having various TDs.

Definition 8. Let $G: (M, N)$ be a NCG. The degree of an edge uv is defined as $d_G(uv) = ((d_{T_C}^-(uv), d_{T_D}^-(uv)), (d_{I_C}^-(uv), d_{I_D}^-(uv)), (d_{F_C}^-(uv), d_{F_D}^-(uv)))$ where

$$d_{T_C}^-(uv) = d_{T_A}^-(u) + d_{T_A}^-(v) - 2\tilde{T}_C(uv), \quad d_{T_D}^-(uv) = d_{T_B}^-(u) + d_{T_B}^-(v) - 2\tilde{T}_D(uv).$$

$$d_{I_C}^-(uv) = d_{I_A}^-(u) + d_{I_A}^-(v) - 2\tilde{I}_C(uv), \quad d_{I_D}^-(uv) = d_{I_B}^-(u) + d_{I_B}^-(v) - 2\tilde{I}_D(uv).$$

$$d_{F_C}^-(uv) = d_{F_A}^-(u) + d_{F_A}^-(v) - 2\tilde{F}_C(uv) \quad \text{and} \quad d_{F_D}^-(uv) = d_{F_B}^-(u) + d_{F_B}^-(v) - 2\tilde{F}_D(uv).$$

Definition 9. Let $G: (M, N)$ be a NCG. The TD of an edge uv is defined as $td_G(uv) = ((td_{T_C}^-(uv), td_{T_D}^-(uv)), (td_{I_C}^-(uv), td_{I_D}^-(uv)), (td_{F_C}^-(uv), td_{F_D}^-(uv)))$ where

$$td_{T_C}^-(uv) = d_{T_A}^-(u) + d_{T_A}^-(v) - \tilde{T}_C(uv) = d_{T_C}^-(uv) + \tilde{T}_C(uv).$$

$$td_{T_D}^-(uv) = d_{T_B}^-(u) + d_{T_B}^-(v) - \tilde{T}_D(uv) = d_{T_D}^-(uv) + \tilde{T}_D(uv).$$

$$td_{I_C}^-(uv) = d_{I_A}^-(u) + d_{I_A}^-(v) - \tilde{I}_C(uv) = d_{I_C}^-(uv) + \tilde{I}_C(uv).$$

$$td_{I_D}^-(uv) = d_{I_B}^-(u) + d_{I_B}^-(v) - \tilde{I}_D(uv) = d_{I_D}^-(uv) + \tilde{I}_D(uv).$$

$$td_{F_C}^-(uv) = d_{F_A}^-(u) + d_{F_A}^-(v) - \tilde{F}_C(uv) = d_{F_C}^-(uv) + \tilde{F}_C(uv).$$

$$td_{F_D}^-(uv) = d_{F_B}^-(u) + d_{F_B}^-(v) - \tilde{F}_D(uv) = d_{F_D}^-(uv) + \tilde{F}_D(uv).$$

Definition 10. Let $G: (M, N)$ be a CNCG on $G^*: (V, E)$. Then, G is called a

- (1) NEI-NCG if each pair of AEs has VDs.
- (2) NETI-NCG if each pair of AEs has various TDs.

Example 1. Consider a graph which is both NEI-NCG and NETI-NCG.

Consider $G^*: (V, E)$ where $V = \{u, v, w, x\}$ and $E = \{uv, vw, wx, xu\}$ are defined as

$$M = \left\langle \begin{array}{l} \{u, ([0.3, 0.5], 0.6), ([0.4, 0.7], 0.3), ([0.6, 0.8], 0.5)\}, \\ \{v, ([0.4, 0.6], 0.4), ([0.2, 0.5], 0.2), ([0.3, 0.7], 0.9)\}, \\ \{w, ([0.3, 0.5], 0.6), ([0.4, 0.7], 0.3), ([0.6, 0.8], 0.5)\}, \\ \{x, ([0.4, 0.6], 0.4), ([0.2, 0.5], 0.2), ([0.3, 0.7], 0.9)\} \end{array} \right\rangle \quad (1)$$

$$N = \left\langle \begin{array}{l} \{uv, ([0.2, 0.4], 0.5), ([0.1, 0.2], 0.3), ([0.3, 0.5], 0.4)\}, \\ \{vw, ([0.1, 0.3], 0.4), ([0.3, 0.4], 0.2), ([0.4, 0.6], 0.3)\}, \\ \{wx, ([0.2, 0.4], 0.5), ([0.1, 0.2], 0.3), ([0.3, 0.5], 0.4)\}, \\ \{xu, ([0.1, 0.3], 0.4), ([0.3, 0.4], 0.2), ([0.4, 0.6], 0.3)\} \end{array} \right\rangle \quad (2)$$

From Figure 1,

$$\begin{aligned} d_G(u) &= d_G(v) = d_G(w) = d_G(x) \\ &= (([0.3, 0.7], 0.9), ([0.4, 0.6], 0.5), ([0.7, 1.1], 0.7)), \end{aligned} \quad (3)$$

$$\begin{aligned} d_G(uv) &= d_G(wx) \\ &= (([0.2, 0.6], 0.8), ([0.6, 0.8], 0.4), ([0.8, 1.2], 0.6)), \\ d_G(vw) &= d_G(xu) \\ &= (([0.4, 0.8], 1.0), ([0.2, 0.4], 0.6), ([0.6, 1.0], 0.8)). \end{aligned} \quad (4)$$

Clearly, G is a NEI-NCG.

$$\begin{aligned} td_G(uv) &= td_G(wx) \\ &= (([0.4, 1.0], 1.3), ([0.7, 1.0], 0.7), ([1.1, 1.7], 1.0)), \\ td_G(vw) &= td_G(xu) \\ &= (([0.5, 1.1], 0.5), ([0.5, 0.8], 0.8), ([1.0, 1.6], 1.1)). \end{aligned} \quad (5)$$

So, G is a NETI-NCG.

Therefore, G is both NEI-NCG and NETI-NCG.

Example 2. NEI-NCG need not to be NETI-NCG.

Let G be a NCG and G^* be a star that includes four nodes where $V = \{u, v, w, x\}$ and $E = \{ux, vx, wx\}$ are defined as

$$M = \left\langle \begin{array}{l} \{u, ([0.3, 0.4], 0.4), ([0.2, 0.3], 0.3), ([0.4, 0.5], 0.6)\}, \\ \{v, ([0.5, 0.6], 0.6), ([0.4, 0.5], 0.5), ([0.6, 0.7], 0.8)\}, \\ \{w, ([0.4, 0.5], 0.5), ([0.5, 0.4], 0.9), ([0.7, 0.8], 0.7)\}, \\ \{x, ([0.2, 0.3], 0.3), ([0.4, 0.5], 0.7), ([0.3, 0.4], 0.9)\} \end{array} \right\rangle \quad (6)$$

$$N = \left\langle \begin{array}{l} \{ux, ([0.2, 0.3], 0.3), ([0.1, 0.2], 0.2), ([0.4, 0.5], 0.6)\}, \\ \{vx, ([0.1, 0.2], 0.2), ([0.4, 0.5], 0.5), ([0.6, 0.7], 0.8)\}, \\ \{wx, ([0.0, 0.1], 0.1), ([0.3, 0.4], 0.6), ([0.5, 0.6], 0.7)\} \end{array} \right\rangle \quad (7)$$

From Figure 2,

$$\begin{aligned} d_G(u) &= (([0.2, 0.3], 0.3), ([0.1, 0.2], 0.2), ([0.4, 0.5], 0.6)), \\ d_G(v) &= (([0.1, 0.2], 0.2), ([0.4, 0.5], 0.5), ([0.6, 0.7], 0.8)), \\ d_G(w) &= (([0.0, 0.1], 0.1), ([0.3, 0.4], 0.6), ([0.5, 0.6], 0.7)), \\ d_G(x) &= (([0.3, 0.6], 0.6), ([0.8, 1.1], 1.3), ([1.5, 1.8], 2.1)), \end{aligned} \quad (8)$$

$$\begin{aligned} d_G(ux) &= (([0.1, 0.3], 0.3), ([0.7, 0.9], 1.1), ([1.1, 1.3], 1.5)), \\ d_G(vx) &= (([0.2, 0.4], 0.4), ([0.4, 0.6], 0.8), ([0.9, 1.1], 1.3)), \\ d_G(wx) &= (([0.3, 0.5], 0.5), ([0.5, 0.7], 0.7), ([0.1, 1.2], 1.4)). \end{aligned} \quad (9)$$

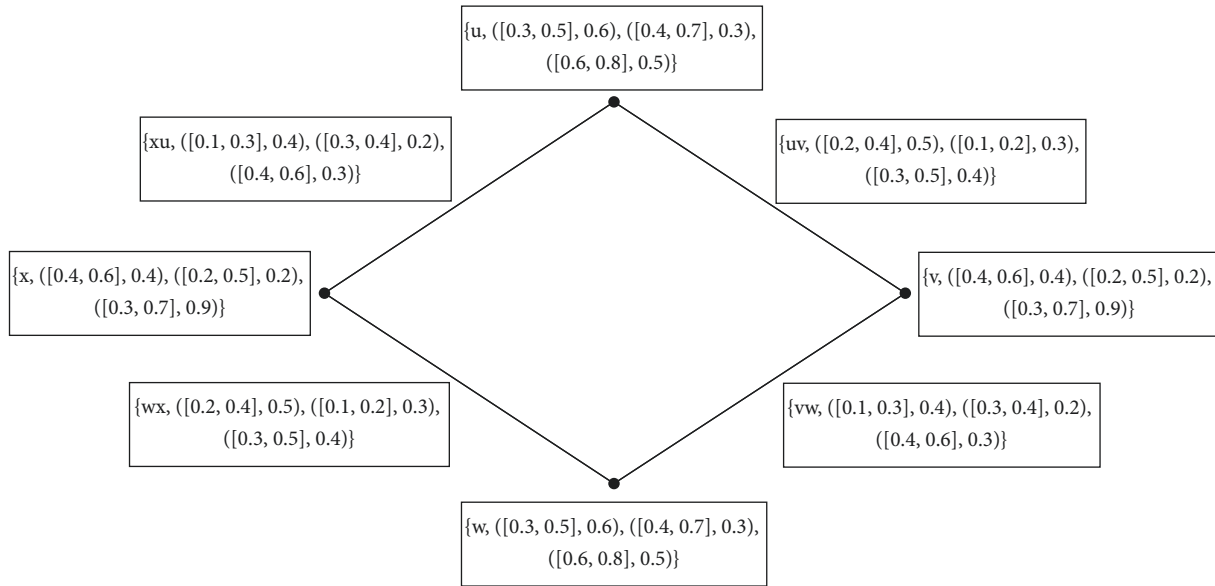


FIGURE 1: NEI-NCG and NETI-NCG.

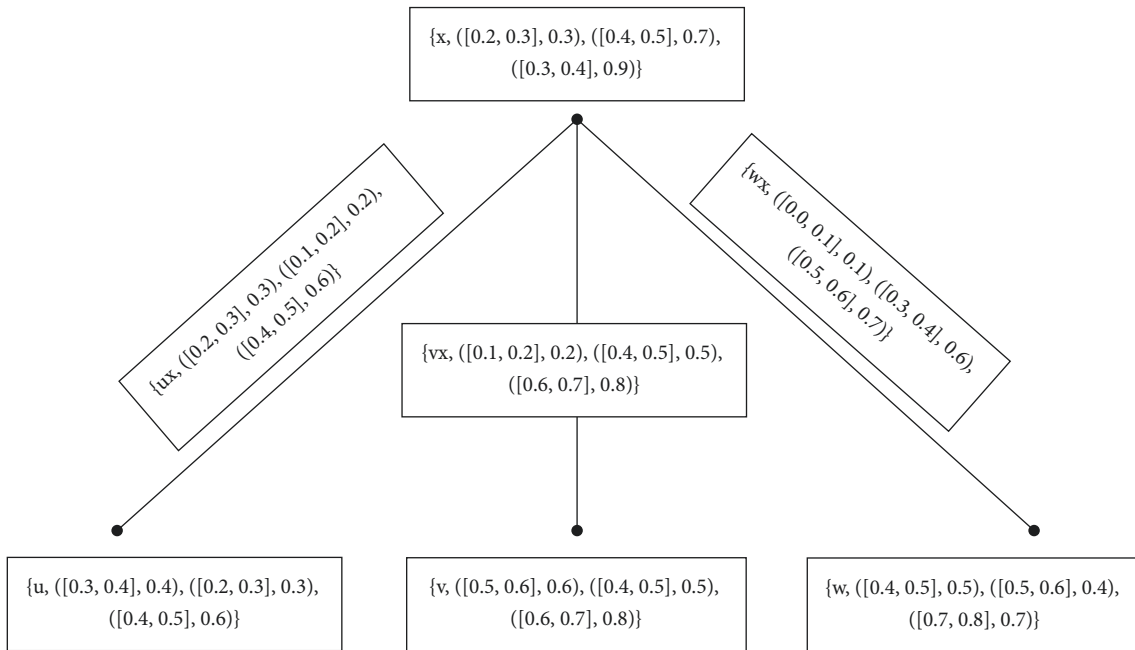


FIGURE 2: G is NEI-NCG but it is not NETI-NCG.

$td_G(ux) = td_G(vx) = td_G(wx) = (([0.3, 0.6], 0.6), ([0.8, 1.1], 1.3), ([1.5, 1.8], 2.1))$.

Here, $d_G(ux) \neq d_G(vx) \neq d_G(wx)$. Hence, G is a NEI-NCG. But G is not a NETI-NCG, since all edges have same TDs.

Example 3. NETI-NCG does not need to be NEI-NCG. The following shows this subject.

Let $G: (M, N)$ be a NCG so that $G^*: (V, E)$ is a path that consists of four nodes where $V = \{u, v, w, x\}$ and $E = \{uv, vw, wx\}$ are defined as

$$M = \left\langle \begin{array}{l} \{u, ([0.4, 0.6], 0.4), ([0.2, 0.3], 0.5), ([0.5, 0.7], 0.2)\}, \\ \{v, ([0.4, 0.7], 0.2), ([0.2, 0.6], 0.3), ([0.7, 0.9], 0.4)\}, \\ \{w, ([0.5, 0.6], 0.7), ([0.3, 0.5], 0.9), ([0.4, 0.7], 0.3)\}, \\ \{x, ([0.3, 0.5], 0.5), ([0.2, 0.4], 0.7), ([0.1, 0.5], 0.6)\} \end{array} \right\rangle, \tag{10}$$

$$N = \left\langle \begin{array}{l} \{uv, ([0.2, 0.3], 0.3), ([0.1, 0.2], 0.4), ([0.3, 0.4], 0.1)\}, \\ \{vw, ([0.4, 0.6], 0.6), ([0.2, 0.4], 0.8), ([0.6, 0.8], 0.2)\}, \\ \{wx, ([0.2, 0.3], 0.3), ([0.1, 0.2], 0.4), ([0.3, 0.4], 0.1)\} \end{array} \right\rangle. \tag{11}$$

From Figure 3,

$$\begin{aligned}
 d_G(u) &= d_G(x) \\
 &= (([0.2, 0.3], 0.3), ([0.1, 0.2], 0.4), \\
 &\quad \cdot ([0.3, 0.4], 0.1)), \tag{12}
 \end{aligned}$$

$$\begin{aligned}
 d_G(v) &= d_G(w) \\
 &= (([0.6, 0.9], 0.9), ([0.3, 0.6], 1.2), \\
 &\quad \cdot ([0.9, 1.2], 0.3)),
 \end{aligned}$$

$$\begin{aligned}
 d_G(uv) &= d_G(vw) = d_G(wx) \\
 &= (([0.4, 0.6], 0.6), ([0.2, 0.4], 0.8), \\
 &\quad \cdot ([0.6, 0.8], 0.2)), \tag{13}
 \end{aligned}$$

$$\begin{aligned}
 td_G(uv) &= td_G(wx) \\
 &= (([0.6, 0.9], 0.9), ([0.3, 0.6], 1.2) \\
 &\quad \cdot ([0.9, 1.2], 0.3)), \\
 td_G(vw) &= (([0.8, 1.2], 1.2), ([0.4, 0.8], 1.6) \\
 &\quad \cdot ([1.2, 1.6], 0.4)). \tag{14}
 \end{aligned}$$

Here, $d_G(uv) = d_G(vw) = d_G(wx)$. Hence, G is not a NEI-NCG. But G is a NETI-NCG, since $td_G(uv) \neq td_G(vw)$ and $td_G(vw) \neq td_G(wx)$.

Theorem 1. Let G be a CNCG on G^* and N be a CF. Then, G is a NEI-NCG, iff G is a NETI-NCG.

Proof. Assume that $N = (C, D) = ((\tilde{T}_C, T_D), (\tilde{I}_C, I_D), (\tilde{F}_C, F_D))$ is a CF, and let $N(uv) = (\tilde{R}, S), \forall uv \in E$, where $(\tilde{R}, S) = ((\tilde{R}_T, S_T), (\tilde{R}_I, S_I), (\tilde{R}_F, S_F))$ is constant.

Let uv and vw be pair of AEs in E . Then,

$$\begin{aligned}
 d_G(uv) \neq d_G(vw) &\iff d_G(uv) + (\tilde{R}, S) \neq d_G(vw) + (\tilde{R}, S) \\
 &\iff \left(\left(d_{\tilde{T}_C}^-(uv), d_{T_D}(uv) \right), \left(d_{\tilde{I}_C}^-(uv), d_{I_D}(uv) \right), \left(d_{\tilde{F}_C}^-(uv), d_{F_D}(uv) \right) \right) \\
 &\quad + ((\tilde{R}_T, S_T), (\tilde{R}_I, S_I), (\tilde{R}_F, S_F)) \neq \left(\left(d_{\tilde{T}_C}^-(vw), d_{T_D}(vw) \right), \left(d_{\tilde{I}_C}^-(vw), d_{I_D}(vw) \right), \left(d_{\tilde{F}_C}^-(vw), d_{F_D}(vw) \right) \right) \\
 &\quad + ((\tilde{R}_T, S_T), (\tilde{R}_I, S_I), (\tilde{R}_F, S_F)) \tag{15}
 \end{aligned}$$

$$\begin{aligned}
 &\iff \left(\left(d_{\tilde{T}_C}^-(uv) + \tilde{R}_T, d_{T_D}(uv) + S_T \right), \left(d_{\tilde{I}_C}^-(uv) + \tilde{R}_I, d_{I_D}(uv) + S_I \right), \left(d_{\tilde{F}_C}^-(uv) + \tilde{R}_F, d_{F_D}(uv) + S_F \right) \right) \\
 &\neq \left(\left(d_{\tilde{T}_C}^-(vw) + \tilde{R}_T, d_{T_D}(vw) + S_T \right), \left(d_{\tilde{I}_C}^-(vw) + \tilde{R}_I, d_{I_D}(vw) + S_I \right), \left(d_{\tilde{F}_C}^-(vw) + \tilde{R}_F, d_{F_D}(vw) + S_F \right) \right) \tag{16}
 \end{aligned}$$

$$\begin{aligned}
 &\iff \left(\left(d_{\tilde{T}_C}^-(uv) + \tilde{T}_C, d_{T_D}(uv) + T_D \right), \left(d_{\tilde{I}_C}^-(uv) + \tilde{I}_C, d_{I_D}(uv) + I_D \right), \left(d_{\tilde{F}_C}^-(uv) + \tilde{F}_C, d_{F_D}(uv) + F_D \right) \right) \\
 &\neq \left(\left(d_{\tilde{T}_C}^-(vw) + \tilde{T}_C, d_{T_D}(vw) + T_D \right), \left(d_{\tilde{I}_C}^-(vw) + \tilde{I}_C, d_{I_D}(vw) + I_D \right), \left(d_{\tilde{F}_C}^-(vw) + \tilde{F}_C, d_{F_D}(vw) + F_D \right) \right) \tag{17}
 \end{aligned}$$

$$\begin{aligned}
 &\iff \left(\left(td_{\tilde{T}_C}^-(uv), td_{T_D}(uv) \right), \left(td_{\tilde{I}_C}^-(uv), td_{I_D}(uv) \right), \left(td_{\tilde{F}_C}^-(uv), td_{F_D}(uv) \right) \right) \\
 &\neq \left(\left(td_{\tilde{T}_C}^-(vw), td_{T_D}(vw) \right), \left(td_{\tilde{I}_C}^-(vw), td_{I_D}(vw) \right), \left(td_{\tilde{F}_C}^-(vw), td_{F_D}(vw) \right) \right) \iff td_G(uv) \neq td_G(vw). \tag{18}
 \end{aligned}$$

Therefore, adjacent edges have various degrees if and only if they have various total degrees. So, G is a NEI-NCG iff G is a NETI-NCG. \square

Remark 1. Let G be a CNCG on G^* . If G is both NEI-NCG and NETI-NCG, then N does not need to be a CF.

Example 4. Let $G: (M, N)$ be a NCG and $G^*: (V, E)$ be a path that consists of four nodes where $V = \{u, v, w, x\}$ and $E = \{uv, vw, wx\}$ are defined as

$$M = \left\langle \begin{aligned} &\{u, ([0.3, 0.5], 0.3), ([0.2, 0.6], 0.6), ([0.4, 0.7], 0.2)\}, \\ &\{v, ([0.2, 0.4], 0.5), ([0.3, 0.5], 0.4), ([0.6, 0.8], 0.3)\}, \\ &\{w, ([0.2, 0.3], 0.2), ([0.4, 0.5], 0.7), ([0.5, 0.6], 0.4)\}, \\ &\{x, ([0.3, 0.6], 0.6), ([0.2, 0.4], 0.1), ([0.7, 0.9], 0.6)\} \end{aligned} \right\rangle, \tag{19}$$

$$N = \left\langle \begin{aligned} &\{uv, ([0.2, 0.3], 0.4), ([0.1, 0.4], 0.5), ([0.6, 0.7], 0.2)\}, \\ &\{vw, ([0.1, 0.2], 0.3), ([0.3, 0.4], 0.6), ([0.5, 0.6], 0.1)\}, \\ &\{wx, ([0.2, 0.3], 0.4), ([0.1, 0.4], 0.5), ([0.6, 0.7], 0.2)\} \end{aligned} \right\rangle. \tag{20}$$

From Figure 4,

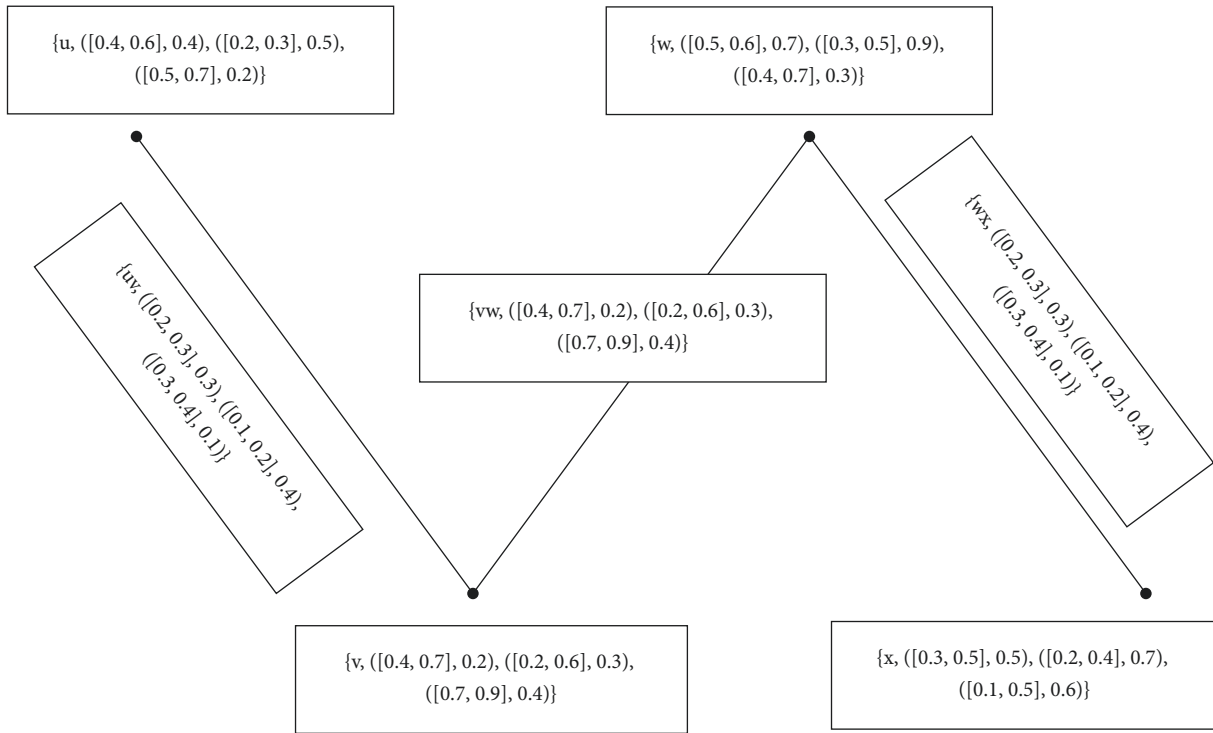


FIGURE 3: G is NETI-NCG but it is not NEI-NCG.

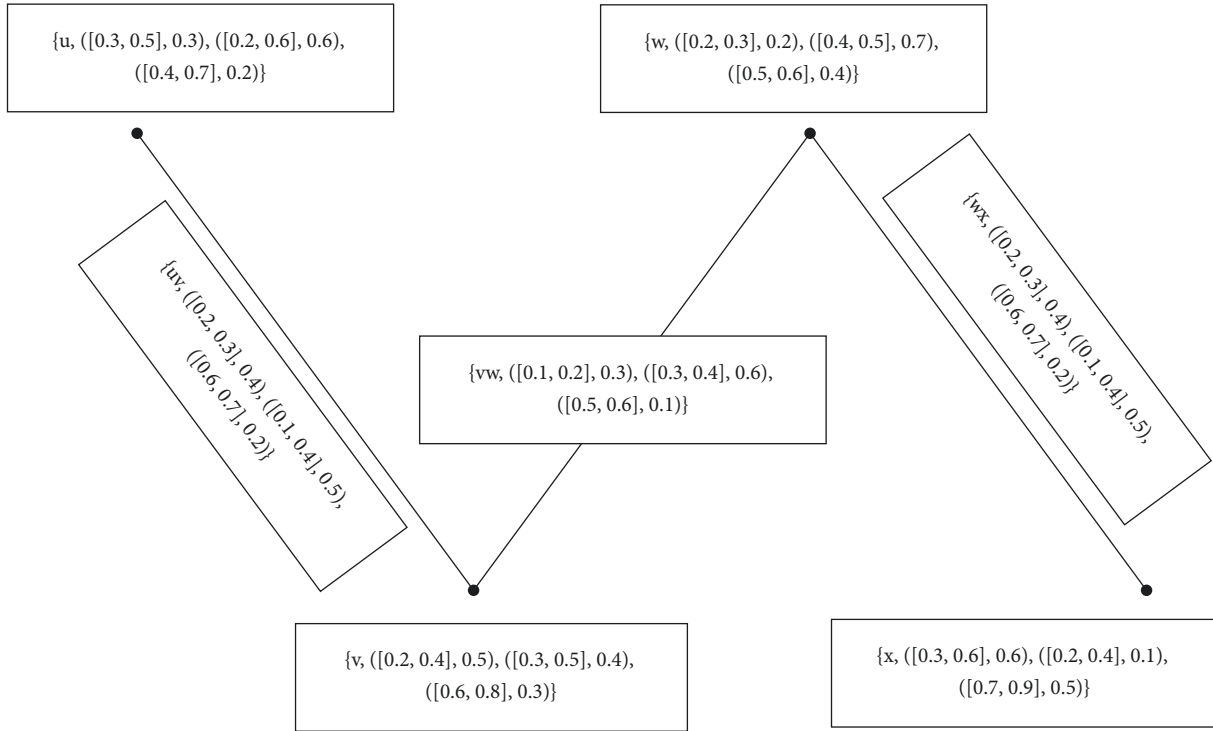


FIGURE 4: N is not a CF.

$$\begin{aligned}
 d_G(u) &= d_G(x) = (([0.2, 0.3], 0.4), ([0.1, 0.4], 0.5), \\
 &\quad \cdot ([0.6, 0.7], 0.2)), \\
 d_G(v) &= d_G(w) = (([0.3, 0.5], 0.7), ([0.4, 0.8], 1.1), \\
 &\quad \cdot ([1.1, 1.3], 0.3)),
 \end{aligned}
 \tag{21}$$

$$\begin{aligned}
 d_G(uv) &= d_G(wx) = (([0.1, 0.2], 0.3), ([0.3, 0.4], 0.6), \\
 &\quad \cdot ([0.5, 0.6], 0.1)), \\
 d_G(vw) &= (([0.4, 0.6], 0.8), ([0.2, 0.8], 1.0), \\
 &\quad \cdot ([1.2, 1.4], 0.4)),
 \end{aligned}
 \tag{22}$$

$$\begin{aligned}
 td_G(uv) &= td_G(wx) = (([0.3, 0.5], 0.7), ([0.4, 0.8], 1.1), \\
 &\quad \cdot ([1.1, 1.3], 0.3)), \\
 td_G(vw) &= (([0.5, 0.8], 1.1), ([0.5, 1.2], 1.7), \\
 &\quad \cdot ([1.7, 2.0], 0.4)).
 \end{aligned}
 \tag{23}$$

Here, $d_G(uv) \neq d_G(vw)$ and $d_G(vw) \neq d_G(wx)$. Hence, G is a NEI-NCG. Also, $td_G(uv) \neq td_G(vw)$ and $td_G(vw) \neq td_G(wx)$. Hence, G is a NETI-NCG. But N is not CF.

Theorem 2. Let G be a CNCG on G^* and N be a CF. If G is a SI-NCG, then G is a NEI-NCG.

Proof. Let $G: (M, N)$ be a CNCG. Assume that $N = (C, D) = ((\tilde{T}_C, T_D), (\tilde{I}_C, I_D), (\tilde{F}_C, F_D))$ is a CF, and let $N(uv) = (\tilde{R}, tS)$, $\forall uv$ in E , where $(\tilde{R}, S) = ((\tilde{R}_T, S_T), (\tilde{R}_I, S_I), (\tilde{R}_F, S_F))$ is constant.

Let uv and vw be any two AEs in G and G be a SI-NCG. Then, each pair of nodes in G has VDs, and hence

$$\begin{aligned}
 d_G(u) \neq d_G(v) \neq d_G(w) &\implies \left((d_{T_A}^-(u), d_{T_B}(u)), (d_{I_A}^-(u), d_{I_B}(u)), (d_{F_A}^-(u), d_{F_B}(u)) \right) \\
 &\neq \left((d_{T_A}^-(v), d_{T_B}(v)), (d_{I_A}^-(v), d_{I_B}(v)), (d_{F_A}^-(v), d_{F_B}(v)) \right)
 \end{aligned}
 \tag{24}$$

$$\begin{aligned}
 &\neq \left((d_{T_A}^-(w), d_{T_B}(w)), (d_{I_A}^-(w), d_{I_B}(w)), (d_{F_A}^-(w), d_{F_B}(w)) \right) \\
 &\implies \left((d_{T_A}^-(u), d_{T_B}(u)), (d_{I_A}^-(u), d_{I_B}(u)), (d_{F_A}^-(u), d_{F_B}(u)) \right) \\
 &\quad + \left((d_{T_A}^-(v), d_{T_B}(v)), t(d_{I_A}^-(v), d_{I_B}(v))n, q(d_{F_A}^-(v), d_{F_B}(v)) \right) - 2((\tilde{R}_T, S_T), t(\tilde{R}_I, S_I)n, q(\tilde{R}_F, S_F))
 \end{aligned}
 \tag{25}$$

$$\begin{aligned}
 &\neq \left((d_{T_A}^-(v), d_{T_B}(v)), (d_{I_A}^-(v), d_{I_B}(v)), (d_{F_A}^-(v), d_{F_B}(v)) \right) \\
 &\quad + \left((d_{T_A}^-(w), d_{T_B}(w)), (d_{I_A}^-(w), d_{I_B}(w)), (d_{F_A}^-(w), d_{F_B}(w)) \right) - 2((\tilde{R}_T, S_T), (\tilde{R}_I, S_I), (\tilde{R}_F, S_F)) \\
 &\implies \left((d_{T_A}^-(u) + d_{T_A}^-(v) - 2\tilde{R}_T, d_{T_B}(u) + d_{T_B}(v) - 2S_T), (d_{I_A}^-(u) + d_{I_A}^-(v) - 2\tilde{R}_I, d_{I_B}(u) + d_{I_B}(v) - 2S_I), \right. \\
 &\quad \left. (d_{F_A}^-(u) + d_{F_A}^-(v) - 2\tilde{R}_F, d_{F_B}(u) + d_{F_B}(v) - 2S_F) \right)
 \end{aligned}
 \tag{26}$$

$$\begin{aligned}
 &\neq \left((d_{T_A}^-(v) + d_{T_A}^-(w) - 2\tilde{R}_T, d_{T_B}(v) + d_{T_B}(w) - 2S_T), (d_{I_A}^-(v) + d_{I_A}^-(w) - 2\tilde{R}_I, d_{I_B}(v) + d_{I_B}(w) - 2S_I), \right. \\
 &\quad \left. (d_{F_A}^-(v) + d_{F_A}^-(w) - 2\tilde{R}_F, d_{F_B}(v) + d_{F_B}(w) - 2S_F) \right)
 \end{aligned}$$

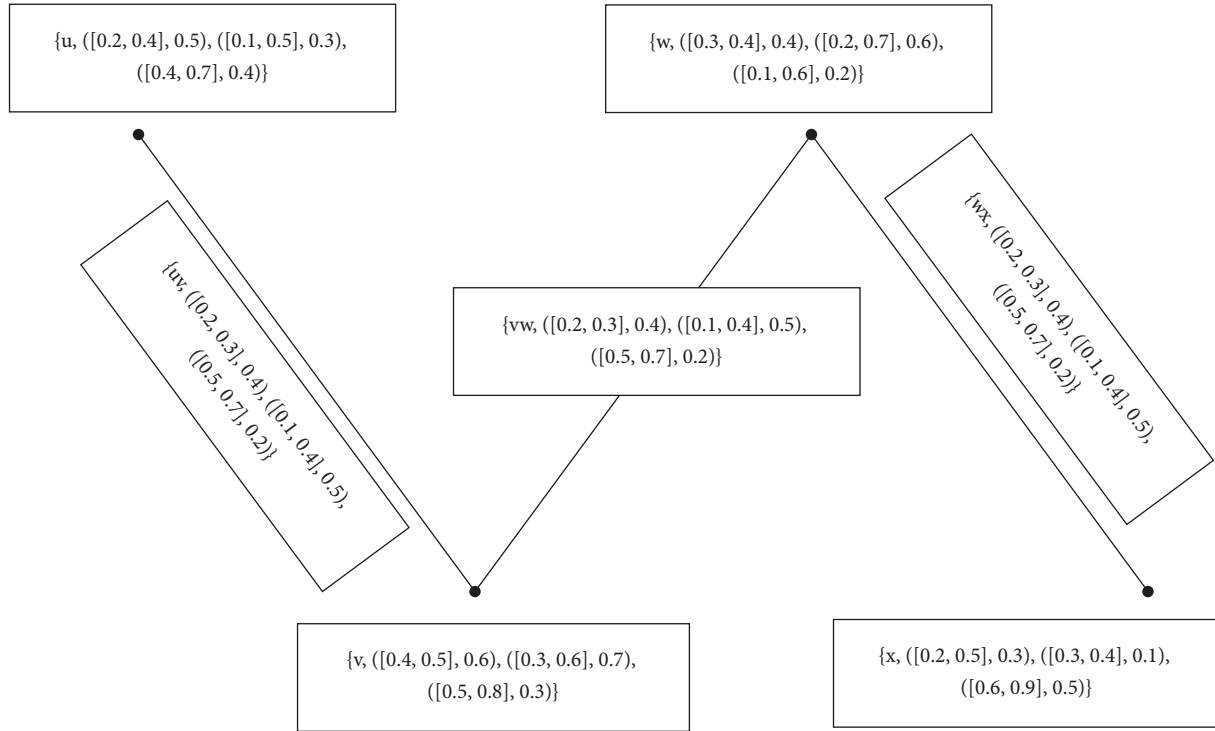
$$\begin{aligned}
 &\implies \left((d_{T_A}^-(u) + d_{T_A}^-(v) - 2\tilde{T}_C(uv), d_{T_B}(u) + d_{T_B}(v) - 2T_D(uv)), \right. \\
 &\quad (d_{I_A}^-(u) + d_{I_A}^-(v) - 2\tilde{I}_C(uv), d_{I_B}(u) + d_{I_B}(v) - 2I_D(uv)), \\
 &\quad \left. (d_{F_A}^-(u) + td_{F_A}^-(v)q - h2\tilde{F}_C x(uv)7, Cd_{F_B};(u) + d_{F_B}(v) - 2F_D(uv)) \right)
 \end{aligned}
 \tag{27}$$

$$\begin{aligned}
 &\neq \left((d_{T_A}^-(v) + d_{T_A}^-(w) - 2\tilde{T}_C(vw), d_{T_B}(v) + d_{T_B}(w) - 2T_D(vw)), \right. \\
 &\quad (d_{I_A}^-(v) + d_{I_A}^-(w) - 2\tilde{I}_C(vw), d_{I_B}(v) + d_{I_B}(w) - 2I_D(vw)), \\
 &\quad \left. (d_{F_A}^-(v) + td_{F_A}^-(w)q - h2\tilde{F}_C x(vw)7, Cd_{F_B};(v) + d_{F_B}(w) - 2F_D(vw)) \right)
 \end{aligned}$$

$$\begin{aligned}
 &\implies \left((d_{T_C}^-(uv), d_{T_D}(uv)), (d_{I_C}^-(uv), d_{I_D}(uv)), (d_{F_C}^-(uv), d_{F_D}(uv)) \right) \\
 &\neq \left((d_{T_C}^-(vw), d_{T_D}(vw)), (d_{I_C}^-(vw), d_{I_D}(vw)), (d_{F_C}^-(vw), d_{F_D}(vw)) \right) \implies d_G(uv) \neq d_G(vw).
 \end{aligned}
 \tag{28}$$

Therefore, each pair of AEs has VDs. Hence, G is a NEI-NCG. \square

Theorem 3. Let G be a CNCG on G^* and N be a CF. If G is a SI-NCG, then G is a NETI-NCG.

FIGURE 5: G is both NEI-NCG and NETI-NCG, but it is not SI-NCG.

Remark 2. Converse of Theorems 3 is not generally true.

Example 5. Let $G: (M, N)$ be a NCG so that $G^*: (V, E)$ is a path on four nodes where $V = \{u, v, w, x\}$ and $E = \{uv, vw, wx\}$ are defined as

$$M = \left\langle \begin{array}{l} \{u, ([0.2, 0.4], 0.5), ([0.1, 0.5], 0.3), ([0.4, 0.7], 0.4)\}, \\ \{v, ([0.4, 0.5], 0.6), ([0.3, 0.6], 0.7), ([0.5, 0.8], 0.3)\}, \\ \{w, ([0.3, 0.4], 0.4), ([0.2, 0.7], 0.6), ([0.1, 0.6], 0.2)\}, \\ \{x, ([0.2, 0.5], 0.3), ([0.3, 0.4], 0.1), ([0.6, 0.9], 0.5)\} \end{array} \right\rangle, \quad (29)$$

$$N = \left\langle \begin{array}{l} \{uv, ([0.2, 0.3], 0.4), ([0.1, 0.4], 0.5), ([0.5, 0.7], 0.2)\}, \\ \{vw, ([0.2, 0.3], 0.4), ([0.1, 0.4], 0.5), ([0.5, 0.7], 0.2)\}, \\ \{wx, ([0.2, 0.3], 0.4), ([0.1, 0.4], 0.5), ([0.5, 0.7], 0.2)\} \end{array} \right\rangle. \quad (30)$$

From Figure 5,

$$\begin{aligned} d_G(u) &= d_G(x) = (([0.2, 0.3], 0.4), ([0.1, 0.4], 0.5), \\ &\quad \cdot ([0.5, 0.7], 0.2)), \\ d_G(v) &= d_G(w) = (([0.4, 0.6], 0.8), ([0.2, 0.8], 1.0), \\ &\quad \cdot ([1.0, 1.4], 0.4)). \end{aligned} \quad (31)$$

Here, G is not a SI-NCG.

$$\begin{aligned} d_G(uv) &= d_G(wx) = (([0.2, 0.3], 0.4), ([0.1, 0.4], 0.5), \\ &\quad ([0.5, 0.7], 0.2)), \\ d_G(vw) &= (([0.4, 0.6], 0.8), ([0.2, 0.8], 1.0), ([1.0, 1.4], \\ &\quad 0.4)), \\ td_G(uv) &= td_G(wx) = ([0.4, 0.6], 0.8), ([0.2, 0.8], 1.0), \\ &\quad ([1.0, 1.4], 0.4)), \\ td_G(vw) &= (([0.6, 0.9], 1.2), ([0.3, 1.2], 1.5), ([1.5, 2.1], \\ &\quad 0.6)). \end{aligned}$$

It is noted that $d_G(uv) \neq d_G(vw)$ and $d_G(vw) \neq d_G(wx)$. Also, $td_G(uv) \neq td_G(vw)$ and $td_G(vw) \neq td_G(wx)$. Hence, G is both NEI-NCG and NETI-NCG. But G is not a SI-NCG.

Theorem 4. Let G be a CNCG and N be a CF. Then, G is a HI-NCG if and only if G is a NEI-NCG.

Proof. Let $G: (M, N)$ be a CNCG. Assume that $N = (C, D) = ((\tilde{T}_C, T_D), (\tilde{I}_C, I_D), (\tilde{F}_C, F_D))$ is a CF, and let $N(uv) = (\tilde{R}, S), \forall uv$ in E , in which $(\tilde{R}, S) = ((\tilde{R}_T, S_T), (\tilde{R}_F, S_F))$ is constant.

Let uv and vw be any two AEs in G . Then,

$$\begin{aligned}
 d_G(u) \neq d_G(w) &\iff \left(\left(d_{T_A}^-(u), d_{T_B}(u) \right), \left(d_{I_A}^-(u), d_{I_B}(u) \right), \left(d_{F_A}^-(u), d_{F_B}(u) \right) \right) \\
 &\neq \left(\left(d_{T_A}^-(w), d_{T_B}(w) \right), \left(d_{I_A}^-(w), d_{I_B}(w) \right), \left(d_{F_A}^-(w), d_{F_B}(w) \right) \right)
 \end{aligned} \tag{32}$$

$$\begin{aligned}
 &\iff \left(\left(d_{T_A}^-(u), d_{T_B}(u) \right), \left(d_{I_A}^-(u), d_{I_B}(u) \right), \left(d_{F_A}^-(u), d_{F_B}(u) \right) \right) \\
 &\quad + \left(\left(d_{T_A}^-(v), d_{T_B}(v) \right), \left(d_{I_A}^-(v), d_{I_B}(v) \right), \left(d_{F_A}^-(v), d_{F_B}(v) \right) \right) - 2((\tilde{R}_T, S_T), (\tilde{R}_I, S_I), (\tilde{R}_F, S_F)) \\
 &\neq \left(\left(d_{T_A}^-(v), d_{T_B}(v) \right), \left(d_{I_A}^-(v), d_{I_B}(v) \right), \left(d_{F_A}^-(v), d_{F_B}(v) \right) \right) \\
 &\quad + \left(\left(d_{T_A}^-(w), d_{T_B}(w) \right), \left(d_{I_A}^-(w), d_{I_B}(w) \right), \left(d_{F_A}^-(w), d_{F_B}(w) \right) \right) \\
 &\quad - 2((\tilde{R}_T, S_T), (\tilde{R}_I, S_I), (\tilde{R}_F, S_F))
 \end{aligned} \tag{33}$$

$$\begin{aligned}
 &\iff \left(\left(d_{T_A}^-(u) + d_{T_A}^-(v) - 2\tilde{R}_T, d_{T_B}(u) + d_{T_B}(v) - 2S_T \right), \left(d_{I_A}^-(u) + d_{I_A}^-(v) - 2\tilde{R}_I, d_{I_B}(u) + d_{I_B}(v) - 2S_I \right), \right. \\
 &\quad \left. \left(d_{F_A}^-(u) + d_{F_A}^-(v) - 2\tilde{R}_F, d_{F_B}(u) + d_{F_B}(v) - 2S_F \right) \right) \\
 &\neq \left(\left(d_{T_A}^-(v) + d_{T_A}^-(w) - 2\tilde{R}_T, d_{T_B}(v) + d_{T_B}(w) - 2S_T \right), \left(d_{I_A}^-(v) + d_{I_A}^-(w) - 2\tilde{R}_I, d_{I_B}(v) + d_{I_B}(w) - 2S_I \right), \right. \\
 &\quad \left. \left(d_{F_A}^-(v) + d_{F_A}^-(w) - 2\tilde{R}_F, d_{F_B}(v) + d_{F_B}(w) - 2S_F \right) \right)
 \end{aligned} \tag{34}$$

$$\begin{aligned}
 &\iff \left(\left(d_{T_A}^-(u) + d_{T_A}^-(v) - 2\tilde{T}_C(uv), d_{T_B}(u) + d_{T_B}(v) - 2T_D(uv) \right), \right. \\
 &\quad \left(d_{I_A}^-(u) + d_{I_A}^-(v) - 2\tilde{I}_C(uv), d_{I_B}(u) + d_{I_B}(v) - 2I_D(uv) \right), \\
 &\quad \left. \left(d_{F_A}^-(u) + td_{F_A}^-(v)q - h2\tilde{F}_C x(uv)7, Cd_{F_B};(u) + d_{F_B}(v) - 2F_D(uv) \right) \right) \\
 &\neq \left(\left(d_{T_A}^-(v) + d_{T_A}^-(w) - 2\tilde{T}_C(vw), d_{T_B}(v) + d_{T_B}(w) - 2T_D(vw) \right), \right. \\
 &\quad \left(d_{I_A}^-(v) + d_{I_A}^-(w) - 2\tilde{I}_C(vw), d_{I_B}(v) + d_{I_B}(w) - 2I_D(vw) \right), \\
 &\quad \left. \left(d_{F_A}^-(v) + td_{F_A}^-(w)q - h2\tilde{F}_C x(vw)7, Cd_{F_B};(v) + d_{F_B}(w) - 2F_D(vw) \right) \right)
 \end{aligned} \tag{35}$$

$$\begin{aligned}
 &\iff \left(\left(d_{T_C}^-(uv), d_{T_D}(uv) \right), \left(d_{I_C}^-(uv), d_{I_D}(uv) \right), \left(d_{F_C}^-(uv), d_{F_D}(uv) \right) \right) \\
 &\neq \left(\left(d_{T_C}^-(vw), d_{T_D}(vw) \right), \left(d_{I_C}^-(vw), d_{I_D}(vw) \right), \left(d_{F_C}^-(vw), d_{F_D}(vw) \right) \right) \\
 &\iff d_G(uv) \neq d_G(vw).
 \end{aligned} \tag{36}$$

Therefore, every pair of AEs has VDs, iff every node neighbor to the nodes has VDs. Hence, G is a HI-NCG, iff G is a NEI-NCG. \square

Theorem 5. Let G be a CNCG and N be a CF. Then, G is HI-NCG iff G is NETI-NCG.

Proof. It is clear. \square

Definition 11. Let G: (M, N) be a CNCG. Then, G is called to be a

- (i) SEI-NCG if each pair of edges has VDs (or no two edges have same degree).

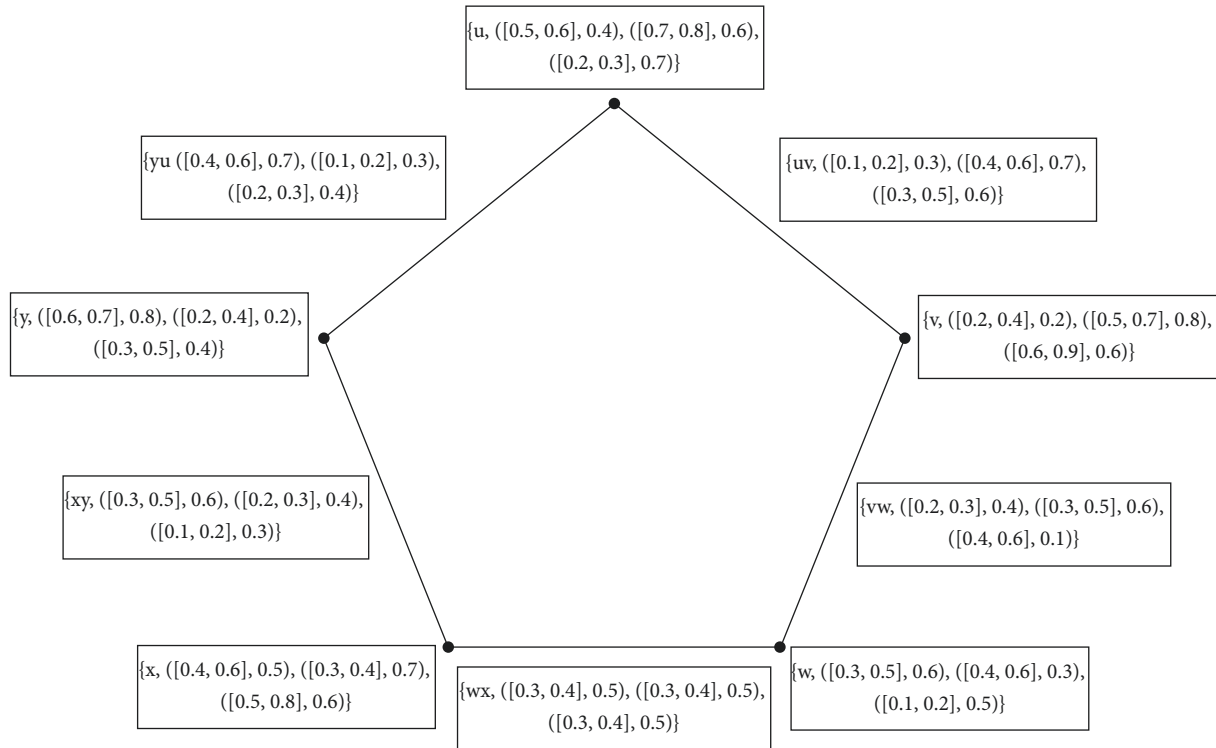


FIGURE 6: G is both SEI-NCG and SETI-NCG.

(ii) SETI-NCG if each pair of edges has various TDs (or no two edges have same TD).

Example 6. Consider a graph that is both SEI-NCG and SETI-NCG.

Let $G: (M, N)$ be a CNCG that is a cycle of length five where $V = \{u, v, w, x, y\}$ and $E = \{uv, vw, wx, xy, yu\}$ are defined as

$$M = \left\langle \begin{aligned} &\{u, ([0.5, 0.6], 0.4), ([0.7, 0.8], 0.6), ([0.2, 0.3], 0.7)\} \\ &\{v, ([0.2, 0.4], 0.2), ([0.5, 0.7], 0.8), ([0.6, 0.9], 0.6)\} \\ &\{w, ([0.3, 0.5], 0.6), ([0.4, 0.6], 0.3), ([0.1, 0.2], 0.5)\} \\ &\{x, ([0.4, 0.6], 0.5), ([0.3, 0.4], 0.7), ([0.5, 0.8], 0.6)\}, \\ &\{y, ([0.6, 0.7], 0.8), ([0.2, 0.4], 0.2), ([0.3, 0.5], 0.4)\}, \end{aligned} \right\rangle \tag{37}$$

$$N = \left\langle \begin{aligned} &\{uv, ([0.1, 0.2], 0.3), ([0.4, 0.6], 0.7), ([0.3, 0.5], 0.6)\} \\ &\{vw, ([0.2, 0.3], 0.4), ([0.3, 0.5], 0.6), ([0.4, 0.6], 0.1)\} \\ &\{wx, ([0.3, 0.4], 0.5), ([0.3, 0.4], 0.5), ([0.3, 0.4], 0.5)\} \\ &\{xy, ([0.3, 0.5], 0.6), ([0.2, 0.3], 0.4), ([0.1, 0.2], 0.3)\} \\ &\{yu, ([0.4, 0.6], 0.7), ([0.1, 0.2], 0.3), ([0.2, 0.3], 0.4)\} \end{aligned} \right\rangle \tag{38}$$

From Figure 6,

$$\begin{aligned} d_G(u) &= (([0.5, 0.3], 1.0), ([0.5, 0.8], 1.0), ([0.5, 0.8], 1.0)), \\ d_G(v) &= (([0.3, 0.5], 0.7), ([0.7, 1.1], 1.3), ([0.7, 1.1], 0.7)), \\ d_G(w) &= (([0.5, 0.7], 0.9), ([0.6, 0.9], 1.1), ([0.7, 1.0], 0.6)), \\ d_G(x) &= (([0.6, 0.9], 1.1), ([0.5, 0.7], 0.9), ([0.4, 0.6], 0.8)), \\ d_G(y) &= (([0.7, 1.1], 1.3), ([0.3, 0.5], 0.7), ([0.3, 0.5], 0.7)), \end{aligned} \tag{39}$$

$$\begin{aligned} d_G(uv) &= (([0.6, 0.9], 1.1), ([0.4, 0.7], 0.9), ([0.6, 0.9], 0.5)), \\ d_G(vw) &= (([0.4, 0.6], 0.8), ([0.7, 1.0], 1.2), ([0.6, 0.9], 1.1)), \\ d_G(wx) &= (([0.5, 0.8], 1.0), ([0.5, 0.8], 1.0), ([0.5, 0.8], 0.4)), \\ d_G(xy) &= (([0.7, 1.0], 1.2), ([0.4, 0.6], 0.8), ([0.5, 0.7], 0.9)), \\ d_G(yu) &= (([0.4, 0.7], 0.9), ([0.6, 0.9], 1.1), ([0.4, 0.7], 0.9)). \end{aligned} \tag{40}$$

So, G is a SEI-NCG.

$$\begin{aligned} td_G(uv) &= (([0.7, 1.1], 1.4), ([0.8, 1.3], 1.6), ([0.9, 1.4], 1.1)), \\ td_G(vw) &= (([0.6, 0.9], 1.2), ([1.0, 1.5], 1.8), ([1.0, 1.5], 1.2)), \\ td_G(wx) &= (([0.8, 1.2], 1.5), ([0.8, 1.2], 1.5), ([0.8, 1.2], 0.9)), \\ td_G(xy) &= (([1.0, 1.5], 1.8), ([0.6, 0.9], 1.2), ([0.6, 0.9], 1.2)), \\ td_G(yu) &= (([0.8, 1.3], 1.6), ([0.7, 1.1], 1.4), ([0.6, 1.0], 1.3)). \end{aligned} \tag{41}$$

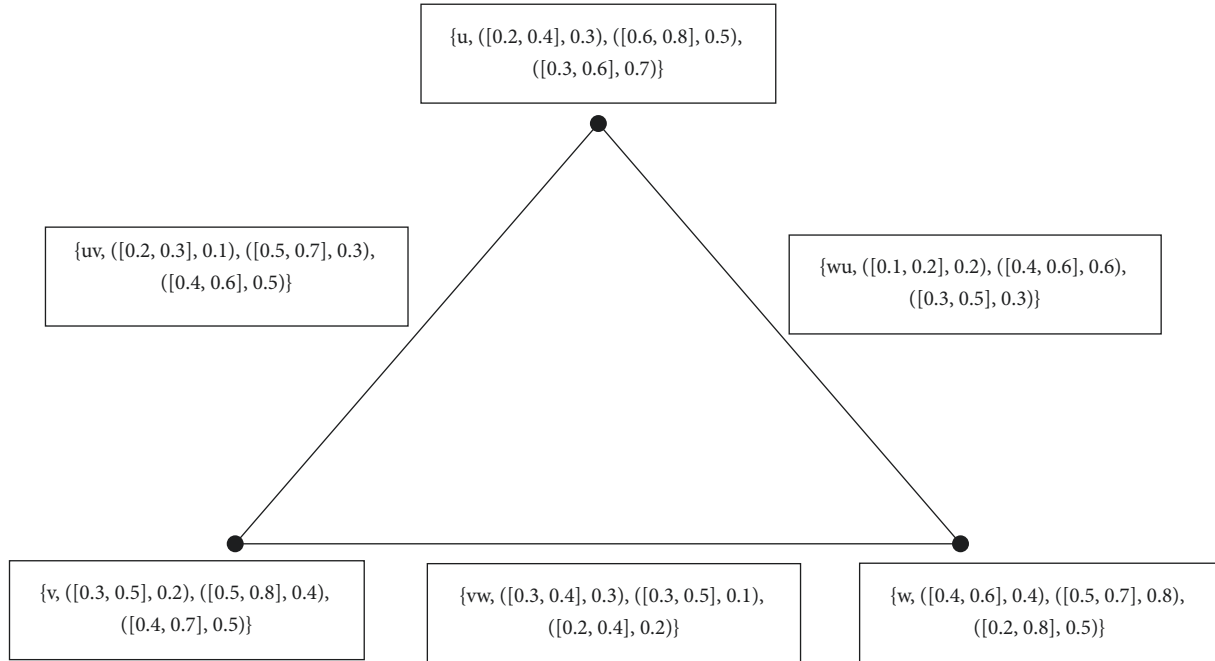


FIGURE 7: G is SEI-NCG, but it is not SETI-NCG.

Thus, G is a SETI-NCG.

Therefore, G is both SEI-NCG and SETI-NCG.

$$td_G(uv) = td_G(wx) = td_G(wu) = (([0.6, 0.9], 0.6), ([1.2, 1.8], 1.0), ([0.9, 1.5], 1.0)). \tag{46}$$

Example 7. SEI-NCG need not be SETI-NCG.

Let $G: (M, N)$ be a NCG on G^* : such that (V, E) is a cycle of length three, $V = \{u, v, w\}$, and $E = \{uv, vw, wu\}$. We define M and N as follows:

$$M = \left\langle \begin{array}{l} \{u, ([0.2, 0.4], 0.3), ([0.6, 0.8], 0.5), ([0.3, 0.6], 0.7)\}, \\ \{v, ([0.3, 0.5], 0.2), ([0.5, 0.8], 0.4), ([0.4, 0.7], 0.5)\}, \\ \{w, ([0.4, 0.6], 0.4), ([0.5, 0.7], 0.8), ([0.2, 0.8], 0.5)\}. \end{array} \right\rangle, \tag{42}$$

$$N = \left\langle \begin{array}{l} \{uv, ([0.2, 0.3], 0.1), ([0.5, 0.7], 0.3), ([0.4, 0.6], 0.5)\}, \\ \{vw, ([0.3, 0.4], 0.3), ([0.3, 0.5], 0.1), ([0.2, 0.4], 0.2)\}, \\ \{wu, ([0.1, 0.2], 0.2), ([0.4, 0.6], 0.6), ([0.3, 0.5], 0.3)\}. \end{array} \right\rangle. \tag{43}$$

From Figure 7,

$$\begin{aligned} d_G(u) &= (([0.3, 0.5], 0.3), ([0.9, 1.3], 0.9), ([0.7, 1.1], 0.8)), \\ d_G(v) &= (([0.5, 0.7], 0.4), ([0.8, 1.2], 0.4), ([0.6, 1.0], 0.7)), \\ d_G(w) &= (([0.4, 0.6], 0.5), ([0.7, 1.1], 0.7), ([0.5, 0.9], 0.5)), \end{aligned} \tag{44}$$

$$\begin{aligned} d_G(uv) &= (([0.4, 0.6], 0.5), ([0.7, 1.1], 0.7), ([0.5, 0.9], 0.5)), \\ d_G(vw) &= (([0.3, 0.5], 0.3), ([0.9, 1.3], 0.9), ([0.7, 1.1], 0.8)), \\ d_G(wu) &= (([0.5, 0.7], 0.4), ([0.8, 1.2], 0.4), ([0.6, 1.0], 0.7)), \end{aligned} \tag{45}$$

Note that G is SEI-NCG, since each pair of edges has VDs. Also, G is not SETI-NCG, since all the edges have same TD. Hence, SEI-NCG need not be SETI-NCG.

Example 8. SETI-NCG need not be SEI-NCG.

Consider $G: (M, N)$ be a NCG so that $G^*: (V, E)$, a cycle of length four where $V = \{u, v, w, x\}$ and $E = \{uv, vw, wx, xu\}$ defined as

$$M = \left\langle \begin{array}{l} \{u, ([0.6, 0.9], 0.8), ([0.4, 0.7], 0.6), ([0.6, 0.8], 0.5)\}, \\ \{v, ([0.4, 0.6], 0.4), ([0.4, 0.5], 0.3), ([0.3, 0.7], 0.9)\}, \\ \{w, ([0.5, 0.7], 0.6), ([0.4, 0.7], 0.4), ([0.6, 0.8], 0.5)\}, \\ \{x, ([0.7, 0.8], 0.7), ([0.2, 0.5], 0.2), ([0.3, 0.7], 0.9)\} \end{array} \right\rangle, \tag{47}$$

$$N = \left\langle \begin{array}{l} \{uv, ([0.1, 0.3], 0.2), ([0.3, 0.4], 0.5), ([0.4, 0.6], 0.3)\}, \\ \{vw, ([0.3, 0.5], 0.4), ([0.4, 0.5], 0.2), ([0.5, 0.7], 0.5)\}, \\ \{wx, ([0.5, 0.7], 0.3), ([0.2, 0.3], 0.2), ([0.4, 0.5], 0.1)\}, \\ \{xu, ([0.6, 0.8], 0.7), ([0.1, 0.3], 0.3), ([0.2, 0.4], 0.1)\} \end{array} \right\rangle. \tag{48}$$

From Figure 8,

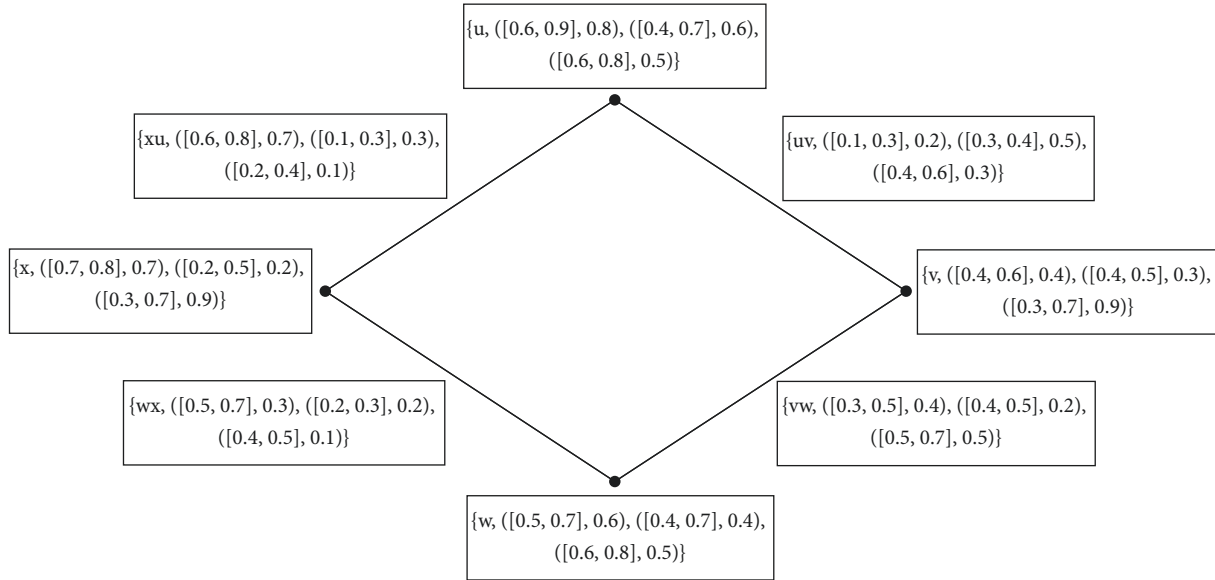


FIGURE 8: G is SETI-NCG but it is not SEI-NCG.

$$\begin{aligned}
 d_G(u) &= (([0.7, 1.1], 0.9), ([0.4, 0.7], 0.8), ([0.6, 1.0], 0.4)), \\
 d_G(v) &= (([0.4, 0.8], 0.6), ([0.7, 0.9], 0.7), ([0.9, 1.3], 0.8)), \\
 d_G(w) &= (([0.8, 1.2], 0.7), ([0.6, 0.8], 0.4), ([0.9, 1.2], 0.6)), \\
 d_G(x) &= (([1.1, 1.5], 1.0), ([0.3, 0.6], 0.5), ([0.6, 0.9], 0.2)),
 \end{aligned}
 \tag{49}$$

$$\begin{aligned}
 d_G(uv) &= d_G(wx) = (([0.9, 1.3], 1.1), ([0.5, 0.8], 0.5), \\
 &\quad \cdot ([0.7, 1.1], 0.6)), \\
 d_G(vw) &= d_G(xu) = (([0.6, 1.0], 0.5), ([0.5, 0.7], 0.7), \\
 &\quad \cdot ([0.8, 1.1], 0.4)),
 \end{aligned}
 \tag{50}$$

$$\begin{aligned}
 td_G(uv) &= (([1.0, 1.6], 1.3), ([0.8, 1.0], 0.9), ([1.1, 1.7], 0.9)), \\
 td_G(vw) &= (([0.9, 1.5], 0.9), ([0.9, 1.2], 0.9), ([1.3, 1.8], 0.9)), \\
 td_G(wx) &= (([1.4, 2.0], 1.4), ([0.7, 1.1], 0.7), ([1.1, 1.6], 0.7)), \\
 d_G(xu) &= (([1.2, 1.8], 1.2), ([0.6, 1.0], 1.0), ([1.0, 1.5], 0.5)).
 \end{aligned}
 \tag{51}$$

Obviously, $d_G(uv) = d_G(wx)$. Hence, G is not SEI-NCG. But G is SETI-NCG, since $td_G(uv) \neq td_G(vw) \neq td_G(wx) \neq td_G(xu)$. Hence, SETI-NCG need not be SEI-NCG.

Theorem 6. Let G be a CNCG on G^* and N be a CF. Then, G is a SEI-NCG, iff G is a SETI-NCG.

Proof. Assume $N = (C, D) = ((\tilde{T}_C, T_D), (\tilde{I}_C, I_D), (\tilde{F}_C, F_D))$ is a CF. Let $N(uv) = (\tilde{R}, S)$, for all uv in E , in which $(\tilde{R}, S) = ((\tilde{R}_T, S_T), (\tilde{R}_I, S_I), (\tilde{R}_F, S_F))$ is constant.

Let uv and xy be any pair of edges in E . Then,

$$\begin{aligned}
 d_G(uv) \neq d_G(xy) &\iff d_G(uv) + \tilde{R} \\
 &\iff \left(\left(d_{\tilde{T}_C}^-(uv), d_{T_D}(uv) \right), \left(d_{\tilde{I}_C}^-(uv), d_{I_D}(uv) \right), \right. \\
 &\quad \left. \left(d_{\tilde{F}_C}^-(uv), d_{F_D}(uv) \right) \right) \\
 &\quad + \left((\tilde{R}_T, S_T), (\tilde{R}_I, S_I), (\tilde{R}_F, S_F) \right) \\
 &\neq \left(\left(d_{\tilde{T}_C}^-(xy), d_{T_D}(xy) \right), \left(d_{\tilde{I}_C}^-(xy), d_{I_D}(xy) \right), \right. \\
 &\quad \left. \left(d_{\tilde{F}_C}^-(xy), d_{F_D}(xy) \right) \right) \\
 &\iff \left(\left(d_{\tilde{T}_C}^-(uv) + \tilde{R}_T, d_{T_D}(uv) + S_T \right), \right. \\
 &\quad \left(d_{\tilde{I}_C}^-(uv) + \tilde{R}_I, d_{I_D}(uv) + S_I \right), \right. \\
 &\quad \left. \left(d_{\tilde{F}_C}^-(uv) + \tilde{R}_F, d_{F_D}(uv) + S_F \right) \right) \\
 &\neq \left(\left(d_{\tilde{T}_C}^-(xy) + \tilde{R}_T, d_{T_D}(xy) + S_T \right), \right. \\
 &\quad \left(d_{\tilde{I}_C}^-(xy) + \tilde{R}_I, d_{I_D}(xy) + S_I \right), \right. \\
 &\quad \left. \left(d_{\tilde{F}_C}^-(xy) + \tilde{R}_F, d_{F_D}(xy) + S_F \right) \right)
 \end{aligned}
 \tag{52}$$

(53)

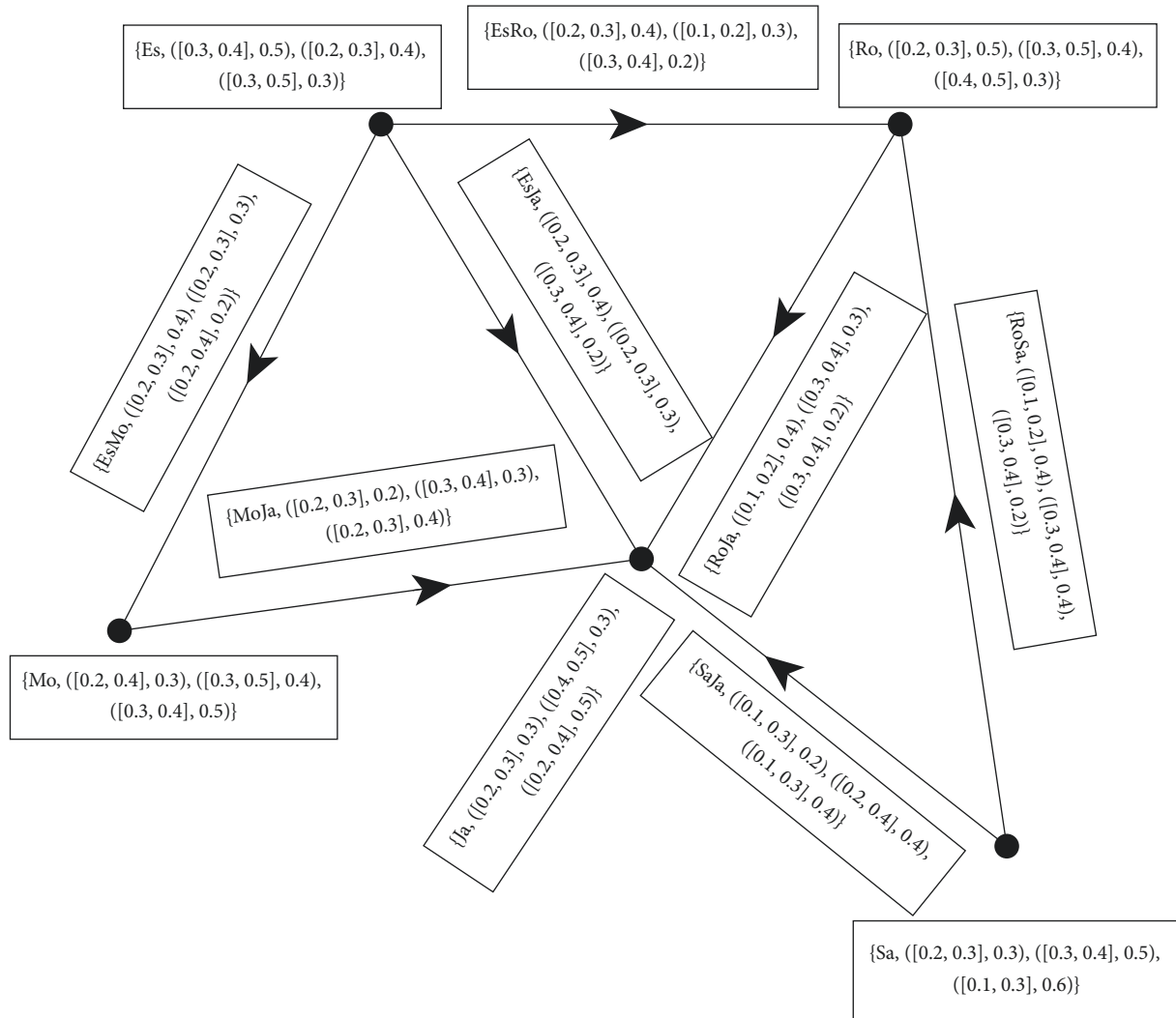


FIGURE 9: NC-digraph (influence graph).

$$\begin{aligned}
 &\Leftrightarrow \left(\left(d_{T_C}^-(uv) + \bar{T}_C, d_{T_D}(uv) + T_D \right), \right. \\
 &\quad \left(d_{I_C}^-(uv) + \bar{I}_C, d_{I_D}(uv) + I_D \right), \\
 &\quad \left. \left(d_{F_C}^-(uv) + \bar{F}_C, d_{F_D}(uv) + F_D \right) \right) \\
 &\neq \left(\left(d_{T_C}^-(xy) + \bar{T}_C, d_{T_D}(xy) + T_D \right), \right. \\
 &\quad \left(d_{I_C}^-(xy) + \bar{I}_C, d_{I_D}(xy) + I_D \right), \\
 &\quad \left. \left(d_{F_C}^-(xy) + \bar{F}_C, d_{F_D}(xy) + F_D \right) \right) \quad (54)
 \end{aligned}$$

$$\begin{aligned}
 &\Leftrightarrow \left(\left(td_{T_C}^-(uv), td_{T_D}(uv) \right), \left(td_{I_C}^-(uv), td_{I_D}(uv) \right), \right. \\
 &\quad \left. \left(td_{F_C}^-(uv), td_{F_D}(uv) \right) \right) \\
 &\neq \left(\left(td_{T_C}^-(xy), td_{T_D}(xy) \right), \left(td_{I_C}^-(xy), td_{I_D}(xy) \right), \right. \\
 &\quad \left. \left(td_{F_C}^-(xy), td_{F_D}(xy) \right) \right) \quad (55) \\
 &\Leftrightarrow td_G(uv) \neq td_G(xy).
 \end{aligned}$$

So, each edge has different degree if and only if it has different total degrees. Hence, G is SEI-NCG iff G is a SETI-NCG. \square

Remark 3. Let G be a CNCG. If G is both SEI-NCG and SETI-NCG, then N need not be a CF.

Example 9. Let $G: (M, N)$ be a NCG so that $G^*: (V, E)$ is graph for Example 6 (Figure 9). As seen in that example, each pair of edges in G has VDs. Hence, G is a SEI-NCG.

Also, note that each pair of edges in G has various TDs. Hence, G is a SETI-NCG. Therefore, G is both SEI-NCG and SETI-NCG. But N is not a CF.

Theorem 7. Let G be a NCG on G^* . If G is a SEI-NCG, then G is a NEI-NCG.

Proof. Let $G: (M, N)$ be a NCG. Assume that G is a SEI-NCG. Then, each pair of edges in G has VDs. So, each pair of AEs has VDs. Hence, G is a NEI-NCG. \square

Theorem 8. Let G be a NCG. If G is a SETI-NCG, then, G is a NETI-NCG.

Proof. Let $G: (M, N)$ be a NCG. Suppose that G is a SETI-NCG; then, each pair of edges in G has various TDs. So, each pair of AEs has various TDs. Hence, G is a NETI-NCG. \square

Remark 4. The inverse of Theorems 7 and 8 is not generally true.

Example 10. Consider $G: (M, N)$ be a NCG so that $G^*: (V, E)$ is graph for Example 4 (Figure 4). As seen in that example, $d_G(uv) \neq d_G(vw)$ and $d_G(vw) \neq d_G(wx)$. Hence, G is a NEI-NCG. But G is not a SEI-NCG, since

$d_G(uv) \neq d_G(wx)$. Also, note that $td_G(uv) \neq td_G(vw)$ and $td_G(vw) \neq td_G(wx)$. Hence, G is a NETI-NCG. But G is not a SETI-NCG, since $td_G(uv) \neq td_G(wx)$.

Theorem 9. Let G be a CNCG and N be a CF. If G is a SEI-NCG, then G is an irregular NCG.

Proof. Let $G: (A, B)$ be a CNCG. Assume that $N = (C, D) = ((\tilde{T}_C, T_D), (\tilde{I}_C, I_D), (\tilde{F}_C, F_D))$ is a CF, and let $N(uv) = (\tilde{R}, S)$, for all uv in E , in which $(\tilde{R}, S) = ((\tilde{R}_T, S_T), (\tilde{R}_I, S_I), (\tilde{R}_F, S_F))$ is constant.

Suppose G is a SEI-NCG. Then, each edge has VD. Let uv and vw be AEs in G having VDs, and hence

$$d_G(uv) \neq d_G(vw) \implies \left((d_{T_C}^-(uv), d_{T_D}^-(uv)), (d_{I_C}^-(uv), d_{I_D}^-(uv)), (d_{F_C}^-(uv), d_{F_D}^-(uv)) \right) \neq \left((d_{T_C}^-(vw), d_{T_D}^-(vw)), (d_{I_C}^-(vw), d_{I_D}^-(vw)), (d_{F_C}^-(vw), d_{F_D}^-(vw)) \right) \quad (56)$$

$$\implies \left((d_{T_A}^-(u) + d_{T_A}^-(v) - 2\tilde{T}_C(uv), d_{T_B}^-(u) + d_{T_B}^-(v) - 2T_D(uv)), (d_{I_A}^-(u) + d_{I_A}^-(v) - 2\tilde{I}_C(uv), d_{I_B}^-(u) + d_{I_B}^-(v) - 2I_D(uv)), (d_{F_A}^-(u) + d_{F_A}^-(v) - 2\tilde{F}_C(uv), d_{F_B}^-(u) + d_{F_B}^-(v) - 2F_D(uv)) \right) \neq \left((d_{T_A}^-(v) + d_{T_A}^-(w) - 2\tilde{T}_C(vw), d_{T_B}^-(v) + d_{T_B}^-(w) - 2T_D(vw)), (d_{I_A}^-(v) + d_{I_A}^-(w) - 2\tilde{I}_C(vw), d_{I_B}^-(v) + d_{I_B}^-(w) - 2I_D(vw)), (d_{F_A}^-(v) + d_{F_A}^-(w) - 2\tilde{F}_C(vw), d_{F_B}^-(v) + d_{F_B}^-(w) - 2F_D(vw)) \right) \quad (57)$$

$$\implies \left((d_{T_A}^-(u) + d_{T_A}^-(v) - 2\tilde{R}_T, d_{T_B}^-(u) + d_{T_B}^-(v) - 2S_T), t(d_{I_A}^-(u) + d_{I_A}^-(v) - 2\tilde{R}_I, d_{I_B}^-(u) + d_{I_B}^-(v) - 2S_I), n, q(d_{F_A}^-(u) + d_{F_A}^-(v) - 2\tilde{R}_F, d_{F_B}^-(u) + d_{F_B}^-(v) - 2S_F) \right) \neq \left((d_{T_A}^-(v) + d_{T_A}^-(w) - 2\tilde{R}_T, d_{T_B}^-(v) + d_{T_B}^-(w) - 2S_T), (d_{I_A}^-(v) + d_{I_A}^-(w) - 2\tilde{R}_I, d_{I_B}^-(v) + d_{I_B}^-(w) - 2S_I), (d_{F_A}^-(v) + d_{F_A}^-(w) - 2\tilde{R}_F, d_{F_B}^-(v) + d_{F_B}^-(w) - 2S_F) \right) \quad (58)$$

$$\implies \left((d_{T_A}^-(u) + d_{T_A}^-(v), d_{T_B}^-(u) + d_{T_B}^-(v)), (d_{I_A}^-(u) + d_{I_A}^-(v), d_{I_B}^-(u) + d_{I_B}^-(v)), (d_{F_A}^-(u) + d_{F_A}^-(v), d_{F_B}^-(u) + d_{F_B}^-(v)) \right) - 2((\tilde{R}_T, S_T), (\tilde{R}_I, S_I), (\tilde{R}_F, S_F)) \neq \left((d_{T_A}^-(v) + d_{T_A}^-(w), d_{T_B}^-(v) + d_{T_B}^-(w)), (d_{I_A}^-(v) + d_{I_A}^-(w), d_{I_B}^-(v) + d_{I_B}^-(w)), (d_{F_A}^-(v) + d_{F_A}^-(w), d_{F_B}^-(v) + d_{F_B}^-(w)) \right) - 2((\tilde{R}_T, S_T), (\tilde{R}_I, S_I), (\tilde{R}_F, S_F)) \quad (59)$$

$$\begin{aligned}
&\implies \left(\left(d_{T_A}^-(u) + d_{T_A}^-(v), d_{T_B}(u) + d_{T_B}(v) \right), \left(d_{I_A}^-(u) + d_{I_A}^-(v), d_{I_B}(u) + d_{I_B}(v) \right), \left(d_{F_A}^-(u) + d_{F_A}^-(v), d_{F_B}(u) + d_{F_B}(v) \right) \right) \\
&\neq \left(\left(d_{T_A}^-(v) + d_{T_A}^-(w), d_{T_B}(v) + d_{T_B}(w) \right), \left(d_{I_A}^-(v) + d_{I_A}^-(w), d_{I_B}(v) + d_{I_B}(w) \right), \left(d_{F_A}^-(v) + d_{F_A}^-(w), d_{F_B}(v) + d_{F_B}(w) \right) \right) \\
&\implies \left(\left(d_{T_A}^-(u), d_{T_B}(u) \right), \left(d_{I_A}^-(u), d_{I_B}(u) \right), \left(d_{F_A}^-(u), d_{F_B}(u) \right) \right) + \left(\left(d_{T_A}^-(v), d_{T_B}(v) \right), \left(d_{I_A}^-(v), d_{I_B}(v) \right), \left(d_{F_A}^-(v), d_{F_B}(v) \right) \right) \\
&\neq \left(\left(d_{T_A}^-(v), d_{T_B}(v) \right), \left(d_{I_A}^-(v), d_{I_B}(v) \right), \left(d_{F_A}^-(v), d_{F_B}(v) \right) \right) \\
&\quad + \left(\left(d_{T_A}^-(w), d_{T_B}(w) \right), \left(d_{I_A}^-(w), d_{I_B}(w) \right), \left(d_{F_A}^-(w), d_{F_B}(w) \right) \right) \\
&\implies d_G(u) + d_G(v) \neq d_G(v) + d_G(w) \implies d_G(u) \neq d_G(w).
\end{aligned} \tag{60}$$

So, there exists a node v which is neighbor to nodes u and w having VDs. Hence, G is an irregular NCG. \square

Theorem 10. Let G be a CNCG and N be a CF. If G is a SETI-NCG, then G is an irregular NCG.

Proof. Proof is similar to Theorem 9. \square

Remark 5. The inverse of Theorems 9 and 10 is not generally true.

Example 11. Let $G: (M, N)$ be a NCG so that $G^*: (V, E)$ is graph for Example 5 (Figure 5). As seen in that example, G is an irregular NCG. Also, it is noted that $d_G(uv) = d_G(wx)$.

Hence, G is not a SEI-NCG. Also, $td_G(uv) = td_G(wx)$. Hence, G is not a SETI-NCG.

Theorem 11. Let G be a CNCG and N be a CF. If G is a SEI-NCG, then G is a HI-NCG.

Proof. Let $G: (M, N)$ be a CNCG. Assume that $N = (C, D) = ((\tilde{T}_C, T_D), (\tilde{I}_C, I_D), (\tilde{F}_C, F_D))$ is a CF. Let $N(uv) = (\tilde{R}, S)$, $\forall uv$ in E , in which $(\tilde{R}, S) = ((\tilde{R}_T, S_T), (\tilde{R}_I, S_I), (\tilde{R}_F, S_F))$ is constant.

Let v be any node neighbor with u, w , and x . Then, uv, vw , and vx are AEs in G . Let G be a SEI-NCG. Then, each pair of edges in G has VDs. So, each pair of AEs in G has VDs. Hence,

$$\begin{aligned}
d_G(uv) \neq d_G(vw) \neq d_G(vx) &\implies \left(\left(d_{T_C}^-(uv), d_{T_D}(uv) \right), \left(d_{I_C}^-(uv), d_{I_D}(uv) \right), \left(d_{F_C}^-(uv), d_{F_D}(uv) \right) \right) \\
&\neq \left(\left(d_{T_C}^-(vw), d_{T_D}(vw) \right), \left(d_{I_C}^-(vw), d_{I_D}(vw) \right), \left(d_{F_C}^-(vw), d_{F_D}(vw) \right) \right) \\
&\neq \left(\left(d_{T_C}^-(vx), d_{T_D}(vx) \right), \left(d_{I_C}^-(vx), d_{I_D}(vx) \right), \left(d_{F_C}^-(vx), d_{F_D}(vx) \right) \right) \\
&\implies \left(\left(d_{T_A}^-(u) + d_{T_A}^-(v) - 2\tilde{T}_C(uv), d_{T_B}(u) + d_{T_B}(v) - 2T_D(uv) \right), \right. \\
&\quad \left(d_{I_A}^-(u) + d_{I_A}^-(v) - 2\tilde{I}_C(uv), d_{I_B}(u) + d_{I_B}(v) - 2I_D(uv) \right), \\
&\quad \left. \left(d_{F_A}^-(u) + d_{F_A}^-(v) - 2\tilde{F}_C(uv), d_{F_B}(u) + d_{F_B}(v) - 2F_D(uv) \right) \right) \\
&\neq \left(\left(d_{T_A}^-(v) + d_{T_A}^-(w) - 2\tilde{T}_C(vw), d_{T_B}(v) + d_{T_B}(w) - 2T_D(vw) \right), \right. \\
&\quad \left(d_{I_A}^-(v) + d_{I_A}^-(w) - 2\tilde{I}_C(vw), d_{I_B}(v) + d_{I_B}(w) - 2I_D(vw) \right), \\
&\quad \left. \left(d_{F_A}^-(v) + d_{F_A}^-(w) - 2\tilde{F}_C(vw), d_{F_B}(v) + d_{F_B}(w) - 2F_D(vw) \right) \right) \\
&\neq \left(\left(d_{T_A}^-(v) + d_{T_A}^-(x) - 2\tilde{T}_C(vx), d_{T_B}(v) + d_{T_B}(x) - 2T_D(vx) \right), \right. \\
&\quad \left. \left(d_{I_A}^-(v) + d_{I_A}^-(x) - 2\tilde{I}_C(vx), d_{I_B}(v) + d_{I_B}(x) - 2I_D(vx) \right), \right.
\end{aligned} \tag{61}$$

$$\begin{aligned}
&\quad \left. \left(d_{F_A}^-(v) + d_{F_A}^-(x) - 2\tilde{F}_C(vx), d_{F_B}(v) + d_{F_B}(x) - 2F_D(vx) \right) \right)
\end{aligned} \tag{62}$$

$$\begin{aligned}
 & \left(d_{F_A}^-(v) + d_{F_A}^-(x) - 2\tilde{F}_C(vx), d_{F_B}^-(v) + d_{F_B}^-(x) - 2F_D(vx) \right) \\
 \implies & \left(\left(d_{T_A}^-(u) + d_{T_A}^-(v) - 2\tilde{R}_T, d_{T_B}^-(u) + d_{T_B}^-(v) - 2S_T \right), \left(d_{I_A}^-(u) + d_{I_A}^-(v) - 2\tilde{R}_I, d_{I_B}^-(u) + d_{I_B}^-(v) - 2S_I \right), \right. \\
 & \left. \left(d_{F_A}^-(u) + d_{F_A}^-(v) - 2\tilde{R}_F, d_{F_B}^-(u) + d_{F_B}^-(v) - 2S_F \right) \right) \\
 \neq & \left(\left(d_{T_A}^-(v) + d_{T_A}^-(w) - 2\tilde{R}_T, d_{T_B}^-(v) + d_{T_B}^-(w) - 2S_T \right), \left(d_{I_A}^-(v) + d_{I_A}^-(w) - 2\tilde{R}_I, d_{I_B}^-(v) + d_{I_B}^-(w) - 2S_I \right), \right. \\
 & \left. \left(d_{F_A}^-(v) + d_{F_A}^-(w) - 2\tilde{R}_F, d_{F_B}^-(v) + d_{F_B}^-(w) - 2S_F \right) \right) \\
 \neq & \left(\left(d_{T_A}^-(v) + d_{T_A}^-(x) - 2\tilde{R}_T, d_{T_B}^-(v) + d_{T_B}^-(x) - 2S_T \right), \left(d_{I_A}^-(v) + d_{I_A}^-(x) - 2\tilde{R}_I, d_{I_B}^-(v) + d_{I_B}^-(x) - 2S_I \right), \right. \\
 & \left. \left(d_{F_A}^-(v) + d_{F_A}^-(x) - 2\tilde{R}_F, d_{F_B}^-(v) + d_{F_B}^-(x) - 2S_F \right) \right)
 \end{aligned} \tag{63}$$

$$\begin{aligned}
 \implies & \left(d_{T_A}^-(u) + d_{T_A}^-(v), d_{T_B}^-(u) + d_{T_B}^-(v), \left(d_{I_A}^-(u) + d_{I_A}^-(v), d_{I_B}^-(u) + d_{I_B}^-(v) \right), \left(d_{F_A}^-(u + d_{F_A}^-(v), d_{F_B}^-(u) + d_{F_B}^-(v)) \right) \right) \\
 \neq & \left(\left(d_{T_A}^-(v) + d_{T_A}^-(w), d_{T_B}^-(v) + d_{T_B}^-(w) \right), \left(d_{I_A}^-(v) + d_{I_A}^-(w), d_{I_B}^-(v) + d_{I_B}^-(w) \right), \left(d_{F_A}^-(v) + d_{F_A}^-(w), d_{F_B}^-(v) + d_{F_B}^-(w) \right) \right) \\
 \neq & \left(\left(d_{T_A}^-(v) + d_{T_A}^-(x), d_{T_B}^-(v) + d_{T_B}^-(x) \right), \left(d_{I_A}^-(v) + d_{I_A}^-(x), d_{I_B}^-(v) + d_{I_B}^-(x) \right), \left(d_{F_A}^-(v) + d_{F_A}^-(x), d_{F_B}^-(v) + d_{F_B}^-(x) \right) \right)
 \end{aligned} \tag{64}$$

$$\begin{aligned}
 \implies & \left(\left(d_{T_A}^-(u), d_{T_B}^-(u) \right), \left(d_{I_A}^-(u), d_{I_B}^-(u) \right), \left(d_{F_A}^-(u), d_{F_B}^-(u) \right) \right) \\
 & + \left(\left(d_{T_A}^-(v), d_{T_B}^-(v) \right), \left(d_{I_A}^-(v), d_{I_B}^-(v) \right), \left(d_{F_A}^-(v), d_{F_B}^-(v) \right) \right) \\
 \neq & \left(\left(d_{T_A}^-(v), d_{T_B}^-(v) \right), \left(d_{I_A}^-(v), d_{I_B}^-(v) \right), \left(d_{F_A}^-(v), d_{F_B}^-(v) \right) \right) \\
 & + \left(\left(d_{T_A}^-(w), d_{T_B}^-(w) \right), \left(d_{I_A}^-(w), d_{I_B}^-(w) \right), \left(d_{F_A}^-(w), d_{F_B}^-(w) \right) \right) \\
 \neq & \left(\left(d_{T_A}^-(v), d_{T_B}^-(v) \right), \left(d_{I_A}^-(v), d_{I_B}^-(v) \right), \left(d_{F_A}^-(v), d_{F_B}^-(v) \right) \right) \\
 & + \left(\left(d_{T_A}^-(x), d_{T_B}^-(x) \right), \left(d_{I_A}^-(x), d_{I_B}^-(x) \right), \left(d_{F_A}^-(x), d_{F_B}^-(x) \right) \right)
 \end{aligned} \tag{65}$$

$$\begin{aligned}
 \implies & \left(\left(d_{T_A}^-(u), d_{T_B}^-(u) \right), \left(d_{I_A}^-(u), d_{I_B}^-(u) \right), \left(d_{F_A}^-(u), d_{F_B}^-(u) \right) \right) \\
 \neq & \left(\left(d_{T_A}^-(w), d_{T_B}^-(w) \right), \left(d_{I_A}^-(w), d_{I_B}^-(w) \right), \left(d_{F_A}^-(w), d_{F_B}^-(w) \right) \right) \\
 \neq & \left(\left(d_{T_A}^-(x), d_{T_B}^-(x) \right), \left(d_{I_A}^-(x), d_{I_B}^-(x) \right), \left(d_{F_A}^-(x), d_{F_B}^-(x) \right) \right) \implies d_G(u) \neq d_G(w) \neq d_G(x).
 \end{aligned} \tag{66}$$

Therefore, the node v is neighbor to the nodes with VDs. Hence, G is a HI-NCG. \square

Theorem 12. Let G be a CNCG and N be a CF. If G is a SETI-NCG, then G is a HI-NCG.

Proof. Proof is similar to Theorem 11. \square

Remark 6. The inverse of Theorems 11 and 12 is not generally true.

Example 12. Let $G: (M, N)$ be a NCG so that $G^*: (V, E)$ is graph for Example 5 (Figure 5). As seen in that example, G is a HI-NCG. Note that $d_G(uv) = d_G(wx)$. So, G is not a SEI-NCG. Also, $td_G(uv) = td_G(wx)$. Thus, G is not a SETI-NCG.

4. Application of Neutrosophic Cubic-Influence Digraph to Find the Most Effective Person in a School

School is one of the most important places for education and training of students. The approved goals of the study courses

TABLE 2: The names of the staff and their specialization in the school.

Name	Services
Eskandari (Es)	Headmaster
Momeni (Mo)	School deputy
Rouhi (Ro)	Educational instructor
Salimi (Sa)	Representative of the teacher’s council
Jafari (Ja)	Representative of the parents and teachers association

are established and managed in accordance with the rules and instructions of the Ministry of Education. At school, a student interacts with his/her classmates and tries to learn the necessary scientific points. The physical, psychological, and educational environment of the school is one of the issues that can have an important and significant reflection on the structure of mental and intellectual growth and development, as well as creativity and mental health of students. The transfer of the basic values of the society is the main focus of the educational system, in such a way that the school commits the students to internalize the values of the society. In schools, values are taught in a variety of subjects, and the effectiveness of teaching values in each subject depends on the teacher’s understanding of the objectives of the subject. By recognizing the possibilities, the teacher equips the educational environment and by recognizing the interests and abilities of the students guides them in the right direction of learning because success in schools requires teachers to accept the opinions of others. Therefore, considering the importance of schools in shaping the personality and behavior of students, we have tried to determine the most effective person in a school according to its performance. To do this, we consider the nodes of this influence graph as the staff of a school and the edges as the influence of one employee on another employee. The names of the staff and their specialization in the school are shown in Table 2. For this school, the staff is as follows:

$$A = \{\text{Eskandari, Momeni, Rouhi, Salimi, Jafari}\}. \quad (67)$$

- (i) Momeni has been working with Salimi for 16 years and values his views on issues.
- (ii) Eskandari has been the head of the school, and not only Salimi but also Jafari is very satisfied with Eskandari’s performance.
- (iii) Taking care of the educational and moral affairs of students is one of the most important issues. Rouhi is an expert for this.
- (iv) Rouhi and Jafari have a long history of conflict.
- (v) Jafari is a very effective person in communicating with students’ parents and teachers in school.

Considering the above points, the influence graph can be very important. But such a graph cannot show the power of employees within a school and the degree of influence of employees on each other. Since power and influence do not

TABLE 3: Employee power.

	Eskandari	Momeni	Rouhi	Salimi	Jafari
\tilde{T}_A	[0.3, 0.4]	[0.2, 0.4]	[0.2, 0.3]	[0.2, 0.3]	[0.2, 0.3]
T_B	0.5	0.3	0.5	0.3	0.3
\tilde{I}_A	[0.2, 0.3]	[0.3, 0.5]	[0.3, 0.5]	[0.3, 0.4]	[0.4, 0.5]
I_B	0.4	0.4	0.4	0.5	0.3
\tilde{F}_A	[0.3, 0.5]	[0.3, 0.4]	[0.4, 0.5]	[0.1, 0.3]	[0.2, 0.4]
F_B	0.3	0.5	0.3	0.6	0.5

have defined boundaries, they can be represented as a neutrosophic cubic set. On the other hand, there can be no fair interpretation of the power and influence of individuals because evaluations are always accompanied by skepticism. So, here we use the neutrosophic cubic degrees, which is very useful for influence and conflicts between employees. The neutrosophic cubic set of employees is shown in Table 3.

We have shown the influence of persons in the NC-digraph with an edge. This graph is shown in Figure 9.

School staff are the vertices of the NC-digraph of Figure 9. The weight of the vertices, respectively, indicates the power of speech, the degree of interaction with students, and the extent of their management in school affairs. For example, Mr. Rouhi has 20% to 30% of eloquence, but does not have 30% of the power to interact with students. Also, his power in processing school affairs is between 20% and 40%. Edges represent the extent of friendship, cultural, and political relationships, respectively. For example, Mr. Momeni has between 20% and 30% friendship with Mr. Jafari, but cultural differences between them are equal to 30%. Similarly, the rate of political relations between them is equal to 20% to 30%.

In Figure 9, it is clear that Mr. Eskandari controls the school deputy, Mr. Momeni, the representative of the parents and teachers association, Mr. Jafari, and educational instructor, Mr. Rouhi. Clearly, Mr. Eskandari has the most influence in the organization because he has an impact on three school staff and also has the highest level of management among other employees.

5. Conclusions

Neutrosophic cubic graphs have various uses in modern science and technology, especially in the fields of neural networks, computer science, operation research, and decision making. Also, they have a wide range of applications in the field of psychological sciences as well as the identification of individuals based on oncological behaviors. Therefore, in this research, some types of EI-NCGs such as NETI-NCGs, SEI-NCGs, and SETI-NCGs are introduced. A comparative study between NEI-NCGs and NETI-NCGs is presented. Finally, an application of neutrosophic cubic digraph to find the most effective person in a school has been introduced. In our future work, we will introduce connectivity index, Wiener index, and Randic index in neutrosophic cubic graphs and investigate some of their properties. Also, we will study some types of edge irregular neutrosophic cubic graphs such as neighborly edge totally irregular, strongly

edge irregular, and strongly edge totally irregular neutrosophic cubic graphs.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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