Hindawi
Journal of Mathematics
Volume 2022, Article ID 6832472, 13 pages
https://doi.org/10.1155/2022/6832472

Research Article

Asymptotical Stability of Riemann-Liouville Nonlinear Fractional Neutral Neural Networks with Time-Varying Delays

Erdal Korkmaz, Abdulhamit Ozdemir, and Kenan Yildirim

Mathematics, Mus Alparslan University, Mus, Turkey

Correspondence should be addressed to Kenan Yildirim; k.yildirim@alparslan.edu.tr

Received 9 May 2022; Accepted 1 September 2022; Published 17 September 2022

Academic Editor: A. Hussain

In this paper, the asymptotic stability of solutions is investigated for a class of nonlinear fractional neutral neural networks with time-dependent delays which are unbounded. By constructing the appropriate Lyapunov functional, sufficient conditions for asymptotic stability of neural networks are obtained with the help of LMI. An example is presented by using the LMI Toolbox to demonstrate the effectiveness of the obtained results.

1. Introduction

Fractional calculus with a history of more than 300 years has been proved to be valuable tools in modeling many phenomena in the various fields of engineering, physics, control systems, diffusion, epidemic model, financial systems, and so on [1–3]. Fractional derivative, which is a generalization of the integer derivative, has different definitions such as Caputo, Riemann-Liouville, Grünwald-Letnikov, and Hadamard, which were defined by different researchers [3]. Qualitative properties such as stability, asymptotic stability, and exponential stability of the solutions of the equations with delay or without delay of phenomena modeled in various fields with fractional differential equations have been studied by many authors. In most of these analyses, sufficient conditions for qualitative properties have been obtained by using the Lyapunov-Krasovskii functional known as the energy function. Some of these studies used LMI because it is one of the most useful tools for showing the derivative of the Lyapunov functional to be less than zero [4]. In [5], making use of the stability theorem of fractional-order systems with multiple time delays, some fractional derivative inequalities, and comparison theorem, several sufficient criteria are established for confirming that the synchronization error of the concerned system can reach zero within a limited time. In [6–8], sufficient conditions are obtained for the stability of solutions of certain fourth-order differential equations by using Lyapunov's second method. In [9–14], sufficient conditions were searched for qualitative properties such as stability, uniform stability, and asymptotic stability of solutions of fractional order delayed, undelayed, or neutral differential equations.

The other popular topic of recent times is neural networks, known as a part of the human brain, which has been the subject of research for more than 1000 years. Artificial neural networks are an information processing technology inspired by the information processing technique of the human brain. Researchers can refer to the references and their references for detailed information on the work done on fractional order modeled neural networks. In [15–38], global asymptotic stability, global stability, and Mittag-Leffler stability analyses of solutions of neural network equations modeled in fractional order are reported, and valuable results are obtained.

When the existing literature is examined, it is seen that the Riemann-Liouville derivative and Caputo derivative are mostly used among the definitions of fractional derivatives for fractional equations and systems of equations. A comparison between these two derivatives shows that the Caputo derivative is used more often because the initial conditions coincide with systems of integer order, which contributes to describing some well-understood properties of the physical process and makes it more applicable to real phenomena. On the other hand, the main advantage of the Riemann-Liouville derivative lies in the composition properties of the Riemann-Liouville fractional derivative and integral. Furthermore, the Riemann-Liouville derivative is a continuous operator of order q [1–3].

In this paper, our main goal is to build an appropriate Lyapunov functional to discuss the asymptotic stability of the Riemann-Liouville fractional-order neural networks with time-varying delays, using the mentioned advantage of the Riemann derivative. The acquired stability criteria are expressed as the matrix inequalities, which are also suitable

and practicable to test the asymptotic stability of the addressed neural networks. Specifically, the original contribution of the present paper to literature is that by employing the Lyapunov functional method, sufficient conditions, for showing the asymptotic stability of solutions for a class of nonlinear fractional neutral neural networks with time-dependent unbounded delays, are derived. In order to indicate the validation of the obtained results, an example is presented. Obtained results show that conditions proved in this paper are sufficient for asymptotic stability of solutions of nonlinear fractional neutral neural networks with time-dependent unbounded delays.

2. Preliminaries

In this section, we introduce some fundamental definitions of fractional calculus together with important lemmas.

Note that n-dimensional Euclidean space is denoted by R^n. The set of entire $n\times n$ real matrices is indicated by $R^{n\times n}$. The Euclidean norm of a real vector x is denoted by $\|x\|$. The spectral norm of matrix A is indicated by $\|A\|$. When $A<0$ (or $A>0$), the symmetric matrix A is negative definite (or positive definite).

Definition 1. The Riemann-Liouville fractional derivative and integral are described as the following, respectively,

$$\begin{aligned} {}_{t_0}D_t^{\alpha}x(t) &= \frac{1}{\Gamma(n-\alpha)}\frac{d^n}{dt^n}\int_{t_0}^{t}\frac{x(s)}{(t-s)^{\alpha+1-n}}ds\,(n-1\le\alpha<n),\\ {}_{t_0}D_t^{-\alpha}x(t) &= \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-s)^{\alpha-1}x(s)ds\,(\alpha>0) \end{aligned} \tag{1}$$

(see [3]).

Lemma 2. *If $\alpha>\beta>0$, then*

$${}_{t_0}D_t^{\beta}\left({}_{t_0}D_t^{-\alpha}x(t)\right)={}_{t_0}D_t^{\beta-\alpha}x(t) \tag{2}$$

holds for "sufficiently good" functions $x(t)$. Particularly, if $x(t)$ is integrable, then this relation holds (see [2]).

Lemma 3. *If $x(t)\in R^n$ is a vector of differentiable function, then the following relationship satisfies*

$$\frac{1}{2}{}_{t_0}D_t^{\alpha}\left(x^T(t)Px(t)\right)\le x^T(t)P{}_{t_0}D_t^{\alpha}x(t),\quad \forall\alpha\in(0,1),\forall t\ge t_0, \tag{3}$$

where $P\in R^{n\times n}$ is a positive semidefinite, symmetric, square, and constant matrix (see [35]).

Lemma 4. *For any positive definite matrix $Q>0$, scalar $\beta>0$, vector function $f(\cdot)$: $[0,\beta]\longrightarrow R^n$ such that the integrations concerned are well defined, the following inequality holds:*

$$\left(\int_0^{\beta}f(s)ds\right)^T Q\left(\int_0^{\beta}f(s)ds\right)\le\beta\left(\int_0^{\beta}f^T(s)Qf(s)ds\right) \tag{4}$$

(see [27]).

3. Main Results

Now, we present the stability of solutions of nonlinear fractional neutral neural networks having time-varying delays. We further consider linear matrix inequality to determine sufficient conditions on the asymptotical stability of solutions of these systems.

Let the fractional nonlinear neutral neural system be given by the following:

$$\begin{aligned} {}_0D_t^{\alpha}x(t) &= Ax(t)+B_1f_1(t,x(t))+B_2f_2(t,x(t-\tau_1(t)))\\ &\quad + C_0D_t^{\alpha}x(t-\tau_2(t))+B_3\int_{t-\tau_3(t)}^{t}f_3(s,x(s))ds, \end{aligned} \tag{5}$$

where $0<\alpha<1$ is a real number, $x=[x_1,x_2,\cdots,x_n]^T\in R^n$ is the state vector, $A,B_1,B_2,B_3,C\in R^{n\times n}$ are known constant matrices, for all $t>t_0$, $\tau_1(t),\tau_2(t),\tau_3(t)>0$ are time-varying delays. $f_j(t,x)(j=1,2,3)$ are vector-valued time-varying nonlinear functions with $f_j(t,0)=0$ and satisfy the following Lipschitz condition for all $(t,x),(t,\hat{x})\in R\times R^n$:

$$\left\|f_j(t,x)-f_j(t,\hat{x})\right\|\le a_j\left\|M_j(x-\hat{x})\right\|,\quad j=1,2,3, \tag{6}$$

where M_j are constant matrices with appropriate dimension and a_j are positive numbers. Consequently, from (6), we have

$$\left\|f_j(t,x)\right\|\le a_j\left\|M_jx\right\|,\quad j=1,2,3. \tag{7}$$

Theorem 5. *The trivial solution of system (5) is asymptotically stable, if $\|C\|<1$, for all $t>t_0$, $\tau_i'(t)\le d_i<1\,(i=1,2,3)$, $|\tau_3(t)|\le\varepsilon$, and there exist positive and symmetric definite matrices P,R, and S such that the following LMI satisfies:*

$$K=\begin{pmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} \\ K_{12}^T & K_{22} & K_{23} & K_{24} & K_{25} \\ K_{13}^T & K_{23}^T & K_{33} & K_{34} & K_{35} \\ K_{14}^T & K_{24}^T & K_{34}^T & K_{44} & K_{45} \\ K_{15}^T & K_{25}^T & K_{35}^T & K_{45}^T & K_{55} \end{pmatrix} < 0, \quad (8)$$

where

$$\begin{aligned} K_{11} &= A^TP + PA + A^T(R+mS)A + a_1^2M_1^TM_1 \\ &\quad + a_2^2M_2^TM_2 + \varepsilon^2a_3^2M_3^TM_3, \\ K_{12} &= PB_1 + A^T(R+mS)B_1, \\ K_{13} &= PB_2 + A^T(R+mS)B_2, \\ K_{14} &= PC + A^T(R+mS)C, \\ K_{15} &= PB_3 + A^T(R+mS)B_3, \\ K_{22} &= B_1^T(R+mS)B_1 - I, \\ K_{23} &= B_1^T(R+mS)B_2, \\ K_{24} &= B_1^T(R+mS)C, \\ K_{25} &= B_1^T(R+mS)B_3, \\ K_{33} &= B_2^T(R+mS)B_2 - (1-d_1)I, \\ K_{34} &= B_2^T(R+mS)C, \\ K_{35} &= B_2^T(R+mS)B_3, \\ K_{44} &= C^T(R+mS)C - (1-d_2)R, \\ K_{45} &= C^T(R+mS)B_3, \\ K_{55} &= B_3^T(R+mS)B_3 - (1-d_3)I, \end{aligned} \quad (9)$$

m and ε are positive constants and I unit matrix.

Proof. Let the Lyapunov-Krasovskii functional be defined by

$$\begin{aligned} V(t) &= {}_{t_0}D_t^{\alpha-1}\left(x^T(t)Px(t)\right) + \int_{-\tau_2(t)}^{0}\left({}_{t_0}D_t^{\alpha}x(t+s)\right)^T \\ &\quad \cdot R\left({}_{t_0}D_t^{\alpha}x(t+s)\right)ds + a_2^2\int_{t-\tau_1(t)}^{t}x^T(s)M_2^TM_2x(s)ds \\ &\quad + \int_{t-m}^{t}\int_{\theta}^{t}\left({}_{t_0}D_t^{\alpha}x(t+s)\right)^TS\left({}_{t_0}D_s^{\alpha}x(s)\right)dsd\theta \\ &\quad + \varepsilon a_3^2\int_{-\tau_3(t)}^{0}\int_{t+s}^{t}x^T(\eta)M_3^TM_3x(\eta)d\eta ds. \end{aligned} \quad (10)$$

Since P, R, and S matrices are positive definite, the Lyapunov-Krasovskii functional $V(t)$ is positive definite.

From (7) inequality,

$$\begin{aligned} &f_j^T(t,x(t))f_j(t,x(t)) \\ &\quad \le a_j^2\left\|M_jx\right\|^2 = a_j^2x^T(t)M_j^TM_jx(t), \quad j=1,2,3, \end{aligned} \quad (11)$$

can be written. Hence, according to Lemma 4 and (11), for all fixed t, we can get

$$\begin{aligned} &-\varepsilon a_3^2\int_{t-\tau_3(t)}^{t}x^T(s)M_3^TM_3x(s)ds \\ &\quad \le -\tau_3(t)a_3^2\int_{t-\tau_3(t)}^{t}x^T(s)M_3^TM_3x(s)ds \\ &\quad \le -\tau_3(t)\int_{t-\tau_3(t)}^{t}f_3^T(s,x(s))f_3(s,x(s))ds \\ &\quad \le -\left(\int_{t-\tau_3(t)}^{t}f_3(s,x(s))ds\right)^T I\int_{t-\tau_3(t)}^{t}f_3(s,x(s))ds. \end{aligned} \quad (12)$$

From (12) and Lemmas 2 and 3, the derivative of $V(t)$ is obtained along the trajectories of system (5) as follows:

$$\begin{aligned} \dot V(t) &= {}_{t_0}D_t^{\alpha}\left(x^T(t)Px(t)\right) + a_1^2x^T(t)M_1^TM_1x(t) \\ &\quad - a_1^2x^T(t)M_1^TM_1x(t) + \left({}_{t_0}D_t^{\alpha}x(t)\right)^TR\left({}_{t_0}D_t^{\alpha}x(t)\right) \\ &\quad - \left(1-\tau_2'(t)\right)\left({}_{t_0}D_t^{\alpha}x(t-\tau_2(t))\right)^TR\left({}_{t_0}D_t^{\alpha}x(t-\tau_2(t))\right) \\ &\quad + a_2^2x^T(t)M_2^TM_2x(t) - \left(1-\tau_1'(t)\right)a_2^2x^T(t-\tau_1(t)) \\ &\quad \cdot M_2^TM_2x(t-\tau_1(t)) + m\left({}_{t_0}D_t^{\alpha}x(t)\right)^TS\left({}_{t_0}D_t^{\alpha}x(t)\right) \\ &\quad - \int_{t-m}^{t}\left({}_{t_0}D_s^{\alpha}x(t)\right)^TS\left({}_{t_0}D_s^{\alpha}x(t)\right)ds + \varepsilon a_3^2\tau_3(t)x^T \\ &\quad \cdot (t)M_3^TM_3x(t) - \varepsilon a_3^2\left(1-\tau_3'(t)\right)\int_{t-\tau_3(t)}^{t}x^T \\ &\quad \cdot (s)M_3^TM_3x(s)ds \le 2x^T(t)PD_t^{\alpha}x(t) + a_1^2x^T \\ &\quad \cdot (t)M_1^TM_1x(t) + a_2^2x^T(t)M_2^TM_2x(t) + \varepsilon^2a_3^2x^T \\ &\quad \cdot (t)M_3^TM_3x(t) + \left({}_{t_0}D_t^{\alpha}x(t)\right)^T(R+mS)\left({}_{t_0}D_t^{\alpha}x(t)\right) \\ &\quad - (1-d_1)f_2^T(t,x(t-\tau_1(t)))If_2(t,x(t-\tau_1(t))) \\ &\quad - f_1^T(t,x(t))If_1(t,x(t)) - (1-d_2) \\ &\quad \cdot \left({}_{t_0}D_t^{\alpha}x(t-\tau_2(t))\right)^TR\left({}_{t_0}D_t^{\alpha}x(t-\tau_2(t))\right) \\ &\quad - (1-d_3)\left(\int_{t-\tau_3(t)}^{t}f_3(s,x(s))ds\right)^T I\int_{t-\tau_3(t)}^{t}f_3(s,x(s))ds \\ &= x^T(t)\left(A^TP + PA + a_1^2M_1^TM_1 + a_2^2M_2^TM_2 + \varepsilon^2a_3^2M_3^TM_3\right. \\ &\quad \left. + A^T(R+mS)A\right)x(t) + 2x^T(t)PB_1f_1(t,x(t)) \\ &\quad + 2x^T(t)PB_2f_2(t,x(t-\tau_1(t))) \\ &\quad + 2x^T(t)PC_{t_0}D_t^{\alpha}x(t-\tau_2(t)) \\ &\quad + 2x^T(t)PB_3\int_{t-\tau_3(t)}^{t}f_3(s,x(s))ds + x^T(t)A^T \\ &\quad \cdot (R+mS)B_1f_1(t,x(t)) + x^T(t)A^T(R+mS) \end{aligned}$$

$$
\begin{aligned}
&\cdot B_2 f_2(t, x(t-\tau_1(t))) + x^T(t) A^T (R+mS) C_{t_0} D_t^\alpha x(t-\tau_2(t)) \\
&+ x^T(t) A^T (R+mS) B_3 \int_{t-\tau_3(t)}^t f_3(s, x(s)) ds \\
&+ (f_1(t, x(t)))^T B_1^T (R+mS) A x(t) \\
&+ (f_1(t, x(t)))^T B_1^T (R+mS) B_1 f_1(t, x(t)) \\
&+ (f_1(t, x(t)))^T B_1^T (R+mS) B_2 f_2(t, x(t-\tau_1(t))) \\
&+ (f_1(t, x(t)))^T B_1^T (R+mS) C_{t_0} D_t^\alpha x(t-\tau_2(t)) \\
&+ (f_1(t, x(t)))^T B_1^T (R+mS) B_3 \int_{t-\tau_3(t)}^t f_3(s, x(s)) ds \\
&+ (f_2(t, x(t-\tau_1(t))))^T B_2^T (R+mS) A x(t) \\
&+ (f_2(t, x(t-\tau_1(t))))^T B_2^T (R+mS) B_1 f_1(t, x(t)) \\
&+ (f_2(t, x(t-\tau_1(t))))^T B_2^T (R+mS) B_2 f_2(t, x(t-\tau_1(t))) \\
&+ (f_2(t, x(t-\tau_1(t))))^T B_2^T (R+mS) C_{t_0} D_t^\alpha x(t-\tau_2(t)) \\
&+ (f_2(t, x(t-\tau_1(t))))^T B_2^T (R+mS) B_3 \int_{t-\tau_3(t)}^t f_3(s, x(s)) ds \\
&+ \left({}_{t_0} D_t^\alpha x(t-\tau_2(t))\right)^T C^T (R+mS) A x(t) \\
&+ \left({}_{t_0} D_t^\alpha x(t-\tau_2(t))\right)^T C^T (R+mS) B_1 f_1(t, x(t)) \\
&+ \left({}_{t_0} D_t^\alpha x(t-\tau_2(t))\right)^T C^T (R+mS) B_2 f_2(t, x(t-\tau_1(t))) \\
&+ \left({}_{t_0} D_t^\alpha x(t-\tau_2(t))\right)^T C^T (R+mS) C_{t_0} D_t^\alpha x(t-\tau_2(t)) \\
&+ \left({}_{t_0} D_t^\alpha x(t-\tau_2(t))\right)^T C^T (R+mS) B_3 \int_{t-\tau_3(t)}^t f_3(s, x(s)) ds \\
&+ \left(\int_{t-\tau_3(t)}^t f_3(s, x(s)) ds\right)^T B_3^T (R+mS) A x(t) \\
&+ \left(\int_{t-\tau_3(t)}^t f_3(s, x(s)) ds\right)^T B_3^T (R+mS) B_1 f_1(t, x(t)) \\
&\cdot \left(\int_{t-\tau_3(t)}^t f_3(s, x(s)) ds\right)^T B_3^T (R+mS) B_2 f_2(t, x(t-\tau_1(t))) \\
&+ \left(\int_{t-\tau_3(t)}^t f_3(s, x(s)) ds\right)^T B_3^T (R+mS) C_{t_0} D_t^\alpha x(t-\tau_2(t)) \\
&+ \left(\int_{t-\tau_3(t)}^t f_3(s, x(s)) ds\right)^T B_3^T (R+mS)\, B_3 \int_{t-\tau_3(t)}^t f_3(s, x(s)) ds \\
&- f_1^T(t, x(t)) I f_1(t, x(t)) - (1-d_1) f_2^T(t, x(t-\tau_1(t))) \\
&\cdot I f_2(t, x(t-\tau_1(t))) - (1-d_2)\left({}_{t_0} D_t^\alpha x(t-\tau_2(t))\right)^T \\
&\cdot R\left({}_{t_0} D_t^\alpha x(t-\tau_2(t))\right) - (1-d_3)\left(\int_{t-\tau_3(t)}^t f_3(s, x(s)) ds\right)^T \\
&\cdot I \int_{t-\tau_3(t)}^t f_3(s, x(s)) ds.
\end{aligned}
\tag{13}
$$

Therefore, we can write

$$
\dot{V}(t) \le \xi^T K \xi, \tag{14}
$$

where

$$
\begin{aligned}
K &= \begin{pmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} \\ K_{12}^T & K_{22} & K_{23} & K_{24} & K_{25} \\ K_{13}^T & K_{23}^T & K_{33} & K_{34} & K_{35} \\ K_{14}^T & K_{24}^T & K_{34}^T & K_{44} & K_{45} \\ K_{15}^T & K_{25}^T & K_{35}^T & K_{45}^T & K_{55} \end{pmatrix}, \\
K_{11} &= A^T P + PA + A^T (R+mS) A \\
&\quad + a_1^2 M_1^T M_1 + a_2^2 M_2^T M_2 + \varepsilon^2 a_3^2 M_3^T M_3, \\
K_{12} &= PB_1 + A^T (R+mS) B_1, \\
K_{13} &= PB_2 + A^T (R+mS) B_2, \\
K_{14} &= PC + A^T (R+mS) C, \\
K_{15} &= PB_3 + A^T (R+mS) B_3, \\
K_{22} &= B_1^T (R+mS) B_1 - I, \\
K_{23} &= B_1^T (R+mS) B_2, \\
K_{24} &= B_1^T (R+mS) C, \\
K_{25} &= B_1^T (R+mS) B_3, \\
K_{33} &= B_2^T (R+mS) B_2 - (1-d_1) I, \\
K_{34} &= B_2^T (R+mS) C, \\
K_{35} &= B_2^T (R+mS) B_3, \\
K_{44} &= C^T (R+mS) C - (1-d_2) R, \\
K_{45} &= C^T (R+mS) B_3, \\
K_{55} &= B_3^T (R+mS) B_3 - (1-d_3) I, \\
\xi &= \Bigg(x^T(t), f_1^T(t, x(t)), f_2^T(t, x(t-\tau_1(t))), {}_{t_0} D_t^\alpha x \\
&\quad \cdot (t-\tau_2(t))^T \left(\int_{t-\tau_3(t)}^t f_3(s, x(s)) ds\right)^T \Bigg)^T.
\end{aligned}
\tag{15}
$$

From axiom (8) of the theorem $\dot{V}(t)$ is negative definite. The trivial solution of system (5) is asymptotically stable. This completes the proof. □

Theorem 6. *The trivial solution of system (5) is asymptotically stable, if $\|C\| < 1$, for all $t > t_0$, $\tau_i'(t) \le d_i < 1$ $(i = 1, 2, 3)$, $|\tau_3(t)| \le \varepsilon$ and there exist positive and symmetric define matrices P, Q_2 and R such that the following LMI satisfies:*

$$
L = \begin{pmatrix} L_{11} & L_{12} & L_{13} & L_{14} & L_{15} \\ L_{12}^T & L_{22} & L_{23} & L_{24} & L_{25} \\ L_{13}^T & L_{23}^T & L_{33} & L_{34} & L_{35} \\ L_{14}^T & L_{24}^T & L_{34}^T & L_{44} & L_{45} \\ L_{15}^T & L_{25}^T & L_{35}^T & L_{45}^T & L_{55} \end{pmatrix} < 0, \tag{16}
$$

where

$$\begin{aligned}
L_{11} &= PA + A^{T}P + a_1^2 M_1^T M_1 + a_2^2 M_2^T M_2 \\
&\quad + \varepsilon^2 a_3^2 M_3^T M_3 + Q_2 + mA^T RA, \\
L_{12} &= PB_1 + mA^T RB_1, \\
L_{13} &= PB_2 + mA^T RB_2, \\
L_{14} &= -A^T PC, \\
L_{15} &= PB_3 + mA^T RB_3, \\
L_{22} &= mB_1^T RB_1 - I, \\
L_{23} &= mB_1^T RB_2, \\
L_{24} &= -B_1^T PC, \\
L_{25} &= mB_1^T RB_3, \\
L_{33} &= mB_2^T RB_2 - (1-d_1)I, \\
L_{34} &= -B_2^T PC, \\
L_{35} &= mB_2^T RB_3, \\
L_{44} &= -(1-d_2)Q_2, \\
L_{45} &= -C^T PB_3, \\
L_{55} &= mB_3^T RB_3 - (1-d_3)I,
\end{aligned} \tag{17}$$

m and ε are positive constants and I unit matrix.

Proof. Let the Lyapunov-Krasovskii function be defined by

$$\begin{aligned}
V(t) &= {}_{t_0}D_t^{\alpha-1}\left((x(t) - Cx(t-\tau_2(t)))^T P(x(t) - Cx(t-\tau_2(t)))\right) \\
&\quad + \int_{t-\tau_2(t)}^{t} x^T(s) Q_2 x(s) ds + a_2^2 \int_{t-\tau_1(t)}^{t} x^T(s) M_2^T M_2 x(s) ds \\
&\quad + \varepsilon a_3^2 \int_{-\tau_3(t)}^{0} \int_{t+s}^{t} x^T(\eta) M_3^T M_3 x(\eta) d\eta ds \\
&\quad + \int_{t-m}^{t} \int_{\theta}^{t} \left({}_{t_0}D_s^{\alpha}(x(s) - Cx(s-\tau_2(s)))\right)^T R\left({}_{t_0}D_s^{\alpha}(x(s) \right. \\
&\quad \left. - Cx(s-\tau_2(s)))\right) ds d\theta.
\end{aligned} \tag{18}$$

Since P, Q_2, and R matrices are positive definite, the functional $V(t)$ is positive definite. From (12) and Lemmas 2 and 3, the derivative of $V(t)$ is obtained along the trajectories of system (5) as follows:

$$\begin{aligned}
\dot{V}(t) &= {}_{t_0}D_t^{\alpha}\left((x(t) - Cx(t-\tau_2(t)))^T P(x(t) - Cx(t-\tau_2(t)))\right) \\
&\quad + a_1^2 x^T(t) M_1^T M_1 x(t) - a_1^2 x^T(t) M_1^T M_1 x(t) \\
&\quad + a_2^2 x^T(t) M_2^T M_2 x(t) - \left(1 - \tau_1'(t)\right) a_2^2 x^T(t-\tau_1(t)) \\
&\quad \cdot M_2^T M_2 x(t-\tau_1(t)) + x^T(t) Q_2 x(t) \\
&\quad - \left(1 - \tau_2'(t)\right) x^T(t-\tau_2(t)) Q_2 x(t-\tau_2(t)) \\
&\quad + \varepsilon a_3^2 \tau_3(t) x^T(t) M_3^T M_3 x(t) - \varepsilon a_3^2 \left(1 - \tau_3'(t)\right) \\
&\quad \cdot \int_{t-\tau_3(t)}^{t} x^T(s) M_3^T M_3 x(s) ds \\
&\quad + m\left({}_{t_0}D_t^{\alpha}(x(t) - Cx(t-\tau_2(t)))\right)^T \\
&\quad \cdot R\left({}_{t_0}D_t^{\alpha}(x(t) - Cx(t-\tau_2(t)))\right) \\
&\quad - \int_{t-m}^{t} \left({}_{t_0}D_s^{\alpha}(x(t) - Cx(t-\tau_2(t)))\right)^T \\
&\quad \cdot R\left({}_{t_0}D_s^{\alpha}(x(t) - Cx(t-\tau_2(t)))\right) ds \\
&\le 2(x(t) - Cx(t-\tau_2(t)))^T P\, {}_{t_0}D_t^{\alpha}(x(t) - Cx(t-\tau_2(t))) \\
&\quad + a_1^2 x^T(t) M_1^T M_1 x(t) + a_2^2 x^T(t) M_2^T M_2 x(t) \\
&\quad + \varepsilon^2 a_3^2 x^T(t) M_3^T M_3 x(t) + x^T(t) Q_2 x(t) \\
&\quad + m\left({}_{t_0}D_t^{\alpha}(x(t) - Cx(t-\tau_2(t)))\right)^T \\
&\quad \cdot R\left({}_{t_0}D_t^{\alpha}(x(t) - Cx(t-\tau_2(t)))\right)^T \\
&\quad - f_1^T(t, x(t)) I f_1(t, x(t)) - (1-d_1) \\
&\quad \cdot f_2^T(t, x(t-\tau_1(t))) I f_2(t, x(t-\tau_1(t))) \\
&\quad - (1-d_2) x^T(t-\tau_2(t)) Q_2 x(t-\tau_2(t)) \\
&\quad - (1-d_3)\left(\int_{t-\tau_3(t)}^{t} f_3(s, x(s)) ds\right)^T I \int_{t-\tau_3(t)}^{t} f_3(s, x(s)) ds \\
&= x^T(t)\left(PA + A^T P + a_1^2 M_1^T M_1 + a_2^2 M_2^T M_2 \right. \\
&\quad \left. + \varepsilon^2 a_3^2 M_3^T M_3 + mA^T RA + Q_2\right) x(t) \\
&\quad - 2x^T(t-\tau_2(t)) C^T PA x(t) + 2x^T(t) PB_1 f_1(t, x(t)) \\
&\quad - 2x^T(t-\tau_2(t)) C^T PB_1 f_1(t, x(t)) \\
&\quad + 2x^T(t) PB_2 f_2(t, x(t-\tau_1(t))) \\
&\quad - 2x^T(t-\tau_2(t)) C^T PB_2 f_2(t, x(t-\tau_1(t))) \\
&\quad + 2x^T(t) PB_3 \int_{t-\tau_3(t)}^{t} f_3(s, x(s)) ds - 2x^T(t-\tau_2(t)) \\
&\quad \cdot C^T PB_3 \int_{t-\tau_3(t)}^{t} f_3(s, x(s)) ds + mx^T(t) A^T RB_1 f_1(t, x(t)) \\
&\quad + mx^T(t) A^T RB_2 f_2(t, x(t-\tau_1(t))) + mx^T(t) A^T RB_3 \\
&\quad \cdot \int_{t-\tau_3(t)}^{t} f_3(s, x(s)) ds + mf_1^T(t, x(t)) B_1^T RA x(t) \\
&\quad + mf_1^T(t, x(t)) B_1^T RB_1 f_1(t, x(t)) + mf_1^T(t, x(t)) \\
&\quad \cdot B_1^T RB_2 f_2(t, x(t-\tau_1(t))) + mf_1^T(t, x(t)) B_1^T RB_3 \\
&\quad \cdot \int_{t-\tau_3(t)}^{t} f_3(s, x(s)) ds + mf_2^T(t, x(t-\tau_1(t))) B_2^T RA x(t) \\
&\quad + mf_2^T(t, x(t-\tau_1(t))) B_2^T RB_1 f_1(t, x(t)) \\
&\quad + mf_2^T(t, x(t-\tau_1(t))) B_2^T RB_2 f_2(t, x(t-\tau_1(t))) \\
&\quad + mf_2^T(t, x(t-\tau_1(t))) B_2^T RB_3 \int_{t-\tau_3(t)}^{t} f_3(s, x(s)) ds \\
&\quad + m\left(\int_{t-\tau_3(t)}^{t} f_3(s, x(s)) ds\right)^T B_3^T RA x(t)
\end{aligned}$$

$$+m\left(\int_{t-\tau_3(t)}^{t} f_3(s,x(s))ds\right)^T B_3^T R B_1 f_1(t,x(t))$$
$$+m\left(\int_{t-\tau_3(t)}^{t} f_3(s,x(s))ds\right)^T B_3^T R B_2 f_2(t,x(t-\tau_1(t)))$$
$$+m\left(\int_{t-\tau_3(t)}^{t} f_3(s,x(s))ds\right)^T B_3^T R B_3 \int_{t-\tau_3(t)}^{t} f_3(s,x(s))ds$$
$$-f_1^T(t,x(t))If_1(t,x(t))-(1-d_1)f_2^T(t,x(t-\tau_1(t)))$$
$$\cdot If_2(t,x(t-\tau_1(t)))-(1-d_2)x^T(t-\tau_2(t))Q_2x(t-\tau_2(t))$$
$$-(1-d_3)\left(\int_{t-\tau_3(t)}^{t} f_3(s,x(s))ds\right)^T I\int_{t-\tau_3(t)}^{t} f_3(s,x(s))ds. \tag{19}$$

Therefore, we can write

$$\dot{V}(t)\le \xi^T L\xi, \tag{20}$$

where

$$L=\begin{pmatrix} L_{11} & L_{12} & L_{13} & L_{14} & L_{15}\\ L_{12}^T & L_{22} & L_{23} & L_{24} & L_{25}\\ L_{13}^T & L_{23}^T & L_{33} & L_{34} & L_{35}\\ L_{14}^T & L_{24}^T & L_{34}^T & L_{44} & L_{45}\\ L_{15}^T & L_{25}^T & L_{35}^T & L_{45}^T & L_{55}\end{pmatrix},$$
$$L_{11}=PA+A^TP+a_1^2M_1^TM_1+a_2^2M_2^TM_2+\varepsilon^2a_3^2M_3^TM_3+Q_2+mA^TRA,\ L_{12}=PB_1+mA^TRB_1,$$
$$L_{13}=PB_2+mA^TRB_2,$$
$$L_{14}=-A^TPC,$$
$$L_{15}=PB_3+mA^TRB_3,$$
$$L_{22}=mB_1^TRB_1-I,$$
$$L_{23}=mB_1^TRB_2,$$
$$L_{24}=-B_1^TPC,$$
$$L_{25}=mB_1^TRB_3,$$
$$L_{33}=mB_2^TRB_2-(1-d_1)I,$$
$$L_{34}=-B_2^TPC,$$
$$L_{35}=mB_2^TRB_3,$$
$$L_{44}=-(1-d_2)Q_2,$$
$$L_{45}=-C^TPB_3,$$
$$L_{55}=mB_3^TRB_3-(1-d_3)I,$$
$$\xi=\left(x^T(t),f_1^T(t,x(t)),f_2^T(t,x(t-\tau_1(t))),\cdot x^T(t-\tau_2(t)),\left(\int_{t-\tau_3(t)}^{t} f_3(s,x(s))ds\right)^T\right)^T. \tag{21}$$

From the axiom (16) of the theorem $\dot{V}(t)$ is negative definite. The trivial solution of system (5) is asymptotically stable. This completes the proof. □

4. Corollary

In this section, some results for the asymptotic stability of the solutions of various variations of the fractional neutral neural network (5) are expressed as a convex optimization problem. Two examples are given to show that the results obtained are applicable. The effectiveness of the results obtained with the help of these examples is discussed.

Corollary 7. *The trivial solution of system (5) is asymptotically stable, if* $\|C\|<1$, *for all* $t>t_0$, $\tau_i'(t)\le d_i<1\ (i=1,2,3)$, $|\tau_3(t)|\le\varepsilon$ *and there exist* a_1^2, a_2^2, *and* a_3^2 *numbers and symmetric matrices* P, R, *and* S *such that the following convex optimization problem on* a_1^2 *number and* P, R, *and* S *matrices is solvable:*

$$\begin{aligned}&\textit{minimize} && -a_1^2\\ &\textit{subject to} && P>0, R>0, S>0, K<0,\end{aligned} \tag{22}$$

where K *matrix defined by (8),* m *and* ε *are positive constants and* I *unit matrix.*

Corollary 8. *The trivial solution of system (5) is asymptotically stable, if* $\|C\|<1$, *for all* $t>t_0$, $\tau_i'(t)\le d_i<1\ (i=1,2,3)$, $|\tau_3(t)|\le\varepsilon$ *and there exist* a_1^2, a_2^2, *and* a_3^2 *numbers and symmetric matrices* P, Q_2, *and* R *such that the following convex optimization problem on* a_1^2 *number and* P, Q_2, *and* R *matrices is solvable:*

$$\begin{aligned}&\textit{minimize} -a_1^2\\ &\textit{subject to}\ P>0, Q_2>0, R>0, L<0,\end{aligned} \tag{23}$$

where L *matrix defined by (16),* m *and* ε *are positive constants and* I *unit matrix.*

If $f_2(t,x(t-\tau_1(t)))=x(t-\tau_1(t))$, *then the nonlinear fractional neutral neural system (5) can be written as the following:*

$${}_{t_0}D_t^{\alpha}x(t)=Ax(t)+B_1f_1(t,x(t))+B_2x(t-\tau_1(t))+C_0D_t^{\alpha}x(t-\tau_2(t))+B_3\int_{t-\tau_3(t)}^{t} f_3(s,x(s))ds, \tag{24}$$

where $0<\alpha<1$ *is a real number,* $x=[x_1,x_2,\cdots,x_n]^T\in R^n$ *is the state vector,* $A, B_1, B_2, B_3, C\in R^{n\times n}$ *are known constant matrices, for all* $t>t_0$, $\tau_1(t),\tau_2(t),\tau_3(t)>0$ *are time-varying delays.* $f_j(t,x)(j=1,3)$ *are vector-valued time-varying nonlinear functions with* $f_j(t,0)=0$ *and satisfy the following Lipschitz condition for all* $(t,x)\in R\times R^n$:

$$\left\| f_j(t,x) \right\| \le a_j \left\| M_j x \right\|, \tag{25}$$

where M_j are constant matrices with appropriate dimension and a_j are positive numbers.

Corollary 9. *The trivial solution of system (24) is asymptotically stable, if $\|C\| < 1$, for all $t > t_0$, $\tau_i'(t) \le d_i < 1 (i = 1, 2, 3)$, $|\tau_3(t)| \le \varepsilon$ and there exist a_1^2 and a_3^2 numbers and symmetric matrices P, Q_1, R, and S such that the following convex optimization problem on a_1^2 number and P, Q_1, R, and S matrices is solvable:*

$$\begin{gathered} \text{minimize} - a_1^2 \\ \text{subject to } P > 0, Q_1 > 0, R > 0, S > 0, \\ \Delta = \begin{pmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & \Delta_{14} & \Delta_{15} \\ \Delta_{12}^T & \Delta_{22} & \Delta_{23} & \Delta_{24} & \Delta_{25} \\ \Delta_{13}^T & \Delta_{23}^T & \Delta_{33} & \Delta_{34} & \Delta_{35} \\ \Delta_{14}^T & \Delta_{24}^T & \Delta_{34}^T & \Delta_{44} & \Delta_{45} \\ \Delta_{15}^T & \Delta_{25}^T & \Delta_{35}^T & \Delta_{45}^T & \Delta_{55} \end{pmatrix} < 0, \end{gathered} \tag{26}$$

where

$$\begin{aligned} \Delta_{11} &= A^T P + PA + A^T (R + mS) A + a_1^2 M_1^T M_1 \\ &\quad + Q_1 + \varepsilon^2 a_3^2 M_3^T M_3, \\ \Delta_{12} &= PB_1 + A^T (R + mS) B_1, \\ \Delta_{13} &= PB_2 + A^T (R + mS) B_2, \\ \Delta_{14} &= PC + A^T (R + mS) C, \\ \Delta_{15} &= PB_3 + A^T (R + mS) B_3, \\ \Delta_{22} &= B_1^T (R + mS) B_1 - I, \\ \Delta_{23} &= B_1^T (R + mS) B_2, \\ \Delta_{24} &= B_1^T (R + mS) C, \\ \Delta_{25} &= B_1^T (R + mS) B_3, \\ \Delta_{33} &= B_2^T (R + mS) B_2 - (1 - d_1) Q_1, \\ \Delta_{34} &= B_2^T (R + mS) C, \\ \Delta_{35} &= B_2^T (R + mS) B_3, \\ \Delta_{44} &= C^T (R + mS) C - (1 - d_2) R, \\ \Delta_{45} &= C^T (R + mS) B_3, \\ \Delta_{55} &= B_3^T (R + mS) B_3 - (1 - d_3) I, \end{aligned} \tag{27}$$

m and ε are positive constants and I unit matrix.

Corollary 10. *The trivial solution of system (24) is asymptotically stable, if $\|C\| < 1$, for all $t > t_0$, $\tau_i'(t) \le d_i < 1 (i = 1, 2, 3)$, $|\tau_3(t)| \le \varepsilon$ and there exist a_1^2 and a_3^2 numbers and symmetric matrices P, Q_1, Q_2, and R such that the following convex optimization problem on a_1^2 number and P, Q_1, Q_2, and R matrices is solvable:*

$$\begin{gathered} \text{minimize} - a_1^2 \\ \text{subject to } P > 0, Q_1 > 0, Q_2 > 0, R > 0, \\ Z = \begin{pmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{15} \\ Z_{12}^T & Z_{22} & Z_{23} & Z_{24} & Z_{25} \\ Z_{13}^T & Z_{23}^T & Z_{33} & Z_{34} & Z_{35} \\ Z_{14}^T & Z_{24}^T & Z_{34}^T & Z_{44} & Z_{45} \\ Z_{15}^T & Z_{25}^T & Z_{35}^T & Z_{45}^T & Z_{55} \end{pmatrix} < 0, \end{gathered} \tag{28}$$

where

$$\begin{aligned} Z_{11} &= PA + A^T P + a_1^2 M_1^T M_1 + \varepsilon^2 a_3^2 M_3^T M_3 \\ &\quad + Q_1 + Q_2 + mA^T RA, \\ Z_{12} &= PB_1 + mA^T RB_1, \\ Z_{13} &= PB_2 + mA^T RB_2, \\ Z_{14} &= -A^T PC, \\ Z_{15} &= PB_3 + mA^T RB_3, \\ Z_{22} &= mB_1^T RB_1 - I, \\ Z_{23} &= mB_1^T RB_2, \\ Z_{24} &= -B_1^T PC, \\ Z_{25} &= mB_1^T RB_3, \\ Z_{33} &= mB_2^T RB_2 - (1 - d_1) Q_1, \\ Z_{34} &= -B_2^T PC, \\ Z_{35} &= mB_2^T RB_3, \\ Z_{44} &= -(1 - d_2) Q_2, \\ Z_{45} &= -C^T PB_3, \\ Z_{55} &= mB_3^T RB_3 - (1 - d_3) I, \end{aligned} \tag{29}$$

m and ε are positive constants and I unit matrix.

If $f_3(t, x(t)) = x(t)$, then the nonlinear fractional neutral neural system (5) can be written as the following:

$$\begin{aligned} {}_{t_0}D_t^{\alpha} x(t) &= Ax(t) + B_1 f_1(t, x(t)) + B_2 f_2(t, x(t - \tau_1(t))) \\ &\quad + C_{t_0} D_t^{\alpha} x(t - \tau_2(t)) + B_3 \int_{t-\tau_3(t)}^{t} x(s) ds, \end{aligned} \tag{30}$$

where $0 < \alpha < 1$ is a real number, $x = [x_1, x_2, \cdots, x_n]^T \in R^n$ is the state vector, $A, B_1, B_2, B_3, C \in R^{n \times n}$ are known constant matrices, for all $t > t_0$, $\tau_1(t), \tau_2(t), \tau_3(t) > 0$ are time-varying delays. $f_j(t, x) (j = 1, 2)$ is vector-valued time-varying nonlinear function with $f_j(t, 0) = 0$ and satisfies the following Lipschitz condition for all $(t, x) \in R \times R^n$:

$$\left\| f_j(t,x) \right\| \le a_j \left\| M_j x \right\|, \tag{31}$$

where M_j are constant matrices with appropriate dimension and a_j are positive numbers.

Corollary 11. *The trivial solution of system (30) is asymptotically stable, if $\|C\| < 1$, for all $t > t_0$, $\tau_i'(t) \le d_i < 1$ $(i = 1, 2, 3)$, $|\tau_3(t)| \le \varepsilon$ and there exist a_1^2 and a_2^2 numbers and symmetric matrices P, Q_3, R, and S such that the following convex optimization problem on a_1^2 number and P, Q_3, R, and S matrices is solvable:*

$$\begin{aligned} &\text{minimize} \quad -a_1^2 \\ &\text{subject to} \quad P > 0, Q_3 > 0, R > 0, S > 0, \\ &O = \begin{pmatrix} O_{11} & O_{12} & O_{13} & O_{14} & O_{15} \\ O_{12}^T & O_{22} & O_{23} & O_{24} & O_{25} \\ O_{13}^T & O_{23}^T & O_{33} & O_{34} & O_{35} \\ O_{14}^T & O_{24}^T & O_{34}^T & O_{44} & O_{45} \\ O_{15}^T & O_{25}^T & O_{35}^T & O_{45}^T & O_{55} \end{pmatrix} < 0, \end{aligned} \tag{32}$$

where

$$\begin{aligned} O_{11} &= A^T P + PA + A^T (R + mS) A \\ &\quad + a_1^2 M_1^T M_1 + a_2^2 M_2^T M_2 + \varepsilon^2 Q_3, \\ O_{12} &= PB_1 + A^T (R + mS) B_1, \\ O_{13} &= PB_2 + A^T (R + mS) B_2, \\ O_{14} &= PC + A^T (R + mS) C, \\ O_{15} &= PB_3 + A^T (R + mS) B_3, \\ O_{22} &= B_1^T (R + mS) B_1 - I, \\ O_{23} &= B_1^T (R + mS) B_2, \\ O_{24} &= B_1^T (R + mS) C, \\ O_{25} &= B_1^T (R + mS) B_3, \\ O_{33} &= B_2^T (R + mS) B_2 - (1 - d_1) I, \\ O_{34} &= B_2^T (R + mS) C, \\ O_{35} &= B_2^T (R + mS) B_3, \\ O_{44} &= C^T (R + mS) C - (1 - d_2) R, \\ O_{45} &= C^T (R + mS) B_3, \\ O_{55} &= B_3^T (R + mS) B_3 - (1 - d_3) Q_3, \end{aligned} \tag{33}$$

m and ε are positive constants and I unit matrix.

Corollary 12. *The trivial solution of system (30) is asymptotically stable, if $\|C\| < 1$, for all $t > t_0$, $\tau_i'(t) \le d_i < 1$ $(i = 1, 2, 3)$, $|\tau_3(t)| \le \varepsilon$ and there exist a_1^2 and a_2^2 numbers and symmetric matrices P, Q_2, Q_3, and R such that the following convex optimization problem on a_1^2 number and P, Q_2, Q_3, and R matrices is solvable:*

$$\begin{aligned} &\text{minimize} \quad -a_1^2 \\ &\text{subject to} \quad P > 0, Q_2 > 0, Q_3 > 0, R > 0, \\ &\Pi = \begin{pmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} \\ \Pi_{12}^T & \Pi_{22} & \Pi_{23} & \Pi_{24} & \Pi_{25} \\ \Pi_{13}^T & \Pi_{23}^T & \Pi_{33} & \Pi_{34} & \Pi_{35} \\ \Pi_{14}^T & \Pi_{24}^T & \Pi_{34}^T & \Pi_{44} & \Pi_{45} \\ \Pi_{15}^T & \Pi_{25}^T & \Pi_{35}^T & \Pi_{45}^T & \Pi_{55} \end{pmatrix} < 0, \end{aligned} \tag{34}$$

where

$$\begin{aligned} \Pi_{11} &= PA + A^T P + a_1^2 M_1^T M_1 + a_2^2 M_2^T M_2 \\ &\quad + \varepsilon^2 Q_3 + Q_2 + mA^T RA, \\ \Pi_{12} &= PB_1 + mA^T RB_1, \\ \Pi_{13} &= PB_2 + mA^T RB_2, \\ \Pi_{14} &= -A^T PC, \\ \Pi_{15} &= PB_3 + mA^T RB_3, \\ \Pi_{22} &= mB_1^T RB_1 - I, \\ \Pi_{23} &= mB_1^T RB_2, \\ \Pi_{24} &= -B_1^T PC, \\ \Pi_{25} &= mB_1^T RB_3, \\ \Pi_{33} &= mB_2^T RB_2 - (1 - d_1) I, \\ \Pi_{34} &= -B_2^T PC, \\ \Pi_{35} &= mB_2^T RB_3, \\ \Pi_{44} &= -(1 - d_2) Q_2, \\ \Pi_{45} &= -C^T PB_3, \\ \Pi_{55} &= mB_3^T RB_3 - (1 - d_3) Q_3, \end{aligned} \tag{35}$$

m and ε are positive constants and I unit matrix.

If $f_2(t, x(t - \tau_1(t)) = x(t - \tau_1(t)$ and $f_3(t, x(t)) = x(t)$, then the nonlinear fractional neutral neural system (5) can be written as the following:

$$\begin{aligned} {}_{t_0}D_t^{\alpha} x(t) &= Ax(t) + B_1 f_1(t, x(t)) + B_2 x(t - \tau_1(t)) \\ &\quad + C\,{}_{t_0}D_t^{\alpha} x(t - \tau_2(t)) + B_3 \int_{t-\tau_3(t)}^{t} x(s)\,ds, \end{aligned} \tag{36}$$

where $0 < \alpha < 1$ is a real number, $x = [x_1, x_2, \cdots, x_n]^T \in R^n$ is the state vector, $A, B_1, B_2, B_3, C \in R^{n\times n}$ are known constant matrices, for all $t > t_0$, $\tau_1(t), \tau_2(t), \tau_3(t) > 0$ are time-varying delays. $f_1(t, x)$ is vector-valued time-varying nonlinear function with $f_1(t, 0) = 0$ and satisfies the following

Lipschitz condition for all $(t, x) \in R \times R^n$:

$$\|f_1(t, x)\| \le a_1 \|M_1 x\|, \tag{37}$$

where M_1 *is a constant matrix with appropriate dimension and* a_1 *is a positive number.*

Corollary 13. *The trivial solution of system (36) is asymptotically stable, if* $\|C\| < 1$, *for all* $t > t_0$, $\tau_i'(t) \le d_i < 1$ $(i = 1, 2, 3)$, $|\tau_3(t)| \le \varepsilon$ *and there exist* a_1^2 *number and symmetric matrices* P, Q_1, Q_3, R, *and* S *such that the following convex optimization problem on* a_1^2 *number and* P, Q_1, Q_3, R, *and* S *matrices is solvable:*

$$\begin{aligned} &\text{minimize} \quad -a_1^2 \\ &\text{subject to} \quad P > 0, Q_1 > 0, Q_3 > 0, R > 0, S > 0, \\ &\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} \\ \Sigma_{12}^T & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} & \Sigma_{25} \\ \Sigma_{13}^T & \Sigma_{23}^T & \Sigma_{33} & \Sigma_{34} & \Sigma_{35} \\ \Sigma_{14}^T & \Sigma_{24}^T & \Sigma_{34}^T & \Sigma_{44} & \Sigma_{45} \\ \Sigma_{15}^T & \Sigma_{25}^T & \Sigma_{35}^T & \Sigma_{45}^T & \Sigma_{55} \end{pmatrix} < 0, \end{aligned} \tag{38}$$

where

$$\begin{aligned} \Sigma_{11} &= A^T P + PA + A^T (R + mS) A \\ &\quad + a_1^2 M_1^T M_1 + Q_1 + \varepsilon^2 Q_3, \\ \Sigma_{12} &= PB_1 + A^T (R + mS) B_1, \\ \Sigma_{13} &= PB_2 + A^T (R + mS) B_2, \\ \Sigma_{14} &= PC + A^T (R + mS) C, \\ \Sigma_{15} &= PB_3 + A^T (R + mS) B_3, \\ \Sigma_{22} &= B_1^T (R + mS) B_1 - I, \\ \Sigma_{23} &= B_1^T (R + mS) B_2, \\ \Sigma_{24} &= B_1^T (R + mS) C, \\ \Sigma_{25} &= B_1^T (R + mS) B_3, \\ \Sigma_{33} &= B_2^T (R + mS) B_2 - (1 - d_1) Q_1, \\ \Sigma_{34} &= B_2^T (R + mS) C, \\ \Sigma_{35} &= B_2^T (R + mS) B_3, \\ \Sigma_{44} &= C^T (R + mS) C - (1 - d_2) R, \\ \Sigma_{45} &= C^T (R + mS) B_3, \\ \Sigma_{55} &= B_3^T (R + mS) B_3 - (1 - d_3) Q_3, \end{aligned} \tag{39}$$

m and ε are positive constants and I unit matrix.

Corollary 14. *The trivial solution of system (36) is asymptotically stable, if* $\|C\| < 1$, *for all* $t > t_0$, $\tau_i'(t) \le d_i < 1$ $(i = 1, 2, 3)$, $|\tau_3(t)| \le \varepsilon$ *and there exist* a_1^2 *number and symmetric matrices* P, Q_1, Q_2, Q_3, *and* R *such that the following convex optimization problem on* a_1^2 *number and* P, Q_1, Q_2, Q_3, *and* R *matrices is solvable:*

$$\begin{aligned} &\text{minimize} \quad -a_1^2 \\ &\text{subject to} \quad P > 0, Q_1 > 0, Q_2 > 0, Q_3 > 0, R > 0, \\ &Y = \begin{pmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} & Y_{15} \\ Y_{12}^T & Y_{22} & Y_{23} & Y_{24} & Y_{25} \\ Y_{13}^T & Y_{23}^T & Y_{33} & Y_{34} & Y_{35} \\ Y_{14}^T & Y_{24}^T & Y_{34}^T & Y_{44} & Y_{45} \\ Y_{15}^T & Y_{25}^T & Y_{35}^T & Y_{45}^T & Y_{55} \end{pmatrix} < 0, \end{aligned} \tag{40}$$

where

$$\begin{aligned} Y_{11} &= PA + A^T P + a_1^2 M_1^T M_1 + \varepsilon^2 Q_3 \\ &\quad + Q_1 + Q_2 + mA^T RA, \\ Y_{12} &= PB_1 + mA^T RB_1, \\ Y_{13} &= PB_2 + mA^T RB_2, \\ Y_{14} &= -A^T PC, \\ Y_{15} &= PB_3 + mA^T RB_3, \\ Y_{22} &= mB_1^T RB_1 - I, \\ Y_{23} &= mB_1^T RB_2, \\ Y_{24} &= -B_1^T PC, \\ Y_{25} &= mB_1^T RB_3, \\ Y_{33} &= mB_2^T RB_2 - (1 - d_1) Q_1, \\ Y_{34} &= -B_2^T PC, \\ Y_{35} &= mB_2^T RB_3, \\ Y_{44} &= -(1 - d_2) Q_2, \\ Y_{45} &= -C^T PB_3, \\ Y_{55} &= mB_3^T RB_3 - (1 - d_3) Q_3, \end{aligned} \tag{41}$$

m and ε are positive constants and I unit matrix.

5. Numerical Examples

Example 1. Let the nonlinear fractional neutral neural system be given by

$$\begin{aligned} {}_{t_0}D_t^{\alpha} x(t) &= Ax(t) + B_1 f_1(t, x(t)) + B_2 f_2(t, x(t - \tau_1(t))) \\ &\quad + C_{t_0} D_t^{\alpha} x(t - \tau_2(t)) + B_3 \int_{t-\tau_3(t)}^{t} f_3(s, x(s)) ds, \end{aligned} \tag{42}$$

Table 1: Numerical solutions for Example 1.

Corollaries	p	q_1	q_2	q_3	r	s	$a_{\max}$	$\lambda_{\max}$
Corollary 7	0.1249	—	—	—	0.0040	0.0283	1.3360	-7.7393×10^{-7}
Corollary 8	0.1515	—	0.5198	—	0.0817	—	1.3832	-0.0059
Corollary 9	0.1337	1.1361	—	—	0.0043	0.0141	0.9778	-4.7656×10^{-7}
Corollary 10	0.1676	1.2112	0.5721	—	0.0467	—	1.0479	-4.4837×10^{-5}
Corollary 11	0.1326	—	—	1.3443	0.0043	0.0125	1.2761	-1.1118×10^{-6}
Corollary 12	0.1626	—	0.5569	1.4256	0.0641	—	1.3285	-2.0478×10^{-4}
Corollary 13	0.1715	1.4609	—	1.7441	0.0056	0.0088	0.9214	-1.2880×10^{-6}
Corollary 14	0.2225	1.6103	0.7581	1.9257	0.0473	—	1.0168	-4.6914×10^{-5}

where

$$A=\begin{bmatrix}-40 & 5 & 3\\ 4 & -20 & 1\\ 1 & 2 & -20\end{bmatrix},\quad B_1=B_2=B_3=\begin{bmatrix}5 & 0 & 0\\ 0 & 5 & 0\\ 0 & 0 & 5\end{bmatrix},\quad C=\begin{bmatrix}0.1 & 0 & 0\\ 0 & 0.2 & 0\\ 0 & 0 & 0.3\end{bmatrix},$$
$$f_1(t,x(t))=a_1\begin{pmatrix}\sin(x_2(t))\\ \sin(x_1(t))\\ \sin(x_3(t))\end{pmatrix},$$
$$f_2(t,x(t-\tau_1(t)))=0.4\begin{pmatrix}e^{-0.5t}\sin(x_1(t-\tau_1(t)))\\ e^{-0.3t}\sin(x_2(t-\tau_1(t)))\\ e^{-0.4t}\sin(x_3(t-\tau_1(t)))\end{pmatrix},$$
$$f_3(t,x(t))=0.6\begin{pmatrix}\frac{1}{\sqrt{2}}\sin(x_1(t)+x_3(t))\\ \sin(x_2(t))\\ \frac{1}{\sqrt{2}}\sin(x_1(t)-x_3(t))\end{pmatrix}, \tag{43}$$

$\alpha\in(1,0)$, $\tau_1(t)=0.4t+0.2\sin(t)$, $\tau_2(t)=0.2t+0.3\cos(t)$, and $\tau_3(t)=0.5+0.04\sin(t)$.

For $f_1(t,x(t))$, we have

$$\begin{aligned}\|f_1(t,x(t))\|^2&=a_1^2\left(\sin^2(x_2(t))+\sin^2(x_1(t))+\sin^2(x_3(t))\right)\\&\le a_1^2\left(x_1^2(t)+x_2^2(t)+x_3^2(t)\right)\\&=a_1^2x^T(t)M_1^TM_1x(t),\end{aligned} \tag{44}$$

where $M_1=I_3$. For $f_2(t,x(t-\tau_1(t)))$, we have

$$\begin{aligned}&\|f_2(t,x(t-\tau_1(t)))\|^2\\&\quad=0.16\big(e^{-t}\sin^2(x_2(t-\tau_1(t)))+e^{-0.6t}\sin^2(x_1(t-\tau_1(t)))\\&\qquad+e^{-0.8t}\sin^2(x_3(t-\tau_1(t)))\big)\\&\quad\le 0.16\left(x_1^2(t)+x_2^2(t)+x_3^2(t)\right)\\&\quad=0.16x^T(t-\tau_1(t))M_2^TM_2x(t-\tau_1(t)),\end{aligned} \tag{45}$$

where $M_2=I_3$. For $f_3(t,x(t))$, we have

$$\begin{aligned}&\|f_3(t,x(t))\|^2\\&\quad=0.36\Big(\frac{1}{2}\sin^2(x_1(t)+x_3(t))\\&\qquad+\sin^2(x_1(t))+\frac{1}{2}\sin^2(x_1(t)-x_3(t))\Big)\\&\quad\le 0.36\Big(\frac{1}{2}(x_1(t)+x_3(t))^2+x_2^2(t)+\frac{1}{2}(x_1(t)-x_3(t))^2\Big)\\&\quad=0.36\left(x_1^2(t)+x_2^2(t)+x_3^2(t)\right)=0.36x^T(t)M_3^TM_3x(t),\end{aligned} \tag{46}$$

where $M_3=I_3$.

Let us choose $d_1=0.6$, $d_2=0.5$, $d_3=0.04$, $\varepsilon=0.54$, $m=0.001$. Let the P,Q_1,Q_2,Q_3,R,S matrices be defined by

$$\begin{aligned}P&=p\times I_3,\\Q_1&=q_1\times I_3,\\Q_2&=q_2\times I_3,\\Q_3&=q_3\times I_3,\\R&=r\times I_3,\\S&=s\times I_3,\end{aligned} \tag{47}$$

for positive numbers p,q_1,q_2,q_3,r,s. $a_{\max}$ is the solution of the convex optimization problem in the corresponding result; $\lambda_{\max}$ is the largest eigenvalue of the matrix in the corresponding result, which must be negative definite.

Table 2: Numerical solutions for Example 2.

Corollaries	p	q_1	q_2	q_3	r	s	$a_{\max}$	$\lambda_{\max}$
Corollary 7	0.2034	—	—	—	0.0037	0.0591	2.1212	-6.6719×10^{-6}
Corollary 8	0.2347	—	0.4821	—	0.2406	—	2.2420	−0.0010
Corollary 9	0.2262	1.4181	—	—	0.0042	0.0264	1.9674	-1.8412×10^{-6}
Corollary 10	0.2649	1.5066	0.5382	—	0.1582	—	2.1105	-1.1406×10^{-4}
Corollary 11	0.5133	—	—	33.2800	0.0093	0.0929	3.3488	-1.6887×10^{-6}
Corollary 12	0.5976	—	1.2075	36.0390	0.2756	—	3.5497	-1.8091×10^{-4}
Corollary 13	3.5778	22.4603	—	277.0393	0.0608	0.2733	7.6140	-2.2252×10^{-6}
Corollary 14	4.3805	25.1945	8.9098	265.5547	3.0956	—	8.3108	-1.1824×10^{-4}

The solutions to the convex optimization problems in the conclusions obtained for the variations of the (42) system are presented in Table 1 to make it easier to compare the conclusions. Matlab-LMI Toolbox has been used to calculate the numerical solutions in this table.

The tolerable bounds ($a_{\max}$) of the results obtained from Theorem 6 are greater than the tolerable bounds of the results obtained from Theorem 5, as shown in Table 1.

Example 2. Let the nonlinear fractional neutral neural system be given by

$$\begin{aligned} {}_{t_0}D_t^{\alpha}x(t) &= Ax(t) + B_1 f_1(t, x(t)) + B_2 f_2(t, x(t-\tau_1(t))) \\ &\quad + C\,{}_{t_0}D_t^{\alpha}x(t-\tau_2(t)) + B_3\int_{t-\tau_3(t)}^{t} f_3(s, x(s))ds, \end{aligned} \tag{48}$$

where

$$A = \begin{bmatrix} -65 & 5 & 1 \\ 2 & -25 & 1 \\ 3 & 1 & -80 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 6 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 9 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.1 \end{bmatrix},$$

$$f_1(t, x(t)) = a_1 \begin{pmatrix} \frac{1}{\sqrt{2}} \sin(x_1(t) + x_2(t)) \\ \sin(x_3(t)) \\ \frac{1}{\sqrt{2}} \sin(x_1(t) - x_2(t)) \end{pmatrix},$$

$$f_2((t-\tau_1(t))) = 0.5 \begin{pmatrix} e^{-0.6t} \sin(x_1(t-\tau_1(t))) \\ e^{-0.5t} \sin(x_2(t-\tau_1(t))) \\ e^{-0.8t} \sin(x_3(t-\tau_1(t))) \end{pmatrix},$$

$$f_3(t, x(t)) = 0.15 \begin{pmatrix} \frac{1}{\sqrt{2}} \sin(x_1(t) + x_3(t)) \\ \sin(x_2(t) + x_3(t)) \\ \frac{1}{\sqrt{2}} \sin(x_1(t) + x_2(t)) \end{pmatrix}, \tag{49}$$

$\alpha \in (1, 0)$, $\tau_1(t) = 30 + 0.2t + 0.1\cos(t)$, $\tau_2(t) = 10 + 0.3t + 0.2\sin(t)$, and $\tau_3(t) = 0.1 + 0.03\sin(t)$. For $f_1(t, x(t))$, we have

$$\begin{aligned} &\|f_1(t, x(t))\|^2 \\ &\quad = a_1^2\left(\frac{1}{2}\sin^2(x_1(t) + x_2(t)) + \sin^2(x_3(t)) \right. \\ &\qquad \left. + \frac{1}{2}\sin^2(x_1(t) - x_2(t))\right) \\ &\quad \le a_1^2\left(\frac{1}{2}(x_1(t) + x_2(t))^2 + x_3^2(t) + \frac{1}{2}(x_1(t) - x_2(t))^2\right) \\ &\quad = a_1^2\left(x_1^2(t) + x_2^2(t) + x_3^2(t)\right) = a_1^2 x^T(t) M_1^T M_1 x(t), \end{aligned} \tag{50}$$

where $M_1 = I_3$. For $f_2(t, x(t-\tau_1(t)))$, we have

$$\begin{aligned}
&\|f_2(t,x(t-\tau_1(t)))\|^2\\
&\quad= 0.25\left(e^{-1.2t}\sin^2(x_2(t-\tau_1(t))) + e^{-t}\sin^2(x_1(t-\tau_1(t)))\right)\\
&\qquad + e^{-1.6t}\sin^2(x_3(t-\tau_1(t))) \le 0.25\left(x_1^2(t)+x_2^2(t)+x_3^2(t)\right)\\
&\quad= 0.25x^T(t-\tau_1(t))M_2^T M_2 x(t-\tau_1(t)),
\end{aligned} \tag{51}$$

where $M_2 = I_3$. For $f_3(t,x(t))$, we have

$$\begin{aligned}
\|f_3(t,x(t))\|^2 &= 0.0225\left(\sin^2(x_1(t)+x_3(t)) + \sin^2(x_2(t)\right.\\
&\quad \left.+ x_3(t)) + \sin^2(x_1(t)+x_2(t))\right)\\
&= 0.0225\left([x_1(t)+x_3(t)]^2 + \right)[x_2(t)+x_3(t)]^2\\
&\quad + [x_1(t)+x_2(t)]^2\big) = 0.09x^T(t)M_3^T M_3 x(t),
\end{aligned} \tag{52}$$

where

$$M_3 = \begin{bmatrix} \frac{4}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{4}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{4}{6} \end{bmatrix}. \tag{53}$$

Let us choose $d_1 = 0.3$, $d_2 = 0.5$, $d_3 = 0.03$, $\varepsilon = 0.13$, $m = 0.001$. Let the P, Q_1, Q_2, Q_3, R, S matrices be defined by

$$\begin{aligned}
P &= p \times I_3,\\
Q_1 &= q_1 \times I_3,\\
Q_2 &= q_2 \times I_3,\\
Q_3 &= q_3 \times I_3,\\
R &= r \times I_3,\\
S &= s \times I_3,
\end{aligned} \tag{54}$$

for positive numbers p, q_1, q_2, q_3, r, s. $a_{\max}$ is the solution of the convex optimization problem in the corresponding result; $\lambda_{\max}$ is the largest eigenvalue of the matrix in the corresponding result, which must be negative definite.

The solutions to the convex optimization problems in the conclusions obtained for the variations of the (48) system are presented in Table 2 to make it easier to compare the conclusions. Matlab-LMI Toolbox has been used to calculate the numerical solutions in this table.

The tolerable bounds ($a_{\max}$) of the results obtained from Theorem 6 are greater than the tolerable bounds of the results obtained from Theorem 5, as shown in Table 2.

6. Conclusion

The asymptotic stability of solutions for a class of nonlinear fractional neutral neural systems with time-dependent delays is obtained using the Lyapunov-Krasovskii method with linear matrix equation inequality. The obtained sufficient conditions are expressed in terms of LMI to find the less conservative criteria.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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