

Research Article

An Exponential Curve Model and Its Application in Forecasting Private Car Ownership of China

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To solve problems of modeling and prediction of nonstationary sequences caused by deterministic factors in the real world, a new type of exponential curve model with first-order polynomial term is proposed in this paper based on the classical exponential curve model and the modified exponential curve model. According to the characteristics of the new model, the piecewise summation method is adopted to determine the analytical expressions of parameters. Moreover, the model is applied to the ownership of private cars in China. Comparing the calculated results with the classical exponential curve model and the modified exponential curve model, it can be seen that the new exponential curve model has higher accuracy in the analysis of the ownership of private cars in China.

1. Introduction

For time series analysis and statistics, there are many trend curve models used to analyze nonstationary sequences caused by deterministic factors. For example, the parabolic curve model, the Logistic curve model, the exponential curve model, and the Gompertz curve model, see [1–3]. When the trend of the phenomenon is not linear but has certain regularity, it is necessary to match the appropriate trend curve models. If the long-term trend of the phenomenon increases or decreases at roughly the same growth rate in each period, it can be fitted with an exponential curve model. If the trend of the phenomenon increases rapidly in the initial stage, then the growth rate decreases gradually, and finally takes a constant as the growth limit, the modified exponential curve model can be used for fitting.

Yin and Miao [4] used the exponential curve model to predict the data of the hepatitis A epidemic in Quzhou cite with raw data from 1990 to 2005. Compared with the actual incidence, the computational results show that the exponential curve model can obtain convincing results. Qin et al. [5] discussed the subgrade settlement problem by using the exponential curve model and combined with some

measured data of the Wuhan–Guangzhou railway passenger dedicated line. Cheng et al. [6] used the exponential curve method to study the structural fatigue safety of in-service storage tanks with an uneven settlement. Zhuo [7] used the data of 1,907 sets of broad-leaved trees collected in Fujian province as modeling data to construct the exponential curve model of broad-leaved tree polymorphic status (grade) in Fujian Province, and the results have small error and high precision. Sun et al. [8] applied an exponential curve to the speed control of a launch vehicle thrust regulation motor, which achieves the purposes of good fast response performance, short speed up and downtime, stable and accurate control.

In 2009, Wu et al. [9] collected the death toll of the Wenchuan earthquake at each time, fitted it with the modified exponential curve model and the exponential curve model. The model accuracy is verified with the data of the Jiji earthquake and the Hanshin earthquake in Japan. The calculation results show that the modified exponential curve model can well estimate the number of earthquake deaths, to provide disaster relief decision-making reference for earthquake relief headquarters at all levels. Sherman and Morrison [10] provided a simplified procedure for fitting the

Gompertz curve model and the modified exponential curve model. Chen et al. [11] compared between modified exponential model and common models of light-response curve. Su et al. [12] improved the conventional exponential curve by using the Taylor expansion method and then calculated the subgrade section of the high-speed railway. The results show that the fitting prediction curve is in good agreement with the actual curve. Ou et al. [13] constructed a new modified exponential curve model based on the existing model. By fitting the static load test data of different types of single piles, it is verified that the new model can fit the P-S curve of a single pile well. Tatik et al. [14] designed the system parameters of the modified exponential models with three parameters. Recently, Tan [15] proposed an exponential curve model with oscillation terms based on the classical modified exponential curve and applied it to the primary energy consumption. Yu et al. [16] studied a new type of exponential curve model and applied it to the tertiary industry of China. The system parameters are estimated by a nonlinear least square method. The minimum residual sum of squares is taken as the objective function to develop a nonlinear unconstrained optimized problem, and the trust region algorithm is used to numerically determine the values of system parameters. However, it is worth noting that in reference [15, 16], the author did not give the analytical expression of the model parameters but used the numerical calculation method to solve the model parameters.

In this paper, inspired by classical exponential curve, modified exponential curve, a new exponential curve model with a first-order polynomial term is proposed, and the three-sum method is extended and applied in this model to solve the analytical expressions of model parameters. Finally, taking private car ownership in China as an example, the calculation results are compared with the traditional exponential model and the modified exponential model. It is obvious from the numerical results that the model with first-order polynomial term in this paper has higher accuracy.

2. The Exponential Curve Model and the Modified Exponential Curve Model

2.1. The Exponential Curve Model. It can be seen from references [1–3] that if the long-term trend of the phenomenon increases or decreases with the same growth rate in each period, it can be fitted with an exponential curve model. The classical exponential curve equation is

$$Y_t = ab^t. \quad (1)$$

In order to estimate the model parameters, generally, logarithms are taken on both sides of equation (1) to obtain

$$\ln Y_t = \ln a + t \ln b. \quad (2)$$

Then, using the least square method and equation (2), the following equation is obtained:

$$\begin{cases} \sum \ln Y = n \ln a + (\sum t) \ln b, \\ \sum t \ln Y = (\sum t) \ln a + (\sum t^2) \ln b. \end{cases} \quad (3)$$

The estimated values of parameters a and b can be obtained by estimating parameters $\ln a$ and $\ln b$, and taking the antilogarithm.

2.2. The Modified Exponential Curve Model. According to the monographs [1–3], if the trend of the phenomenon increases rapidly in the initial stage, then the growth rate decreases gradually, and finally takes a constant as the growth limit, the modified exponential curve can be used for modeling and fitting. By adding a constant c on the formula of the classical exponential curve, the equation of the modified exponential curve model is outlined as

$$Y_t = ab^t + c, \quad (4)$$

where a, b, c are the unknown parameters, $a \neq 0, b \in (0, 1) \cup (1, \infty), c \in (0, \infty)$.

The basic idea of estimating parameters a, b, c is the three-sum method. The detailed calculation process is as follows: the length of the observations is divided into three equal arrays, each group has m items. The three parameters are determined according to the three local summation of trend value \bar{Y}_t equal to the three local summation of observation value Y_t of the original series. Specifically, let three local summations of the observed values be S_1, S_2, S_3 , which are

$$\begin{aligned} S_1 &= \sum_{t=1}^m Y_t, \\ S_2 &= \sum_{t=m+1}^{2m} Y_t, \\ S_3 &= \sum_{t=2m+1}^{3m} Y_t. \end{aligned} \quad (5)$$

The following set of equations is obtained by the three-sum method:

$$\begin{cases} S_1 = mc + a + ab + ab^2 + \dots + ab^{m-1}, \\ S_2 = mc + ab^m + ab^{m+1} + \dots + ab^{2m-1}, \\ S_3 = mc + ab^{2m} + ab^{2m+1} + \dots + ab^{3m-1}. \end{cases} \quad (6)$$

By solving equation (6), we obtain

$$\begin{cases} b = \left(\frac{S_3 - S_2}{S_2 - S_1} \right)^{1/m}, \\ a = (S_2 - S_1) \frac{b - 1}{(b^m - 1)^2}, \\ c = \frac{1}{m} \left(S_1 - \frac{a(b^m - 1)}{b - 1} \right). \end{cases} \quad (7)$$

3. An Exponential Curve Model with First-Order Polynomial Term

Inspired by the classical exponential curve and the modified exponential curve models, an exponential curve model with first-order polynomial term is proposed. The three sum method used in the modified exponential curve is extended and applied to this model for solving model parameters. The general equation of the new exponential curve model proposed in this paper is

$$Y_t = ab^t + ct + d. \tag{8}$$

As can be seen from equation (8), if $c = 0, d = 0$, the new exponential curve model with the first-order polynomial term degenerates into the classical exponential curve model. If $c = 0$, the new exponential curve model with first-order polynomial term degenerates into the modified exponential curve model.

Next, the specific expressions of parameters a, b, c, d in the system are derived by using the idea of three sum method. First, we divide the data used for modeling into four equal groups, each group has m items. Actually, the lengths

of the original data are the following four cases, namely, $4m, 4m + 1, 4m + 2, 4m + 3$, respectively. To derive system parameters, we choose a subsequence with $4m$ data from a sequence. The four parameters are determined according to the four local summation of trend value \hat{Y}_t equal to the four local summation of observation value Y_t of the original series. Specifically, let four local summations of the observed values be S_1, S_2, S_3, S_4 , which are

$$\begin{aligned} S_1 &= \sum_{t=1}^m Y_t, \\ S_2 &= \sum_{t=m+1}^{2m} Y_t, \\ S_3 &= \sum_{t=2m+1}^{3m} Y_t, \\ S_4 &= \sum_{t=3m+1}^{4m} Y_t. \end{aligned} \tag{9}$$

Then, the following equations are obtained, respectively:

$$\left\{ \begin{aligned} S_1 &= \sum_{t=1}^m ab^t + c \sum_{t=1}^m t + \sum_{t=1}^m d = \frac{ab(1-b^m)}{1-b} + c \frac{1+m}{2} m + md, \\ S_2 &= \sum_{t=m+1}^{2m} ab^t + c \sum_{t=m+1}^{2m} t + \sum_{t=m+1}^{2m} d = \frac{ab^{m+1}(1-b^m)}{1-b} + c \frac{1+3m}{2} m + md, \\ S_3 &= \sum_{t=2m+1}^{3m} ab^t + c \sum_{t=2m+1}^{3m} t + \sum_{t=2m+1}^{3m} d = \frac{ab^{2m+1}(1-b^m)}{1-b} + c \frac{1+5m}{2} m + md, \\ S_4 &= \sum_{t=3m+1}^{4m} ab^t + c \sum_{t=3m+1}^{4m} t + \sum_{t=3m+1}^{4m} d = \frac{ab^{3m+1}(1-b^m)}{1-b} + c \frac{1+7m}{2} m + md. \end{aligned} \right. \tag{10}$$

Furthermore, we set $Z_i = S_{i+1} - S_i, i = 1, 2, 3$; there are

$$\left\{ \begin{aligned} Z_1 &= S_2 - S_1 = \frac{ab(1-b^m)}{1-b} (b^m - 1) + cm^2, \\ Z_2 &= S_3 - S_2 = \frac{ab^{m+1}(1-b^m)}{1-b} (b^m - 1) + cm^2, \\ Z_3 &= S_4 - S_3 = \frac{ab^{2m+1}(1-b^m)}{1-b} (b^m - 1) + cm^2. \end{aligned} \right. \tag{11}$$

Similarly, we set $W_i = Z_{i+1} - Z_i, i = 1, 2$, and we have that

$$\left\{ \begin{aligned} W_1 &= Z_2 - Z_1 = \frac{ab(1-b^m)(b^m - 1)}{1-b} (b^m - 1), \\ W_2 &= Z_3 - Z_2 = \frac{ab^{m+1}(1-b^m)(b^m - 1)}{1-b} (b^m - 1). \end{aligned} \right. \tag{12}$$

By solving equation (12), we obtain that

TABLE 1: Calculation results of private car ownership data by exponential curve, modified exponential curve, and the new exponential curve of first order polynomial term.

Year	Actual value	Exponential curve	APE (%)	Modified exponential curve	APE (%)	New exponential curve	APE (%)
2003	1219.23	1353.2936	10.9958	1151.4744	5.5572	1371.5434	12.4926
2004	1481.66	1659.5926	12.0090	1480.2007	0.0985	1451.6567	2.0250
2005	1848.07	2035.2180	10.1267	1880.9844	1.7810	1725.7599	6.6183
2006	2333.32	2495.8609	6.9661	2369.6205	1.5557	2190.4378	6.1236
2007	2876.22	3060.7638	6.4162	2965.3666	3.0994	2842.3354	1.1781
2008	3501.39	3753.5246	7.2010	3691.7013	5.4353	3678.1568	5.0485
2009	4574.91	4603.0820	0.6158	4577.2499	0.0511	4694.6642	2.6176
2010	5938.71	5644.9248	4.9470	5656.9122	4.7451	5888.6766	0.8425
2011	7326.79	6922.5741	5.5170	6973.2385	4.8255	7257.0692	0.9516
2012	8838.60	8489.4012	3.9508	8578.1058	2.9472	8796.7721	0.4732
2013	10501.68	10410.8576	0.8648	10534.7629	0.3150	10504.7694	0.0294
2014	12339.36	12767.2086	3.4673	12920.3228	4.7082	12378.0985	0.3139
2015	14099.10	15656.8864	11.0488	15828.8015	12.2682	14413.8487	2.2324
2016	16330.20	19200.6021	17.5773	19374.8241	18.6441	16609.1607	1.7083
2017	18515.10	23546.3879	27.1740	23698.1410	27.9936	18961.2255	2.4095
2018	20574.93	28875.7812	40.3445	28969.1368	40.7982	21467.2836	4.3371
2019		35411.4076		35395.5441		24124.6239	
2020		43426.2809		43230.6314		26930.5833	
MAPE _{simu} (%)		5.6438		2.6875		2.3838	
MAPE _{fit} (%)		24.0361		24.9260		2.6718	
MAPE _{total} (%)		10.5484		8.6178		2.4606	

$$\left\{ \begin{aligned} b &= \left(\frac{W_2}{W_1} \right)^{1/m}, \\ a &= \frac{(1-b)W_1}{b(1-b^m)(b^m-1)^2}, \\ c &= \frac{Z_1}{m^2} - \frac{ab(1-b^m)(b^m-1)}{m^2(1-b)}, \\ d &= \frac{S_1}{m} - \frac{ab(1-b^m)}{m(1-b)} - \frac{m+1}{2}c. \end{aligned} \right. \quad (13)$$

So far, we have obtained the specific expressions of model parameters through the analytical method. Once the original sequence is given, it can be analyzed by the new model.

4. Numerical Results

Firstly, we give some expressions to measure the accuracy of different models. According to the calculated values and the actual values, the absolute percentage error (APE) and mean absolute percentage error (MAPE) are determined as follows:

$$\begin{aligned} APE &= \left| \frac{Y_t - \hat{Y}_t}{Y_t} \right| \times 100\%, t = 1, 2, \dots, n, \\ MAPE &= \frac{1}{v-l+1} \sum_{t=l}^v \left| \frac{Y_t - \hat{Y}_t}{Y_t} \right| \times 100\%, v \leq n, \end{aligned} \quad (14)$$

where when $l = 2, v = 4m$, the MAPE is named as MAPE_{simu}, when $l = 4m + 1, v = n$, the MAPE is named as MAPE_{fit},

when $l = 2, v = n$, the MAPE is named as MAPE_{total}, the \hat{Y}_t is the calculated values, Y_t is the raw data, $4m$ is the number of samples used for modeling, and n is the total number of samples.

Next, the statistical data of private car ownership in China from 2003 to 2018 are used for research by combined with MATLAB programming software and exponential curve model. First, the exponential curve models are established based on the first 12 data from 2003 to 2014, and are verified by the left 4 data from 2015 to 2018, and further are predicted China's private car ownership in 2019 and 2020. The numerical calculation results of the classical exponential curve model, the modified exponential curve model, and the exponential curve model with first-order polynomial term are shown in Table 1, and the corresponding graphs are shown in Figures 1 and 2, respectively.

The calculation results show that the exponential curve model with the first-order polynomial term is closer to the real data. The mean absolute modeling percentage error computed by the first 14 data from 2013 to 2014, the mean absolute fitting percentage error computed by the last 4 data from 2015 to 2018, and the total mean absolute percentage error computed by all data from 2003 to 2018 of exponential curve are 5.6438%, 24.0361%, and 10.5484%, respectively. The modeling error, fitting error, and total error of the modified exponential curve are 2.6875%, 24.9260%, and 8.6178%, respectively. The modeling error, fitting error, and total error of the exponential curve with the first-order polynomial term are 2.3838%, 2.6718%, and 2.4606%, respectively. The effect of the exponential curve model and the modified exponential curve model on data fitting is very poor, which is far worse than the accuracy of first-order polynomial exponential curve. It can be seen from Table 1 and Figures 1 and 2 that the exponential curve with first-

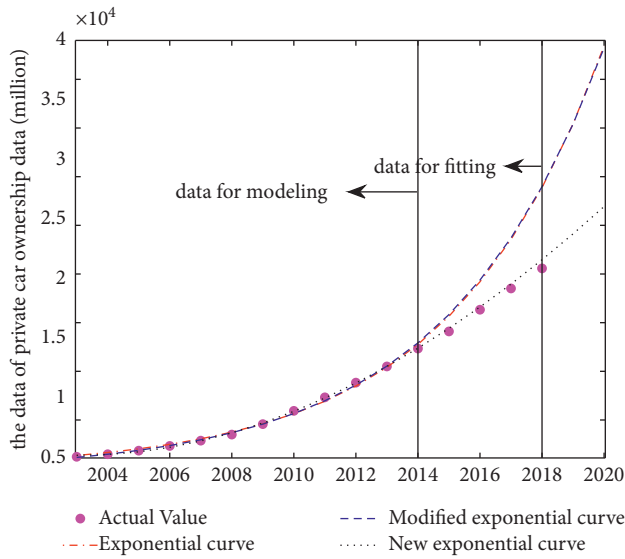


FIGURE 1: Fitting and prediction of private car ownership data by exponential curve, modified exponential curve, and new exponential curve with first-order polynomial term.

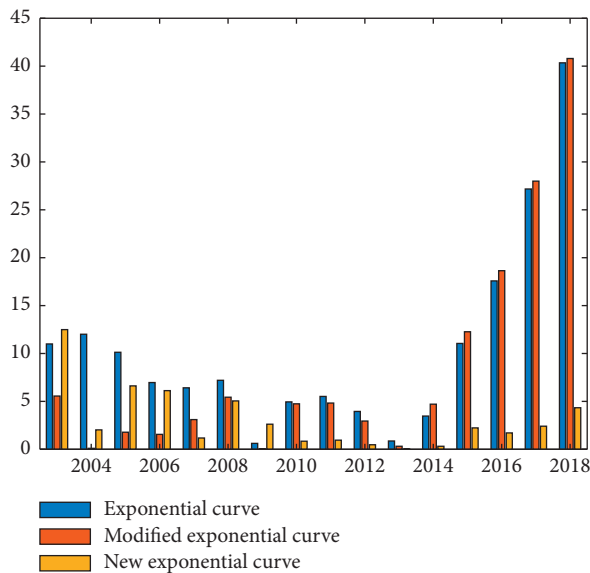


FIGURE 2: Absolute percentage error of private car ownership data by exponential curve, modified exponential curve, and new exponential curve with first-order polynomial term.

order polynomial term has higher accuracy for private car ownership data.

5. Conclusion

In this paper, a new exponential curve model with first-order polynomial term is discussed, and the specific expressions of four parameters in the model are given by making full use of the idea of piecewise summation. Finally, taking the ownership of private cars in China as an example, it shows that the modeling accuracy and fitting accuracy of this model

under some data are higher than the classical exponential curve and the modified exponential curve models.

It follows from the structure of the exponential curve model with first-order polynomial term that this is a non-linear forecasting model. In most cases, the analytical expressions of system parameters cannot be deduced. Therefore, some numerical calculation methods are selected to numerical determine the values of system parameters. In this work, we novelty used a piecewise summation method to derive the expressions of these parameters. It is worth mentioning that the idea of piecewise summation in this paper is simple and feasible. In the future research, this method can be applied to the parameter solution of other similar models.

On the other hand, this piecewise summation has its own disadvantages. We only use $4m$ raw observations to construct a set of equations to derive system parameters, and the left $n-4m$ data are used for fitting. This means that there are certain requirements for the length of data for modeling when deriving model parameters. Thus, the revision of this method is also an important work to be considered in the future.

Data Availability

All data generated or used during the study are available within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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