Hindawi
Journal of Mathematics
Volume 2022, Article ID 6892342, 11 pages
https://doi.org/10.1155/2022/6892342

Research Article

Solving Stochastic Fuzzy Transportation Problem with Mixed Constraints Using the Weibull Distribution

T.K. Buvaneshwari and D. Anuradha

Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore, TamilNadu, India

Correspondence should be addressed to D. Anuradha; anuradhadhanapal1981@gmail.com

Received 8 March 2022; Revised 22 June 2022; Accepted 16 July 2022; Published 22 September 2022

Academic Editor: M. M. Bhatti

Copyright © 2022 T.K. Buvaneshwari and D. Anuradha. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The development in the industries has necessitated the growth of transportation methods. Due to the variation in the transportation systems, many problems seem to arise in the present times. One such problem is the stochastic fuzzy transportation problem (SFTP). The SFTP is a chance-constrained programming (CCP) problem with probabilistic constraints where supply and demand are randomness, and the objective function is in fuzzy nature. In this paper, we have developed three models for the SFTP, where the constraints are mixed type following Weibull distribution (WD). The aim of the research work is to optimize the transportation cost in FTP under probabilistic mixed constraints. In order to achieve this, the cost coefficient of the fuzzy objective function is converted to alpha cut representation, and the probabilistic mixed constraints are converted to deterministic form by using the WD. The proposed models are illustrated by providing a numerical example, and the problem is solved using Lingo software. It is worth pointing out that these models are constructed from different points of view. The decision-maker's (DM's) preference has the final say in the usage of the models. A sensitivity analysis (SA) is performed to explore the sensitivity of the parameters in the proposed model.

1. Introduction

Decision-making plays a vital role in many domains, including Economics, Psychology, Philosophy, Mathematics, and Statistics. The importance of transportation needs to be recognized as a necessary part of distribution networks. The primary objective of the transportation problem (TP) is to lower the cost of moving products and materials among customers and producers to facilitate the manufacturer in meeting the requirement of the customers. Cost, supply, and demand are the parameters of the TP. Although several modes of transportation are available for shipments of commodities in a transportation system, we may convey items from sources to destinations using distinct modes of transportation to save money or fulfill deadlines. The usual transportation system is transformed into an unequal transportation problem with mixed constraints (TPMC) if the capacity of a supply is appreciably expanded/reduced and the requirement of a demand is also appreciably expanded/decreased. The special type of transportation problem with mixed constraints has been meticulously studied first by Brigden [1]. Different approaches to solving different models of the TP with mixed constraints abound in the literature [2–7]. Gupta et al. [8] extended multiobjective capacitated TPMC in a fuzzy environment. In the real-world situation, the two main factors of uncertainties are randomness and fuzziness (or vagueness). The stochastic variability of all possible outcomes of a situation is referred as randomness, which may be completely and quantitatively characterized by using a random variable (RV) in probability theory. Fuzziness, on the other hand, arises from the imprecision of subjective human knowledge and manifests objectively in a range of contexts, including data gathering and processing, hazy parameter limits, expertise applications, and a lack of accurate information. Stochastic transportation problems (STP) and fuzzy transportation problems (FTP) are the names given to TP that are modeled in such conditions. It is observed that the randomness and fuzziness in information govern the decision-making problems in the real world. The information on weather forecasting, predicting the stock market, the study of the economic status of the industrial sector, etc., is not certain. The uncertainty in data is such that fuzzy programming problems and stochastic programming problems cannot be used in isolation to find the appropriate mathematical model. Hence, we consider both fuzziness and randomness while dealing with fuzzy stochastic programming problem. Since most of the mathematical programming models are governed by real-life decision-making problems having multiple numbers of conflicting functions, in our study, we have considered a fuzzy stochastic programming problem.

Decision-making in a fuzzy environment is first developed by Zadeh. The fuzzy set theory has been used to handle a number of real-world problems, such as financial engineering and risk management. The reason seems to be that the strategy enables the DM to study and deal with the unknown elements that are present. Then, employing fuzzy quantities and/or fuzzy restrictions, the imperfect knowledge of asset returns, and the uncertainty associated with capital market behavior can be included. Fuzzy numbers, which are fuzzy subsets of the real line, can be used to represent fuzzy numerical data. Zimmermann [9] discussed fuzzy programming and linear programming with a number of different objective functions. Behera and Nayak [10] have solved the multiobjective mathematical programming in the fuzzy approach with the alpha cut. In stochastic models, the random variables are considered for each problem parameter.

A stochastic programming (SP) problem arises when the probability is present in a mathematical programming problem. It is concerned with decision-making in which some or all parameters are traced as RVs to represent uncertainty. Researchers such as Kataoka, Williams, and Agarwal [5–7] have undertaken numerous studies on the topic of random transportation. Moreover, DMs are confronted with scenarios in which they must select a value from a group of options. In this case, mathematical programming comes to the DMs assistance. The chance-constrained technique is one of the most effective methods to handle optimization problems (OP) with numerous uncertainties. It is the formulation of an OP that ensures that the probability of satisfying a given constraint is greater than a certain benchmark. In other words, it restricts the feasible region so that the solution has a high level of confidence. Although the chance-constrained technique is a rather robust strategy, it can be challenging to implement. If the linear constraints of a TP are probabilistic, it becomes a CCP problem. This paper discusses the parameters used to address uncertainty using both probability theory and fuzzy set theory.

1.1. Literature Review. This section contains a survey of the literature on TP and FTP in SP. Charnes and Copper [11] established TP, and Kataoka [12] discussed the SP method for a TP. Spoerl and Wood [13] analysed the stochastic generalized assignment problem. Williams [14] defined a stochastic variable for demand using a joint cumulative distribution function and then investigated a stochastic TP with a supply and a demand. Agrawal and Ganesh [15] proposed a TP solution incorporating stochastic demand and nonlinear multichoice costs. Powell and Topaloglu [16] studied SP in transportation and logistics. Anholcer [17] proposed a stochastic generalized TP with a discrete distribution of demand. Ojha et al. [18] designed a stochastic discounted multiobjective STP with demand as a stochastic parameter, converting stochastic variables to deterministic ones using the expected value criteria. Holmberg and Tuy [19] demonstrated a branch-and-bound method for solving the TP when demand is stochastic and production costs are concave. However, in real-world situations, both supply and demand are stochastic. Mahapatra et al. [20] focused their efforts on inequality constraints in a multiobjective STP, where all supply and demand are log-normal RVs, and the objectives are incommensurable and contradictory. Agrawal [21] solved the STP by using an artificial intelligence approach in a stochastic environment. The following authors discussed articles that involve both fuzziness and randomness. A fuzzy goal programming method for STPs with restricted resources was discussed by Aruna Chalam [22]. Giri et al. [23] proposed fuzzy stochastic solid TP using a fuzzy goal programming approach. Acharya et al. [24] demonstrated how to compute a multiobjective fuzzy STP. Gessesse [25] developed a fuzzy programming approach based on a genetic algorithm for multiobjective linear fractional stochastic TP employing a four-parameter Burr distribution. Maity et al. [26]studied the optimal intervention in transportation networks using fuzzy stochastic multimodal systems.

In this paper, the WD function could be employed to define the stochastic variable. We express supply and demand as stochastic variables. Weibull [27], a Swedish researcher, invented the WD in the 1950s, and it has since become a helpful and informative data treatment tool in reliability studies of components and electronic systems. The WD is both theoretically and practically pertinent to Fok's [28] studies of future probability estimates under a given loading. Since the WD has flexibility in location, shape, and scale, it can design an extensive range of failure rates. The WD is defined mathematically as follows: the probability density function of an RV y is described by $f = (\chi/\lambda)((y-\xi)/(\lambda))^{\chi-1} e^{-(((y-\xi)/(\lambda))^{\chi})}$, where $\chi > 0, \lambda > 0, y \geq \xi$ andξ, χ and λ are the location, shape, and scale parameters, and then, random variable y is called WD. In dependability applications, the WD has been shown to be beneficial for characterizing cost times and lifetimes. Distribution has received the greatest attention in recent decades. Although the WD is a good choice for describing data on lifespan or strength, its performance falls below its competitors in several practical situations. Furthermore, the WD has applications in geosciences and transportation among other fields. Klakattawi [29] investigated the features and uses of the Weibull-gamma distribution. Mahapatra [30] investigated the characteristics and applications of the Weibull-gamma distribution.

1.2. Research Gap, Motivation, and the Proposed Contribution. The incidence of mixed constraint in TPs is a common phenomenon that may be seen in reality. The mixed constraint paradox in a TP occurs when it is possible to ship more total goods for less (or equal) total cost, while transporting the same amount or more from each origin and to each destination and keeping all the shipment costs nonnegative. However, there may be situations where cost coefficients, availability, and demand quantities are uncertain due to some uncontrollable factors. Uncertainty can be mainly classified into two types, namely fuzzy TP and stochastic TP. The TP in which the transportation cost, supply, and demand are fuzzy quantities is called fuzzy TP. The objective of the FTP is to determine the shipping schedule that minimizes the total fuzzy transportation cost while satisfying fuzzy supply and fuzzy demand limits. The situation where the parameters are imprecise in a stochastic sense and described by random variables with known probability distributions is called stochastic TP. The main benefits of fuzziness and randomness techniques are that they do not require prior predictable regularities and can deal with imprecise input information incorporating feelings and emotions quantified based on the DM's subjective evaluation. The STP and stochastic fuzzy transportation problems (SFTP) with equality constraints have recently received significant attention, with various models and solution methods applied to stochastic environments. The literature search (Table 1) reveals no systematic model for addressing stochastic fuzzy transportation problem with mixed constraints (SFTPMC). But in reality, imprecise mixed constraints that supply at origins may be uncertain, since any trouble or delay may occur for various reasons and demand requirement uncertainty cannot be avoided often due to inexact forecasting or requirement volatility. This has motivated us to develop the model for FTP mixed constraint under a stochastic environment.

Based on this motivation, the main contribution of this study is summarized as follows: the goal of this paper is as follows: (1) to propose a mathematical model that deals with fuzziness and randomness under one roof, (2) to consider the fuzzy objective value and probabilistic constraints, and (3) to present a simplified computation conversion of probabilistic constraints to their equivalent deterministic constraint using WD. In order to achieve this, the fuzzy objective is converted to alpha cut representation, and probabilistic constraints are converted to deterministic form by using the WD and then solved using Lingo software. To investigate the sensitivity of the parameters in the proposed model, a sensitivity analysis is performed.

1.3. Managerial Insights Based on the Research. The last decade has seen the growth in various fields such as economics, trade, health care, transportation, cultivation, army, engineering, and technology. It has increased the importance of stochastic fuzzy optimization in solving scientific managerial decision-making problems in every field. The uncertainty factors which make the problem more complex have not spared the transportation process. The objective of this paper is to optimize the total transportation cost under uncertain conditions by utilizing the SFTPMC model. Generally, uncertainty is described using fuzzy or probability theory. However, it is not always reasonable to use fuzzy theory or probability theory to illustrate all the indeterminacies, as it requires adequate information. So, we prefer to describe the uncertainty with both fuzzy and probability parameters. In fact, low-frequency events occur in our daily lives causing the uncertainty theory to take effect. Under these circumstances, scheduling a proper transportation plan to lower the total costs becomes a challenge. This study provides an applicable model for the DM to deal with some uncertain factors without losing reliability from customers. This research could also be extended to other aspects, in which a decision/plan needs to be made in uncertain conditions.

The rest of this article is structured as follows. Section 2 presents the preliminaries that are related to this article. Section 3 deals with the assumptions and notation used in this paper. Section 4 investigates and converts the mathematical programming models of SFTPMC and its variants to the comparable deterministic models. Section 5 presents the proposed solution method with an illustration. Section 6 presents and analyses the computational experiences of SA. Finally, Section 7 summarizes the research outcome and limitations and suggests future directions.

2. Some Preliminaries

Definition 1 (Fuzzy set, Zadeh., [38]). Let R be a collection of sets and $\mu_R(x)$ be a membership function from R to $[0,1]$. A fuzzy set $\widetilde{R}$ with the membership function $\mu_R(x)$ is defined by $\widetilde{R} = \{(x, \mu_R(x))/\ x \in R \text{ and } \mu_R(x) \in [0,1]\}$.

Definition 2 (Fuzzy number, Zadeh., [38]). A real fuzzy number $\widetilde{r} = (r_1, r_2, r_3)$ is a fuzzy subset of the real line R with the membership function $\mu_{\widetilde{r}}(r)$ satisfying the following conditions:

(i) $\mu_{\widetilde{r}}(r)$: $R \longrightarrow [0,1]$ is continuous
(ii) $\mu_{\widetilde{r}}(r) = 0$ for all $(-\infty, r_1] \cup [r_3, \infty)$
(iii) $\mu_{\widetilde{r}}(r)$ is strictly increasing on $[r_1, r_2]$ and strictly decreasing on $[r_2, r_3]$
(iv) $\mu_{\widetilde{r}}(r) = 1$ for all $r \in r_2$ where $r_1 \le r_2 \le r_3$

Definition 3 (Triangular fuzzy number, Zadeh., [38]).A fuzzy number $\widetilde{R}$ is denoted as a triangular fuzzy number by (r_1, r_2, r_3) where a_1, a_2, and a_3 are real numbers, and its membership function $\mu_{\widetilde{A}}(x)$ is given as follows:

$$\mu_{\widetilde{R}}(x) = \begin{cases} \dfrac{x - r_1}{r_2 - r_1}, & r_1 \le x \le r_2, \\ \dfrac{x - r_3}{r_2 - r_3}, & r_2 \le x \le r_3, \\ 0, & \text{otherwise}. \end{cases} \tag{1}$$

Table 1: Comparison of the approach to the present models.

Authors	Problem type	Stochastic parameters	Distribution	Methodology
Gessesse [31]	Fractional TP	Demand & supply	Normal distribution	Simulation-based genetic algorithm
Jerbi [32]	TP	Demand & supply	The power law distribution	Fuzzy programming approach
Al Qahtani et al. [33]	TP	Demand & supply	Extreme value distribution	Goal programming approach
Nasseri and Bavandi [34]	TP	Demand & supply	Expectation value model	Fuzzy programming approach
Dutta et al. [35]	Fuzzy TP	Demand & supply	Fuzzy lognormal distribution and the confidence levels are treated as fuzzy numbers	Genetic algorithm approach
Mahapatra et al. [36]	TP	Demand & supply	Logistic distribution	A transformation technique is presented for manipulating cost coefficients involving multichoice for binary variables with auxiliary constraints and solved by Lingo software
Agrawal and Ganesh [15]	TP	Demand	Galton distribution	Parameters are replaced by Newton's divided difference interpolating polynomial and solved by Lingo software
Das and Lee [37]	Solid TP	Demand, supply, and capacity	Weibull distribution	Global criterion method and fuzzy goal programming approach
This article	SFTPMC	Demand & supply	Weibull distribution	Alpha cut representation for cost function and WD for constraints and then solved by Lingo software

Definition 4 (Alpha cut, Zadeh., [38]). The alpha cut of a fuzzy number $R(x)$ is defined as $R(\alpha) = \{((x)/\mu(x)) \geq \alpha, \quad \alpha \in [0,1]\}$.

Definition 5 (Linear membership function, [10]). A linear membership function can be defined as $\mu_R(X) = \begin{cases} 0 & \text{if } x_{ij} < \underline{x}_{ij} \\ ((\overline{x}_{ij} - x_{ij})/\overline{x}_{ij} - \underline{x}_{ij}) & \text{if } \underline{x}_{ij} < x_{ij} < \overline{x}_{ij} \\ 1 & \text{if } x_{ij} > \overline{x}_{ij} \end{cases}.$

In order to transform the fuzzy system to a deterministic set, the alpha cut representation using linear membership function is $((\overline{x}_{ij} - x_{ij})/(\overline{x}_{ij} - \underline{x}_{ij})) = \alpha$ such that $x_{ij} = (1-\alpha)\overline{x}_{ij} + \alpha\underline{x}_{ij}$ for all $\alpha \in [0, 1]$.

Definition 6 (Feasible solution). Any set of $\{x_{ij} \geq 0, i = 1,2, \ldots, m;\ j = 1,2, \ldots n\}$ that satisfies all the constraints is called a feasible solution to the problem.

Definition 7 (Optimal solution). A feasible solution to the problem which minimizes the total shipping cost is called an optimal solution to the problem.

3. Assumptions and Notations

An SFTPMC is associated with costs, supply, and demand. The direct value is the cost of transportation per unit amount. The mixed constraint occurs for the transportation activity between a source and a destination. The following notations are introduced in order to develop the mathematical model for the SFTPMCs:

a_i: the amount of homogeneous product availability at the source i

b_j: the amount of homogeneous product demand at the destination j

$\tilde{c}_{ij}$: fuzzy transportation cost per unit for carrying one unit of goods from source i to destination j

x_{ij}: the amount shipped from source i to destination j

p_{a_i}: probabilities for a_i

p_{b_j}: probabilities for b_j

χ_{a_i}: shape parameter for a_i

χ_{b_j}: shape parameter for b_j

λ_{a_i}: scale parameter for a_i

λ_{b_j}: scale parameter for b_j

ξ_{a_i}: location parameter for a_i

ξ_{b_j}: location parameter for b_j

4. Problem Formulation and Solution Concepts

The objective value and constraints play an important role for SFTPMC. Minimizing the total transportation costs is our main objective. In different real-life situations, parameters (cost, supply, and demand) become uncertain in nature; then, DM faces difficulty to make the optimal decision. This situation can be handled with fuzzy and random variables. In this model, we consider cost as triangular fuzzy variables and constraints as random variables. The uncertainty in supply or demand constraints may or may not occur, depending on the situation of DM. Therefore, we formulate three SFTPMC models based on the uncertainty in constraints. The model explained below shows the formulation of an SFTPMC model with m sources and n destinations.

4.1. Chance-Constrained Programming Model for SFTPMC. To obtain the quantiles of a probability distribution function in a closed form, it is necessary to apply the constraints in the suggested SP model to the deterministic constraints. Another reason to adopt the WD is that it has the closed form of the quantiles. The CCP paradigm for SFTPMC is described in this paper as follows:

$$(P)\,\text{Minimize}\,\tilde{z} = \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}\,x_{ij}. \tag{2}$$

Subject to

$$P\left(\sum_{j=1}^{n}x_{ij}\geq a_i\right)\geq p_{a_i},\quad i\in q_1, \tag{3}$$

$$P\left(\sum_{j=1}^{n}x_{ij}= a_i\right)\geq p_{a_i},\quad i\in q_2, \tag{4}$$

$$P\left(\sum_{j=1}^{n}x_{ij}\leq a_i\right)\geq p_{a_i},\quad i\in q_3, \tag{5}$$

$$P\left(\sum_{i=1}^{m}x_{ij}\geq b_j\right)\geq p_{b_j},\quad j\in r_1, \tag{6}$$

$$P\left(\sum_{i=1}^{m}x_{ij}= b_j\right)\geq p_{b_j},\quad j\in r_2, \tag{7}$$

$$P\left(\sum_{i=1}^{m}x_{ij}\leq b_j\right)\geq p_{b_j},\quad j\in r_3, \tag{8}$$

$$x_{ij}\geq 0,\quad i=1,2,\ldots,m,\quad j=1,2,\ldots,n, \tag{9}$$

where p_{a_i} and p_{b_j} are probabilities given. It is observed that RVs a_i and b_j represent supply and demand, following the WD, respectively. The WD for a_i has three parameters ξ_{a_i}, χ_{a_i}, and λ_{a_i} which serve as location, scale, and shape parameters. Similarly, the parameters for b_j are defined as ξ_{b_j}, χ_{b_j}, andλ_{b_j}. Constraints (3)–(5) are the probabilistic constraint for the quantity of supply at origin i, which ensures with a given probability p_{a_i}, the sum of quantities of shipments from supply source i must distribute exactly a_i units of supply or it must distribute at least a_i units of supply or it must distribute at most a_i units of supply which $\{q_1, q_2,$ and $q_3\}$ are partitions of $i=\{1, 2, 3, \ldots, \text{m}\}$. In a similar manner, constraints (6)–(8) can be construed for the demand at destination j, must receive exactly b_j units, or it must receive at least b_j units or it receives at most b_j units of demand which $\{r_1, r_2,$ and $r_3\}$ are partitions of $j=\{1, 2, 3, \ldots, \text{n}\}$. $\tilde{c}_{ij}$ is the fuzzy transportation cost (1) per unit for carrying one unit of goods from sources i to destination j. The aim is to describe the quantity x_{ij} transported from origin i to destination j while minimizing the total transport cost when satisfying mixed type supply and demand constraints. For the general cases of TPMC type, in which a single random variable among a_i and b_j is uncertain, the transformation of the probabilistic parameter into a deterministic parameter is studied in Case I and II.

4.1.1. Case (I) Only a_i is Uncertain

(i) For $P(\sum_{j=1}^{n} x_{ij}\geq a_i)\ \geq p_{a_i},\quad i\in q_1$ the proof is shown below

It is considered that a_i $(i\in q_1 =1, 2, m_1)$ are independent RVs with the WD and three known parameters, ξ_{a_i}, χ_{a_i}, andλ_{a_i}. Equation (3) can therefore be transformed as follows:

$$P\left(a_i\leq\sum_{j=1}^{n}x_{ij}\right)\geq p_{a_i},\quad i\in q_1. \tag{10}$$

Let us consider $\sum_{j=1}^{n}x_{ij}=\delta_{a_i}$

$P(a_i\leq\delta_{a_i})\geq p_{a_i},\quad i\in q_1$, by using WD for the probabilistic function,

$$\int_{-\infty}^{\delta_{a_i}}\frac{\chi_{a_i}}{\lambda_{a_i}}\left(\frac{a_i-\xi_{a_i}}{\lambda_{a_i}}\right)^{\chi_{a_i}}{}^{-1}e^{-\left(\left(a_i-\xi_{a_i}/\lambda_{a_i}\right)^{\chi_{a_i}}\right)}\mathrm{d}a_i\geq p_{a_i},\quad i\in q_1. \tag{11}$$

Simultaneously, $a_i\geq\xi_{a_i}i\in q_1$, and the integration of equation (11) gives the following form:

$$\int_{\xi_{a_i}}^{\delta_{a_i}}\frac{\chi_{a_i}}{\lambda_{a_i}}\left(\frac{a_i-\xi_{a_i}}{\lambda_{a_i}}\right)^{\chi_{a_i}^{-1}}e^{-\left(\left(a_i-\xi_{a_i}/\lambda_{a_i}\right)^{\chi_{a_i}}\right)}\mathrm{d}a_i\geq p_{a_i}. \tag{12}$$

After integrating equation (12), we get

$$1-e^{-\left(\left(\delta_{a_i}-a_i/\lambda_{a_i}\right)^{\chi_{a_i}}\right)}\geq p_{a_i}. \tag{13}$$

Equation (13) can be further simplified by applying the logarithm as,

$\delta_{a_i}\geq\xi_{a_i}+\lambda_{a_i}\{-1n(1-p_{a_i})\}^{1/\chi_{a_i}},\quad i\in q_1$. Lastly, this can be stated as a deterministic constraint in the equivalent terms: $\sum_{j=1}^{n}x_{ij}\geq\xi_{a_i}+\lambda_{a_i}\{-1n(1-p_{a_i})\}^{1/\chi_{a_i}},\quad i\in q_1$.

(ii) For $P(\sum_{j=1}^{n}x_{ij}\leq a_i)\geq p_{a_i},\quad i\in q_3\quad (i=m_2+1, m_2+2,\ldots, m)$ the proof of converting the probabilistic values into deterministic values by using WD is obtained in [37].

4.1.2. Case (II) Only b_j is Uncertain

(i) For $P(\sum_{i=1}^{m} x_{ij}\geq b_j)\ \geq p_{b_j},\quad j\in r_1$, $(j=1,2,\ldots, n_1)$ the proof is shown in [37].

(ii) For $P(\sum_{i=1}^{m}x_{ij}\leq b_j)\geq p_{b_j}$, $j\in r_3$ the proof is shown as follows.

It is considered that b_j $(j = n_2+1,\ n_2+2,\ldots,n)$ are independent RVs with the WD and three known parameters ξ_{b_j}, χ_{b_j}, and λ_{b_j}. Equation (8) can therefore be transformed as follows:

$$P\left(b_j \geq \sum_{i=1}^{m} x_{ij}\right) \geq p_{b_j}, \quad j \in r_3. \tag{14}$$

Let us consider $\sum_{i=1}^{m} x_{ij} = \delta_{b_j}$,

$$P\left(b_j \geq \delta_{b_j}\right) \geq p_{b_j}, \quad j \in r_3, \tag{15}$$

$$\int_{\delta_{b_j}}^{\infty} \frac{\chi_{b_j}}{\lambda_{b_j}}\left(\frac{b_j - \xi_{b_j}}{\lambda_{b_j}}\right)^{\chi_{b_j}-1} e^{-\left(\left(b_j - \xi_{b_j}/\lambda_{b_j}\right)^{\chi_{b_j}}\right)} \mathrm{d}b_j \geq p_{b_j}, \quad j \in r_3. \tag{16}$$

Simultaneously, $b_j \geq \xi_{b_j} j \in r_3$, and the integration of equation (16) gives the following form:

$$\int_{\delta_{b_j}}^{\xi_{b_j}} \frac{\chi_{b_j}}{\lambda_{b_j}}\left(\frac{b_j - \xi_{b_j}}{\lambda_{b_j}}\right)^{\chi_{b_j}-1} e^{-\left(\left(b_j - \xi_{b_j}/\lambda_{b_j}\right)^{\chi_{b_j}}\right)} \mathrm{d}b_j \geq p_{b_j}. \tag{17}$$

After integrating equation (17), we get

$$1 - e^{-\left(\left(\delta_{b_j} - \xi_{b_j}/\lambda_{b_j}\right)^{\chi_{b_j}}\right)} \geq p_{b_j}. \tag{18}$$

Equation (18) can be further simplified by applying the logarithm as $\delta_{b_j} \leq \xi_{b_j} + \lambda_{b_j}\left\{-1n\left(p_{b_j}\right)\right\}^{\left(1/\chi_{b_j}\right)}, \quad j \in r_3$. Lastly, this can be stated as a deterministic constraint in equivalent terms $\sum_{i=1}^{m} x_{ij} \leq \xi_{b_j} + \lambda_{b_j}\left\{-\ln\left(p_{b_j}\right)\right\}^{\left(1/\chi_{b_j}\right)}, \quad j \in r_3$.

4.1.3. Remarks. Let $P\left(\sum_{j=1}^{n} x_{ij} = a_i\right) \geq p_{a_i}$, $i \in q_2 = m_1 + 1, m_1 + 2, \ldots, m_2$, and $P\left(\sum_{i=1}^{m} x_{ij} = b_j\right) \geq p_{b_j}$, $j \in r_2 = n_1 + 1, n_1 + 2, \ldots, n_2$ be the equality type supply and demand constraints. Then, change equality type to inequality type for the supply constraint as $P\left(\sum_{j=1}^{n} x_{ij} \leq a_i\right) \geq p_{a_i}$ and $P\left(\sum_{j=1}^{n} x_{ij} \geq a_i\right) \geq p_{a_i}$ for the demand constraint as $P\left(\sum_{i=1}^{m} x_{ij} \leq b_j\right) \geq p_{b_j}$ and $P\left(\sum_{i=1}^{m} x_{ij} \geq b_j\right) \geq p_{b_j}$. Choosing the probability value at 50% level in the supply and demand inequality constraint by using WD, we obtain the same deterministic value for both inequality types for supply and demand constraints. So, the deterministic values of equality constraint for the supply as $\sum_{j=1}^{n} x_{ij} = \xi_{a_i} + \lambda_{a_i}\left\{-\ln\ \left(1 - p_{a_i}\right)^{\left(1/\chi_{a_i}\right)}\right\}, \quad i \in q_2$ or $\sum_{j=1}^{n} x_{ij} = \xi_{a_i} + \lambda_{a_i}\left\{-ln\left(p_{a_i}\right)\right\}^{\left(1/\chi_{a_i}\right)}, \quad i \in q_2$ and the demand as $\sum_{i=1}^{m} x_{ij} = \xi_{b_j} + \lambda_{b_j}\left\{-1n\left(p_{b_j}\right\}^{\left(1/\chi_{b_j}\right)}, \quad j \in r_2$ or $\sum_{i=1}^{m} x_{ij} = \xi_{b_j} + \lambda_{b_j}\left\{-1n\left(1 - p_{b_j}\right)\right\}^{\left(1/\chi_{b_j}\right)}, \quad j \in r_2$.

Deterministic mathematical programming models for SFTPMC are presented in this part, which use the quantiles of the WD as constraints and the fuzzy linear membership function for the cost function. In a real scenario, only some aspects of supply and demand may be uncertain, while others are certain. As a result, SFTPMC deterministic can be modified as needed according to the situation. Three models, in which any one of the parameters are a_i or b_j are uncertain, are studied in Model 1 and 2, respectively. Model 3 presents a general model in which all RVs are uncertain. Commercially available solvers can be used to solve this deterministic mathematical programming for the optimal solution.

Model 1. For deterministic modeling of SFTPMC, a linear membership function [10] is used for the cost function, a quantile of the Weibull distribution is used for probabilistic supply constraint, and demand constraint remains certain.

$$(\mathrm{D}_1)\ \text{Minimize}\, z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\left((1-\alpha)\overline{x_{ij}} + \alpha \underline{x}_{ij}\right). \tag{19}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} \geq \xi_{a_i} + \lambda_{a_i}\left\{-1n\left(1 - p_{a_i}\right)\right\}^{\left(1/\chi_{a_i}\right)}, \quad i \in q_1, \tag{20}$$

$$\sum_{j=1}^{n} x_{ij} = \xi_{a_i} + \lambda_{a_i}\left\{-1n\left(1 - p_{a_i}\right)\right\}^{\left(1/\chi_{a_i}\right)}, \quad i \in q_2, \tag{21}$$

$$\sum_{j=1}^{n} x_{ij} \leq \xi_{a_i} + \lambda_{a_i}\left\{-1n\left(p_{a_i}\right)\right\}^{\left(1/\chi_{a_i}\right)}, \quad i \in q_3, \tag{22}$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j \quad, j \in r_1, \tag{23}$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad, j \in r_2, \tag{24}$$

$$\sum_{i=1}^{m} x_{ij} \leq b_j \quad, j \in r_3, \tag{25}$$

$$x_{ij} \geq 0. \tag{26}$$

Model 2. For deterministic modeling of SFTPMC, a linear membership function [10] is used for the cost function, a quantile of the Weibull distribution is used for demand constraint, and supply constraint remains certain.

$$(\mathrm{D}_2)\ \text{Minimize}\, z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\left((1-\alpha)\overline{x}_{ij} + \alpha \underline{x}_{ij}\right). \tag{27}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} \geq a_i, \quad i \in q_1, \tag{28}$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i \in q_2, \tag{29}$$

$$\sum_{j=1}^{n} x_{ij} \le a_i, \quad i \in q_3, \tag{30}$$

$$\sum_{i=1}^{m} x_{ij} \ge \xi_{b_j} + \lambda_{b_j}\left\{-1n\left(1-p_{b_j}\right)\right\}^{\left(1/\chi_{b_j}\right)}, \quad j \in r_1, \tag{31}$$

$$\sum_{i=1}^{m} x_{ij} = \xi_{b_j} + \lambda_{b_j}\left\{-1n\left\{p_{b_j}\right\}\right\}^{\left(1/\chi_{b_j}\right)}, \quad j \in r_2, \tag{32}$$

$$\sum_{i=1}^{m} x_{ij} \le \xi_{b_j} + \lambda_{b_j}\left\{-1n\left(p_{b_j}\right)\right\}^{\left(1/\chi_{b_j}\right)}, \quad j \in r_3, \tag{33}$$

$$x_{ij} \ge 0. \tag{34}$$

Model 3. For deterministic modeling of SFTPMC, a linear membership function [10] is used for the cost function, and a quantile of the Weibull distribution is used for supply and demand constraints.

$$D_3 \text{ Minimize } z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\left((1-\alpha)\overline{x}_{ij} + \alpha\underline{x}_{ij}\right). \tag{35}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} \ge \xi_{a_i} + \lambda_{a_i}\left\{-1n\left(1-p_{a_i}\right)\right\}^{\left(1/\chi_{a_i}\right)}, \quad i \in q_1, \tag{36}$$

$$\sum_{j=1}^{n} x_{ij} = \xi_{a_i} + \lambda_{a_i}\left\{-1n\left(1-p_{a_i}\right)\right\}^{\left(1/\chi_{a_i}\right)}, \quad i \in q_2, \tag{37}$$

$$\sum_{j=1}^{n} x_{ij} \le \xi_{a_i} + \lambda_{a_i}\left\{-1n\left(p_{a_i}\right)\right\}^{\left(1/\chi_{a_i}\right)}, \quad i \in q_3, \tag{38}$$

$$\sum_{i=1}^{m} x_{ij} \ge \xi_{b_j} + \lambda_{b_j}\left\{-1n\left(1-p_{b_j}\right)\right\}^{\left(1/\chi_{b_j}\right)}, \quad j \in r_1, \tag{39}$$

$$\sum_{i=1}^{m} x_{ij} = \xi_{b_j} + \lambda_{b_j}\left\{-1n\left(p_{b_j}\right)\right\}^{\left(1/\chi_{b_j}\right)}, \quad j \in r_2, \tag{40}$$

$$\sum_{i=1}^{m} x_{ij} \le \xi_{b_j} + \lambda_{b_j}\left\{-1n\left(p_{b_j}\right)\right\}^{\left(1/\chi_{b_j}\right)}, \quad j \in r_3, \tag{41}$$

$$x_{ij} \ge 0. \tag{42}$$

5. Solution Method

The solution method for the model SFTPMC is given as follows:

Step 1. Convert the given triangular fuzzy cost of problem (P) into an equivalent deterministic cost (17) by using alpha cut.

Step 2. Then, problem (P) is transformed into 3 models depending on its constraint.

Step 3. (i) For model 1, a_i is an uncertain probabilistic supply constraint, and b_j remains precise values. For a_i, we use WD to reduce the probabilistic supply constraint into an equivalent deterministic supply constraint (20)–(22). Then, construct the problem (D_1) with the reduced deterministic cost (17), deterministic supply constraint (20)–(22), and precise demand constraint (23)–(25). (ii) At alpha level 0, solve the problem (D_1) to obtain the optimal transportation cost and unit flow, using the Lingo software.

Step 4. (i) For model 2, only b_j is uncertain, and a_i remains precise values. For b_j, we use WD to reduce the probabilistic demand constraint into an equivalent deterministic demand constraint (31)–(33). Then, construct the problem (D_2) with the reduced deterministic cost (17), deterministic demand constraint (31)–(33), and precise supply constraint (28)–(30). (ii) At alpha level 0, solve the problem (D_2) to obtain the optimal transportation cost and unit flow, using the Lingo software.

Step 5. (i) For model 3, a_i and b_j are uncertain values, and we use WD to reduce the probabilistic supply and demand constraint into an equivalent deterministic supply (36)–(41). Then, construct the problem (D_3) with the reduced deterministic cost (17) and deterministic constraint (36)–(41). (ii) At alpha level 0, solve the problem (D_3) to obtain the optimal transportation cost and unit flow, using the Lingo software.

Step 6. Repeat the above steps (3–5) to obtain the optimal transportation cost and unit flow for different alpha levels.

5.1. Numerical Experiment and Discussions. This segment provides an illustration to demonstrate the efficacy and applicability of SFTPMC and its variants. The coal plant generates a homogeneous product, and there are three plants and four repositories. Coal plant A has a manufacturing capacity of exactly a_1 units, coal plant B has a production capacity of at least a_2 units, and coal plant C has a manufacturing capacity of the most a_3 units. Likewise, repository 1 has a capacity of demands at least b_1 units; repository 2 has a capacity of demands at the most b_2 units; repository 3 has a capacity of demands of at least b_3 units. Repository 4 has a capacity of demands exactly b_4 units. If transporting per unit cost from each coal plant to each repository is $\tilde{c}_{ij}$, it is given in triangular fuzzy cost data as shown in Table 2.

In the numerical experiment, nominal values of certain constants are supplied as $a_1 = 20$, $a_2 = 16a_3 = 25$, and demand as $b_1 = 11$, $b_2 = 13$, $b_3 = 17$, $b_4 = 14$ in the following sections. Furthermore, the following arbitrary probabilities are given as $P_{a_1} = 0.50$, $P_{a_2} = 0.96$, $P_{a_3} = 0.95$, $P_{b_1} = 0.26$, $P_{b_2} = 0.29$, $P_{b_3} = 0.25$, $P_{b_4} = 0.28$. Since a_i and b_j are presumed to follow WD, the distinct values for the parameters are $\xi_{a_1} = 19$, $\xi_{a_2} = 13$, $\xi_{a_3} = 24$, $\xi_{b_1} = 10$, $\xi_{b_2} = 11$, $\xi_{b_3} = 16$,

Table 2: Stochastic fuzzy transportation problem mixed constraint (P).

Repositories		1	2	3	4	a_i
Coal plant	A	(0,0.5,1)	(2,4,6)	(1.5,2,3)	(2,4,5)	$= a_1$
	B	(3,5,7)	(1.5,2,3)	(0,0.5,1)	(4,5.8,6)	$\geq a_2$
	C	(7,8.5,9)	(2.5,3,4)	(3,4,5)	(2,3,4)	$\leq a_3$
	b_j	$\geq b_1$	$\leq b_2$	$\geq b_3$	$= b_4$	

$\xi_{b_4} = 13$, $\chi_{a_i} = \chi_{b_j} = 2$, $\lambda_{a_i} = \lambda_{b_j} = 2$. The probabilistic constraints can simply be transformed to deterministic forms by utilizing equations (36)–(41). By using Step 1, we have to convert the given triangular fuzzy cost of the problem (P) into deterministic cost using the linear membership function. Table 3 shows the alpha cut representation for the cost function of a given problem (P).

Now, using Step 2, problem (P) is transformed into 3 models depending on its constraint. By Step 3, only supply constraints are uncertain, and demand remains certain for model 1. At alpha level 0 on the cost function, WD is used for the probabilistic supply constraint. Here, supply is $a_1 = 20.67$, $a_2 = 16.59$, and $a_3 = 24.45$, and demand is $b_1 = 11$, $b_2 = 13$, $b_3 = 17$, and $b_4 = 14$. By using the Lingo software, we obtain the optimal transportation cost as 93.67 and $x_{11} = 11$, $x_{14} = 9.67$, $x_{23} = 17$, $x_{34} = 4.33$ and unit flow as 42.

By Step 4, only demand constraints are uncertain, and supply remains at certain values for model 2. At alpha level 0 on the cost function, WD is used for the demand constraint. Here, supply is $a_1 = 20$, $a_2 = 16$, $a_3 = 25$, and demand is $b_1 = 11.1$, $b_2 = 13.23$, $b_3 = 17.07$, $b_4 = 14.67$, and we obtain the optimal transportation cost as 95.75 and $x_{11} = 11.10$, $x_{14} = 8.9$, $x_{23} = 17.07$, $x_{34} = 5.77$ and unit flow as 42.84.

By Step 5, both supply and demand constraints are uncertain for model 3. Here, supply is $a_1 = 20.67$, $a_2 = 16.59$, $a_3 = 24.45$, and demand is $b_1 = 11.1$, $b_2 = 13.23b_3 = 17.07$, $b_4 = 14.67$. Similarly, we obtain the optimal transportation cost as 96.42 and $x_{11} = 11.10$, $x_{14} = 9.57$, $x_{23} = 17.07$, $x_{34} = 5.1$ and unit flow as 42.84. In the same manner, we have to solve different alpha levels for the 3 cases of constraints. Computational results for 3 models are shown in Table 4. Using Step 6, we can solve for different alpha levels.

Three new models have been constructed for SFTPMC. While model 1 presents supply as a probabilistic value and demand as a certain value, model 2 presents demand as a probabilistic value and supply as a certain value, and model 3 presents supply and demand as probabilistic values. The cost coefficient of the objective function is transformed to alpha cut representation, and the stochastic mixed constraints are transformed to deterministic form by using the WD for solving the models. The fact that these models are constructed from different points of view is worth noting. Since the usage of the models is dependent on the DM's preference, we cannot conclude which model is the best in the process of decision-making.

6. Sensitivity Analysis and Discussion

In this section, SFTPMC performed a SA of optimality in terms of variations in probabilities on uncertain parameters such as source and demand. For the SA, we used the model 3 problem and changed the probability from ($0 \leq P \leq 1$), where P is the probability on a_i or b_j. We analysed the problem by holding any one parameter of probabilities (p_{ai} or p_{bj}) as constant at 0.5 and changed the probabilities of the other parameter of probabilities (p_{ai} or p_{bj}). Using the remark (4.1.3), the probability of the particular parameter remains 0.5 for equality constraint. In optimal solutions for model 3 in each stochastic parameter and fuzzy cost, both transportation cost and total shipping units (Flow) were obtained and listed. To handle several objective functions in this SA, the Lingo software was employed to solve this optimization problem. The SA findings for the probability for demand b_j are shown in Table 5. These analysis's graphical representations for transportation cost and unit flow in relation to the probability for b_j are shown in Figures 1 and 2. The transportation cost gradually increases with the probability for b_j, as shown in Figure 1. It is worth noting that transportation costs are sensitive to variations in demand requirements probability. In Figure 2, the unit flow decreases gradually depending on variations in demand requirements probability. Similarly, we can find the SA for a_i. The SA of the probability reveals some intriguing patterns. DM's can get knowledge and capacity to develop the transportation system by gaining an understanding of the sensitivity patterns of probability for uncertain parameters.

The SA of SFTPMC is obtained by varying the probability of demand constraint in Table 5, and it is also depicted using the graph, and the outcomes of optimal transportation cost and unit flow are shown in Figures 1 and 2.

In this study, the proposed SFTPMC models obtain the best solutions in an unpredictable situation, according to the results. It is easy to contemplate more conservative decisions in extremely uncertain conditions. In the results, it is noticed that the varying probabilities for b_j are different values of transportation cost and unit flow for this problem. It is common to move toward more conservative solutions when there is greater uncertainty in the optimization problem. It is observed that when modeling SFTPMC with the uncertainty introduced by the probability for a_i and b_j, more conservative solutions are selected as the optimal. This analysis, on the other hand, demonstrates the need of grasping the sensitivity of mixed constraints in the face of rising uncertainty. It helps a DM

Table 3: Alpha cut representation of cost function for problem (P).

	1	2	3	4	a_i
A	$(1-\alpha)1\overline{x}_{11}+0\alpha\underline{x}_{11}$	$(1-\alpha)6\overline{x}_{12}+2\alpha\underline{x}_{12}$	$(1-\alpha)3\overline{x}_{13}+1.5\alpha\underline{x}_{13}$	$(1-\alpha)5\overline{x}_{14}+2\alpha\underline{x}_{14}$	$=a_1$
B	$(1-\alpha)7\overline{x}_{21}+3\alpha\underline{x}_{21}$	$(1-\alpha)3\overline{x}_{22}+1.5\alpha\underline{x}_{22}$	$(1-\alpha)1\overline{x}_{23}+0\alpha\underline{x}_{23}$	$(1-\alpha)6\overline{x}_{24}+4\alpha\underline{x}_{24}$	$\geq a_2$
C	$(1-\alpha)9\overline{x}_{31}+7\alpha\underline{x}_{31}$	$(1-\alpha)4\overline{x}_{32}+2.5\alpha\underline{x}_{32}$	$(1-\alpha)5\overline{x}_{33}+3\alpha\underline{x}_{33}$	$(1-\alpha)4\overline{x}_{34}+2\alpha\underline{x}_{34}$	$\leq a_3$
b_j	$\geq b_1$	$\leq b_2$	$\geq b_3$	$=b_4$	

Table 4: Computational results for 3 models.

Optimization method (Lingo software)	Model 1	Model 2	Model 3
Optimal transportation cost	93.67	95.75	96.42
Units in flow	42	42.84	42.84

Table 5: Sensitivity analysis of SFTPMC by the varying probability of (p_{b_j}).

Probabilities for $b_j\,(P_{b_j})$	Probabilities $a_i\,(p_{a_i})$	Optimal transportation cost	Unit flow
0.99		99.64	49.25
0.95		98.81	47.59
0.9		98.38	46.73
0.85		98.1	46.17
0.8		97.89	45.74
0.75		97.7	45.05
0.7		97.54	45.05
0.65		97.4	44.77
0.6		97.26	44.37
0.55		97.14	44.25
0.5	0.5	97.02	44.01
0.45		96.9	43.77
0.4		96.78	43.53
0.35		96.66	43.29
0.3		96.54	43.05
0.25		96.42	43.41
0.2		96.29	42.55
0.15		96.16	42.29
0.1		96	41.97
0.05		95.8	41.57

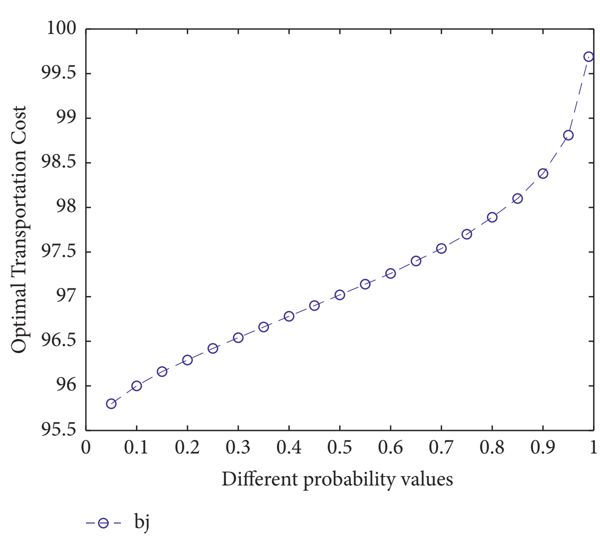

Figure 1: SA of the optimal transportation cost.

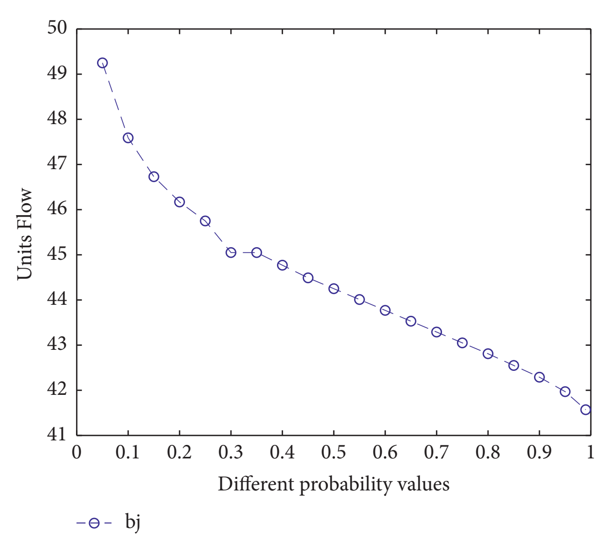

Figure 2: SA of the unit's flow.

choose the right level of uncertainty for uncertain parameters.

7. Conclusions and Future Scope

This article describes a model for solving an SFTPMC that includes the cost coefficient of the objective function as a fuzzy number and probabilistic constraints accompanying the WD. The fuzzy objective value is converted into an equivalent deterministic objective function using alpha cut representation, and all stochastic constraints are converted into an equivalent deterministic constraint using WD. Three models, models 1, 2, and 3, of SFTPMC are also established, and the optimal solution for each model has been obtained by using Lingo software. A numerical example is given to show the performance of the models. The SA results are presented for model 3 supply and demand parameters. A pictorial representation (Figures 1 and 2) of model 3 on SA is to illustrate the effect of the change in demand parameters on total transportation costs and unit flow. Likewise, perform SA for the supply parameters. In addition, this analysis demonstrates the need of grasping the sensitivity of mixed constraints in the face of rising uncertainty. It assists a DM to choose the right level of uncertainty for uncertain parameters. The computed results clearly indicate that the designed model is robust with respect to the different parameters. As this is the problem, SFTPMC plays a vital role in many cases of managerial decision-making situations such as the planning of many complex resource allocation problems in the areas of industrial production, in which demand and supply are random variables in nature. In such situations, this model will serve as an efficient tool for optimal planning.

The limitations of the current study are presented. This research has considered an FTP with mixed constraints under a stochastic environment, where the single objective that is transportation cost is considered. In reality, the decision-making process handling with the complex organizational situation cannot depend exclusively on a single condition. So, we must understand the presence of numerous criteria that can improve multicriteria decision-making. In view of this, in our future study, we plan to investigate SFTPMC scenarios by considering multi-objective with multiitem parameters. Using the approach presented in this work, supply and demand variations can be considered in the economic order quantity model. In our forthcoming research, we plan to get real-world data from proper authorities and employ statistical regularity criteria to derive its probability distribution. In this case, the WD has a broad range of applications.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The authors wish to thank the management of Vellore Institute of Technology, Vellore- 632014, India, for their continuous support and encouragement to carry out this research work.

References

[1] M. E. B. Brigden, “A Variant of the transportation problem in which the constraints are of mixed Type,” *Operational Research Society*, vol. 25, no. 3, pp. 437–445, 1977.

[2] V. Adlakha, K. Kowalski, and B. Lev, “Solving transportation problems with mixed constraints,” *International Journal of Management Science and Engineering Management*, vol. 1, no. 1, pp. 47–52, 2006.

[3] A. Das, U. K. Bera, and B. Das, “A solid transportation problem with mixed constraint in different environment,” *Journal of Applied Analysis & Computation*, vol. 6, no. 1, pp. 179–195, 2016.

[4] S. Agarwal and S. Sharma, “A Shootout method for time minimizing transportation problem with mixed constraints,” *American Journal of Mathematical and Management Sciences*, vol. 39, no. 4, pp. 299–314, 2020.

[5] S. Gupta, I. Ali, and A. Ahmed, “Multi-objective capacitated transportation problem with mixed constraint: a case study of certain and uncertain environment,” *Opsearch*, vol. 55, no. 2, pp. 447–477, 2018.

[6] V. Vidhya, P. Uma Maheswari, and K. Ganesan, “An alternate method for finding more for less solution to fuzzy transportation problem with mixed constraints,” *Soft Computing*, vol. 25, no. 18, 2021.

[7] F. Rashid, A. R. Khan, and S. Uddin, “Mixed constraints cost minimization transportation problem an effective algorithmic approach,” *American Journal of Operations Research*, vol. 11, no. 1, 2021.

[8] S. Gupta, I. Ali, and A. Ahmed, “An extended multi-objective capacitated transportation problem with mixed constraints in fuzzy environment,” *International Journal of Operational Research*, vol. 37, no. 3, pp. 345–376, 2020.

[9] H. J. Zimmermann, “Fuzzy programming and linear programming with several objective functions,” *Fuzzy Sets and Systems*, vol. 1, no. 1, pp. 45–55, 1978.

[10] S. K. Behera and J. R. Nayak, “Solution of multi-objective mathematical programming problems in fuzzy approach,” *International Journal of Computer Science and Engineering*, vol. 3, no. 12, 2011.

[11] A. Charnes and W. W. Cooper, “Chance-constrained programming,” *Management Science*, vol. 6, no. 1, pp. 73–79, 1959.

[12] S. Kataoka, “A stochastic programming model,” *Econometrica*, vol. 31, 1963.

[13] D. R. Spoerl and R. K. Wood, “A stochastic generalized assignment aroblem,” 2004, https://www.tdx.cat/bitstream/handle/10803/6510/06Mas06de10.pdf;jsessionid=ACC3D914155E30DACEC7B5AF86645D52?sequence=6.

[14] A. C. Williams, “A stochastic transportation problem,” *Operations Research*, vol. 11, no. 5, pp. 759–770, 1963.

[15] P. Agrawal and T. Ganesh, “Solving transportation problem with stochastic demand and non-linear multi-choice cost,” 2019, https://www.researchgate.net/publication/335077560_Solving_transportation_problem_with_stochastic_demand_and_non-linear_multi-choice_cost.

[16] W. B. Powell and H. Topaloglu, "Stochastic programming in transportation and logistics," *Handbooks OR &MS*, vol. 10, 2003.
[17] M. Anholcer, "Stochastic generalized transportation problem with discrete distribution of demand," *Operations. Researc-h.and Decisions*, vol. 4, no. 4, 2013.
[18] A. Ojha, B. Das, S. K. Mondal, and M. Maiti, "A transportation problem with fuzzy-stochastic cost," *Applied Mathematical Modelling*, vol. 38, no. 4, 2014.
[19] K. Holmberg and H. Tuy, "A production-transportation problem with stochastic demand and concave production costs," *Mathematical Programming*, vol. 85, no. 1, pp. 157–179, 1999.
[20] M. Mahapatra, S. K. Roy, and D. R. Biswal, "Multi-objective stochastic transportation problem involving log-normal," *Journal of Physical Science*, vol. 14, no. 6, pp. 63–76, 2010.
[21] P. Agrawal, K. Alnowibet, T. Ganesh, A. F Alrasheedi, H. Ahmad, and A. Wagdy Mohamed, "An artificial intelligence approach for solving stochastic transportation problems," *Computers, Materials & Continua*, vol. 70, no. 1, pp. 817–829, 2022.
[22] G. Aruna Chalam, "Fuzzy goal programming (FGP) approach to a stochastic transportation problem under budgetary constraint," *Fuzzy Sets and Systems*, vol. 66, no. 3, pp. 293–299, 1994.
[23] P. K. Giri, M. K. Maiti, and M. Maiti, "Fuzzy stochastic solid transportation problem using fuzzy goal programming approach," *Computers & Industrial Engineering*, vol. 72, pp. 160–168, 2014.
[24] S. Acharya, N. Ranarahu, J. K. Dash, and M. M. Acharya, "Computation of a multi-objective fuzzy stochastic transportation problem," *International Journal of Fuzzy Computation and Modelling*, vol. 1, no. 2, p. 212, 2014.
[25] A. Abebaw Gessesse, R. Mishra, M. M. Acharya, and K. N. Das, "Genetic algorithm based fuzzy programming approach for multi- objective linear fractional stochastic transportation problem involving four-parameter Burr distribution," *International Journal of System Assurance Engineering and Management*, vol. 11, no. 1, pp. 93–109, 2020.
[26] G. Maity, V. F. Yu, and S. K. Roy, "Optimum Intervention in Transportation Networks Using Multimodal System under Fuzzy Stochastic Environment," *Jounal of Advanced Transportation*, vol. 2022, Article ID 3997396, 14 pages, 2022.
[27] W. Weibull, "A statistical distribution function of wide applicability," *Journal of Applied Mechanics*, vol. 18, no. 3, pp. 293–297, 1951.
[28] S. L. Fok and J. Smart, "The accuracy of failure predictions based on Weibull statistics," *Journal of the European Ceramic Society*, vol. 15, no. 9, pp. 905–908, 1995.
[29] H. S. Klakattawi, "The Weibull-Gamma distribution: properties and applications," *Entropy*, vol. 21, no. 5, pp. 438–515, 2019.
[30] D. R. Mahapatra, "Multi-choice stochastic transportation problem involving weibull distribution," *An International Journal of Optimization and Control: Theories & Applications*, vol. 4, no. 1, pp. 45–55, 2013.
[31] A. A. Gessesse, R. Mishra, and M. M. Acharya, "Solving multi-objective linear fractional stochastic transportation problems involving normal distribution using simulation-based genetic algorithm," *International Journal of Engineering and Advanced Technology*, vol. 9, no. 2, pp. 9–17, 2019.
[32] B. Jerbi, "A fuzzy multi-objective polynomial time algorithm to solve the stochastic transportation formulation of a hospital bed rearrangement problem," *Journal of Multi-Criteria Decision Analysis*, vol. 28, pp. 34–44, 2021.
[33] H. Al Qahtani, A. El-Hefnawy, M. M. El-Ashram, and A. Fayomi, "A goal programming approach to multichoice multiobjective stochastic transportation problems with extreme value distribution," *Advances in Operations Research*, vol. 2019, Article ID 9714137, 6 pages, 2019.
[34] S. H. Nasseri and S. Bavandi, "Solving multi-objective multi-choice stochastic transportation problem with fuzzy programming approach," *2020 8th Iranian Joint Congress on Fuzzy and intelligent Systems (CFIS)*, vol. 2020, Article ID 9238695, 2020.
[35] S. Dutta, S. Acharya, and R. Mishra, "Genetic algorithm based fuzzy stochastic transportation programming problem with continuous random variables," *Opsearch*, vol. 53, no. 4, pp. 835–872, 2016.
[36] D. R. Mahapatra, S. Panda, and S. S. Sana, "Multi-choice and stochastic programming for transportation problem involved in supply of foods and medicines to hospitals with consideration of logistic distribution," *RAIRO—Operations Research*, vol. 54, no. 4, 2020.
[37] A. Das and G. M. Lee, "A multi-objective stochastic solid transportation problem with the supply, demand, and conveyance capacity following the weibull distribution," *Mathematics*, vol. 9, no. 15, 2021.
[38] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.