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Research Article

Parametric Identification for the Biased Ship Roll Motion Model Using Genocchi Polynomials

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Roll motion is one of the key motions related to a vessel's dynamic stability. It is essential for the dynamic stability of ships in the realistic sea. For this research study, we have investigated the parameters involved in damping of the ship. In general, mathematical modelling of the rolling response of a ship can be formulated by the linear, nonlinear, and fractional differential equations because the amplitude of oscillation is increased. An efficient Genocchi polynomial approximation method (GPAM) is successfully applied for the biased ship roll motion model. The basic idea of the collocation method together with the operational matrices of derivatives used for nonlinear differential equation and convert it into a system of algebraic equations. The convergence and error analysis of the proposed method are also discussed. A few numerical experiments are carried out for some specific and important types of problems including the biased roll motion equations. The results are compared to those produced using the Legendre wavelet method (LWM) and the homotopy perturbation method (HPM). It is observed that the proposed spectral algorithm is robust, accurate, and easy to apply.

1. Introduction

Several research papers have been studied on the ship roll motion models because they are highly nonlinear differential equations. It is an essential for the dynamic stability of ships in realistic sea. For this research study, GPAM has been applied to estimate the parameters involved in damping of the ship. In general, mathematical modelling of the rolling response of a ship can be formulated by the nonlinear differential equations because the amplitude of oscillation is increased. In ship roll motion models, the nonlinear term occurs due to the damping and restoring moments.

In pioneering work, Spyrou [1] established the ship roll motion model for large regular waves. Several research papers have been published related to ship dynamics models

[2, 3]. Nonlinear mathematical formulation of the ship roll model is discussed in [4]. In general, the linear differential model of the motion has been violated by the nonlinear features of the motion.

The theoretical models of ship dynamics having nonlinear terms such as damping and restoring moments are discussed in [2, 5]. A few mathematical methods had been established to investigate the approximate solutions of nonlinear models.

During the last decades, Appell polynomials are employed in analytic number theory as well as asymptotic approximation theory. They discovered differential equations of Bernoulli and Euler polynomials as a special case. For Appell polynomials, a collection of finite-order differential equations were established by [6]. The 2 D Bernoulli and 2 D Euler polynomials were introduced for solving differential, partial differential, and integrodifferential equations. Hermite polynomials based Appell polynomial were presented by [7] and some of the references related with Appell polynomials are [6, 8–15]. Several analytical and approximation schemes are proposed to solve a system of differential equations in nonlinear and fractional order [16, 17]. Hariharan et al. [18, 19] had developed the wavelet method for solving reaction-diffusion problems and integral equations.

Nonlinear problem research has advanced at a rapid and intensive rate during the forecast period. For example, approaches based on operational matrices for integration of large numbers of polynomials and functions can be mentioned. Over the last four decades, numerical techniques based on operational matrices of integration (especially for orthogonal polynomials and functions) have gained a lot of attention for dealing with variational problems.

The authors of [20, 21] have recognised the significance of using operational approaches in the study of special functions and their applications. The majority of the attention is focused on operational identities related to ordinary and multivariable versions of Hermite and Laguerre polynomials.

A few researcher developed Frobenius–Euler and Genocchi polynomials and its application introduced to solve nonlinear differential equations. The novel identities of umbral calculus application majorly using the Genocchi numbers and polynomials which were derived by [8]. Recently, Swaminathan et al. [22] established the Genocchi polynomial approximation method for Bratu-type differential equations. Some of the researcher solved fractional-order differential equations [23] and nonlinear reaction-diffusion problems [24]. And also he derived some more special polynomials and function arising in application of fractional calculus [23, 25].

This study provides a numerical analysis of ship roll motion in terms of concentrated parameters within the time domain using a Genocchi polynomial operational matrix method.

This study is summarized as follows. Ship roll motion in biased type is discussed in Section 2. Genocchi polynomial and its properties are given in Section 3. In Section 4, a few illustrative examples are presented to show the efficiency and accuracy of the proposed Genocchi polynomial method. Conclusion is given in Section 5.

2. Ship Roll Motion in Biased Type

The biased roll motion is described theoretically in [26]:

$$\theta'' + \beta\theta' + \theta(1-\theta)(1+a\theta) = F\sin(\omega t), \tag{1}$$

where a is a measure of the amount of bias, and its parameter lies between 0 and 1. Due to large angles of roll, the hull may affected by waves, winds, or cargo shifting. The perspective of biased roll motion is to approximate the ship. There is no bias in the Falzarano model [27]. If $a = 1$, then the restoring moment is a cubic polynomial of symmetric type. If $a = 0$, then the restoring moment becomes the second-order polynomial. Here, we consider the mathematical model in extreme case of bias. The ship can only capsize from one side. Then, it is called Helmholtz–Thompson equation, and Spyrou et al. [26] pointed out the value of a is closer to 1 in realistic situation. Therefore, $(1-a)$ is a small number. Equation (1) becomes

$$\theta'' + \theta - \theta^3 + \beta\theta' = (1-a)\left(\theta^2 - \theta^3\right) + F\sin(\omega t). \tag{2}$$

3. Genocchi Polynomial and Its Properties

Genocchi polynomials have been used in various fields of mathematics. The classical Genocchi polynomials is denoted by $G_m(x)$, and it is defined as [8, 28]

$$\frac{2te^{xt}}{e^t+1} = \sum_{m=0}^{\infty} G_m(x)\frac{t^n}{n!}, \quad (|t| < \pi), \tag{3}$$

where $G_m(x)$ is the m^{th} degree Genocchi polynomial and is given by

$$G_m(x) = \sum_{p=0}^{m} \binom{m}{p} G_{m-p} x^p. \tag{4}$$

G_p is called the Genocchi number. Differentiate (4), with respect to x; then,

$$\begin{aligned} \frac{dG_m(x)}{dx} &= mG_{m-1}(x), \quad m \geq 1, \\ G_m(1) + G_m(0) &= 0, \quad m > 1. \end{aligned} \tag{5}$$

The above equality (6) is an important property. Let $G(x) = [G_1(x), G_2(x), G_3(x), \ldots, G_N(x)]$ be the Gennochi vectors, then its derivative $(G\prime(x))$, by the reference of equation (5), can be expressed by

$$G\prime(x)^T = SG^T(x), \tag{6}$$

where

$$G\prime(x)^T = \begin{pmatrix} G_1'(x) \\ G_2'(x) \\ G_3'(x) \\ G_4'(x) \\ \cdots \\ G_{R-1}'(x) \\ G_R'(x) \end{pmatrix},$$

$$E = \begin{pmatrix} 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 2 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & 3 & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & 4 & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ldots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & R-1 & 0 & 0 \\ 0 & 0 & 0 & \ldots & 0 & R & 0 \end{pmatrix}, \tag{7}$$

$$G(x)^T = \begin{pmatrix} G_1(x) \\ G_2(x) \\ G_3(x) \\ G_4(x) \\ \cdots \\ G_{R-1}(x) \\ G_R(x) \end{pmatrix},$$

where S operational matrix of derivative of order $R \times R$. Based on that procedure, the p^{th} derivative of $G(x)$ can be expressed as

$$G^{(p)}(x) = G(x)\left(E^T\right)^p. \tag{8}$$

3.1. Function Approximation. Let $M = L^2[0,1]$ [23] (section 2.4, page 3039) and $\{G_1(x), G_2(x), \ldots, G_R(x)\} \subset M$ be the set of all Genocchi polynomials, and the $\text{Span}\{G_1(x), G_2(x), \ldots, G_R(x)\} = X$. Consider g be an arbitrary element of M, since X is a finite dimensional vector space, and g^* be an unique better approximation in R such that

$$\left\|g - g^*\right\|_2 \leq \|g - x\|_2 \quad \forall x \in X. \tag{9}$$

Since $g^* \in N$, then there exist an unique coefficients $c_1, c_2, \ldots, c_R$ such that

$$g \approx g^* = \sum_{r=1}^{R} c_r G_r(x) = CG(x), \tag{10}$$

where

$$\begin{aligned} C &= \left[c_1, c_2, \ldots, c_R\right]^T, \\ G(x) &= \left[G_1(x), G_2(x), \ldots, G_R(x)\right]. \end{aligned} \tag{11}$$

The coefficients c_n can be derived using the following lemma.

Lemma 1. *Let $g \in M = L^2[0,1]$ be an arbitrary function approximated by the truncated Genocchi series $\sum_{r=1}^{N} c_r G_r(x)$; then, the coefficients c_r, for $r = 1, 2, \ldots, N$, can be calculated from the following relation:*

$$c_r = \frac{1}{2r!}\left(g^{(r-1)}(1) + g^{(r-1)}(0)\right), \tag{12}$$

where $g^{(r-1)}(x)$ denotes the $(r-1)^{th}$ derivative of g.

Proof. See Lemma 4.1, page 2125 in [29]. □

Theorem 1. *Consider $g(x) \in C^\infty[0,1]$ and $g^*(x)$ be the approximated value of $g(x)$ by using Genocchi polynomials; then, the error bound can be expressed as*

$$\|\text{error}(g(x))\|_\infty \leq \frac{1}{R!} G_R F_R, \tag{13}$$

where G_R and F_R denote the maximum value of $G_R(x)$ and $g^{(r-1)}(x)$ for all $x \in [0,1]$, respectively.

Proof. The proof is obvious by using the above lemma. See [29] (Theorem 2, page 2125). □

4. Illustrative Examples

Example 1. Consider the equations with initial conditions:

$$\begin{aligned} &\theta\prime\prime + 1.8882267\theta - 0.081886\cos(0.527t) = 0, \quad t \in (0,1], \\ &\theta(0) = 1, \\ &\theta(0) = 0. \end{aligned} \tag{14}$$

By using the procedure of Section 3 for $R = 12$. The general collocation points of Gennochi polynomial is $t_i = i/(R-2)$, $\quad i = 1, 2, \ldots, R-2$. We find the approximated values of $\theta\prime\prime(t), \theta\prime(t)$, and $\theta(t)$ was substituted in (15); we obtain

$$G(t)\left(E^T\right)^2 C + 1.888267[G(t)C] - 0.081886\cos(0.527t) = 0, \tag{15}$$

with the initial conditions,

$$\begin{aligned} G(0)C &= 1, \\ G(0)\left(E^T\right)C &= 0. \end{aligned} \tag{16}$$

Apply the collocation point (16) at $t_i = i/10$; we get ten different algebraic equations. The obtained equations are solved together with (17) for the values of c_j, $\quad j = 1, 2, \ldots, 12$, and we obtain

$$\begin{aligned} c_1 &= 0.6340705, \\ c_2 &= -0.3659295, \\ c_3 &= -0.2439530, \\ c_4 &= c_5 = c_6 = c_7 = c_8 = c_9 = c_{10} = c_{11} = c_{12} = 0. \end{aligned} \tag{17}$$

Hence,

Table 1: Comparative Study of Genocchi polynomial, LWM, and HPM for roll angle with respect to the time t (in sec).

Time	Ex:1			Ex:2			Ex:3		
t	Genocchi polynomial	LWM	HPM	Genocchi polynomial	LWM	HPM	Genocchi polynomial	LWM	HPM
0.0	1	1	1	1	1	1	1	1	1
0.1	0.9926814	0.9926814	0.9930814	0.9992989	0.9992989	0.9993359	0.9993105	0.9993105	0.9993365
0.2	0.9707256	0.9707256	0.9723256	0.9971956	0.9971956	0.9973436	0.9972423	0.9972423	0.9973463
0.3	0.9341326	0.9341326	0.9377326	0.9936902	0.9936902	0.9940232	0.9937953	0.9937953	0.9940293
0.4	0.8829025	0.8829024	0.8893024	0.9887826	0.9887826	0.9893746	0.9889695	0.9889695	0.9893855
0.5	0.8170352	0.8170350	0.8270350	0.9824729	0.9824729	0.9833979	0.9827648	0.9827648	0.9834148
0.6	0.7365307	0.7365304	0.7509304	0.9747610	0.9747610	0.9760930	0.9751813	0.9751814	0.9761173
0.7	0.6413890	0.6413886	0.6609886	0.9656469	0.9656469	0.9674599	0.9662191	0.9662191	0.9674931
0.8	0.5316102	0.5316096	0.5572096	0.9551307	0.9551307	0.9574987	0.9558780	0.9558780	0.9575420
0.9	0.4071942	0.4071934	0.4395934	0.9432123	0.9432123	0.9462093	0.9441581	0.9441581	0.9462641
1.0	0.2681410	0.2681404	0.3081402	0.9298918	0.9298918	0.9335918	0.9310594	0.9310595	0.9336594

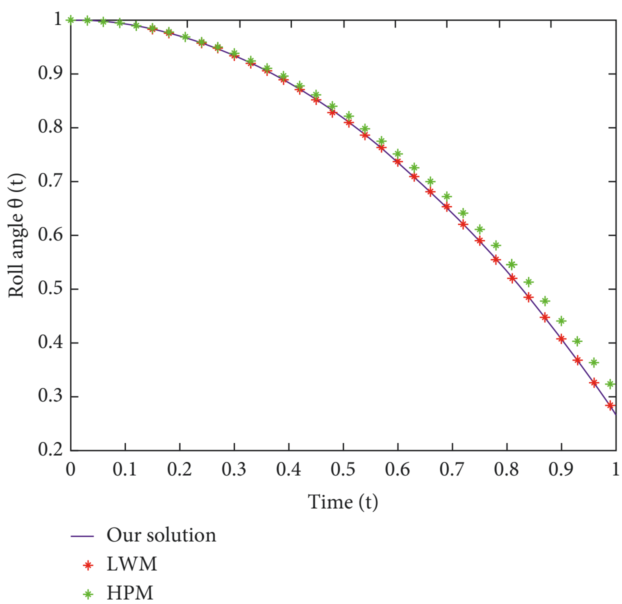

Figure 1: Comparative study of Genocchi polynomial, LWM, and HPM for roll angle with respect to the time t (in sec) for Ex.1.

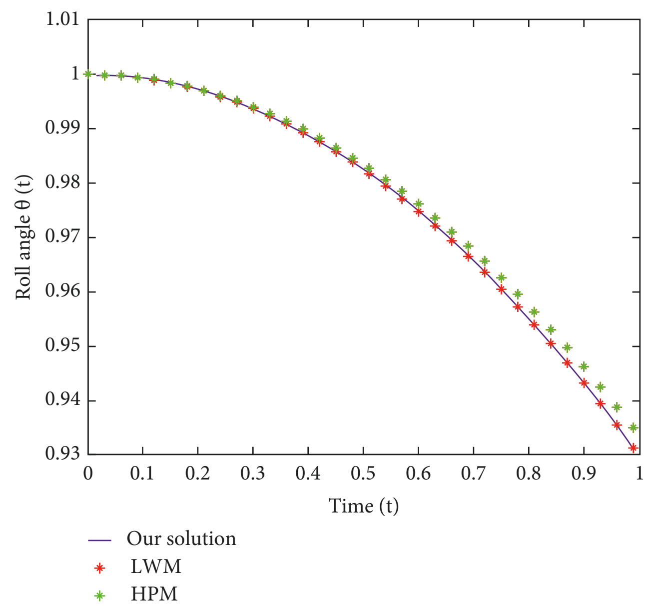

Figure 2: Comparative study of Genocchi polynomial, LWM, and HPM for roll angle with respect to the time t (in sec) for Ex.2.

$$\theta(t) = 1 - 0.731859t^2. \tag{18}$$

The Genocchi polynomial results are compared with HPM and LWM in Table 1. The approximate solution has been illustrated in Figure 1.

Example 2. Consider the equation with initial conditions:

$$\begin{gathered}\theta'' + 1.8882267\theta - 0.044582\cos(0.527t)\theta = 0, \quad t \in (0,1],\\ \theta(0) = 1,\\ \theta'(0) = 0.\end{gathered} \tag{19}$$

By Section 3 procedure, this example have been done with $N = 12$.

After simplification, we obtain

$$\theta(t) = 1 - 0.0701082t^2. \tag{20}$$

The Genocchi polynomial results are compared with HPM and LWM in Table 1. The approximate solution has been illustrated in Figure 2.

Example 3. Consider the equation with initial conditions:

$$\begin{gathered}\theta'' + 0.034480\theta' + 0.1857761\theta - 0.044582\cos(0.527t)\theta = 0, \quad t \in (0,1],\\ \theta(0) = 1,\\ \theta'(0) = 0.\end{gathered} \tag{21}$$

By Section 3 procedure, this example has been done with $N = 12$.

After simplification, we obtain

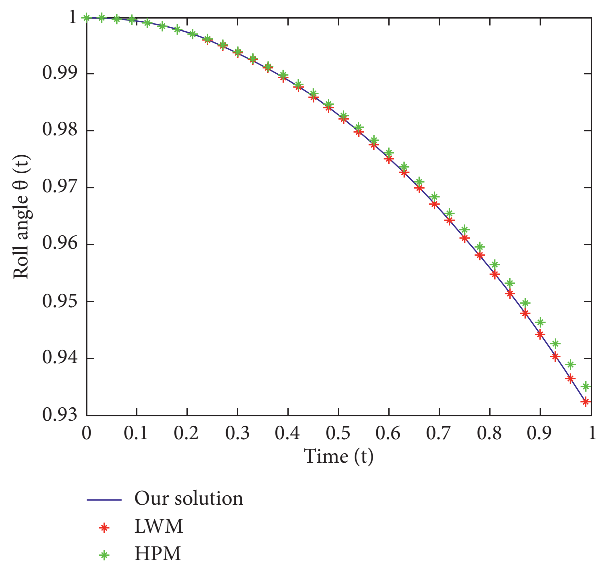

Figure 3: Comparative study of Genocchi polynomial, LWM, and HPM for roll angle with respect to the time t values (in sec) for Ex.3.

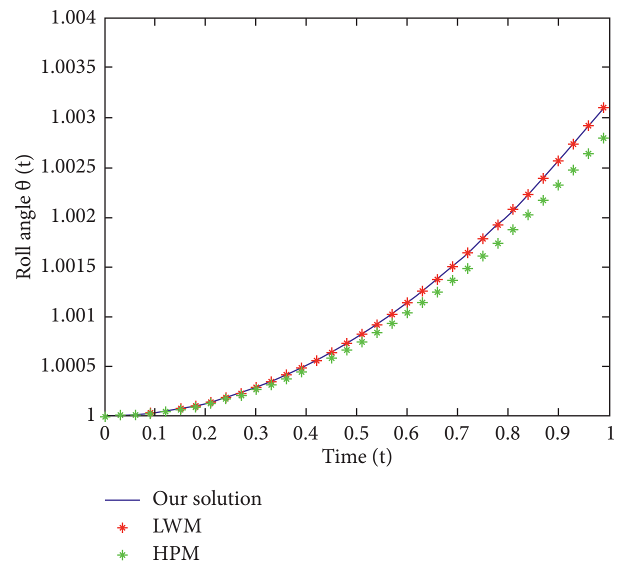

Figure 4: Comparative study of Genocchi polynomial, LWM, and HPM for roll angle with respect to the time t values (in sec) for Ex.4.

$$\theta(t) = 1 - 0.0689406t^2. \tag{22}$$

The Genocchi polynomial results are compared with LWM and HPM in Table 1. The approximate solution has been illustrated in Figure 3.

Example 4. Consider the equation with initial conditions:

$$\begin{aligned} \theta'' + 0.01\theta' + \theta - 0.025\theta^2 - 0.975\theta^3 - 0.0195\sin(0.527t) &= 0, \quad t \in (0,1], \\ \theta(0) &= 1, \\ \theta'(0) &= 0. \end{aligned} \tag{23}$$

By Section 3 procedure, this example has been carried out with $N = 12$.

After simplification, we obtain

$$\theta(t) = 1 + 0.00316317t^2. \tag{24}$$

The Genocchi polynomial results are compared with LWM and HPM in Table 1. The approximate solution has been illustrated in Figure 4.

Example 5. Consider the equation with initial conditions:

$$\begin{aligned} \theta'' + 0.01\theta' + \theta - 0.05\theta^2 - 0.95\theta^3 - 0.0195\sin(0.527t) &= 0, \quad t \in (0,1], \\ \theta(0) &= 1, \\ \theta'(0) &= 0. \end{aligned} \tag{25}$$

By using Section 3 procedure, this example have been carried out with $N = 12$.

After simplification, we obtain

$$\theta(t) = 1 + 0.00315087t^2. \tag{26}$$

The Genocchi polynomial results are compared with LWM and HPM in Table 2. The approximate solution has been illustrated in Figure 5.

Example 6. Consider the equation with initial conditions:

Table 2: Comparative Study of Genocchi polynomial, LWM, and HPM for roll angle with respect to the time t (in sec.).

Time	Ex:4			Ex:5			Ex:6			Ex:7		
t	Genocchi polynomial	LWM	HPM	Genocchi polynomial	LWM	HPM	Genocchi polynomial	LWM	HPM	Genocchi polynomial	LWM	HPM
0.0	1	1	1	1	1	1	1	1	1	1	1	1
0.1	1.000031	1.000031	1.000028	1.000031	1.000031	1.000028	1.000032	1.000032	1.000034	1.000032	1.000032	1.000034
0.2	1.000126	1.000126	1.000114	1.000126	1.000126	1.000114	1.000130	1.000130	1.000138	1.000130	1.000130	1.000138
0.3	1.000284	1.000284	1.000257	1.000283	1.000283	1.000257	1.000293	1.000293	1.000311	1.000292	1.000292	1.000310
0.4	1.000506	1.000506	1.000458	1.000504	1.000504	1.000457	1.000522	1.000522	1.000554	1.000520	1.000520	1.000552
0.5	1.000790	1.000790	1.000715	1.000787	1.000787	1.000715	1.000816	1.000816	1.000866	1.000812	1.000812	1.000862
0.6	1.001138	1.001138	1.001030	1.001134	1.001134	1.001029	1.001175	1.001175	1.001247	1.001170	1.001170	1.001242
0.7	1.001549	1.001549	1.001402	1.001543	1.001543	1.001401	1.001599	1.001599	1.001697	1.001593	1.001593	1.001691
0.8	1.002024	1.002024	1.001832	1.002016	1.002016	1.001830	1.002089	1.002089	1.002217	1.002081	1.002081	1.002209
0.9	1.002562	1.002562	1.002319	1.002552	1.002552	1.002317	1.002644	1.002644	1.002806	1.002633	1.002633	1.002795
1.0	1.003163	1.003163	1.002863	1.003150	1.003150	1.002860	1.003264	1.003264	1.003464	1.003251	1.003251	1.003451

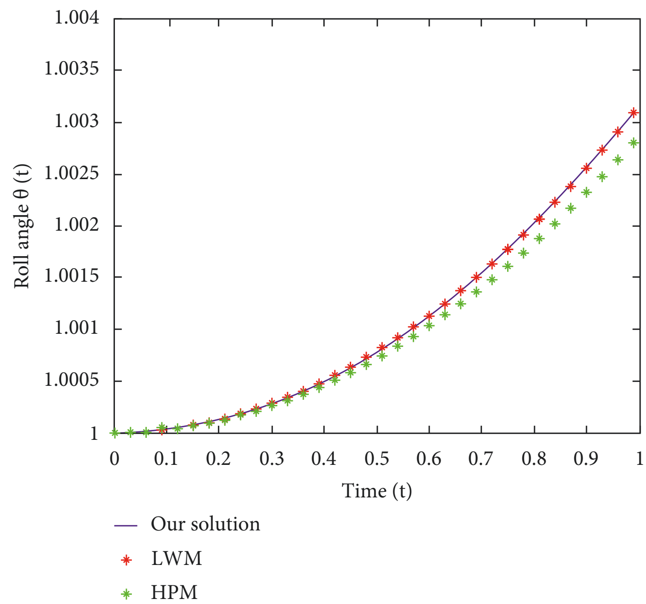

FIGURE 5: Comparative Study of Genocchi polynomial, LWM, and HPM for roll angle with respect to the time t (in sec) for Ex.5.

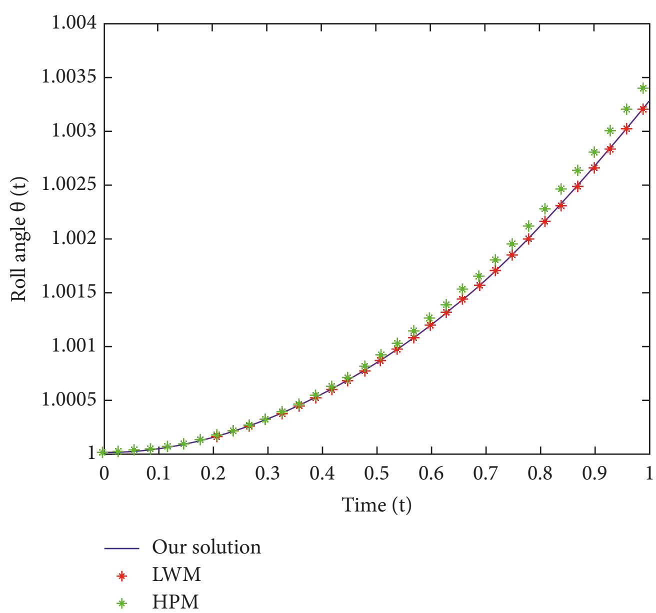

FIGURE 6: Comparative Study of Genocchi polynomial, LWM, and HPM for roll angle with respect to the time t (in sec) for Ex.6.

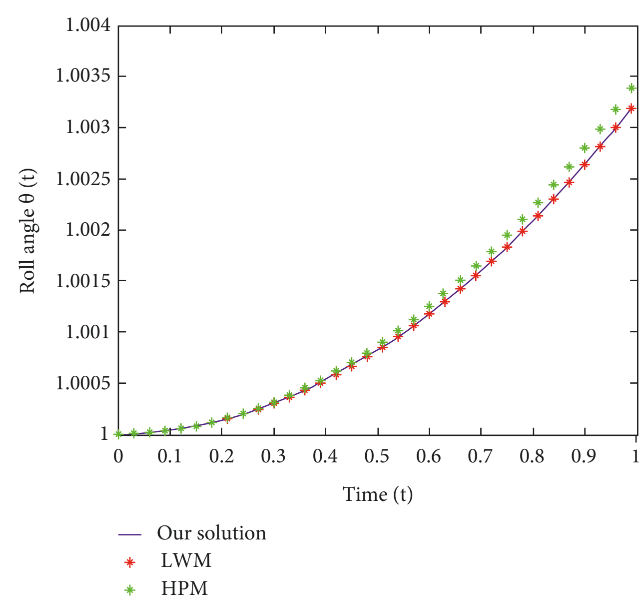

FIGURE 7: Comparative Study of Genocchi polynomial, LWM, and HPM for roll angle with respect to the time t (in sec) for Ex.7.

$$\begin{aligned}&\theta'' + 0.05\theta' + \theta - 0.025\theta^2 - 0.975\theta^3 \\ &\quad - 0.0195\sin(0.527t] = 0, \quad t \in (0,1],\end{aligned} \tag{27}$$

and with initial conditions,

$$\begin{aligned}\theta(0) &= 1, \\ \theta'(0) &= 0.\end{aligned} \tag{28}$$

By using Section 3 procedure, this example have been carried out with $N = 12$.

After simplification, we obtain

$$\theta(t) = 1 + 0.00326487t^2. \tag{29}$$

The Genocchi polynomial results are compared with LWM and HPM in Table 2. The approximate solution has been illustrated in Figure 6.

Example 7. Consider the equation with initial conditions:

Table 3: Absolute error of Genocchi polynomial for roll angle with respect to the time t (in sec).

Time	Ex.1	Ex.2	Ex.3	Ex.4	Ex.5	Ex.6	Ex.7
0.0	0	0	0	0	0	0	0
0.1	0	0.0000037	0.000026	0.000003	0.000003	0.000002	0.000002
0.2	0	0.0001480	0.000104	0.000008	0.000012	0.000008	0.000008
0.3	0.0036	0.000333	0.000234	0.000027	0.000026	0.000018	0.000018
0.4	0.00639	0.000592	0.000416	0.000048	0.000047	0.000032	0.000032
0.5	0.00999	0.000925	0.00065	0.000075	0.000072	0.00005	0.00005
0.6	0.01439	0.001332	0.000936	0.000092	0.000105	0.000072	0.000072
0.7	0.019608	0.001813	0.001274	0.000147	0.000142	0.000098	0.000098
0.8	0.0255994	0.002368	0.001664	0.000192	0.000186	0.000128	0.000128
0.9	0.0323992	0.002997	0.002106	0.000243	0.000235	0.000162	0.000162
1.0	0.0399992	0.0037	0.0026	0.0003	0.000290	0.0002	0.0003

$$\begin{aligned}&\theta'' + 0.05\theta' + \theta - 0.05\theta^2 - 0.95\theta^3\\&\quad - 0.0195\,\sin(0.527t) = 0,\quad t \in (0,1],\end{aligned}\tag{30}$$

which subject to

$$\begin{aligned}\theta(0) &= 1,\\ \theta'(0) &= 0.\end{aligned}\tag{31}$$

By using Section 3 procedure, this example have been carried out with $N = 12$.

After simplification, we obtain

$$\theta(t) = 1 + 0.00325179t^2.\tag{32}$$

The Genocchi polynomial results are compared with LWM and HPM in Table 2. The approximate solution has been illustrated in Figure 7.

In the tables, we have introduced the Genocchi polynomial-based spectral methods for the estimation of the parameters in an equivalent ship roll motion model using only the roll motion response (roll angle and roll velocity), which can be easily identified for a ship sailing at sea. There are several advantages of the proposed Genocchi polynomial approach method by comparison to other existing methods in the literature. The absolute error has been calculated for all the above examples, and it was listed in Table 3.

5. Conclusion

In this study, the nonlinear equations in the roll motion of ships are solved by the Genocchi polynomial operational matrix approach with respect to the suitable collocation points. We successfully applied the proposed Genocchi polynomial operational matrix approach for estimating the ship roll angle. It has been observed that the proposed method is reliable to solve the biased ship roll motion equations approximately. Also, the obtained results have been validated with the results of HPM and LWM. It can also be concluded that the Genocchi polynomial operational matrix approach will be powerful for finding the approximation solutions for a large class of nonlinear differential equations. To avoid the computation complexity, we impose the collocation points to form algebraic equations. Satisfactory agreement with the results and this method is computationally attractive and effective while comparing with other methods. The advantage of this method is that highly accurate approximate solutions are achieved using a few numbers of terms of the approximate expansion. Comparative study of Genocchi polynomial, LWM, and HPM for roll angle and its figure give the accuracy by increasing the value of R. The proposed method can also be extended to solve other types of ODEs and PDEs arising in science and engineering.

Data Availability

No data were used to support the findings of the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors contributed equally to this work. And all the authors have read and approved the final version manuscript.

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