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Research Article

Two Computational Strategies for the Approximate Solution of the Nonlinear Gas Dynamic Equations

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In this article, we propose an idea of Sawi homotopy perturbation transform method (SHPTM) to derive the analytical results of nonlinear gas dynamic (GD) equations. The implementation of this numerical scheme is straightforward and produces the results directly without any assumptions and hypothesis in the recurrence relation. Sawi transform (ST) has an advantage of reducing the computational work and the error of estimated results towards the precise solution. The results obtained with this approach are in the shape of an iteration that converges to the precise solution very gradually. We provide the validity and accuracy of this scheme with the help of illustrated examples and their graphical results. This scheme has shown to be the simplest approach for achieving the analytical results of nonlinear problems in science and engineering.

1. Introduction

In recent decades, nonlinear models are particularly describing various physical phenomena in engineering, physics, chemistry, and other sciences. Numerous analytical and numerical schemes have been broadly applied to these nonlinear problems. The procedure of obtaining the precise results for the nonlinear problems is very complicated, and it is still a challenging issue to solve these nonlinear PDEs in most of the cases; besides this, there are various strategies for their solution. As a result, various researchers and scientists have studied multiple novel methods for getting the analytical solution that are reasonably close to the precise solutions such as the Jacobi elliptic function method [1], Exp $(-\Phi(\eta))$-expansion method [2], new Kudryashov's method [3], rank upgrading technique [4], modified exponential rational method [5], Hermite-Ritz method [6], residual power series (RPS) method [7], and Adomian decomposition method [8, 9].

He [10, 11] developed an idea of homotopy perturbation method (HPM) to obtain the analytical solution of differential problems. Later, Khuri and Sayfy [12] combined Laplace transform with HPM for the analytical results of differential problems. Nadeem and Li [13] presented a combined approach of Laplace transform with HPM for dealing the analytical work of nonlinear vibration systems and nonlinear wave problems. HPM provides the significant results to solve linear and nonlinear equations of reaction-diffusion equations [14], heat transfer model [15], delay differential equations [16], integro-differential equation [17], and Schrödinger equations [18].

Gas dynamic equations are mathematically modeled by various physical laws such as energy, mass, and momentum conservation. The study of gas motion and its impact on structures using the principles of fluid dynamics and fluid mechanics is known as "gas dynamic," and it belongs to the discipline of fluid dynamics [19–21]. Jafari [22] presented the idea of variational iteration method (VIM) on the basis of Lagrange multipliers to investigate the analytical solution of nonlinear gas dynamic equation and Stefan equation. Later, Matinfar et al. [23] used a simple procedure using He's polynomials to obtain the analytical results of GD equation and provided the efficient results to show that the suggested algorithm is quite suitable for such problems.

Kumar and Rashidi [24] formulated a scheme based on Laplace transform and the homotopy analysis scheme for handling the time-fractional GD equations. Singh et al. [25] provided the approximate solution of GD equation and showed that HPM presents the excellent performance in various nonlinear problems. Singh and Aggarwal [26] introduced Sawi transform for population growth and decay problems. Many authors provided that this transform has an excellent performance in various differential problems [27–29].

In this article, we combined Sawi transform and HPM to formulate the idea of SHPTM and obtain the analytical results of GD equations. HPM is used to handle nonlinear components. Sawi transform has an advantage of reducing the computational work and minimizing the error of the estimated results towards the precise results. We observe that HPM is very efficient technique in solving the nonlinear phenomena. Results show that this strategy is very unique and easy to implement than other approaches. This article is presented as follows: in Section 2, we report the concept of Sawi transform with some property functions. In Section 3, a basic idea of HPM is revealed to overcome the nonlinear components. Section 4 demonstrates the basic idea of SHPTM to handle the nonlinear problems. We illustrate two numerical examples to sow the performance of SHPTM and present the conclusion in Sections 5 and 6, respectively.

2. Sawi Transform

Definition 1. Consider $f(t)$ be a function with $t \geq 0$, so

$$\mathscr{L}\{f(t)\} = F(s) = \theta \int_0^\infty f(t) e^{-st} dt \tag{1}$$

is said to be Laplace transform.

Definition 2. Sawi transform is represented by $S(.)$ for a function $\vartheta(\theta)$

$$S[\vartheta(t)] = R(\theta) = \frac{1}{\theta^2} \int_0^\infty \vartheta(t) e^{-t/\theta} dt, t \geq 0, k_1 \leq \theta \leq k_2. \tag{2}$$

Here, S is termed as Sawi transform and if $R(\theta)$ is the Sawi transform of a function $\vartheta(t)$. then $\vartheta(t)$ is the inverse of $R(\theta)$ so that $S^{-1}[R(\theta)] = \vartheta(t), S^{-1}$ is said to be inverse Sawi transform.

Properties. If $S\{g(t)\} = R(\theta)$, the following differential properties yield [26, 28]:

(a) $S\{g'(t)\} = (R(\theta)/\theta) - (G(0)/\theta^2)$

(b) $S\{g''(t)\} = (R(\theta)/\theta^2) - (G(0)/\theta^3) - (G'(0)/\theta^2)$

(c) $S\{g^m(t)\} = (R(\theta)/\theta^m) - (G(0)/\theta^{m+1}) - (G'(0)/\theta^m) - \cdots - (G^{m-1}(0)/\theta^2)$

3. Fundamental Concept of HPM

This sector presents the strategy of HPM with the consideration of a nonlinear functional equation [13]. Consider

$$T(\vartheta) - g(h) = 0, h \in \Omega, \tag{3}$$

with conditions

$$S\left(\vartheta, \frac{\partial \vartheta}{\partial n}\right) = 0, h \in \Gamma. \tag{4}$$

Here, T is a general function and S is the boundary operator, and $g(h)$ is source term. We can now split T such that T_1 is said to be a linear and T_2 be a nonlinear operator. Thus, we can write Equation (3) as

$$T_1(\vartheta) + T_2(\vartheta) - g(h) = 0. \tag{5}$$

Consider $\vartheta(h, \theta)$: $\Omega \times [0, 1] \longrightarrow \mathbb{H}$ such that it is suitable for

$$H(\vartheta, \theta) = (1 - \theta)[T_1(\vartheta) - T_1(\vartheta_0)] + \theta[T_1(\vartheta) - T_2(\vartheta) - g(h)], \tag{6}$$

or

$$H(\vartheta, \theta) = T_1(\vartheta) - T_1(\vartheta_0) + qL(\vartheta_0) + \theta[T_2(\vartheta) - g(h)] = 0. \tag{7}$$

Here, $\theta \in [0, 1]$ is homotopy element and ϑ_0 is the starting approximation of Equation (3). The study of HPM declares that θ is assumed as a minimal factor and the result of Equation (3) can be expressed in the shape of θ.

$$\vartheta = \vartheta_0 + \theta\vartheta_1 + \theta^2\vartheta_2 + \theta^3\vartheta_3 + \cdots = \sum_{i=0}^\infty \theta^i \vartheta_i. \tag{8}$$

Considering $\theta = 1$, we get particular of Equation (3) as

$$\vartheta = \lim_{\theta \longrightarrow 1} \vartheta = \vartheta_0 + \vartheta_1 + \vartheta_2 + \vartheta_3 + \cdots = \sum_{i=0}^\infty \vartheta_i. \tag{9}$$

The nonlinear terms are obtained as

$$T_2\vartheta(x, t) = \sum_{n=0}^\infty \theta^n H_n(\vartheta), \tag{10}$$

where $H_n(\vartheta)$ is defined as

$$H_n(\vartheta_0 + \vartheta_1 + \cdots + \vartheta_n) = \frac{1}{n!} \frac{\partial^n}{\partial \theta^n} \left(T_2 \left(\sum_{i=0}^\infty \theta^i \vartheta_i \right) \right)_{\theta=0}, n = 0, 1, 2, \cdots. \tag{11}$$

This result in Equation (10) generally converges as the rate of convergence depends on the nonlinear operator T_2.

4. Formulation of SHPTM

This section reveals the construction of SHPTM for achieving the analytical results GD equation. Consider a nonlinear differential problem such as

$$\vartheta'(x,t)+\vartheta(x,t)+g(\vartheta)=g(x,t), \tag{12}$$

with initial condition

$$\vartheta(x,0)=a, \tag{13}$$

where ϑ is a function in time domain t, $g(\vartheta)$ represents nonlinear component, $g(x,t)$ is known, and a is the constant. Now, Equation (12) can reconsider as

$$\vartheta'(x,t)=-\vartheta(x,t)-g(\vartheta)+g(x,t). \tag{14}$$

Operating ST on Equation (14), we get

$$S\left[\vartheta'(x,t)\right]=S[-\vartheta(x,t)-g(\vartheta)+g(x,t)]. \tag{15}$$

Implementing the properties of ST, it yields

$$\frac{R(\theta)}{\theta}-\frac{G(0)}{\theta^2}=-S[\vartheta(x,t)+g(\vartheta)-g(x,t)]. \tag{16}$$

Thus, $R(\theta)$ is found from Equation (16) as

$$R[\theta]=\frac{G(0)}{\theta}-\theta S[\vartheta(x,t)+g(\vartheta)-g(x,t)]. \tag{17}$$

Applying inverse ST on Equation (17), it yields

$$\vartheta(x,t)=G(x,t)-S^{-1}[\theta S[\vartheta(x,t)+g(\vartheta)]], \tag{18}$$

Equation (18) is called the recurrence relation of Equation (12) where

$$G(x,t)=S^{-1}\left[\frac{G(0)}{\theta}+\theta g(x,t)\right]. \tag{19}$$

According to the strategy of HPM, consider

$$\vartheta(t)=\sum_{i=0}^{\infty}p^i\vartheta_i(n)=\vartheta_0+p^1\vartheta_1+p^2\vartheta_2+\cdots, \tag{20}$$

and nonlinear terms $g(\vartheta)$ can be determined using an algorithm

$$g(\vartheta)=\sum_{i=0}^{\infty}p^iH_i(\vartheta)=H_0+p^1H_1+p^2H_2+\cdots, \tag{21}$$

where $H_n's$ is He's polynomial, and we calculate them by using the following procedure.

$$H_n(\vartheta_0+\vartheta_1+\cdots+\vartheta_n)=\frac{1}{n!}\frac{\partial^n}{\partial p^n}\left(g\left(\sum_{i=0}^{\infty}p^i\vartheta_i\right)\right)_{p=0}, n=0,1,2,\cdots. \tag{22}$$

Putting Equations (20), (21), and (22) in Equation (18) and equating the same components of p, we obtain the following iterations

$$\begin{aligned}p^0 &: \vartheta_0(x,t)=G(x,t),\\ p^1 &: \vartheta_1(x,t)=-S^{-1}[\theta S\{\vartheta_0(x,t)+H_0(\vartheta)\}],\\ p^2 &: \vartheta_2(x,t)=-S^{-1}[\theta S\{\vartheta_1(x,t)+H_1(\vartheta)\}],\\ p^3 &: \vartheta_3(x,t)=-S^{-1}[\theta S\{\vartheta_2(x,t)+H_2(\vartheta)\}],\\ &\vdots\end{aligned} \tag{23}$$

By repeating the same manner, we can sum up this series to obtain the analytical results such that

$$\vartheta(x,t)=\vartheta_0+\vartheta_1+\vartheta_2+\cdots=\sum_{i=0}^{\infty}\vartheta_i. \tag{24}$$

Thus, Equation (24) yields as an analytical result of differential problem of Equation (12).

5. Numerical Applications

In this portion, we implement the idea of SHPTM in order to obtain the analytical solution of nonlinear GD equations. The solution series converges to the exact solution with few iterations which shows the significance of this approach.

5.1. Example 1. Consider the homogenous and nonlinear GD equation

$$\frac{\partial\vartheta}{\partial t}+\vartheta\frac{\partial\vartheta}{\partial x}-\vartheta(1-\vartheta)=0, \tag{25}$$

with initial condition

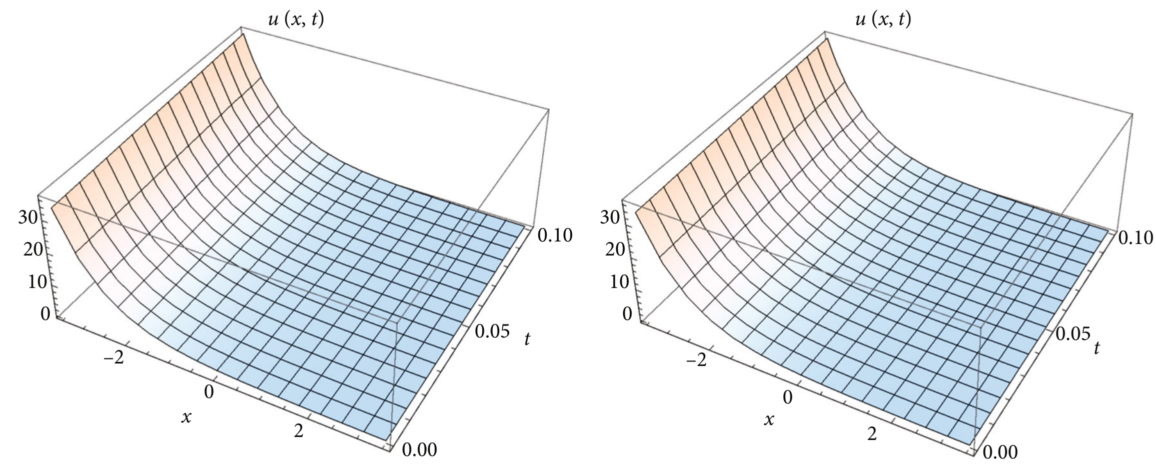
$$\vartheta(x,0)=e^{-x}. \tag{26}$$

Taking the Sawi transform of Equation (25), we get

$$\begin{aligned}S\left[\frac{\partial\vartheta}{\partial t}+\vartheta\frac{\partial\vartheta}{\partial x}-\vartheta(1-\vartheta)\right]&=0,\\ S\left[\frac{\partial\vartheta}{\partial t}\right]=-S\left[\vartheta\frac{\partial\vartheta}{\partial x}-\vartheta(1-\vartheta)\right]&=0.\end{aligned} \tag{27}$$

Employing the properties of Sawi transform, we get

$$\frac{\vartheta(x,\theta)}{\theta}-\frac{\vartheta(x,0)}{\theta^2}=-S\left[\vartheta\frac{\partial\vartheta}{\partial x}-\vartheta(1-\vartheta)\right], \tag{28}$$



(a) The analytical solution of $\vartheta(x, t)$

(b) The exact solution of $\vartheta(x, t)$

Figure 1: The surface solution of GD equation for Example 1.

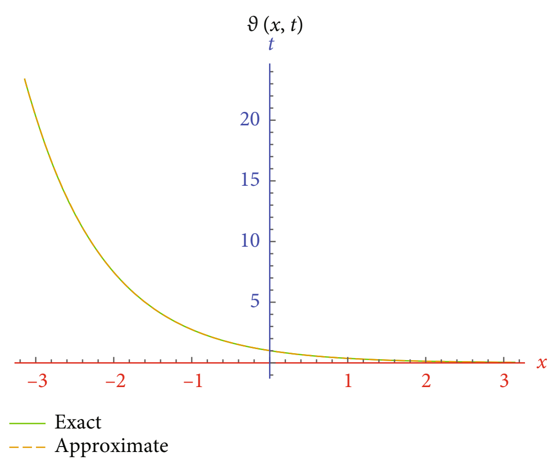


Figure 2: 2D plot for $\vartheta(x, t)$ with various parameter of t.

which may be solved further as

$$\vartheta(x, \theta) = \frac{\vartheta(x, 0)}{\theta} - \theta S\left\{\vartheta \frac{\partial \vartheta}{\partial x} - \vartheta + \vartheta^2)\right\}. \tag{29}$$

Applying inverse Sawi transform, we get

$$\vartheta(x, t) = \vartheta(x, 0) - S^{-1}\left[\theta S\left\{\vartheta \frac{\partial \vartheta}{\partial x} - \vartheta + \vartheta^2)\right\}\right]. \tag{30}$$

Utilizing HPM on Equation (30), we get

$$\sum_{n=0}^{\infty} p^n \vartheta_n(x, t) = \vartheta(x, 0) - pS^{-1} \left[\theta S\left\{\sum_{n=0}^{\infty} p^n \vartheta_n(x, t) \frac{\partial}{\partial x} \sum_{n=0}^{\infty} p^n \vartheta_n(x, t) - \sum_{n=0}^{\infty} p^n \vartheta_n(x, t) + \sum_{n=0}^{\infty} p^n \vartheta_n^2(x, t)\right\}\right]. \tag{31}$$

In comparing, the following iterations can be obtained:

$$\begin{aligned}
&p^0 : \vartheta_0(x, t) = e^{-x},\\
&p^1 : \vartheta_1(x, t) = -S^{-1}\left[\theta S\left\{\vartheta_0 \frac{\partial \vartheta_0}{\partial x} - \vartheta_0 + \vartheta_0^2\right\}\right] = e^{-x} t\\
&p^2 : \vartheta_2(x, t) = -S^{-1}\left[\theta S\left\{\vartheta_0 \frac{\partial \vartheta_1}{\partial x} + \vartheta_1 \frac{\partial \vartheta_0}{\partial x} - \vartheta_1 + 2\vartheta_0\vartheta_1\right\}\right] = e^{-x} \frac{t^2}{2!},\\
&p^3 : \vartheta_3(x, t) = -S^{-1}\left[\theta S\left\{\vartheta_0 \frac{\partial \vartheta_2}{\partial x} + \vartheta_1 \frac{\partial \vartheta_1}{\partial x} + \vartheta_2 \frac{\partial \vartheta_0}{\partial x} - \vartheta_2 + \vartheta_1^2 + 2\vartheta_0\vartheta_2\right\}\right] = e^{-x} \frac{t^3}{3!},\\
&\vdots
\end{aligned} \tag{32}$$

Hence, the solution can be expressed as

$$\begin{aligned}
\vartheta(x, t) &= \vartheta_0(x, t) + \vartheta_1(x, t) + \vartheta_2(x, t) + \vartheta_3(x, t) + \cdots,\\
\vartheta(x, t) &= e^{-x} + e^{-x} t + e^{-x} \frac{t^2}{2!} + e^{-x} \frac{t^3}{3!} + \cdots,\\
\vartheta(x, t) &= e^{t-x}.
\end{aligned} \tag{33}$$

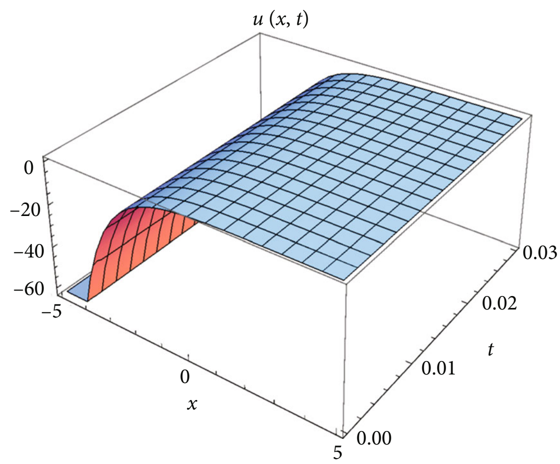
In Figure 1, we show the analytical and exact solution graphs of Problem 1 at $-3.5 \le x \le 3.5$ and $0 \le t \le 0.1$. The graphical results show that the analytical solution and the exact solutions are very close to each other. In addition, Figure 2 presents the graphical error with $-\pi \le x \le \pi$ at $t = 0.01$, and it seems that the suggested approach is very efficient and authentic for finding the analytical solution of nonlinear GD equations.

5.2. Example 2. Consider the nonhomogenous and nonlinear GD equation

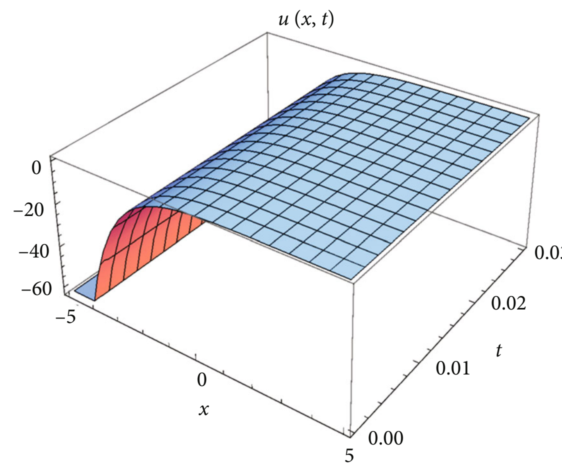
$$\frac{\partial \vartheta}{\partial t} + \vartheta \frac{\partial \vartheta}{\partial x} - \vartheta(1 - \vartheta) = -e^{t-x}, \tag{34}$$

with initial condition

$$\vartheta(x, 0) = 1 - e^{-x}. \tag{35}$$



(a) The analytical solution of $\vartheta(x,t)$



(b) The exact solution of $\vartheta(x,t)$

FIGURE 3: The surface solution of GD equation for Example 2.

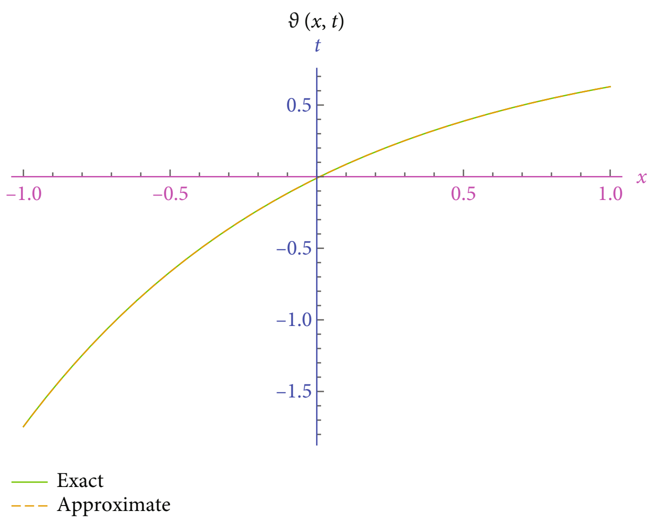


FIGURE 4: 2D plot for $\vartheta(x,t)$ with various parameter of t.

Taking the Sawi transform of Equation (34), we get

$$S\left[\frac{\partial\vartheta}{\partial t}+\vartheta\frac{\partial\vartheta}{\partial x}-\vartheta(1-\vartheta)\right]=-S\left[e^{t-x}\right],$$
$$S\left[\frac{\partial\vartheta}{\partial t}\right]=-S\left[e^{t-x}\right]-S\left[\vartheta\frac{\partial\vartheta}{\partial x}-\vartheta(1-\vartheta)\right]. \quad (36)$$

Employing the properties of Sawi transform, we get

$$\frac{\vartheta(x,\theta)}{\theta}-\frac{\vartheta(x,0)}{\theta^2}=-\frac{e^{-x}}{\theta(1-\theta)}-S\left[\vartheta\frac{\partial\vartheta}{\partial x}-\vartheta(1-\vartheta)\right], \quad (37)$$

which may be solved further as

$$\vartheta(x,\theta)=\frac{\vartheta(x,0)}{\theta}-\frac{e^{-x}}{1-\theta}-\theta S\left\{\vartheta\frac{\partial\vartheta}{\partial x}-\vartheta+\vartheta^2\right\}. \quad (38)$$

Applying inverse Sawi transform, we get

$$\vartheta(x,t)=\vartheta(x,0)-e^{-x}S^{-1}\left[\frac{1}{1-\theta}\right]-S^{-1}\left[\theta S\left\{\vartheta\frac{\partial\vartheta}{\partial x}-\vartheta+\vartheta^2\right\}\right]. \quad (39)$$

Utilizing HPM on Equation (39), we get

$$\sum_{n=0}^{\infty}p^n\vartheta_n(x,t)=1-e^{t-x}-pS^{-1}\left[\theta S\left\{\sum_{n=0}^{\infty}p^n\vartheta_n(x,t)\frac{\partial}{\partial x}\sum_{n=0}^{\infty}p^n\vartheta_n(x,t)-\sum_{n=0}^{\infty}p^n\vartheta_n(x,t)+\sum_{n=0}^{\infty}p^n\vartheta_n^2(x,t)\right\}\right]. \quad (40)$$

In comparing, the following iterations can be obtained:

$$p^0:\vartheta_0(x,t)=1-e^{t-x},$$
$$p^1:\vartheta_1(x,t)=S^{-1}\left[\theta S\left\{\vartheta_0\frac{\partial\vartheta_0}{\partial x}-\vartheta_0+\vartheta_0^2\right\}\right]=0$$
$$p^2:\vartheta_2(x,t)=S^{-1}\left[\theta S\left\{\vartheta_0\frac{\partial\vartheta_1}{\partial x}+\vartheta_1\frac{\partial\vartheta_0}{\partial x}-\vartheta_1+2\vartheta_0\vartheta_1\right\}\right]=0,$$
$$p^3:\vartheta_3(x,t)=S^{-1}\left[\theta S\left\{\vartheta_0\frac{\partial\vartheta_2}{\partial x}+\vartheta_1\frac{\partial\vartheta_1}{\partial x}+\vartheta_2\frac{\partial\vartheta_0}{\partial x}-\vartheta_2+\vartheta_1^2+2\vartheta_0\vartheta_2\right\}\right]=0,$$
$$\vdots \quad (41)$$

Hence, the solution can be expressed as

$$\vartheta(x,t)=\vartheta_0(x,t)+\vartheta_1(x,t)+\vartheta_2(x,t)+\vartheta_3(x,t)+\cdots,$$
$$\vartheta(x,t)=1-e^{t-x}+0+0+\cdots,$$
$$\vartheta(x,t)=1-e^{t-x}. \quad (42)$$

In Figure 3, we show the analytical and exact solution graphs of Problem 1 at $-5\le x\le 5$ and $0\le t\le 0.03$. The graphical results show that the analytical solution and the exact solutions are very close to each other. In addition, Figure 4 presents the graphical error with $-1\le x1$ at $t=$

0.01, and it seems that the suggested approach is very efficient and authentic for finding the analytical solution of nonlinear GD equations.

6. Conclusion

In this paper, we constructed a SHPTM to obtain the analytical solution of nonlinear GD equations. The conservation characteristics of the numerical scheme are demonstrated by theoretical analysis. Additionally, we determined the error estimates to show that the obtained results are in quick convergence. One observation is that if Sawi transform is used with HPM, we do not need to digitize the GD equations which leads to a high number of restrictions and assumptions. This is because Sawi transform is independent of restrictive variable and considered as a direct approach for the conservation law in both linear and nonlinear problems. We use Mathematica software 11.0.1 for the numerical analysis and computation of the iterations of series solutions. One can use this scheme for other nonlinear numerical problems to obtain the excellent results that are stable and accurate. However, our work can easily be modified to study the theory of fractional calculus in science and engineering.

Data Availability

This article contains all the data.

Conflicts of Interest

This article has no conflicts of interest.

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