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Retraction

Retracted: Novel Concepts in Bipolar Fuzzy Graphs with Applications

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Journal of Mathematics has retracted the article titled "Novel Concepts in Bipolar Fuzzy Graphs with Applications" [1] due to concerns that the peer review process has been compromised.

Following an investigation conducted by the Hindawi Research Integrity team [2], significant concerns were identified with the peer reviewers assigned to this article; the investigation has concluded that the peer review process was compromised. We therefore can no longer trust the peer review process, and the article is being retracted with the agreement of the Chief Editor.

The authors do not agree to the retraction.

References

[1] C. Wan, F. Deng, S. Li, S. Omidbakhsh Amiri, A. A. Talebi, and H. Rashmanlou, "Novel Concepts in Bipolar Fuzzy Graphs with Applications," *Journal of Mathematics*, vol. 2022, Article ID 8162474, 9 pages, 2022.

[2] L. Ferguson, "Advancing Research Integrity Collaboratively and with Vigour," 2022, https://www.hindawi.com/post/advancing-research-integrity-collaboratively-and-vigour/.

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Research Article

Novel Concepts in Bipolar Fuzzy Graphs with Applications

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Many problems of practical interest can be modeled and solved by using bipolar graph algorithms. Bipolar fuzzy graph (BFG), belonging to fuzzy graphs (FGs) family, has good capabilities when facing with problems that cannot be expressed by FGs. Hence, in this paper, we introduce the notion of (ϑ, δ)−homomorphism of BFGs and classify homomorphisms (HMs), weak isomorphisms (WIs), and co-weak isomorphisms (CWIs) of BFGs by (ϑ, δ)−HMs. Also, an application of homomorphism of BFGs has been presented by using coloring-FG. Universities are very important organizations whose existence is directly related to the general health of the society. Since the management in each department of the university is very important, therefore, we have tried to determine the most effective person in a university based on the performance of its staff.

1. Introduction

Graphs from ancient times to the present day have played a very important role in various fields, including computer science and social networks, so that with the help of the vertices and edges of a graph, the relationships between objects and elements in a social group can be easily introduced. However, there are some phenomena around our lives that have a wide range of complexities that make it impossible for us to express certainty. These complexities and ambiguities were reduced with the introduction of FSs by Zadeh [1]. The FS focuses on the membership degree of an object in a particular set. However, membership alone could not solve the complexities in different cases, so the need for a degree of membership was felt. To solve this problem, Zhang [2] defined the concept of bipolar fuzzy sets (BFSs) as a generalization of fuzzy sets (FSs). BFSs are an extension of FSs whose membership degree range is [−1, 1]. The first definition of FGs was proposed by Kafmann [3] in 1993, from Zade's fuzzy relations [4, 5]. However, Rosenfeld [6] introduced another elaborated definition including fuzzy vertex and fuzzy edges and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness, and so on. Akram et al. [7, 8] introduced BFGs and cayley-BFGs. Rashmanlou et al. [9] investigated categorical properties in intuitionistic fuzzy graphs. Bhattacharya [10] gave some remarks on FGs, and some operations of FGs were introduced by Mordeson and Peng [11]. The concept of weak isomorphism, co-weak isomorphism, and isomorphism between FGs was introduced by Bhutani in [12]. Liu [13] defined domination number in maximal outer planar graphs. Borzooei [14] introduced domination in vague graphs. Ghorai and Pal [15] studied some isomorphic properties of m-polar FGs. Krishna et al. [16] presented new concept in cubic graph. Shao et al. [17] investigated strong equality of roman and perfect roman domination in trees. Mordeson and Nair [18] introduced the concept of complement of fuzzy graph and studied some operations on fuzzy graphs. The complement of FGs was studied by Sunitha and Vijayakumar [19]. Nagoorgani and Malarvizhi [20] investigated isomorphism properties on FGs. Ezhilmaran et al. [21] studied morphism of bipolar intuitionistic

fuzzy graphs. Muhiuddin et al. [22, 23] introduced new concepts of cubic graphs. Rao et al. [24–26] presented dominating set, equitable dominating set, and isolated vertex of vague graphs. Telebi and Rashmanlou [27] described complement and isomorphism on bipolar fuzzy graphs. Shi et al. [28, 29] introduced total dominating set and global dominating set in product vague graphs. Kosari et al. [30] defined vague graph structure with an application in medical sciences. Kou et al. [31] investigated g-eccentric node and vague detour g-boundary nodes in vague graphs. Ramprasad et al. [32] introduced morphism of m-Polar fuzzy graph. Tahmasbpour et al. [33] presented f-morphism on bipolar fuzzy graphs.

A BFG is a generalized structure of an FG that provides more exactness, adaptability, and compatibility to a system when matched with systems run on FGs. Also, a BFG is able to concentrate on determining the uncertainty coupled with the inconsistent and indeterminate information of any real-world problems, where FGs may not lead to adequate results. With the help of BFGs, the most efficient person in an organization can be identified according to the important factors that can be useful for an institution. Homomorphisms provide a way of simplifying the structure of objects one wishes to study while preserving much of it that is of significance. It is not surprising that homomorphisms also appeared in graph theory, and that they have proven useful in many areas. Hence, in this paper, we defined the notion of (ϑ, δ)–homomorphism of BFGs and classify homomorphisms (HMs), weak isomorphisms (WIs), and co-weak isomorphisms (CWIs) of BFGs by (ϑ, δ)–HMs. Finally, we introduced the application of homomorphism of BFGs by using coloring-FG, and an application of bipolar fuzzy influence digraph has also been presented.

2. Preliminaries

In this section, we give some necessary concepts of bipolar fuzzy graphs and bipolar fuzzy subgroups.

Definition 1. Let V be a finite nonempty set. A graph $G = (V, E)$ on V consists of a vertex set V and an edge set E, where an edge is an unordered pair of distinct vertices of G. We will use xy rather than $\{x, y\}$ to denote an edge. If xy is an edge, then we say that x and y are adjacent. A graph is called complete if every pair of vertices is adjacent.

Definition 2. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be graphs. A mapping $g\colon V_1 \longrightarrow V_2$ is a homomorphism from G_1 to G_2 if $g(r)$ and $g(s)$ are neighbor whenever r and s are neighbor.

Definition 3. Two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic if $\exists$ a bijective mapping $\psi\colon V_1 \longrightarrow V_2$ so that r and s are neighbor in G_1 if and only if $\psi(r)$ and $\psi(s)$ are neighbor in G_2, ψ is named isomorphism from G_1 to G_2. An isomorphism from a graph G to itself is named an automorphism of G. The set of all automorphisms of G forms a group, which is called the automorphism group of G and denoted by $\mathrm{Aut}(G)$.

Definition 4 (see [2]). Let V be a nonempty set. A BFS B in V is an object having the form as follows:

$$B = \left\{\langle r, \mu_B^P(r), \mu_B^N(r)\rangle | r \in V\right\}, \tag{1}$$

where $\mu_B^P\colon V \longrightarrow [0,1]$ and $\mu_B^N\colon V \longrightarrow [-1,0]$ are mappings.

For the sake of simplicity, we shall use the symbol $B = (\mu_B^P, \mu_B^N)$ for the BFS.

$$B = \left\{\langle r, \mu_B^P(r), \mu_B^N(r)\rangle | r \in V\right\}. \tag{2}$$

The family of all BFSs on V is written as $BFS[V]$.

Definition 5. Let $P_* = \{(p, q)\colon q \in [-1, 0], p \in [0, 1]\}$. For any $(q_1, p_1), (q_2, p_2) \in P_*$, the orders $\leq$ and $<$ on P_* are defined as

$$\begin{aligned}&(q_1, p_1) \leq (q_2, p_2) \Leftrightarrow q_1 \geq q_2 \text{ and } p_1 \leq p_2,\\ &(q_1, p_1) < (q_2, p_2) \Leftrightarrow (q_1, p_1) \leq (q_2, p_2) \text{ and } q_1 > q_2 \textit{ or } p_1 < p_2.\end{aligned} \tag{3}$$

By Definition 5, it is easy to see that $(P_*, \leq)$ constitutes a complete lattice with minimum element $(0, 0)$ and maximum element $(1, -1)$.

Definition 6. Let $B = (\mu_B^P, \mu_B^N)$ be a BFS. For each $(p, q) \in P_*$, we describe

$$B_{(p,q)} = \left\{r \in V\colon \mu_B^P(r) \geq p, \mu_B^N(r) \leq q\right\}. \tag{4}$$

Then, $B_{(p,q)}$ is named (q, p)-level set. The set $\{r | r \in V, \mu_A^P(r) \neq 0, \mu_A^N(r) \neq 0\}$ is named the support A and is shown by A^*.

Let V be a finite nonempty set. Denote by $\widetilde{V}^2$ the set of all 2-element subsets of V. A graph on V is a pair (V, E) where $E \subseteq \widetilde{V}^2$, V and E are called vertex set and edge set, respectively.

Definition 7. Let V be a finite nonempty set, $A \in \mathrm{BFS}[V]$, and $B \in \mathrm{BFS}[\widetilde{V}^2]$. The triple $X = (V, A, B)$ is named a BFG on V, if for each $(r, s) \in \widetilde{V}^2$,

$$\mu_B^P(r, s) \leq \mu_A^P(r) \wedge \mu_A^P(s) \text{ and } \mu_B^N(r, s) \geq \mu_A^N(r) \vee \mu_A^N(s). \tag{5}$$

Definition 8. A BFG $X = (V, A, B)$ is called a strong bipolar fuzzy graph (SBFG) if for each $(r, s) \in \widetilde{V}^2$,

$$\mu_B^P(r, s) = \mu_A^P(r) \wedge \mu_A^P(s), \mu_B^N(r, s) = \mu_A^N(r) \vee \mu_A^N(s), \tag{6}$$

that $(\mu_B^P(r, s), \mu_B^N(r, s)) \neq (0, 0)$ and is called complete bipolar fuzzy graph (CBFG), if for each $(r, s) \in \widetilde{V}^2$, we have

$$\mu_B^P(r, s) = \mu_A^P(r) \wedge \mu_A^P(s) \text{ and } \mu_B^N(r, s) = \mu_A^N(r) \vee \mu_A^N(s). \tag{7}$$

A complete bipolar fuzzy graph $X = (V, A, B)$ with n nodes is shown by $K_{n,A}$.

If $X = (V, A, B)$ is a BFG, then it is easy to see that $X^* = (A^*, B^*)$ is a graph and it is called underlying graph of X.

The set of all BFG on V is denoted by $\mathrm{BFG}[V]$. For given $X = (V, A, B) \in \mathrm{BFG}[V]$, in this study, suppose that $A^* = V$.

Definition 9. Let $X_1 = (V_1, A_1, B_1)$ and $X_2 = (V_2, A_2, B_2)$ be two BFGs. Then,

(1) A mapping $\psi: V_1 \longrightarrow V_2$ is a homomorphism from X_1 to X_2 if

(i) $\mu^P_{A_1}(r) \le \mu^P_{A_2}(\psi(r)), \mu^N_{A_1}(r) \ge \mu^N_{A_2}(\psi(r))$, for all $r \in V_1$

(ii) $\mu^P_{B_1}(rs) \le \mu^P_{B_2}(\psi(r)\psi(s)), \mu^N_{B_1}(rs) \ge \mu^N_{B_2}(\psi(r)\psi(s))$, for all $rs \in \widetilde{V}^2$

(2) A mapping $\psi: V_1 \longrightarrow V_2$ is a weak isomorphism from X_1 to X_2 if ψ is a bijective homomorphism from X_1 to X_2 and

$$\mu^P_{A_1}(r) = \mu^P_{A_2}(\psi(r)), \mu^N_{A_1}(r) = \mu^N_{A_2}(\psi(r)), \text{for all } r \in V_1. \tag{8}$$

(3) A mapping $\psi: V_1 \longrightarrow V_2$ is a co-weak isomorphism from X_1 to X_2 if ψ is a bijective homomorphism from X_1 to X_2 and

$$\mu^P_{B_1}(rs) = \mu^P_{B_2}(\psi(r)\psi(s)), \mu^N_{B_1}(rs) = \mu^N_{B_2}(\psi(r)\psi(s)), \text{for all } rs \in \widetilde{V}^2. \tag{9}$$

(4) An isomorphism from X_1 to X_2 is a bijective mapping $\psi: V_1 \longrightarrow V_2$ so that

(i) $\mu^P_{A_1}(r) = \mu^P_{A_2}(\psi(r)), \mu^N_{A_1}(r) = \mu^N_{A_2}(\psi(r))$, for all $r \in V_1$

(ii) $\mu^P_{B_1}(rs) = \mu^P_{B_2}(\psi(r)\psi(s)), \mu^N_{B_1}(rs) = \mu^N_{B_2}(\psi(r)\psi(s))$, for all $rs \in \widetilde{V}^2$

Definition 10. Suppose that $X = (V, A, B)$ and $Y = (V, A', B')$ be two BFGs. Then, X is a BFSG of Y if $A \subseteq A'$ and $B \subseteq B'$.

Definition 11. Let $X = (V, A, B)$ be a BFG and $W \subseteq V$. Then, the BFG $Y = (W, A', B')$ so that

$$\begin{aligned} &\mu^P_{A'}(r) = \mu^P_A(r), \mu^N_{A'}(r) = \mu^N_A(r), \text{for all } r \in W, \\ &\mu^P_{B'}(rs) = \mu^P_B(rs), \mu^N_{B'}(rs) = \mu^N_B(rs), \text{for all } rs \in \widetilde{W}^2, \end{aligned} \tag{10}$$

is called the induced BFSG by W and shown by $X[W]$.

Definition 12. A family $\Gamma = \{\mu_1, \mu_2, \ldots, \mu_k\}$ of BFSs on V is named a k-coloring of BFG $X = (V, A, B)$ if

(i) $\vee\Gamma = A$.

(ii) $\mu_i \wedge \mu_j = 0$, for $1 \le i, j \le k$.

(iii) For each strong edge rs of X, $\min\{\mu_i(r), \mu_i(s)\} = 0$, for $1 \le i \le k$. We say that a graph is k-colorable if it can be colored with k colors.

All the basic notations are shown in Table 1.

3. Homomorphisms and Isomorphisms of Bipolar Fuzzy Graphs

In this section, we discuss homomorphisms and isomorphisms of bipolar fuzzy graphs by homomorphism of level graphs in bipolar fuzzy graphs.

Theorem 1. *Let V be a finite nonempty set, $A \in BFS(V)$ and $B \in BFS(\widetilde{V}^2)$. Then, $X = (V, A, B) \in BFG(V)$ if and only if $X_{(\vartheta,\delta)} = (A_{(\vartheta,\delta)}, B_{(\vartheta,\delta)})$ is a graph, for all $(\vartheta, \delta) \in P_*$, $A_{(\vartheta,\delta)} \ne \varnothing$.*

Proof. Let $X = (V, A, B)$ be a BFG. For each $(\vartheta, \delta) \in P_*$, $A_{(\vartheta,\delta)} \ne \varnothing$, suppose that $rs \in B_{(\vartheta,\delta)}$. Then, $\mu^P_B(rs) \ge \vartheta$ and $\mu^N_B(rs) \le \delta$. Because X is a BFG,

$$\vartheta \le \mu^P_B(rs) \le \mu^P_A(r) \wedge \mu^P_A(s) \text{ and } \delta \ge \mu^N_B(rs) \ge \mu^N_A(r) \vee \mu^N_A(s). \tag{11}$$

It follows that $r, s \in A_{(\vartheta,\delta)}$. Therefore, $(A_{(\vartheta,\delta)}, B_{(\vartheta,\delta)})$ is a graph.

Conversely, let $X_{(\vartheta,\delta)} = (A_{(\vartheta,\delta)}, B_{(\vartheta,\delta)})$ is a graph, for all $(\vartheta, \delta) \in P_*$, $A_{(\vartheta,\delta)} \ne \varnothing$. For each $rs \in \widetilde{V}^2$, let $\mu^P_B(rs) = \vartheta, \mu^N_B(rs) = \delta$. Then, $rs \in B_{(\vartheta,\delta)}$. Hence, $r, s \in A_{(\vartheta,\delta)}$. Thus, $\mu^P_A(r) \ge \vartheta, \mu^P_A(s) \ge \vartheta, \mu^N_A(r) \le \delta$, and $\mu^N_A(s) \le \delta$. This implies that $\mu^P_A(r) \wedge \mu^P_A(s) \ge \vartheta = \mu^P_B(rs)$ and $\mu^N_A(r) \vee \mu^N_A(s) \le \delta = \mu^N_B(rs)$. Therefore, $X = (V, A, B)$ is a BFG. □

Definition 13. Let $X = (V, A, B)$ and $Y = (W, A\prime, B\prime)$ be two BFGs, $g: V \longrightarrow W$ a mapping. Then, for any $(\vartheta, \delta) \in P_*$, $A_{(\vartheta,\delta)} \ne \varnothing$, if g is a homomorphism from $X_{(\vartheta,\delta)} = (A_{(\vartheta,\delta)}, B_{(\vartheta,\delta)})$ to $Y_{(\vartheta,\delta)} = (A'_{(\vartheta,\delta)}, B'_{(\vartheta,\delta)})$, g is named (ϑ, δ)–homomorphism mapping from X to Y.

Theorem 2. *Let $X = (V, A, B)$ and $Y = (W, A', B')$ be two BFGs. Then, $g: X \longrightarrow Y$ is a homomorphism from X to Y if and only if g is (ϑ, δ)–homomorphism from X to Y.*

Proof. Assume that $g: X \longrightarrow Y$ is a homomorphism from X to Y. Let $A_{(\vartheta,\delta)} \ne \varnothing$, $(\vartheta, \delta) \in P_*$. If $r \in A_{(\vartheta,\delta)}$, then

$$\mu^P_{A'}(g(r)) \ge \mu^P_A(r) \ge \vartheta, \mu^N_{A'}(g(r)) \le \mu^N_A(r) \le \delta. \tag{12}$$

Hence, $g(r) \in A'_{(\vartheta,\delta)}$ implying g is a mapping from $A_{(\vartheta,\delta)}$ to $A'_{(\vartheta,\delta)}$. For $r, s \in A_{(\vartheta,\delta)}$, let $rs \in B_{(\vartheta,\delta)}$. Then,

$$\begin{aligned} &\mu^P_B(rs) \ge \vartheta, \\ &\mu^N_B(rs) \le \delta. \end{aligned} \tag{13}$$

Hence,

$$\mu^P_{B'}(g(r)g(s)) \ge \mu^P_B(rs) \ge \vartheta, \mu^N_{B'}(g(r)g(s)) \le \mu^N_B(rs) \le \delta, \tag{14}$$

which implies $g(r)g(s) \in B'_{(\vartheta,\delta)}$. Therefore, g is a homomorphism from $X_{(\vartheta,\delta)}$ to $Y_{(\vartheta,\delta)}$.

Table 1: Some basic notations.

Notation	Meaning
FG	Fuzzy graph
BFS	Bipolar fuzzy set
FS	Fuzzy set
BFG	Bipolar fuzzy graph
CBFG	Complete bipolar fuzzy graph
SBFG	Strong bipolar fuzzy graph
BM	Bijective mapping
HM	Homomorphism
WI	Weak isomorphism
CWI	Co-weak isomorphism
BH	Bijective homomorphism
SG	Subgraph
CG	Complete graph
BFSG	Bipolar fuzzy subgraph

Conversely, let $g\colon V \longrightarrow W$ be a (ϑ,δ)–homomorphism from X to Y. For arbitrary element $r \in X$, let $\mu_A^P(r) = c$, $\mu_A^N(r) = \mathrm{d}$. Then, $r \in A_{(c,d)}$. Hence, $g(r) \in A_{(c,d)}{}'$ because g is a homomorphism from $(A_{(c,d)}, B_{(c,d)})$ to $(A_{(c,d)}{}', B_{(c,d)}{}')$. It follows that

$$\mu_{A'}^P(g(r)) \geq c, \mu_{A'}^N(g(r)) \leq d, \tag{15}$$

that is,

$$\mu_{A'}^P(g(r)) \geq \mu_A^P(r), \mu_{A'}^N(g(r)) \leq \mu_A^N(r). \tag{16}$$

Now, for arbitraries $r, s \in V$, let $\mu_B^P(rs) = e$, $\mu_B^N(rs) = f$. Then,

$$\begin{aligned} e &= \mu_B^P(rs) \leq \mu_A^P(r) \wedge \mu_A^P(s), \\ f &= \mu_B^N(rs) \geq \mu_A^N(r) \vee \mu_A^N(s). \end{aligned} \tag{17}$$

Hence, $r, s \in A_{(e,f)}$ and $rs \in B_{(e,f)}$. Because g is a homomorphism from $X_{(e,f)} = (A_{(e,f)}, B_{(e,f)})$ to $Y_{(e,f)} = (A_{(e,f)}{}', B_{(e,f)}{}')$, we conclude that $g(r), g(s) \in A_{(e,f)}{}'$ and $g(r)g(s) \in B_{(e,f)}{}'$. Therefore,

$$\mu_{B'}^P(g(r)g(s)) \geq e = \mu_B^P(rs), \mu_{B'}^N(g(r)g(s)) \leq f = \mu_B^N(rs). \tag{18}$$

□

Theorem 3. *Let $X = (V, A, B)$ and $Y = (W, A', B')$ be two BFGs. Then, $g\colon V \longrightarrow W$ is a WI from X to Y if and only if g is a bijective (ϑ,δ)–homomorphism from X to Y and*

$$\mu_A^P(r) = \mu_{A'}^P(g(r)), \mu_A^N(r) = \mu_{A'}^N(g(r)) \text{ for all } r \in V. \tag{19}$$

Proof. Let f be a WI from X to Y. From the definition of homomorphism, g is a bijective homomorphism from X to Y. By Theorem 2, f is a bijective (ϑ,δ)–homomorphism from X to Y, and also by the definition of WI, we have

$$\mu_A^P(r) = \mu_{A'}^P(g(r)), \mu_A^N(r) = \mu_{A'}^N(g(r)) \text{ for all } r \in V. \tag{20}$$

Conversely, from hypothesis, $g\colon A_{(0,1)} = V \longrightarrow A_{(0,1)}{}' = W$ is a bijective mapping and

$$\mu_A^P(r) = \mu_{A'}^P(g(r)), \mu_A^N(r) = \mu_{A'}^N(g(r)) \text{ for all } r \in V. \tag{21}$$

For $r, s \in V$, let $\mu_B^P(rs) = e$, $\mu_B^N(rs) = f$. Then,

$$e = \mu_B^P(rs) \leq \mu_A^P(r) \wedge \mu_A^P(s), f = \mu_B^N(rs) \geq \mu_A^N(r) \vee \mu_A^N(s), \tag{22}$$

which implies $r, s \in A_{(e,f)}$ and $rs \in B_{(e,f)}$. Because g is a homomorphism from $(A_{(e,f)}, B_{(e,f)})$ to $(A_{(e,f)}{}', B_{(e,f)}{}')$, we have $g(r), g(s) \in A_{(e,f)}{}'$ and $g(r)g(s) \in B_{(e,f)}{}'$. Hence,

$$\mu_{B'}^P(g(r)g(s)) \geq e = \mu_B^P(rs), \mu_{B'}^N(g(r)g(s)) \leq f = \mu_B^N(rs), \tag{23}$$

which completes the proof. □

Theorem 4. *Let $X = (V, A, B)$ and $Y = (W, A', B')$ be two BFGs. Then, $g\colon V \longrightarrow W$ is a co-weak isomorphism from X to Y if and only if g is a bijective (ϑ,δ)–homomorphism from X to Y and*

$$\mu_B^P(rs) = \mu_{B'}^P(g(r)g(s)), \mu_B^N(rs) = \mu_{B'}^N(g(r)g(s)) \text{for all } rs \in \tilde{V}^2. \tag{24}$$

Proof. Let $g\colon V \longrightarrow W$ be a co-weak isomorphism from X to Y. Then, g is a bijective homomorphism from X to Y. By Theorem 2, f is a bijective (ϑ,δ)–homomorphism from X to Y. Also, by the definition of co-weak isomorphism,

$$\mu_B^P(rs) = \mu_{B'}^P(g(r)g(s)), \mu_B^N(rs) = \mu_{B'}^N(g(r)g(s)) \text{for all } rs \in \tilde{V}^2. \tag{25}$$

Conversely, from hypothesis, we know that $f\colon A_{(0,1)} = V \longrightarrow A_{(0,1)}{}' = W$ is a bijective mapping and

$$\mu_B^P(rs) = \mu_{B'}^P(g(r)g(s)), \mu_B^N(rs) = \mu_{B'}^N(g(r)g(s)). \tag{26}$$

For arbitrary element $r \in V$, suppose that $\mu_A^P(r) = c$, $\mu_A^N(r) = d$. Then, we have $r \in A_{(c,d)}$. Now, because g is a homomorphism from $(A_{(c,d)}, B_{(c,d)})$ to $(A'_{(c,d)}, B'_{(c,d)})$, $g(r) \in A'_{(c,db)}$. Thus, $\mu_{A'}^P(g(r)) \geq c = \mu_A^P(r)$ and $\mu_{A'}^N(g(r)) \leq d = \mu_A^N(r)$, which implies g is a co-weak isomorphism from X to Y. □

Corollary 1. *Let $X = (V, A, B) \in BFG(V)$, $Y = (W, A', B') \in BFG(W)$. If $g\colon V \longrightarrow W$ is a co-weak*

isomorphism from X to Y, then g is an injective homomorphism from $X_{(\vartheta,\delta)}$ to $Y_{(\vartheta,\delta)}$, for all $(\vartheta,\delta) \in P_*$, $A_{(\vartheta,\delta)} \neq \varnothing$.

From the following example, we conclude that the converse of Corollary 1 does not need to be true.

Example 1. Let $X = (V, A, B)$ and $Y = (W, A', B')$ be two BFGs as shown in Figure 1. Consider the mapping $g: V \longrightarrow W$, defined by $g(v_i) = w_i$, $1 \leq i \leq 4$. In view of the (ϑ,δ)–level graphs of X and Y in Figure 1, it is easy to see that if $A_{(\vartheta,\delta)} \neq \varnothing$, then g is an injective homomorphism from $X_{(\vartheta,\delta)}$ to $Y_{(\vartheta,\delta)}$, but g is not a co-weak isomorphism.

Theorem 5. *Let $X = (V, A, B) \in BFG(V)$, $Y = (W, A', B') \in BFG(W)$, and $g: V \longrightarrow W$ be a mapping. For each $(\vartheta,\delta) \in P_*$, $A_{(\vartheta,\delta)} \neq \varnothing$, if g is an isomorphism from $X_{(\vartheta,\delta)}$ to a subgraph of $Y_{(\vartheta,\delta)}$, then g is a co-weak isomorphism from X to an induced BFSG of Y.*

Proof. The mapping g is an isomorphism from $X_{(0,1)} = (V, B_{(0,1)})$ to a subgraph $Y_{(0,1)} = (W, B'_{(0,1)})$. So, $g: V \longrightarrow W$ is an injective mapping. For arbitrary $r \in V$, suppose that $\mu_A^P(r) = \vartheta$, $\mu_A^N(r) = \delta$. Then, $r \in A_{(\vartheta,\delta)}$, and so $g(r) \in A'_{(\vartheta,\delta)}$. Hence, $\mu_{A'}^P(g(r)) \geq \vartheta = \mu_A^P(r)$ and $\mu_{A'}^N(g(r)) \leq \delta = \mu_A^N(r)$. For $r, s \in V$, let $\mu_B^P(rs) = \vartheta$ and $\mu_B^N(rs) = \delta$. Then, $\vartheta \leq \mu_A^P(r)$, $\vartheta \leq \mu_A^P(s)$, $\delta \geq \mu_A^N(r)$, $\delta \geq \mu_A^N(s)$, and $rs \in B_{(\vartheta,\delta)}$. Hence, $r, s \in A_{(\vartheta,\delta)}$ and $rs \in B_{(\vartheta,\delta)}$. Since g isomorphism from $X_{(\vartheta,\delta)}$ to $Y_{(\vartheta,\delta)}$, we get $g(r), g(s) \in A'_{(\vartheta,\delta)}$ and $g(r), g(s) \in B'_{(\vartheta,\delta)}$. Therefore,

$$\mu_{B'}^P(g(r)g(s)) \geq \vartheta = \mu_B^P(rs), \mu_{B'}^N(g(r)g(s)) \leq \delta = \mu_B^N(rs). (I). \tag{27}$$

Now, let $\mu_{B'}^P(g(r)g(s)) = t$, $\mu_{B'}^N(g(r)g(s)) = s$. Then, $g(r)g(s) \in A_{(t,s)}$. Because g is injective and an isomorphism from $X_{(p,q)}$ to a subgraph of $Y_{(p,q)}$, we have $p, q \in A_{(p,q)}$ and $xy \in B_{(p,q)}$. Therefore,

$$\mu_B^P(s) \geq p = \mu_{B'}^P(g(r)g(s)), \mu_B^N(rs) \leq q = \mu_{B'}^N(g(r)g(s)). (II). \tag{28}$$

Now, by (I) and (II), we conclude that

$$\mu_{B'}^P(g(r)g(s)) = \mu_B^P(rs), \mu_{B'}^N(g(r)g(s)) = \mu_B^N(rs). \tag{29}$$

□

Corollary 2. *Let $X = (V, A, B)$ and $Y = (W, A', B')$ be two BFGs with $|V| = |W|$, and $g: V \longrightarrow W$ a mapping. For $(\vartheta,\delta) \in P_*$, $A_{(\vartheta,\delta)} \neq \varnothing$, if g is an isomorphism from $X_{(\vartheta,\delta)}$ to a subgraph of $Y_{(\vartheta,\delta)}$, then g is a co-weak isomorphism from X to Y.*

Theorem 6. *Let $X = (V, A, B)$ and $Y = (W, A', B')$ be two BFGs, $f: V \longrightarrow W$ be a bijective mapping. If for each $(\vartheta,\delta) \in P_*$, g is an isomorphism from $X_{(\vartheta,\delta)}$ to $Y_{(\vartheta,\delta)}$ and then g is an isomorphism from X to Y.*

Proof. From hypothesis, $g^{-1}: W \longrightarrow V$ is a bijective mapping and an isomorphism from $Y_{(\vartheta,\delta)}$ to $X_{(\vartheta,\delta)}$. By Theorem 5, g is a co-weak isomorphism from X to Y and g^{-1} is a co-weak isomorphism from Y to X. Therefore, g is an isomorphism from X to Y. □

Corollary 3. *Let $X = (V, A, B)$ be a BFG and $g: V \longrightarrow V$ a bijective mapping. Then, g is an automorphism of X if and only if $f_{|A_{(\vartheta,\delta)}}$ is an automorphism of $X_{(\vartheta,\delta)}$, from an $(\vartheta,\delta) \in P_*$, $A_{(\vartheta,\delta)} \neq \varnothing$.*

Theorem 7. *Let $X = (V, A, B)$ be a BFG. Then, X is a complete bipolar fuzzy graph if and only if $X_{(\vartheta,\delta)} = (A_{(\vartheta,\delta)}, B_{(\vartheta,\delta)})$ is a complete graph (CG) for $(\vartheta,\delta) \in P_*$.*

Proof. Proof. If $X = (V, A, B)$ is a complete bipolar fuzzy graph and for $(\vartheta,\delta) \in P_*$, $A_{(\vartheta,\delta)} \neq \varnothing$, $r, s \in A_{(\vartheta,\delta)}$, then $\mu_A^P(r) \geq \vartheta$, $\mu_A^P(s) \geq \vartheta$, $\mu_A^N(r) \leq \delta$, $\mu_A^N(s) \leq \delta$, and so

$$\mu_B^P(rs) = \mu_A^P(r) \wedge \mu_A^P(s) \geq \vartheta, \mu_B^N(rs) = \mu_A^N(r) \vee \mu_A^N(s) \leq \delta. \tag{30}$$

Hence, $rs \in B_{(\vartheta,\delta)}$. It follows that $X_{(\vartheta,\delta)}$ is a CG.

Conversely, suppose that $X = (V, A, B)$ is not a complete bipolar fuzzy graph. Then, there are $r, s \in V$ so that $\mu_B^P(rs) < \mu_A^P(r) \wedge \mu_A^P(s)$ or $\mu_B^P(rs) < \mu_A^P(r) \vee \mu_A^P(s)$. Let $\mu_B^P(rs) < \mu_A^P(r) \wedge \mu_A^P(s)$ and $\mu_A^P(r) \wedge \mu_A^P(s) = \vartheta$, for $\vartheta \in (0, 1]$. Then, $\mu_A^P(r) \geq \vartheta$ and $\mu_A^P(s) \geq \vartheta$. Hence, $r, s \in A_{(\vartheta,\delta)}$, for a $\delta \in [0, 1]$, but $rs \notin B_{(\vartheta,\delta)}$. This implies that $X_{(\vartheta,\delta)}$ is not a CG. For the case $\mu_B^N(rs) > \mu_A^N(r) \vee \mu_A^N(s)$, it follows similarly. □

Theorem 8. *Let $X = (V, A, B) \in BFG(V)$. Then, $X_{(\vartheta,\delta)}$ has not IV, for each $(\vartheta,\delta) \in P_*$, $A_{(\vartheta,\delta)} \neq \varnothing$ if and only if for each $r \in V$, there exists $s \in V$ so that $\mu_B^P(rs) = \mu_A^P(r)$ and $\mu_B^N(rs) = \mu_A^N(r)$.*

Proof. Suppose that for each $(\vartheta,\delta) \in P_*$, $A_{(\vartheta,\delta)} \neq \varnothing$, graph $X_{(\vartheta,\delta)}$ has not IV and there is a vertex $r \in V$ so that for each $s \in V$, $\mu_B^P(rs) < \mu_A^P(r)$ or $\mu_B^N(rs) > \mu_A^N(r)$. Let $\mu_B^P(rs) < \mu_A^P(r)$ and $\mu_A^P(r) = \vartheta$, $\mu_A^N(r) = \delta$, for $(\vartheta,\delta) \in P_*$. Then, $r \in A_{(\vartheta,\delta)}$ and for each $s \in V$, $s \neq r$, $rs \notin B_{(\vartheta,\delta)}$. Therefore, r is an IV in the graph $X_{(\vartheta,\delta)} = (A_{(\vartheta,\delta)}, B_{(\vartheta,\delta)})$, which is a contradiction.

Now, suppose that for $(\vartheta,\delta) \in P_*$, $A_{(\vartheta,\delta)} \neq \varnothing$, vertex $r \in A_{(\vartheta,\delta)}$ is an IV in $X_{(\vartheta,\delta)}$. If $s \notin A_{(\vartheta,\delta)}$, then $\mu_B^P(rs) \leq \mu_A^P(s) < \vartheta \leq \mu_A^P(r)$ or $\mu_B^N(rs) \geq \mu_A^N(s) > \delta \geq \mu_A^N(r)$, and if $s \in A_{(\vartheta,\delta)}$, it is trivial that $rs \notin B_{(\vartheta,\delta)}$. Hence, $\mu_B^P(rs) < \vartheta \leq \mu_A^P(r)$ or $\mu_B^N(rs) > \delta \geq \mu_A^N(r)$. Therefore, for each $s \in V$, $\mu_B^P(rs) \neq \mu_A^P(r)$, $\mu_B^N(rs) \neq \mu_A^N(r)$. □

Theorem 9. *A BFG $X = (V, A, B)$ is r-colorable $\Leftrightarrow$ there exists a homomorphism from X to $K_{r,A'}$.*

Proof. Assume that X be r-colorable with r colors labeled $\Gamma = \{\mu_1, \mu_2, \ldots, \mu_r\}$. Let $V_i = \{v \in V | \mu_i(v) \neq 0\}$. We define complete bipolar fuzzy graph $K_{r,A'}$ with vertices set $\{1, 2, \ldots, r\}$, so that the degree of positive membership vertex i is $\mu_{A'}^P(i) = \max\{\mu_A^P(v) | v \in V_i\}$ and the degree of negative membership vertex i is $\mu_{A'}^N(i) = \min\{\mu_A^N(v) | v \in V_i\}$. Now, the mapping $g: X \longrightarrow K_{r,A'}$ defined by $g(v) = i, v \in V_i$ is a graph homomorphism because for $v \in V_i$

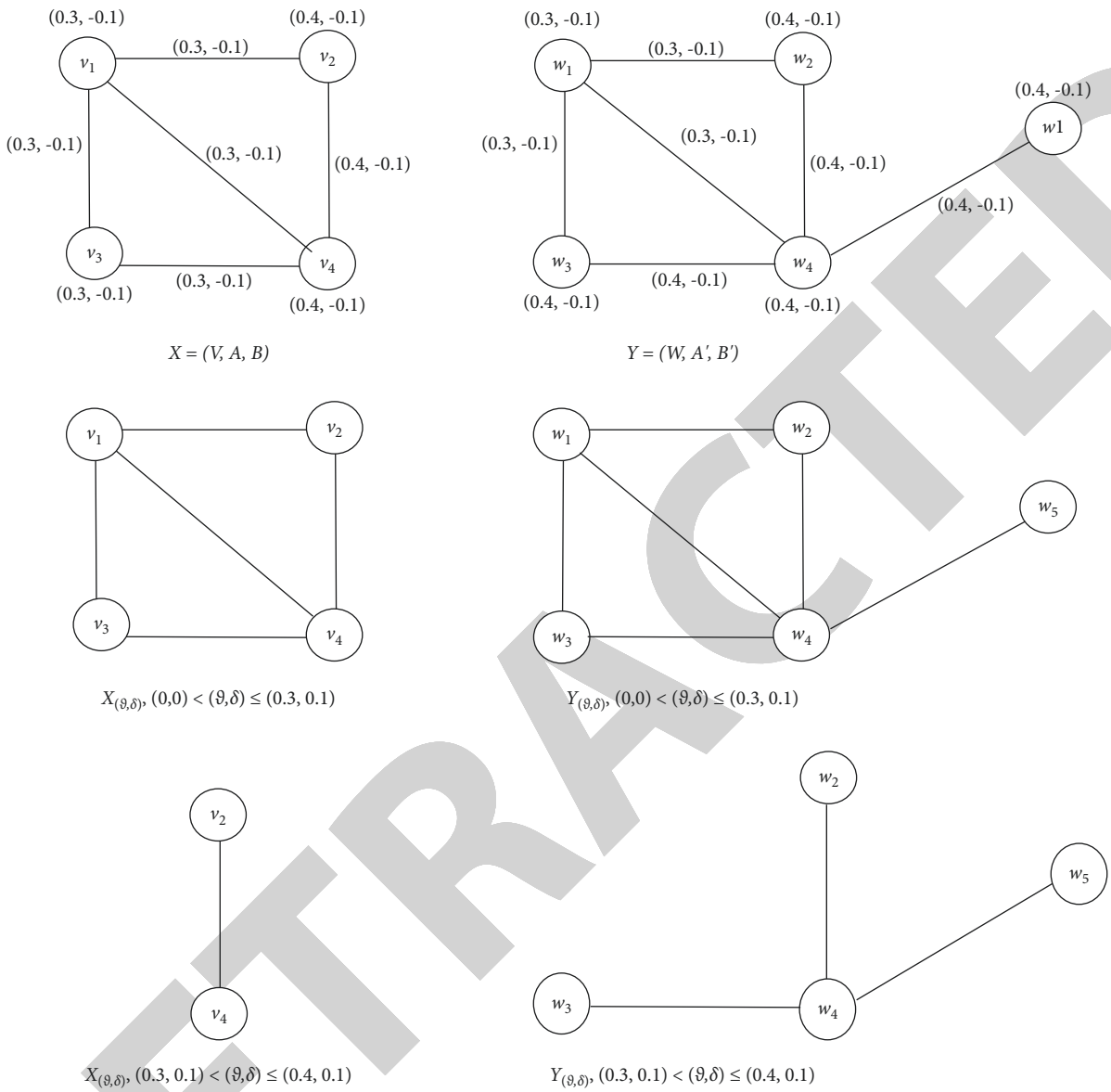

Figure 1: BFGs X, Y and the mapping g: $V_i \longrightarrow W_i$ which is not a co-weak isomorphism.

(i)
$$\mu_A^P(v) \leq \max\{\mu_A^P(w) | w \in V_i\} = \mu_{A'}^P(i) = \mu_{A'}^P(g(v)),$$
$$\mu_A^N(v) \geq \min\{\mu_A^N(w) | w \in V_i\} = \mu_{A'}^N(i) = \mu_{A'}^N(g(v)). \tag{31}$$

(ii) According to the definition of complete bipolar fuzzy graph, for $u \in V_i$ and $v \in V_j$, we have

$$\begin{aligned} \mu_B^P(uv) &\leq \mu_A^P(u) \wedge \mu_A^P(v) \leq \mu_{A'}^P(i) \wedge \mu_{A'}^P(j) = \mu_{A'}^P(g(u)) \wedge \mu_{A'}^P(g(v)), \\ \mu_B^N(uv) &\geq \mu_A^N(u) \vee \mu_A^N(v) \geq \mu_{A'}^N(i) \vee \mu_{A'}^N(j) = \mu_{A'}^N(g(u)) \vee \mu_{A'}^N(g(v)), \end{aligned} \tag{32}$$

then $\mu_B^P(uv) \leq \mu_{B'}^P(g(u)g(v))$ and $\mu_B^N(uv) \geq \mu_{B'}^N(g(u)g((v)]$, for all $uv \in \widetilde{V}^2$.

Conversely, let g: $X \longrightarrow K_{r,A'}$ be a homomorphism. For a given $k \in V(K_{r,A'})$, define the set $g^{-1}(k) \subseteq V$ to be

$$g^{-1}(k) = \{x \in V | g(x) = k\}. \tag{33}$$

If $v \in g^{-1}(k)$, let $\mu_k(v) = (\mu_{\mu_k}^P(v), \mu_{\mu_k}(v)) = (t_A(v), f_A(v))$; otherwise, $\mu_k(v) = 0$. Therefore, the fuzzy bipolar graph X is r-colorable with coloring set $\{\mu_1, \mu_2, \ldots, \mu_r\}$. □

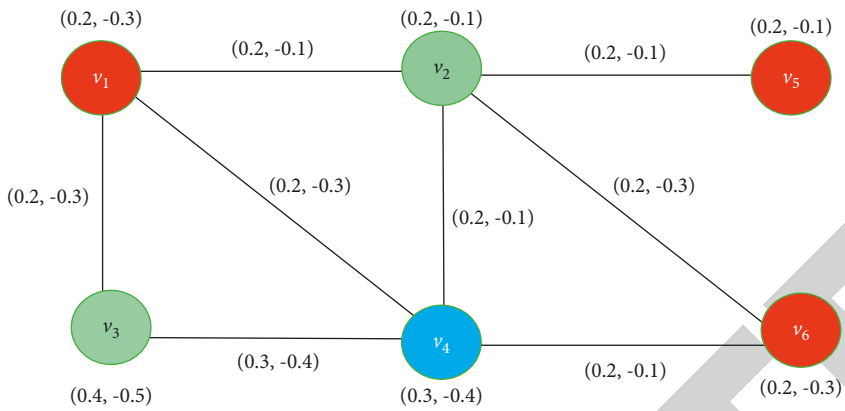

Figure 2: Bipolar fuzzy graph. $X = (V, A, B)$.

4. Application

Nowadays, the issue of coloring is very important in the theory of fuzzy graphs because it has many applications in controlling intercity traffic, coloring geographical maps, as well as finding areas with high population density. Therefore, in this section, we have tried to present an application of the coloring of vertices in a BFG.

Example 2. We obtain a BFG $X = (V, A, B)$ on the vertex set $\{v_1, v_2, v_3, v_4, v_5, v_6\}$ by joining two vertices with respect to the effect they have one another (see Figure 2). Let v_1v_2, v_1v_3, v_1v_4, v_2v_4, v_2v_6, v_2v_5, v_3v_4, and v_6v_4 be edges of graph X. The positive membership and negative membership values (μ_A^P, μ_A^N) of the vertices are the good and bad quality, respectively. Also, the positive membership and negative membership values (μ_B^P, μ_B^N) of the edges are compatible and incompatible materials, respectively. We want to see that how to put the materials in such a way that they do not have any effect on each other. Now, by Theorem 9, there is a homomorphism from X to CG with $n = 3$. Therefore, we need at least 3 parts (3-colors) to put the materials.

In the next example, we want to identify the most effective employee of a university with the help of a bipolar influence digraph.

Example 3. The emergence of science and knowledge is equal to the creation of man, and man has always sought to understand and comprehend. Science and knowledge have a special place in human life. The role of science in human life is to teach human beings the path to happiness, evolution, and construction. Science enables man to build the future the way he wants. Science is given as a tool at the will of man and makes nature as man wants and commands. Science and knowledge are two wings with which man can fly indefinitely. All the tools and instruments that we use today and cause the fundamental difference between past and present life are the result of effort and science and knowledge that man has discovered and used. Thanks to science and knowledge that many patients are saved from death, earthquake-proof buildings are built, and man can see the whole planet from above. Science and knowledge are the result of discovering hidden secrets in the heart of nature and secrets that human beings have endured many hardships to discover so that we can now easily use them. Although knowledge plays a very important role in human life and causes evolution and progress, sometimes it may also bring dangers to the human race, and this is if man uses what he has learned in the wrong way, science and knowledge need to know how to use it properly so that man is always on the right path. So, universities should hire the best teachers and staff to do the work of the students and provide the necessary conditions for their education. Therefore, in this section, we try to identify the most effective employees in a university according to their performance. Hence, we consider the vertices of the bipolar influence graph as the head of each ward of the university and the edges of the graph as the degree of interaction and influence of each other. For this university, the set of staff is $B = \{$Alavi, Rasooli, Tabari, Omrani, Razavi, Salehi, Taghavi$\}$:

Table 2: Name of employees in a university and their services.

Name	Services
Alavi	Head of library
Rasooli	Head of postgraduate education
Tabari	Head of informatics department
Omrani	Head of security
Razavi	Head of university
Salehi	Head of welfare services
Taghavi	Head of research department

(a) Rasooli has been working with Omrani for 11 years and values his views on issues.

(b) Alavi has been the head of library for a long time, and not only Rasooli but also Omrani is very satisfied with Alavi's performance.

(c) In a university, preserving educational documents as well as taking care of university services is a very important task. Omrani is the most suitable person for this responsibility.

(d) Tabari and Salehi have a long history of conflict.

Table 3: The level of staff capability.

	Alavi	Rasooli	Tabari	Omrani	Razavi	Salehi	Taghavi
μ_B^P	0.4	0.9	0.6	0.7	0.8	0.9	0.5
μ_B^N	−0.4	−0.3	−0.2	−0.3	−0.2	−0.2	−0.4

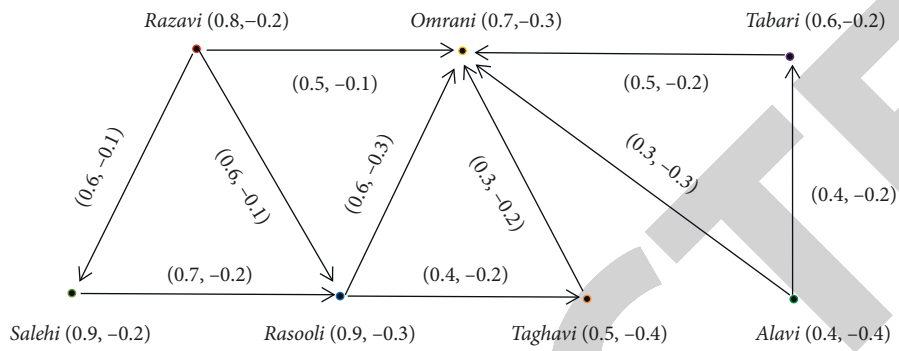

Figure 3: Bipolar influence digraph.

Table 4: Adjacency matrix corresponding to Figure 3.

	Alavi	Rasooli	Tabari	Omrani	Razavi	Salehi	Taghavi
Alavi	(0, 0)	(0, 0)	(0.4, −0.2)	(0.3, −0.3)	(0, 0)	(0, 0)	(0, 0)
Rasooli	(0, 0)	(0, 0)	(0, 0)	(0.6, −0.3)	(0, 0)	(0, 0)	(0.4, −0.2)
Tabari	(0, 0)	(0, 0)	(0, 0)	(0.5, −0.2)	(0, 0)	(0, 0)	(0, 0)
Omrani	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
Razavi	(0, 0)	(0.6, −0.1)	(0, 0)	(0.5, −0.1)	(0, 0)	(0.6, −0.1)	(0, 0)
Salehi	(0, 0)	(0.7, −0.2)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
Taghavi	(0, 0)	(0, 0)	(0, 0)	(0.3, −0.2)	(0, 0)	(0, 0)	(0, 0)

(e) Tabari has an important role in the informatics department.

Given the above, we consider a bipolar influence graph. The vertices represent each of the university staff. Note that each staff member has the desired ability as well as shortcomings in the performance of their duties. Therefore, we use of BFS to express the weight of the vertices. The positive membership indicates the efficiency of the employee, and the negative membership shows the lack of management and shortcomings of each staff. However, the edges describe the level of relationships and friendships between employees that the positive membership shows a friendly relationship between both employees and the negative membership shows the degree of conflict between the two officials. Name of employees and level of staff capability are shown in Tables 2and 3. The adjacency matrix corresponding to Figure 3 is shown in Table 4.

Figure 3 shows that Salehi has 90% of the power needed to do the university work as the head of welfare services but does not have the 20% knowledge needed to be the boss. The directional edge Rasooli–Omrani shows that there is 60% friendship among these two employees, and unfortunately they have 30% conflict. Clearly, Razavi has dominion over both Salehi and Rasooli, and his dominance over both is 60%. It is clear that Razavi is the most influential employee of the university because he controls both the head of welfare services and head of postgraduate education, who have 90% of the power in the university.

5. Conclusion

BFGs have a wide range of applications in the field of psychological sciences as well as the identification of individuals based on oncological behaviors. With the help of BFGs, the most efficient person in an organization can be identified according to the important factors that can be useful for an institution. Hence, in this paper, we introduced the notion of (ϑ, δ)–homomorphism of BFGs and classify homomorphisms, weak isomorphisms, and co-weak isomorphisms of BFGs by (ϑ, δ)–homomorphisms. We also investigated the level graphs of BFGs to characterize some BFGs. Finally, we presented two applications of BFGs in coloring problem and also finding effective person in a university. In our future work, we will introduce new concepts of connectivity in BFGs and investigate some of their properties. Also, we will study new results of global dominating set, restrain dominating set, connected perfect

dominating set, regular perfect dominating set, and independent perfect dominating set on BFGs.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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