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Research Article

Fault-Tolerant Resolvability of Swapped Optical Transpose Interconnection System

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Interconnection systems in computer science and information technology are mainly represented by graphs. One such instance is of swapped network simulated by the optical transpose interconnection system (OTIS). Fault tolerance has become a vital feature of optoelectronic systems. Among multiple types of faults that may take place in an interconnection system, two significant kinds are either due to malfunctioning of a node (processor in case of O_G) or collapse of communication between nodes (failure of interprocessor transmission). To prevail over these faults, the unique recognition of every node is essential. In graph-theoretic interpretation, this leads to instigating the metric dimension $\beta(O_G)$ and fault-metric dimension $\beta'(O_G)$ of the graph O_G obtained from the interconnection system. This paper explores OTIS over base graph P_m (path graph over m vertices) for resolvability and fault-tolerant resolvability. Furthermore, bounds for $\beta(O_G)$ and $\beta\prime(O_G)$ are also imparted over $G = P_m$.

1. Introduction

In computerized broadcasting, data and information are transmitted by utilizing the connectivity of nodes. The data transmission is carried out either over physical media like wires, or aerial mode whose widely used illustration is WiFi. G. Marsden in 1993 [16] put forward an abstraction of OTIS with optoelectronic (optical and electronic) working pattern. This optoelectronic approach escalates the outcome of an electrical system by minimizing power consumption and enhancing bandwidth. Due to the combined effect of electrical and wireless technology, OTIS is brought about to establish a worthwhile network to make modern optoelectronic computers' high yielding.

In OTIS, nodes/processors are set out in the form of clumps. Within every bunch of processors, the communication medium is electronic, while the optical medium is employed among processor clumps. Krishnamoorthy proved in [11] that if the number of processor nodes in every cluster are chosen equal to the number of clusters, the efficiency of OTIS is boosted. A pictorial representation of a swapped OTIS OP_m (see Definition 1 in section 2.2) with m processors lined up in each cluster and comprising of m clusters is shown in Figure 1 for $m = 6$.

1.1. Background and Related Work. Slater [20] and Harary and Melter [6] separately asserted the abstraction of the metric dimension. Many authors worked for determining the metric dimension of various graphs as in [1, 10, 13]. The graph parameters such as metric dimension, partition dimension, fault-tolerant metric dimension, and fault-tolerant partition dimension have come through with imperative applicability in numerous areas of study including, network
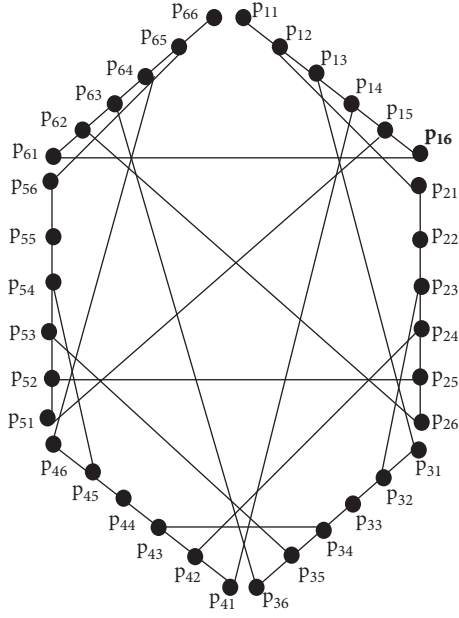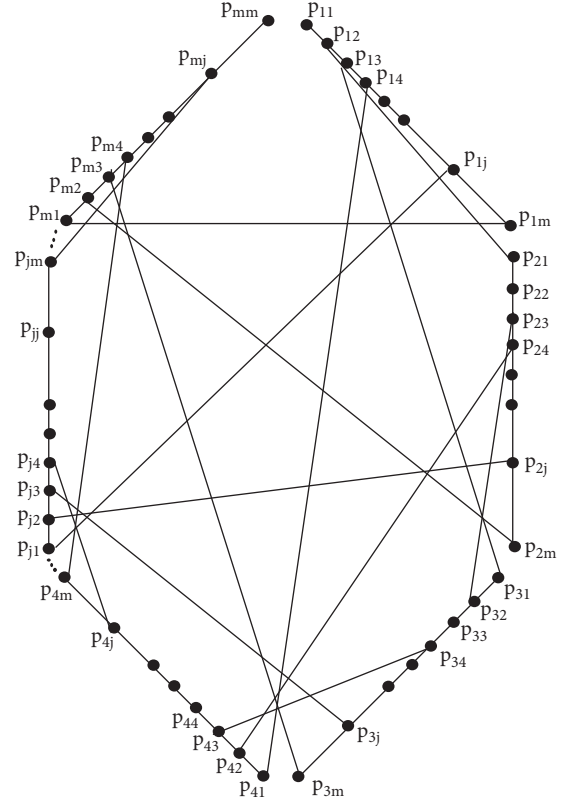
Figure 1: Graph of swapped OTIS OP_6.

analysis [3], robot navigation [12], chemistry [4], and geographical routing protocols [15]. The depiction of an interconnection network by a graph is eventually a popular tool to probe connectivity and alliance of objects on the network.

The graphical representation of interconnection networks has motivated researchers to compute graph invariants such as diameter, resolving sets, fault-tolerant resolving set for such interconnection graphs. Hernando [8] initiates the abstraction of fault-tolerant metric dimension. Later, this concept is explored for several interconnection networks such as oxide interconnection system [21], crystalline structure [14], convex polytopes [9, 19], butterfly, benes, and silicate networks [7, 22]. Afterwards, fault-tolerant partition resolvability of cyclic networks [2] and Toeplitz networks [17] is investigated. Recently, this fault-tolerant resolvability is reinvestigated in [18] for multistage interconnection networks, previously computed by [7]. As a continuation of this idea, in this paper, we investigate swapped OTIS OP_m for the metric dimension and fault-tolerant metric dimension and provide a bound for these invariants.

The rest of the article is organized as follows: Section 2 comprises introductory notions of graphs and the swapped optical transpose interconnection system. Section 3 provides a bound for the metric dimension and the fault-tolerant metric dimension of OP_m. In Section 4, the applications of resolvability and fault-tolerant resolvability are discussed.

2. Introductory Notions

2.1. Basics of Graph Theory.
A graph G is a structure (V, E) comprising a set $V(G)$ termed as vertex set and an edge set $E(G) = \{(m, n)|m, n \in V\}$. For $m, n \in V$, (m, n) is an edge in G. The distance from a vertex m to n, given by $d(m, n)$, is the number of edges in the shortest possible route from m to n.

Figure 2: Graph of swapped OTIS OP_m.

For $a \in V(G)$ and an ordered set $N = \{n_1, n_2, n_3, \ldots, n_k\} \subset V(G)$, the metric code of a for N is given below:

$$c_a(N) = (d(a, n_1), d(a, n_2), d(a, n_3), \ldots, d(a, n_k)), \quad (1)$$

where N is known as resolving set if for $m, n \in V$, $m \neq n \Rightarrow c_m(N) \neq c_n(N)$ [6, 20]. The smallest resolving set in G is basis for G and the cardinality of basis is termed as G's metric dimension indicated by $\beta(G)$. For $M \subseteq V(G)$ and $n \in V(G)$, $d(n, M)$ is defined as $d(n, M) = \min\{d(n, m)|m \in M\}$. A set M is termed as fault-tolerant resolving set if M is itself a resolving set, and for every $m \in M$, $M \setminus \{m\}$ is also a resolving set. The number of elements in the smallest fault-tolerant resolving set is termed the fault-tolerant metric dimension of G, given by $\beta\prime(G)$ [10]. The numbers $\beta(G)$ and $\beta\prime(G)$ are related by the following expression:

$$\beta'(G) \geq \beta(G) + 1. \quad (2)$$

2.2. The Swapped Optical Transpose Interconnection System (OTIS)

Definition 1 (see [16, 23, 24]). The swapped optical transpose interconnection system (OTIS) is a graph O_G, derived from a base network G, with $V(O_G)$ and $E(O_G)$ as given below:

TABLE 1: Metric codes $c_{p_{1i}}(M_{jj})$ of vertices in the first cluster for $1 \le i \le m$.

Vertex	$d(p_{1i}, p_{11})$	$d(p_{1i}, p_{22})$	$d(p_{1i}, p_{33})$	$d(p_{1i}, p_{44})$	...	$d(p_{1i}, p_{j-1j-1})$	$d(p_{1i}, p_{j+1j+1})$	...	$d(p_{1i}, p_{m-1m-1})$	$d(p_{1i}, p_{mm})$
p_{11}	0	3	5	7	...	$2j-3$	$2j+1$	...	$2m-3$	$2m-1$
p_{12}	1	2	4	6	...	$2j-4$	$2j$	...	$2m-4$	$2m-2$
p_{13}	2	3	3	5	...	$2j-5$	$2j-1$	...	$2m-5$	$2m-3$
p_{14}	3	4	4	4	...	$2j-6$	$2j-2$	...	$2m-6$	$2m-4$
p_{15}	4	5	5	5	...	$2j-7$	$2j-3$	...	$2m-7$	$2m-5$
...	...	...	...	...	...	...	...	...	...	...
p_{1j-1}	$j-2$	$j-1$	$j-1$	$j-1$	...	$j-1$	$j+3$	...	$2m-j-1$	$2m-j+1$
p_{1j}	$j-1$	j	j	j	...	j	$j+2$	...	$2m-j-2$	$2m-j$
p_{1j+1}	j	$j+1$	$j+1$	$j+1$	...	$j+1$	$j+1$	...	$2m-j-3$	$2m-j-1$
...	...	...	...	...	...	...	...	...	...	...
p_{1m-1}	$m-2$	$m-1$	$m-1$	$m-1$	...	$m-1$	$m-1$	...	$m-1$	$m-1$
p_{1m}	$m-1$	m	m	m	...	m	m	...	m	m

$$V(O_G) = \{p_{ij} | p_{ij} \in V(G), i, j \in N\},$$
$$E(O_G) = \{(p_{ij}, p_{lm}) | (p_{ij}, p_{lm}) \in E(G)\} \qquad (3)$$
$$\cup \{(p_{uv}, p_{vu}) | p_{uv}, p_{vu} \in V(G), u \neq v\}.$$

The graph G in swapped (OTIS) O_G is nominated as a base graph or factor graph. If $|V(G)| = m$, then O_G comprises of m copies of G, and each copy is known as a subnetwork (cluster or clump) in O_G. We shall denote each vertex in O_G as p_{uv} where uv is the address of vertex v in the cluster u. In swapped network O_G, the intercluster edges are only between p_{uv} and p_{vu} when $u \neq v$. The node p_{uu} specifies the processor u in the subnetwork u, and there is no edge incident to p_{uu} from a cluster other than u.

3. New Results

This section comprises of new results related to resolvability and fault-tolerant resolvability.

3.1. Bound for Metric Dimension of OP_m

Theorem 1. *For all $m \ge 1$, $\beta(OP_m) \le m - 1$.*

Proof. We take a partition of vertex set of OP_m (see Figure 2) as

$$V(OP_m) = \{p_{11}, p_{12}, p_{13}, \ldots, p_{1m}\} \cup \{p_{21}, p_{22}, p_{23}, \ldots, p_{2m}\}$$
$$\cup \{p_{31}, p_{32}, p_{33}, \ldots, p_{3m}\}$$
$$\cup \{p_{41}, p_{42}, p_{43}, \ldots, p_{4m}\} \cup \cdots$$
$$\cup \{p_{j1}, p_{j2}, p_{j3}, \ldots, p_{jm}\} \cup \cdots$$
$$\cup \{p_{m1}, p_{m2}, p_{m3}, \ldots, p_{mm}\}. \qquad (4)$$

Let us take $M = \{p_{11}, p_{22}, p_{33}, \ldots, p_{m-1m-1}, p_{mm}\}$. This is the set of all nodes p_{jj}, for all $1 \le j \le m$. We denote $m-1$ cardinality subsets of M defined as

$$M_{jj} = M \setminus \{p_{jj}\} = \{p_{11}, p_{22}, p_{33}, \ldots, p_{j-1j-1}, \\ p_{j+1j+1}, \ldots, p_{m-1m-1}, p_{mm}\}. \qquad (5)$$

To prove our claim, we need to show that M_{jj} is a resolving set for OP_m. We shall give metric codes of all processor nodes for M_{jj}.

The metric codes of vertices in the first and second cluster are stated in Table 1 and 2, respectively.

The metric codes of vertices in the third cluster are given in Table 3.

Upon continuation in this manner, we have the metric codes of the mth cluster in Table 4.

By Tables 1, 2, 3 and 4, we can conclude that

$$c_{p_{1j}}(M) = \left(j-1, j, j, j, \ldots, \overbrace{j}^{\text{distance from } p_{j-1j-1}}, \overbrace{j+2}^{\text{distance from } p_{j+1j+1}}, j+4, \ldots, 2m-j-2, 2m-j \right),$$

$$c_{p_{2j}}(M) = \left(j+1, j-2, j-1, j-1, \ldots, \overbrace{j-1}^{\text{distance from } p_{j-1j-1}}, \overbrace{j+1}^{\text{distance from } p_{j+1j+1}}, j+3, \ldots, 2m-j-3, 2m-j-1 \right), \qquad (6)$$

$$c_{p_{3j}}(M) = \left(j+2, j, j-3, j-2, \ldots, \overbrace{j-2}^{\text{distance from } p_{j-1j-1}}, \overbrace{j+2}^{\text{distance from } p_{j+11j+1}}, j+4, \ldots, 2m-j-4, 2m-j-2 \right),$$

and continuing in this manner, we have

TABLE 2: Metric codes $c_{p_{2i}}(M_{jj})$ of vertices in the second cluster for $1 \le i \le m$.

Vertex	$d(p_{2i}, p_{11})$	$d(p_{2i}, p_{22})$	$d(p_{2i}, p_{33})$	$d(p_{2i}, p_{44})$	...	$d(p_{2i}, p_{j-1j-1})$	$d(p_{2i}, p_{j+1j+1})$	...	$d(p_{2i}, p_{m-1m-1})$	$d(p_{2i}, p_{mm})$
p_{21}	2	1	4	6	...	$2j-4$	$2j$	...	$2m-4$	$2m-2$
p_{22}	3	0	3	5	...	$2j-5$	$2j-1$	...	$2m-5$	$2m-3$
p_{23}	4	1	2	4	...	$2j-6$	$2j-2$	...	$2m-6$	$2m-4$
p_{24}	5	2	3	3	...	$2j-7$	$2j-3$	...	$2m-7$	$2m-5$
p_{25}	6	3	4	4	...	$2j-8$	$2j-4$	...	$2m-8$	$2m-6$
...	...	...	...	...	...	...	...	...	...	...
p_{2j-1}	j	$j-3$	$j-2$	$j-2$	...	$j-2$	$j+2$	...	$2m-j-2$	$2m-j$
p_{2j}	$j+1$	$j-2$	$j-1$	$j-1$	...	$j-1$	$j+1$	...	$2m-j-3$	$2m-j-1$
p_{2j+1}	$j+2$	$j-1$	j	j	...	j	j	...	$2m-j-4$	$2m-j-2$
...	...	...	...	...	...	...	...	...	...	...
p_{2m-1}	m	$m-3$	$m-2$	$m-2$	...	$m-2$	$m-2$	...	$m-2$	m
p_{2m}	$m+1$	$m-2$	$m-1$	$m-1$	...	$m-1$	$m-1$	...	$m-1$	$m-1$

TABLE 3: Metric codes $c_{p_{3i}}(M_{jj})$ of vertices in the third cluster for $1 \le i \le m$.

Vertex	$d(p_{3i}, p_{11})$	$d(p_{3i}, p_{22})$	$d(p_{3i}, p_{33})$	$d(p_{3i}, p_{44})$	...	$d(p_{3i}, p_{j-1j-1})$	$d(p_{3i}, p_{j+1j+1})$	...	$d(p_{3i}, p_{m-1m-1})$	$d(p_{3i}, p_{mm})$
p_{31}	3	3	2	5	...	$2j-5$	$2j-1$	...	$2m-5$	$2m-3$
p_{32}	4	2	1	4	...	$2j-6$	$2j-2$	...	$2m-6$	$2m-4$
p_{33}	5	3	0	3	...	$2j-7$	$2j-3$	...	$2m-7$	$2m-5$
p_{34}	6	4	1	2	...	$2j-8$	$2j-4$	...	$2m-8$	$2m-6$
p_{35}	7	5	2	3	...	$2j-9$	$2j-5$	...	$2m-9$	$2m-7$
...	...	...	...	...	...	...	...	...	...	...
p_{3j-1}	$j+1$	$j-1$	$j-4$	$j-3$	...	$j-3$	$j+1$	...	$2m-j-3$	$2m-j-1$
p_{3j}	$j+2$	j	$j-3$	$j-2$	...	$j-2$	$j+2$	...	$2m-j-4$	$2m-j-2$
p_{3j+1}	$j+3$	$j+1$	$j-2$	$j-1$	...	$j-1$	$j+3$	...	$2m-j-5$	$2m-j-3$
...	...	...	...	...	...	...	...	...	...	...
p_{3m-1}	$m+1$	$m-1$	$m-4$	$m-3$	...	$m-3$	$m-3$	...	$m-3$	$m-3$
p_{3m}	$m+2$	m	$m-3$	$m-2$	...	$m-2$	$m-2$	...	$m-2$	$m-2$

TABLE 4: Metric codes $c_{p_{mi}}(M_{jj})$ of vertices in the third cluster for $1 \le i \le m$.

Vertex	$d(p_{mi}, p_{11})$	$d(p_{mi}, p_{22})$	$d(p_{mi}, p_{33})$	$d(p_{mi}, p_{44})$	...	$d(p_{mi}, p_{j-1j-1})$	$d(p_{mi}, p_{j+1j+1})$	...	$d(p_{mi}, p_{m-1m-1})$	$d(p_{mi}, p_{mm})$
p_{m1}	m	m	m	m	...	m	m	...	m	$m-1$
p_{m2}	$m+1$	$m-1$	$m-1$	$m-1$	...	$m-1$	$m-1$	...	$m-1$	$m-2$
p_{m3}	$m+2$	m	$m-2$	$m-2$	...	$m-2$	$m-2$	...	$m-2$	$m-3$
p_{m4}	$m+3$	$m+1$	$m-1$	$m-3$	...	$m-3$	$m-3$	...	$m-3$	$m-4$
p_{m5}	$m+4$	$m+2$	m	$m-2$	...	$m-4$	$m-4$	...	$m-4$	$m-5$
...	...	...	...	...	...	...	...	...	...	...
p_{mj-1}	$m+j-2$	$m+j-4$	$m+j-6$	$m+j-8$	...	$m-j+2$	$m-j+2$	...	$m-j+2$	$m-j+1$
p_{mj}	$m+j-1$	$m+j-3$	$m+j-5$	$m+j-7$	...	$m-j+3$	$m-j+1$	...	$m-j+1$	$m-j$
p_{mj+1}	$m+j$	$m+j-2$	$m+j-4$	$m+j-6$	...	$m-j+4$	$m-j$	...	$m-j$	$m-j-1$
...	...	...	...	...	...	...	...	...	...	...
p_{mm-1}	$2m-2$	$2m-4$	$2m-6$	$2m-8$	...	$2m-2j+2$	$2m-2j-2$	...	2	1
p_{mm}	$2m-1$	$2m-3$	$2m-5$	$2m-7$	...	$2m-2j+3$	$2m-2j-1$	...	3	0

$$c_{p_{mj}}(M) = \left(m+j-1, m+j-3, m+j-5, m+j-7, m+j-9, m+j-11, \ldots, \overbrace{m-j+3}^{\text{distance from } p_{j-1j-1}}, \overbrace{m-j+1}^{\text{distance from } p_{j+1j+1}}, m-j+1, \ldots, m-j+1, m-j \right).$$

$$(7)$$

Clearly, the metric codes for all nodes in each cluster differ by at least one component. This gives that M_{jj}, for all $1 \le j \le m$, is a resolving set for OP_m and $\beta(OP_m) \le m-1$.   □

3.2. Bound for Fault-Tolerant Metric Dimension of OP_m

Theorem 2. *For all $m \geq 1$, $\beta'(OP_m) \leq m$.*

Proof. We take a partition of vertex set of OP_m as

$$
\begin{aligned}
V(OP_m) = &\{p_{11}, p_{12}, p_{13}, \ldots, p_{1m}\} \cup \{p_{21}, p_{22}, p_{23}, \ldots, p_{2m}\} \\
&\cup \{p_{31}, p_{32}, p_{33}, \ldots, p_{3m}\} \\
&\cup \{p_{41}, p_{42}, p_{43}, \ldots, p_{4m}\} \cup \cdots \\
&\cup \{p_{j1}, p_{j2}, p_{j3}, \ldots, p_{jm}\} \cup \cdots \\
&\cup \{p_{m1}, p_{m2}, p_{m3}, \ldots, p_{mm}\}.
\end{aligned}
\tag{8}
$$

By Theorem 1, for $M = \{p_{11}, p_{22}, p_{33}, \ldots, p_{mm}\}$, every m-1 cardinality set $M_{jj} = M \backslash \{p_{jj}\}$ is a resolving set for OP_m. The metric codes for $M = M_{jj} \cup p_{jj}$ are m-tuples obtained from metric codes of M_{jj} and $d(p_{uv}, p_{jj})$. By carefully observing metric codes in Theorem 1, we conclude that $m - 1$ tuple metric codes differ in more than one co-ordinate. Consequently, m-tuple metric codes are different in at least one coordinate for all vertices, and M is a resolving set. By definition of fault-tolerant resolving set, M is a fault-tolerant resolving set for OP_m. Consequently, $\beta\prime(OP_m) \leq m$. $\square$

4. Applications

The significance of resolvability and fault-tolerance in resolvability, in diverse areas of study, has stimulated analysts and researchers to scrutinize their application aspects. Congenitally, the abstraction of metric dimension is analogous to the working pattern of the global positioning system termed as trilateration in which the position of every object on Earth is specified by its unique distances from three satellites surrounding the earth in orbit.

Chartrand [5] put forward the idea of using members of a resolving set as sensors. When a sensor is out of order and fails to track down an intruder (a thief, fire etc.), the system may not work correctly due to data loss caused by the intruder. This issue is resolved by fault-tolerant resolving set that guarantees an intruder's recognition even a sensor is failed to work. Consequently, fault-tolerant metric dimension enhances the applicability of metric dimension.

Another application of resolvability is a navigation system. A navigation system is used to navigate an object such as a ship, an aircraft, a submarine, and a robot, using some signal transmission medium to control that object. A navigation system determines the location of an object through sensors or some landmarks at some specified position. An instance of such a system is robot navigation. In a graph framework, the robot identifies its position by determining its distances from specified landmarks. The minimum set of landmarks used for robot navigation gives a resolving set. A fault-tolerant resolving set still locates the robot when any landmark is neglected in this scenario. So, a fault-tolerant resolving set is superior to the resolving set by its application.

Multiprocessor interconnection networks consist of thousands of processors where interprocessor communication is via optical or electrical signals. Multiprocessor interconnection networks are popular because microprocessors and memory chips are inexpensive and widely available. Among such systems, one is the optical transpose interconnection system OP_m over the base graph P_m, consisting of m^2 nodes (processors). Each processor serves as a source for data transmission to the other processors in the same cluster (path in this case) and intercluster communication. The idea of the metric dimension of OP_m gives an estimate of the minimum number of processors required for complete and flawless data transmission throughout the system. A malfunctioning processor results in the failure of complete information transfer and may cause data loss. Another factor affecting the performance of the interconnection system is a faulty optoelectronic connection: either due to defective electrical network within base paths or inoperative optical transmission among paths within the system.

The concept of fault-tolerant resolving set and the fault-tolerant metric dimension has significant considerations to subjugate the faulty communication system. The processors in the fault-tolerant resolving set ensure flawless data transmission even if a processor is out of order or any transmission line is interrupted due to some unavoidable factors. It makes a fault-tolerant resolving set applicable in an interconnection network framework.

Data Availability

All the supporting data are contained in the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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