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Research Article

Subclass of Analytic Functions Related with Pascal Distribution Series

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The prime purpose of this article is to derive a necessary and sufficient condition for a linear operator associated with the Pascal distribution series to be in the class $\mathfrak{T}\mathscr{S}(\mu,\sigma,\delta)$ of analytic functions. Moreover, inclusion relation and an integral operator linked to the Pascal distribution series is considered. We have also provided some results as corollaries of our theorems.

1. Introduction and Class Definition

Let $\mathbb{C}$ be the set of all complex numbers. We denote by $\mathfrak{G}$, the set of all analytic functions in the unit disk $\Delta=\{\xi\in\mathbb{C}\colon |\xi|<1\}$ that have the series of the form.

$$f(\xi)=\xi+\sum_{n=2}^{\infty}a_n\xi^n. \tag{1}$$

For functions $f\in\mathfrak{G}$ given by (1) and $g\in\mathfrak{G}$ given by $g(\xi)=\xi+\sum_{n=2}^{\infty}b_n\xi^n$, we recall that the well-known Hadamard product of f and g is given by.

$$f(\xi)*g(\xi)\coloneqq\xi+\sum_{n=2}^{\infty}a_nb_n\xi^n,\quad \xi\in\Delta. \tag{2}$$

For $\varepsilon\in\mathbb{C}-\{0\}$ and $-1\le\mathfrak{B}<\mathfrak{A}\le 1$, we say that a function $f\in\mathfrak{G}$ lies in the class $\mathscr{R}^{\varepsilon}(\mathfrak{A},\mathfrak{B})$ if

$$\left|\frac{f'(\xi)-1}{(\mathfrak{A}-\mathfrak{B})\varepsilon-\mathfrak{B}[f'(\xi)-1]}\right|<1,\quad \xi\in\Delta. \tag{3}$$

The function class $\mathscr{R}^{\varepsilon}(\mathfrak{A},\mathfrak{B})$ was introduced in [1].

In [2], Magesh and Prameela defined the following class:

Definition 1. A function $f\in G$ lies in the class $\mathscr{S}(\mu,\sigma,\delta)$ if the following inequality is satisfied.

$$\mathfrak{R}\left\{\frac{\xi f'(\xi)+\mu(2\mu-1)\xi^2f''(\xi)}{4\mu(1-\mu)\xi+\mu(2\mu-1)\xi f'(\xi)+\left(2\mu^2-3\mu+1\right)f(\xi)}-\sigma\right\}$$
$$>\delta\left|\frac{\xi f'(\xi)+\mu(2\mu-1)\xi^2f''(\xi)}{4\mu(1-\mu)\xi+\mu(2\mu-1)\xi f'(\xi)+\left(2\mu^2-3\mu+1\right)f(\xi)}-1\right|, \tag{4}$$

where $\delta\ge 0, -1\le\sigma<1,\ 0\le\mu\le 1$, and $\xi\in\Delta$.

We also let

$$\mathfrak{T}\mathscr{S}(\mu,\sigma,\delta)=\mathscr{S}(\mu,\sigma,\delta)\cap\mathfrak{T}, \tag{5}$$

where $\mathfrak{T}$ is be a subclass of $\mathfrak{G}$ consisting of functions of the form

$$f(\xi)=\xi-\sum_{n=2}^{\infty}|a_n|\xi^n,\quad (\xi\in\Delta). \tag{6}$$

We note that, by specializing the parameters μ,σ and δ, we obtain the following subclasses studied by various authors.

(1) $\mathfrak{T}\mathcal{S}(0,\sigma,0)=\mathfrak{T}(\sigma)$ and $\mathfrak{T}\mathcal{S}(1,\sigma,0)=\mathcal{K}(\sigma)$ (Silverman [3])
(2) $\mathfrak{T}\mathcal{S}((1/2),\sigma,0)=\mathcal{P}(\sigma)$ (Al-Amiri [4], Gupta and Jain [5] and Sarangi and Uralegaddi [6])
(3) $\mathfrak{T}\mathcal{S}((1/2),\sigma,\delta)=\mathfrak{T}\mathcal{R}(\sigma,\delta)$ (Rosy [7] and Stephen and Subramanian [8])
(4) $\mathfrak{T}\mathcal{S}(0,\sigma,\delta)=\mathfrak{T}\mathcal{S}(\sigma,\delta)$ and $\mathfrak{T}\mathcal{S}(1,\sigma,\delta)=\mathcal{UCV}(\sigma,\delta)$ (Bharati et al. [9])
(5) $\mathfrak{T}\mathcal{S}(0,0,\delta)=\mathfrak{T}\mathcal{S}_p(\delta)$ (Subramanian et al. [10])
(6) $\mathfrak{T}\mathcal{S}(1,0,\delta)=\mathcal{UCV}(\delta)$ (Subramanian et al. [11])

In [12], El-Deeb et al. provided a power series expansion whose coefficients are related to the probabilities of the Pascal distribution.

$$\Phi_r^t(\xi)=\xi+\sum_{n=2}^{\infty}\binom{n+t-2}{t-1}r^{n-1}(1-r)^t\xi^n,\quad \xi\in\Delta, \tag{7}$$

where $t\geq 1$; $0\leq r\leq 1$. We also define the series

$$\Upsilon_r^t(\xi)=2\xi-\Phi_r^t(\xi)=\xi-\sum_{n=2}^{\infty}\binom{n+t-2}{t-1}r^{n-1}(1-r)^t\xi^n,\quad \xi\in\Delta. \tag{8}$$

Next, let us consider the linear operator

$$\mathfrak{I}_r^t f\colon \mathfrak{G}\longrightarrow\mathfrak{G}, \tag{9}$$

defined by means of the convolution or Hadamard product.

$$\begin{aligned}\mathfrak{I}_r^t f(\xi)&=\Phi_r^t(\xi)*f(\xi)\\&=\xi+\sum_{n=2}^{\infty}\binom{n+t-2}{t-1}r^{n-1}(1-r)^t a_n\xi^n,\quad \xi\in\Delta.\end{aligned} \tag{10}$$

Several results on the engagements between different subclasses of analytic univalent functions and special functions or distribution series have been studied in the literature (see for example, [13–27]). Herein, we provide a necessary and sufficient condition for $\Upsilon_r^t(\xi)$ to be in the aforementioned class $\mathfrak{T}\mathcal{S}(\mu,\sigma,\delta)$ and investigate an inclusion property of the class $\mathfrak{T}\mathcal{S}(\mu,\sigma,\delta)$ associated with the operator $\mathfrak{I}_r^t f$. Ultimately, we provide conditions for the integral operator $\mathcal{G}_r^t f(\xi)=\int_0^{\xi}\Upsilon_r^t(t)/t\,dt$ to be in the investigated class $\mathfrak{T}\mathcal{S}(\mu,\sigma,\delta)$.

We shall need the following lemmas to state and prove our results.

Lemma 1 (see [2]). *Given that $\delta\geq 0,-1\leq\sigma<1$ and $(1/2)\leq\mu<1$. A function $f\in\mathfrak{T}$ belongs to the class $\mathfrak{T}\mathcal{S}(\mu,\sigma,\delta)$ if and only if the following inequality is satisfied.*

$$\sum_{n=2}^{\infty}\left[\digamma(\mu,\delta)n^2+\Theta(\mu,\sigma,\delta)n+\Lambda(\mu,\sigma,\delta)\right]\left|a_n\right|\leq 1-\sigma, \tag{11}$$

where

$$\begin{aligned}\digamma(\mu,\delta)&=\left(2\mu^2-\mu\right)(1+\delta),\\ \Theta(\mu,\sigma,\delta)&=\left(\mu-2\mu^2\right)(1+\sigma+2\delta)+1+\delta,\\ \Lambda(\mu,\sigma,\delta)&=\left(3\mu-1-2\mu^2\right)(\sigma+\delta).\end{aligned} \tag{12}$$

The result is sharp for the function.

$$f(\xi)=\xi-\frac{1-\sigma}{\digamma(\mu,\delta)n^2+\Theta(\mu,\sigma,\delta)n+\Lambda(\mu,\sigma,\delta)}\xi^n. \tag{13}$$

Lemma 2 (see [1]). *If $f\in\mathfrak{G}$ lies in the class $\mathcal{R}^{\varepsilon}(\mathfrak{A},\mathfrak{B})$, then*

$$\left|a_n\right|\leq(\mathfrak{A}-\mathfrak{B})\frac{|\varepsilon|}{n},\quad n\in\mathbb{N}-\{1\}. \tag{14}$$

The result is sharp.

2. The Necessary and Sufficient Condition

Throughout this paper, we use significantly the following identities for $0\leq r<1$ and $t\geq 1$.

$$\begin{aligned}\sum_{n=2}^{\infty}\binom{n+t-2}{t-1}r^{n-1}&=\frac{1}{(1-r)^t}-1,\\ \sum_{n=2}^{\infty}(n-1)\binom{n+t-2}{t-1}r^{n-1}&=\frac{rt}{(1-r)^{t+1}},\\ \sum_{n=3}^{\infty}(n-1)(n-2)\binom{n+t-2}{t-1}r^{n-1}&=\frac{r^2t(t+1)}{(1-r)^{t+2}}.\end{aligned} \tag{15}$$

Using Lemma 1, we get the following necessary and sufficient condition for $\Upsilon_r^t(\xi)$ to be in the investigated class $\mathfrak{T}\mathcal{S}(\mu,\sigma,\delta)$.

Theorem 1. *For $t\geq 1$, $\Upsilon_r^t(\xi)\in\mathfrak{T}\mathcal{S}(\mu,\sigma,\delta)$ if and only if*

$$\begin{aligned}&\frac{\digamma(\mu,\delta)r^2t(t+1)}{(1-r)^2}+(3\digamma(\mu,\delta)+\Theta(\mu,\sigma,\delta))\frac{rt}{1-r}\\&+(\digamma(\mu,\delta)+\Theta(\mu,\sigma,\delta)+\Lambda(\mu,\sigma,\delta))\left(1-(1-r)^t\right)\leq 1-\sigma.\end{aligned} \tag{16}$$

Proof. From Lemma 1, we only need to show that

$$\begin{aligned}&\sum_{n=2}^{\infty}\left[\digamma(\mu,\delta)n^2+\Theta(\mu,\sigma,\delta)n+\Lambda(\mu,\sigma,\delta)\right]\\&\binom{n+t-2}{t-1}r^{n-1}(1-r)^t\leq 1-\sigma.\end{aligned} \tag{17}$$

Putting $n=(n-1)+1$ and $n^2=(n-1)(n-2)+3(n-1)+1$, in (17), we have

$$\sum_{n=2}^{\infty}\left[\mathsf{F}(\mu,\delta)n^2+\Theta(\mu,\sigma,\delta)n+\Lambda(\mu,\sigma,\delta)\right]\binom{n+t-2}{t-1}r^{n-1}(1-r)^t$$
$$=(1-r)^t\left[\begin{array}{c}\sum_{n=3}^{\infty}\mathsf{F}(\mu,\delta)(n-1)(n-2)\binom{n+t-2}{t-1}r^{n-1}\\ +\sum_{n=2}^{\infty}[3\mathsf{F}(\mu,\delta)+\Theta(\mu,\sigma,\delta)](n-1)\binom{n+t-2}{t-1}r^{n-1}\\ +\sum_{n=2}^{\infty}[\mathsf{F}(\mu,\delta)+\Theta(\mu,\sigma,\delta)+\Lambda(\mu,\sigma,\delta)]\binom{n+t-2}{t-1}r^{n-1}\end{array}\right]$$
$$=\frac{\mathsf{F}(\mu,\delta)r^2t(t+1)}{(1-r)^2}+(3\mathsf{F}(\mu,\delta)+\Theta(\mu,\sigma,\delta))\cdot\frac{rt}{1-r}+(\mathsf{F}(\mu,\delta)+\Theta(\mu,\sigma,\delta)+\Lambda(\mu,\sigma,\delta))\left(1-(1-r)^t\right). \quad (18)$$

But this last expression is bounded previously by $1-\sigma$ if and only if (16) is satisfied. □

3. Inclusion Properties

Next, we show that $\mathfrak{I}_r^t(\mathscr{R}^{\varepsilon}(\mathfrak{A},\mathfrak{B}))\subset\mathfrak{T}\mathscr{S}(\mu,\sigma,\delta)$.

Theorem 2. *For $t>1$ and $f\in\mathscr{R}^{\varepsilon}(\mathfrak{A},\mathfrak{B}),\mathfrak{I}_r^t f(\xi)\in\mathfrak{T}\mathscr{S}(\mu,\sigma,\delta)$ if*

$$(\mathfrak{A}-\mathfrak{B})|\varepsilon|\left[\frac{\mathsf{F}(\mu,\delta)rt}{1-r}+(\mathsf{F}(\mu,\delta)+\Theta(\mu,\sigma,\delta))\left(\frac{1}{(1-r)^t}-1\right)+\frac{\Lambda(\mu,\sigma,\delta)}{r(t-1)}\left[(1-r)-(1-r)^t-r(t-1)(1-r)^t\right]\right]\le 1-\sigma. \quad (19)$$

Proof. According to Lemma 1, it suffices to prove that

$$\sum_{n=2}^{\infty}\left[\mathsf{F}(\mu,\delta)n^2+\Theta(\mu,\sigma,\delta)n+\Lambda(\mu,\sigma,\delta)\right]\binom{n+t-2}{t-1}r^{n-1}(1-r)^t|a_n|\le 1-\sigma. \quad (20)$$

Applying Lemma 2, we find from (14) and (20) that

$$\sum_{n=2}^{\infty}\left[\mathsf{F}(\mu,\delta)n^2+\Theta(\mu,\sigma,\delta)n+\Lambda(\mu,\sigma,\delta)\right]\binom{n+t-2}{t-1}r^{n-1}(1-r)^t|a_n|$$
$$\le(\mathfrak{A}-\mathfrak{B})|\varepsilon|(1-r)^t\left[\sum_{n=2}^{\infty}\left[\mathsf{F}(\mu,\delta)n+\Theta(\mu,\sigma,\delta)+\frac{\Lambda(\mu,\sigma,\delta)}{n}\right]\binom{n+t-2}{t-1}r^{n-1}\right]$$

$$
\begin{aligned}
&= (\mathfrak{A}-\mathfrak{B})|\varepsilon|(1-r)^t \left[\begin{array}{c}\sum_{n=2}^{\infty}[\digamma(\mu,\delta)(n-1)+\digamma(\mu,\delta)+\Theta(\mu,\sigma,\delta)]\binom{n+t-2}{t-1}r^{n-1}\\ \sum_{n=2}^{\infty}\frac{\Lambda(\mu,\sigma,\delta)}{n}\binom{n+t-2}{t-1}r^{n-1}\end{array}\right]\\
&= (\mathfrak{A}-\mathfrak{B})|\varepsilon|\left[\frac{\digamma(\mu,\delta)rt}{1-r}+(\digamma(\mu,\delta)+\Theta(\mu,\sigma,\delta))\left(\frac{1}{(1-r)^t}-1\right)\right.\\
&\quad\left.+\frac{\Lambda(\mu,\sigma,\delta)}{r(t-1)}\left[(1-r)-(1-r)^t-r(t-1)(1-r)^t\right]\right].
\end{aligned} \tag{21}
$$

Thus, the proof is completed since the RHS of the above inequality is bounded by $1-\sigma$. □

4. An Integral Operator

In this section, we will consider the following integral operator.

$$
\mathcal{G}_r^t f(\xi)=\int_0^{\xi}\frac{\Upsilon_r^t(t)}{t}\mathrm{d}t. \tag{22}
$$

Theorem 3. *For $t>1$, $\mathcal{G}_r^t f(\xi)\in\mathfrak{T}\mathcal{S}(\mu,\sigma,\delta)$ if and only if*

$$
\frac{\digamma(\mu,\delta)rt}{1-r}+(\digamma(\mu,\delta)+\Theta(\mu,\sigma,\delta))\left(\frac{1}{(1-r)^t}-1\right)+\frac{\Lambda(\mu,\sigma,\delta)}{r(t-1)}\left[(1-r)-(1-r)^t-r(t-1)(1-r)^t\right]\leq 1-\sigma. \tag{23}
$$

Proof. From (8) and (22), we easily get

$$
\mathcal{G}_r^t f(\xi)=\xi-\sum_{n=2}^{\infty}\binom{n+t-2}{t-1}r^{n-1}(1-r)^t\frac{\xi^n}{n}. \tag{24}
$$

Then using Lemma 1, we only need to prove that.

$$
\begin{aligned}
&\sum_{n=2}^{\infty}\left[\digamma(\mu,\delta)n^2+\Theta(\mu,\sigma,\delta)n+\Lambda(\mu,\sigma,\delta)\right]\\
&\quad\times\frac{1}{n}\binom{n+t-2}{t-1}r^{n-1}(1-r)^t\leq 1-\sigma.
\end{aligned} \tag{25}
$$

Clearly, we have

$$
\sum_{n=2}^{\infty}[\digamma(\mu,\delta)n+\Theta(\mu,\sigma,\delta)]\binom{n+t-2}{t-1}r^{n-1}(1-r)^t+\sum_{n=2}^{\infty}\frac{\Lambda(\mu,\sigma,\delta)}{n}\binom{n+t-2}{t-1}r^{n-1}(1-r)^t\leq 1-\sigma. \tag{26}
$$

The rest of the proof can be made similar to that of Theorem 2. □

5. Corollaries and Consequences

Letting $\mu=1/2$ in Theorems 1–3, one can get the following corollaries.

Corollary 1. *For $t\geq 1$, $\Upsilon_r^t(\xi)\in\mathfrak{T}\mathcal{R}(\sigma,\delta)$ if and only if*

$$
(1+\delta)\left[\frac{rt}{1-r}+\left(1-(1-r)^t\right)\right]\leq 1-\sigma. \tag{27}
$$

Corollary 2. *for $t>1$ and $f\in\mathcal{R}^{\varepsilon}(\mathfrak{A},\mathfrak{B})$, $\mathfrak{I}_r^t f(\xi)\in\mathfrak{T}\mathcal{R}(\sigma,\delta)$ if*

$$
(\mathfrak{A}-\mathfrak{B})|\varepsilon|\left[(1+\delta)\left(\frac{1}{(1-r)^t}-1\right)\right]\leq 1-\sigma. \tag{28}
$$

Corollary 3. *For $t > 1$, the integral operator $\mathscr{G}_r^t f(\xi)$ given by (22) is in the class $\mathfrak{T}\mathscr{R}(\sigma, \delta)$ if and only if*

$$(1+\delta)\left(\frac{1}{(1-r)^t} - 1\right) \leq 1 - \sigma. \tag{29}$$

6. Conclusions

In the present paper and due the earlier works (see, for example, [12, 15, 21]), we find a necessary and sufficient condition and inclusion relation for Pascal distribution series to be in a class of analytic functions with negative coefficients. Furthermore, we consider an integral operator related to Pascal distribution series. Some interesting corollaries and applications of the results are also discussed.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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