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*Research Article*

# Analytical Soliton Solutions of the Coupled Radhakrishnan-Kundu-Lakshmanan Equation via Three Techniques

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The analytical soliton solutions to the coupled Radhakrishnan-Kundu-Lakshmanan (RKL) model are greatly important for birefringent fibers without the effect of four-wave mixing (4WM). A significant number of general and standard analytical soliton solutions to this model have been extracted using three powerful techniques, namely the generalized Kudryashov's method, the extended tanh method, and the $(G'/G)$-expansion method in this article. The schematic profiles of the solitons are sketched using the symbolic mathematical program Mathematica and are presented in two and three dimensions. The reported solutions might be helpful in explaining the RKL equation's physical significance as well as some other related nonlinear phenomena that appear in engineering and nonlinear sciences.

## 1. Introduction

The theory and investigation of soliton solutions is one of the important research fields relating to nonlinear partial differential equations ascending in telecommunication engineering, optics, mathematical physics, and other domains of nonlinear sciences. Therefore, diverse academics and researchers developed a number of numerical and analytical techniques, namely, the $(m+1/G')$-expansion technique [1], the truncated $M$-fractional derivative scheme [2], the $q$-homotopy analysis technique [3], Atangana-Baleanu operator scheme [4], the improved Bernoulli subequation function process [5], the sine-Gordon expansion approach [6], the Haar wavelet technique [7], the biframelet systems process [8], the Lie symmetry technique [9], the generalized exponential rational function mode [10], the Painlevé analysis [11], the extended subequation method [12], the improved $(G'/G)$-expansion scheme [13], the Hirota simplified method [14], the one-dimensional subalgebra system [15], Painlevé analysis and multi-soliton solutions technique [16], the one-parameter Lie group of transformations approach [17], etc.

Optical solitons are one of the kinds of solitary waves where the waves propagate without scattering across a vast distance. The optical solitons were discovered since 1971 when Zhakarov and Sabat investigated the nonlinear Schrödinger (NLS) equation using the inverse scattering method. Hasegawa and Tappert realized in 1973 that the same NLS equation governs pulse propagation inside optical fibers. Fiber-optic solitons are light pulses achieve stabilization in their shape when the balance between the fibers dispersion and its nonlinearity occur; this enables them to be convenient as signalling light pulses in optical data transmission. The Radhakrishnan-Kundu-Lakshmanan (RKL) model represents dispersive nonlinear waves in polarization-preserving fibers with the Kerr law

$$\mathrm{i}p_t + \mathrm{a}p_{xx} + b|p|^2 p = i\lambda\left(|p|^2 p\right)_x - i\gamma p_{xxx}, \tag{1}$$

where $p(x,t)$ represent complex-valued wave profile, $a$ is the coefficient of chromatic dispersion (CD), $\lambda$ is the coefficient of the self-steepening considered for short pulses to eschew the formation of shock waves, $\gamma$ is the coefficient of third order dispersion (3OD), and $b$ is Kerr nonlinearity. The Radhakrishnan-Kundu-Lakshmanan equation is one of the well-known models to deal with dispersive optical solitons with Kerr law nonlinearity. Most of the optical fibers that have become reliable at present obey this law of nonlinearity. Also, this medium indicates itself as self-phase modulation, a self-encouraged phase, and frequency-shift of a pulse of light as it travels toward a fiber nonlinearity. Therefore, in this work, we study the couple Radhakrishnan-Kundu-Lakshmanan model for birefringent fibers without the effect of four wave mixing (4WM) named by the basic case of fiber nonlinearity through three efficient methods, notably the $(G'/G)$-expansion method, the generalized Kudryashov method, and the extended tanh method to find different types of soliton solutions. In the following, it is considered the couple Radhakrishnan-Kundu-Lakshmanan model [18, 19]:

$$\begin{aligned} ip_t + a_1 p_{xx} + \left(b_1|p|^2 + c_1|q|^2\right)p &= i\left(\lambda_1\left(|p|^2 p\right)_x + \gamma_1\left(|q|^2 p\right)_x\right) - i\beta_1 p_{xxx}, \\ iq_t + a_2 q_{xx} + \left(b_2|q|^2 + c_2|p|^2\right)q &= i\left(\lambda_2\left(|q|^2 q\right)_x + \gamma_2\left(|p|^2 q\right)_x\right) - i\beta_2 q_{xxx}. \end{aligned} \tag{2}$$

The complex valued wave potentials $p(x, t)$ and $q(x, t)$ present the wave profiles, $b_j$ accounts for self-phase modulation (SPM), $c_j$ is the cross-phase modulation terms, and $\lambda_j$ and $\gamma_j$ associated with self steepening terms where the impact of four-wave mixing is dismissed, for $j = 1, 2$. Some strategies for solving the RKL model are found in the literature, such as the extended rational sine-cosine and sinh-cosh techniques applied by Rehman and Ahmad [20], Bilal et al. [21] used the generalized exponential rational function method, the advanced generalized auxiliary equation method (NGAEM) was used by Abbagari et al. [22], Seadawy et al. applied the Fan-extended subequation (FESE) approach [23], the Riccati equation method, the Sine-Gordon equation method, the functional variable technique, the F-expansion principle, and the exp-expansion function are applied by Yildirim et al. [18], the extended auxiliary equation scheme and the unified Riccati equation approaches were implemented by Zayed et al. [24], and Yildirim et al. [19] used the trial equation method and the modified simple equation method.

This work is arranged as, in Section 1, we analyze briefly about the fundamental techniques. The $(G'/G)$-expansion method [25–27], the general form of Kudryashov method [28, 29], and the extended tanh method [30–34], the analysis of solutions is given in Section 3, in Section 4, we present some illustrative graphs, and a definitive conclusion is presented in Section 5.

## 2. Fundamental Techniques

*2.1. The $(G'/G)$-Expansion Method.* We take into account the governing equation in the form

$$F(u, u_x, u_t, u_{xx}, u_{tt}, u_{xt}) = 0, \tag{3}$$

where $F$ is a polynomial $u = u(x, t)$ and its partial derivatives. Applying the traveling wave transformation, Equation (3) can be converted to an ordinary differential equation:

$$H\left(v, v', v'', v''', v''''\right) = 0. \tag{4}$$

The essential steps of the $(G'/G)$-expansion method are:

*Step 1.* Assume the solution of (4) as follows:

$$v(\eta) = \sum_{i=0}^{N}\left(f_i\left(\frac{G'}{G}\right)^i\right), \tag{5}$$

where $G = G(\eta)$ fulfill the linear ODE:

$$G''(\eta) + \lambda G'(\eta) + \mu G(\eta) = 0, \tag{6}$$

where $f_i (i = 0, 1, 2, \cdots, N), f_N \neq 0, \lambda$ and $\mu$ are constants to be determined.

*Step 2.* In (5), $N$ is a positive integer to be calculated by balancing the highest order derivative term with the highest power nonlinear term in (4).

*Step 3.* We acquire three different cases of solutions of (5):

*Case 1.* Hyperbolic function solutions, when $\lambda^2 - 4\mu > 0$,

$$\frac{G'}{G} = \frac{-\lambda}{2} + \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\,\frac{h_1 \sinh 1/2\sqrt{\lambda^2 - 4\mu}\eta + h_2 \cosh 1/2\sqrt{\lambda^2 - 4\mu}\eta}{h_1 \cosh 1/2\sqrt{\lambda^2 - 4\mu}\eta + h_2 \sinh 1/2\sqrt{\lambda^2 - 4\mu}\eta}. \tag{7}$$

*Case 2.* Trigonometric function solutions, when $\lambda^2 - 4\mu < 0$,

$$\frac{G'}{G} = \frac{-\lambda}{2} + \frac{1}{2}\sqrt{4\mu - \lambda^2}\,\frac{-h_1 \sin 1/2\sqrt{4\mu - \lambda^2}\eta + h_2 \cos 1/2\sqrt{4\mu - \lambda^2}\eta}{h_1 \cos 1/2\sqrt{4\mu - \lambda^2}\eta + h_2 \sin 1/2\sqrt{4\mu - \lambda^2}\eta}. \tag{8}$$

*Case 3.* Rational function solutions, when $\lambda^2 - 4\mu = 0$,

$$\frac{G'}{G} = \frac{-\lambda}{2} + \frac{h_2}{h_1 + h_2\eta}. \tag{9}$$

*Step 4.* Putting (5) into (4), and using (6), gathering all terms with the like power of $(G'/G)$ with each other and set each coefficient equal to zero, we get a system of algebraic equations, which can be solved by the aid of Mathematica program.

*2.2. The Generalized Kudryashov Method.* In this section, we give a brush up of the steps of the generalized Kudryashov method.

*Step 1*. Suppose the exact solutions of (4) can be expressed as follows:

$$v(\eta) = \sum_{i=0}^{N} \left(A_i (Q(\eta))^i\right), \tag{10}$$

where $A_i (i = 0, 1, 2, \cdots, N), A_N \neq 0$ are constants to be determined with $(A_i \neq 0$. $N$ will be determined by homogeneous balance principle.

*Step 2*. $Q(\eta)$ has the following definition:

$$Q(\eta) = \frac{\delta\phi(\eta)}{\delta + \rho(\phi(\eta) - 1)}, \rho \neq 0, \tag{11}$$

where $A_0, A_1, A_2, \cdots, A_N, \delta$ and $\rho$ are arbitrary constants to be calculated. The function $\phi(\eta)$ fulfill the next differential equation:

$$\frac{d\phi}{d\eta} = b \log(m)(\phi(\eta) - 1)\phi(\eta). \tag{12}$$

The solution of (12) is given by

$$\phi(\eta) = \frac{1}{1 + \mathrm{dm}^{b\eta}}. \tag{13}$$

*Step 3*. Putting (10), (11), and (13) into (4), we acquire a polynomial of $Q(\eta)$. Gathering all terms with the like powers of $Q(\eta)$ with each other's and setting each coefficient to zero, we get a set of algebraic equations.

*Step 4*. Solving this system with the aid of Mathematica program, we attain the exact soliton solution of (4).

*2.3. The Extended Tanh-Function Method*

*Step 1*. The modified extended tanh method present the solution of (4) by the following finite series:

$$v(\eta) = A_0 + \sum_{i=1}^{N} \left(A_i \phi(\eta)^i + B_i \phi(\eta)^{-i}\right), \tag{14}$$

where $\phi = \phi(\eta)$ satisfy the Riccati equation

$$\frac{d\phi}{d\eta} = w_1 + \phi^2. \tag{15}$$

The solution of the Riccati Equation (15) has the following cases of solutions:

If $\omega_1 < 0$, then

$$\begin{aligned} \phi(\eta) &= -\sqrt{-\omega_1} \tanh\left(\sqrt{-\omega_1}\eta\right), \\ \phi(\eta) &= -\sqrt{-\omega_1} \coth\left(\sqrt{-\omega_1}\eta\right). \end{aligned} \tag{16}$$

If $\omega_1 = 0$, then

$$\phi(\eta) = -\frac{1}{\eta}. \tag{17}$$

If $\omega_1 > 0$, then

$$\begin{aligned} \phi(\eta) &= \sqrt{\omega_1} \tan\left(\sqrt{\omega_1}\eta\right), \\ \phi(\eta) &= -\sqrt{\omega_1} \cot\left(\sqrt{\omega_1}\eta\right). \end{aligned} \tag{18}$$

*Step 2*. $N$ can be determined by balancing the highest order derivative term with the highest power nonlinear term in (4).

*Step 3*. Substituting (14) and (15) into (4), then gather all coefficients of the same powers of $\phi(\eta)^i$ and put them equal to zero, we get a system of algebraic equations for $\omega_1, A_0, \cdots, A_N, B_1, \cdots, B_N$, solving this equations we get all constants.

## 3. Analysis of Solutions

We first propose the following assumptions in order to generate the traveling wave solutions to the RKLE (2):

$$\begin{aligned} p(x,t) &= e^{i\xi} v_1(\eta), \\ q(x,t) &= e^{i\xi} v_2(\eta), \end{aligned} \tag{19}$$

where

$$\xi = -\mathrm{kx} + \omega t, \eta = x - \alpha t. \tag{20}$$

where $v_j(\eta)$ is the amplitude $(j = 1, 2)$, $\alpha$ gives the soliton velocity, $k$ is the frequency of the soliton, and $\omega$ is the wave number of the soliton.

Applying the assumptions (19) and (20) into (2), we acquire real and imaginary parts as:

$$-a_j k^2 v_j - k^3 \beta_j v_j - \omega v_j + b_j v_j^3 + c_j v_j v_{\bar{j}}^2 - k\lambda_j v_j^3 - k\gamma_j v_j v_{\bar{j}}^2 + a_j v_j'' + 3k\beta_j v_j'' = 0, \tag{21}$$

$$-2a_j k v_j' - \alpha v_j' - 3k^2 \beta_j v_j' - 3\lambda_j v_j^2 v_j' - \gamma_j v_{\bar{j}}^2 v_j' - 2\gamma_j v_j v_{\bar{j}} v_{\bar{j}}' + \beta_j v_j''' = 0. \tag{22}$$

For both $j = 1, 2$ and $\hat{j} = 3 - j$ with the balance principle $v_j = v_{\bar{j}}$, Equations (21) and (22) become:

$$-a_j k^2 v_j - k^3 \beta_j v_j - \omega v_j + b_j v_j^3 + c_j v_j^3 - k\lambda_j v_j^3 - k\gamma_j v_j^3 + a_j v_j'' + 3k\beta_j v_j'' = 0, \tag{23}$$

$$-2a_j k v_j' - \alpha v_j' - 3k^2 \beta_j v_j' - 3\lambda_j v_j^2 v_j' - 3\gamma_j v_j^2 v_j' + \beta_j v_j''' = 0. \tag{24}$$

Integrating (24) with respect to $\eta$ and putting the constant of the integration equal to zero, we obtain

$$-\left(2a_j k + \alpha + 3k^2 \beta_j\right) v_j - \left(\lambda_j + \gamma_j\right) v_j^3 + \beta_j v_j'' = 0. \tag{25}$$

The function $v_j$ satisfies both Equations (23) and (25) in the following restriction relation:

$$\frac{a_j + 3k\beta_j}{\beta_j} = \frac{-\left(a_j k^2 + k^3\beta_j + \omega\right)}{-\left(2a_j k + \alpha + 3k^2\beta_j\right)} = \frac{\left(b_j + c_j - k\lambda_j - k\gamma_j\right)}{-\left(\lambda_j + \gamma_j\right)}, \tag{26}$$

where

$$\begin{aligned} b_j &= -\frac{c_j\beta_j + a_j\lambda_j + 2k\beta_j\lambda_j + a_j\gamma_j + 2k\beta_j\gamma_j}{\beta_j}, \\ \omega &= \frac{2a_j^2 k + a_j\alpha + 8a_j k^2\beta_j + 3k\alpha\beta_j + 8k^3\beta_j^2}{\beta_j}. \end{aligned} \tag{27}$$

Applying the balance principle in (23) between $v_j''$ and $v_j^3$, we get $N + 2 = 3N \Rightarrow N = 1$.

*3.1. Solutions through the $(G'/G)$-Expansion Method.* From (5), the solution of (23) can be presented as:

$$v_j(\eta) = f_0 + f_1\left(\frac{G'}{G}\right). \tag{28}$$

Substituting (28) into (23), setting the coefficient of like power of $G'/G$ equal to zero, we acquire the following system:

$$\begin{aligned} &-2a_j^2 k f_0 - a_j\alpha f_0 - 9a_j k^2\beta_j f_0 - 3k\alpha\beta_j f_0 - 9k^3\beta_j^2 f_0 - a_j\lambda_j f_0^3 - 3k\beta_j\lambda_j f_0^3 - a_j\gamma_j f_0^3 - 3k\beta_j\gamma_j f_0^3 + a_j\beta_j\lambda\mu f_1 + 3k\beta_j^2\lambda\mu f_1 = 0, \\ &-2a_j^2 k f_1 - a_j\alpha f_1 - 9a_j k^2\beta_j f_1 - 3k\alpha\beta_j f_1 - 9k^3\beta_j^2 f_1 + a_j\lambda^2 f_1 + 3k\beta_j^2\lambda^2 f_1 + 2a_j\beta_j\mu f_1 + 6k\beta_j^2\mu f_1 - 3a_j\lambda_j f_0^2 f_1 - 9k\beta_j\lambda_j f_0^2 f_1 - 3a_j\gamma_j f_0^2 f_1 - 9k\beta_j\gamma_j f_0^2 f_1 = 0, \\ &3a_j\beta_j\lambda f_1 + 9k\beta_j^2\lambda f_1 - 3a_j\lambda_j f_0 f_1^2 - 9k\beta_j\lambda_j f_0 f_1^2 - 3a_j\gamma_j f_0 f_1^2 - 9k\beta_j\gamma_j f_0 f_1^2 = 0, \\ &2a_j\beta_j f_1 + 6k\beta_j^2 f_1 - a_j\lambda_j f_1^3 - 3k\beta_j\lambda_j f_1^3 - a_j\gamma_j f_1^3 - 3k\beta_j\gamma_j f_1^3 = 0. \end{aligned} \tag{29}$$

Solving the upwards set of equations with the aid of Mathematica program, we obtain the following solutions. Substituting (7), (8), and (9) with the different classes of solutions into (28), the traveling wave solutions of (2) are listed in the following:

Class 1:

$$f_0 = -\frac{\lambda\sqrt{\beta_j}}{\sqrt{2\left(\lambda_j + \gamma_j\right)}}, f_1 = -\frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j + \gamma_j}}, a_j = \frac{-2\alpha - 6k^2\beta_j - \beta_j\lambda^2 + 4\mu\beta_j}{4k}. \tag{30}$$

Hyperbolic function solutions, when $\lambda^2 - 4\mu > 0$,

$$p(x,t) = q(x,t) = e^{i(-\mathrm{kx}+\omega t)}\left(-\frac{\lambda\sqrt{\beta_j}}{\sqrt{2\left(\lambda_j + \gamma_j\right)}} - \frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j + \gamma_j}}\left(\frac{-\lambda}{2} + \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\frac{h_1 \sinh 1/2\sqrt{\lambda^2 - 4\mu}(x - \alpha t) + h_2 \cosh 1/2\sqrt{\lambda^2 - 4\mu}(x - \alpha t)}{h_1 \cosh 1/2\sqrt{\lambda^2 - 4\mu}(x - \alpha t) + h_2 \sinh 1/2\sqrt{\lambda^2 - 4\mu}(x - \alpha t)}\right)\right). \tag{31}$$

Trigonometric function solutions, when $\lambda^2 - 4\mu < 0$,

$$p(x,t) = q(x,t) = e^{i(-\mathrm{k}x+\omega t)}\left(-\frac{\lambda\sqrt{\beta_j}}{\sqrt{2\left(\lambda_j+\gamma_j\right)}} - \frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j+\gamma_j}}\left(\frac{-\lambda}{2} + \frac{1}{2}\sqrt{4\mu-\lambda^2}\,\frac{-h_1\sin 1/2\sqrt{4\mu-\lambda^2}(x-\alpha t) + h_2\cos 1/2\sqrt{4\mu-\lambda^2}(x-\alpha t)}{h_1\cos 1/2\sqrt{4\mu-\lambda^2}(x-\alpha t) + h_2\sin 1/2\sqrt{4\mu-\lambda^2}(x-\alpha t)}\right)\right). \quad (32)$$

Rational function solutions, when $\lambda^2 - 4\mu = 0$,

$$p(x,t) = q(x,t) = e^{i(-\mathrm{k}x+\omega t)}\left(-\frac{\lambda\sqrt{\beta_j}}{\sqrt{2\left(\lambda_j+\gamma_j\right)}} - \frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j+\gamma_j}}\left(\frac{-\lambda}{2} + \frac{h_2}{h_1 + h_2(x-\alpha t)}\right)\right). \quad (33)$$

Class 2:

$$f_0 = \frac{\lambda\sqrt{\beta_j}}{\sqrt{2\left(\lambda_j+\gamma_j\right)}}, f_1 = \frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j+\gamma_j}}, a_j = \frac{-2\alpha - 6k^2\beta_j - \beta_j\lambda^2 + 4\mu\beta_j}{4k}, \quad (34)$$

Hyperbolic function solutions, when $\lambda^2 - 4\mu > 0$,

$$p(x,t) = q(x,t) = e^{i(-\mathrm{k}x+\omega t)}\left(\frac{\lambda\sqrt{\beta_j}}{\sqrt{2\left(\lambda_j+\gamma_j\right)}} + \frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j+\gamma_j}}\left(\frac{-\lambda}{2} + \frac{1}{2}\sqrt{\lambda^2-4\mu}\,\frac{h_1\sinh 1/2\sqrt{\lambda^2-4\mu}(x-\alpha t) + h_2\cosh 1/2\sqrt{\lambda^2-4\mu}(x-\alpha t)}{h_1\cosh 1/2\sqrt{\lambda^2-4\mu}(x-\alpha t) + h_2\sinh 1/2\sqrt{\lambda^2-4\mu}(x-\alpha t)}\right)\right). \quad (35)$$

Trigonometric function solutions, when $\lambda^2 - 4\mu < 0$,

$$p(x,t) = q(x,t) = e^{i(-\mathrm{k}x+\omega t)}\left(\frac{\lambda\sqrt{\beta_j}}{\sqrt{2\left(\lambda_j+\gamma_j\right)}} + \frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j+\gamma_j}}\left(\frac{-\lambda}{2} + \frac{1}{2}\sqrt{4\mu-\lambda^2}\,\frac{-h_1\sin 1/2\sqrt{4\mu-\lambda^2}(x-\alpha t) + h_2\cos 1/2\sqrt{4\mu-\lambda^2}(x-\alpha t)}{h_1\cos 1/2\sqrt{4\mu-\lambda^2}(x-\alpha t) + h_2\sin 1/2\sqrt{4\mu-\lambda^2}(x-\alpha t)}\right)\right). \quad (36)$$

Rational function solutions, when $\lambda^2 - 4\mu = 0$,

$$p(x,t) = q(x,t) = e^{i(-\mathrm{k}x+\omega t)}\left(\frac{\lambda\sqrt{\beta_j}}{\sqrt{2\left(\lambda_j+\gamma_j\right)}} + \frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j+\gamma_j}}\left(\frac{-\lambda}{2} + \frac{h_2}{h_1 + h_2(x-\alpha t)}\right)\right). \quad (37)$$

*3.2. Solutions through the Generalized Kudryashov Method.* From (10), the solution of (23) can be written in the form:

$$v(\eta) = A_0 + A_1 Q(\eta). \tag{38}$$

Substituting (38) into (23), setting the coefficient of like power of $Q(\eta)$ equal to zero, we acquire the following set of equations:

$$\begin{gathered}
2a_j k\left(-a_j - 3k\beta_j\right)A_0 + \alpha\left(-a_j - 3k\beta_j\right)A_0 + 3k^2\beta_j\left(-a_j - 3k\beta_j\right)A_0 + \left(-a_j - 3k\beta_j\right)\left(\lambda_j + \gamma_j\right)A_0^3 = 0,\\
2a_j k\left(-a_j - 3k\beta_j\right)A_1 + \alpha\left(-a_j - 3k\beta_j\right)A_1 + 3k^2\beta_j\left(-a_j - 3k\beta_j\right)A_1 - b^2\beta_j\left(-a_j - 3k\beta_j\right)\log(m)^2 A_1 + 3\left(-a_j - 3k\beta_j\right)\left(\lambda_j + \gamma_j\right)A_0^2 A_1 = 0,\\
3b^2\beta_j\left(-a_j - 3k\beta_j\right)\log(m)^2 A_1 + 3\left(-a_j - 3k\beta_j\right)\left(\lambda_j + \gamma_j\right)A_0 A_1^2 = 0,\\
-2b^2\beta_j\left(-a_j - 3k\beta_j\right)\log(m)^2 A_1 + \left(-a_j - 3k\beta_j\right)\left(\lambda_j + \gamma_j\right)A_1^3 = 0.
\end{gathered} \tag{39}$$

Solving this system, we get the following solution classes:

Class 1:

$$A_0 = -\frac{b\log(m)\sqrt{\beta_j}}{\sqrt{2\left(\lambda_j + \gamma_j\right)}}, A_1 = \frac{b\log(m)\sqrt{2\beta_j}}{\sqrt{\lambda_j + \gamma_j}}, a_j = \frac{-2\alpha - 6k^2\beta_j - b^2\beta_j\log(m)^2}{4k}. \tag{40}$$

Substituting (40) in (38) with (11),(13), and (21), we obtain the following solution of (2):

$$p(x,t) = q(x,t) = e^{i(-\mathrm{kx}+\omega t)}\left(-\frac{b\log(m)\sqrt{\beta_j}}{\sqrt{2\left(\lambda_j + \gamma_j\right)}} + \frac{b\log(m)\sqrt{2\beta_j}}{\sqrt{\lambda_j + \gamma_j}} \times \left(\frac{\delta\left(1/1 + \mathrm{dm}^{b(x-\alpha t)}\right)}{\delta + \rho\left(\left(1/1 + \mathrm{dm}^{b(x-\alpha t)}\right) - 1\right)}\right)\right). \tag{41}$$

Class 2:

$$A_0 = \frac{b\log(m)\sqrt{\beta_j}}{\sqrt{2\left(\lambda_j + \gamma_j\right)}}, A_1 = -\frac{b\log(m)\sqrt{2\beta_j}}{\sqrt{\lambda_j + \gamma_j}}, a_j = \frac{-2\alpha - 6k^2\beta_j - b^2\beta_j\log(m)^2}{4k}. \tag{42}$$

We obtain the following solution:

$$p(x,t) = q(x,t) = e^{i(-\mathrm{kx}+\omega t)}\left(\frac{b\log(m)\sqrt{\beta_j}}{\sqrt{2\left(\lambda_j + \gamma_j\right)}} - \frac{b\log(m)\sqrt{2\beta_j}}{\sqrt{\lambda_j + \gamma_j}} \times \left(\frac{\delta\left(1/1 + \mathrm{dm}^{b(x-\alpha t)}\right)}{\delta + \rho\left(\left(1/1 + \mathrm{dm}^{b(x-\alpha t)}\right) - 1\right)}\right)\right). \tag{43}$$

*3.3. Solutions through the Extended Tanh-Function Method.* From (15), the solutions of (24) are given in the form

$$v_j(\eta) = A_0 + A_1\phi(\eta) + \frac{B_1}{\phi(\eta)}. \tag{44}$$

Substituting from (44) into (24) and use (16), grouping the coefficients of the same $\phi^i(\eta)$, we obtain the following system:

$$\begin{gathered}
-2a_j^2kA_0 - a_j\alpha A_0 - 9a_jk^2\beta_jA_0 - 3k\alpha\beta_jA_0 - 9k^3\beta_j^2A_0 - a_j\lambda_jA_0^3 - 3k\beta_j\lambda_jA_0^3 - a_j\gamma_jA_0^3 - 3k\beta_j\gamma_jA_0^3 - 6a_j\lambda_jA_0A_1B_1 - 18k\beta_j\lambda_jA_0A_1B_1 - 6a_j\gamma_jA_0A_1B_1 - 18k\beta_j\gamma_jA_0A_1B_1 = 0,\\
2a_j\beta_j\omega_1^2B_1 + 6k\beta_j^2\omega_1^2B_1 - a_j\lambda_jB_1^3 - 3k\beta_j\lambda_jB_1^3 - a_j\gamma_jB_1^3 - 3k\beta_j\gamma_jB_1^3 = 0,\\
3a_j\lambda_jA_0B_1^2 + 9k\beta_j\lambda_jA_0B_1^2 + 3a_j\gamma_jA_0B_1^2 + 9k\beta_j\gamma_jA_0B_1^2 = 0,\\
-2a_j^2kB_1 - a_j\alpha B_1 - 9a_jk^2\beta_jB_1 - 3k\alpha\beta_jB_1 - 9k^3\beta_j^2B_1 + 2a_j\beta_j\omega_1B_1 + 6k\beta_j^2\omega_1B_1 - 3a_j\lambda_jA_0^2B_1 - 9k\beta_j\lambda_jA_0^2B_1 - 3a_j\gamma_jA_0^2B_1 - 9k\beta_j\gamma_jA_0^2B_1 - 3a_j\lambda_jA_1B_1^2 - 9k\beta_j\lambda_jA_1B_1^2 - 3a_j\gamma_jA_1B_1^2 - 9k\beta_j\gamma_jA_1B_1^2 = 0,\\
-2a_j^2kA_1 - a_j\alpha A_1 - 9a_jk^2\beta_jA_1 - 3k\alpha\beta_jA_1 - 9k^3\beta_j^2A_1 + 2a_j\beta_j\omega_1A_1 + 6k\beta_j^2\omega_1A_1 - 3a_j\lambda_jA_0^2A_1 - 9k\beta_j\lambda_jA_0^2A_1 - 3a_j\gamma_jA_0^2A_1 - 9k\beta_j\gamma_jA_0^2A_1 - 3a_j\lambda_jA_1^2B_1 - 9k\beta_j\lambda_jA_1^2B_1 - 3a_j\gamma_jA_1^2B_1 - 9k\beta_j\gamma_jA_1^2B_1 = 0,\\
-3a_j\lambda_jA_0A_1^2 - 9k\beta_j\lambda_jA_0A_1^2 - 3a_j\gamma_jA_0A_1^2 - 9k\beta_j\gamma_jA_0A_1^2 = 0,\\
2a_j\beta_jA_1 + 6k\beta_j^2A_1 - a_j\lambda_jA_1^3 - 3k\beta_j\lambda_jA_1^3 - a_j\gamma_jA_1^3 - 3k\beta_j\gamma_jA_1^3 = 0.
\end{gathered} \tag{45}$$

Setting the coefficients equal to zero, we obtain the following results after solving the system:

Class 1:

$$A_1 = \pm\frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j+\gamma_j}}, B_1 = 0, A_0 = 0, a_j = \frac{-\alpha - 3k^2\beta_j + 2\beta_j\omega_1}{2k}. \tag{46}$$

From (46) into (44) with (20) and (21), we get the following solutions:

If $\omega_1 < 0$, then

$$p(x,t) = q(x,t) = e^{i(-\mathrm{kx}+\omega t)}\left(\mp\frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j+\gamma_j}}\sqrt{-\omega_1}\,\coth\left(\sqrt{-\omega_1}((x-\alpha t))\right)\right), \tag{47}$$

$$p(x,t) = q(x,t) = e^{i(-\mathrm{kx}+\omega t)}\left(\mp\frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j+\gamma_j}}\sqrt{-\omega_1}\,\tanh\left(\sqrt{-\omega_1}((x-\alpha t))\right)\right). \tag{48}$$

If $\omega_1 > 0$, then

$$p(x,t) = q(x,t) = e^{i(-\mathrm{kx}+\omega t)}\left(\mp\frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j+\gamma_j}}\sqrt{\omega_1}\,\cot\left(\sqrt{\omega_1}((x-\alpha t))\right)\right), \tag{49}$$

$$p(x,t) = q(x,t) = e^{i(-\mathrm{kx}+\omega t)}\left(\pm\frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j+\gamma_j}}\sqrt{\omega_1}\,\tan\left(\sqrt{\omega_1}((x-\alpha t))\right)\right). \tag{50}$$

Class 2:

$$A_1 = \pm\frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j+\gamma_j}}, B_1 = \pm\frac{\sqrt{2\beta_j}\omega_1}{\sqrt{\lambda_j+\gamma_j}}, A_0 = 0, a_j = \frac{-\alpha - 3k^2\beta_j - 4\beta_j\omega_1}{2k}. \tag{51}$$

If $\omega_1 < 0$, then

$$\begin{gathered}
p(x,t) = q(x,t) = e^{i(-\mathrm{kx}+\omega t)}\left(\mp\frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j+\gamma_j}}\sqrt{-\omega_1}\,\coth\left(\sqrt{-\omega_1}((x-\alpha t))\right) \mp \frac{\sqrt{2\beta_j}\omega_1}{\sqrt{\lambda_j+\gamma_j}\sqrt{-\omega_1}}\tanh\left(\sqrt{-\omega_1}((x-\alpha t))\right)\right),\\
p(x,t) = q(x,t) = e^{i(-\mathrm{kx}+\omega t)}\left(\mp\frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j+\gamma_j}}\sqrt{-\omega_1}\,\tanh\left(\sqrt{-\omega_1}((x-\alpha t))\right) \mp \frac{\sqrt{2\beta_j}\omega_1}{\sqrt{\lambda_j+\gamma_j}\sqrt{-\omega_1}}\coth\left(\sqrt{-\omega_1}((x-\alpha t))\right)\right).
\end{gathered} \tag{52}$$

If $\omega_1 > 0$, then

$$p(x,t) = q(x,t) = e^{i(-\mathrm{kx}+\omega t)}\left(\mp\frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j+\gamma_j}}\sqrt{\omega_1}\cot\left(\sqrt{\omega_1}((x-\alpha t)\right)\mp\frac{\sqrt{2\beta_j}\omega_1}{\sqrt{\lambda_j+\gamma_j\sqrt{\omega_1}}}\tan\left(\sqrt{\omega_1}((x-\alpha t))\right)\right),$$
$$p(x,t) = q(x,t) = e^{i(-\mathrm{kx}+\omega t)}\left(\pm\frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j+\gamma_j}}\sqrt{\omega_1}\tan\left(\sqrt{\omega_1}((x-\alpha t)\right)\pm\frac{\sqrt{2\beta_j}\omega_1}{\sqrt{\lambda_j+\gamma_j\sqrt{\omega_1}}}\cot\left(\sqrt{\omega_1}((x-\alpha t))\right)\right). \quad (53)$$

Class 3:

$$A_1 = \pm\frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j+\gamma_j}}, B_1 = \mp\frac{\sqrt{2\beta_j\omega_1}}{\sqrt{\lambda_j+\gamma_j}}, A_0 = 0, a_j = \frac{-\alpha-3k^2\beta_j+8\beta_j\omega_1}{2k}. \quad (54)$$

If $\omega_1 < 0$, then

$$p(x,t) = q(x,t) = e^{i(-\mathrm{kx}+\omega t)}\left(\mp\frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j+\gamma_j}}\sqrt{-\omega_1}\coth\left(\sqrt{-\omega_1}((x-\alpha t)\right)\pm\frac{\sqrt{2\beta_j}\omega_1}{\sqrt{\lambda_j+\gamma_j\sqrt{-\omega_1}}}\tanh\left(\sqrt{-\omega_1}((x-\alpha t))\right)\right),$$
$$p(x,t) = q(x,t) = e^{i(-\mathrm{kx}+\omega t+\theta_1)}\left(\mp\frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j+\gamma_j}}\sqrt{-\omega_1}\tanh\left(\sqrt{-\omega_1}((x-\alpha t))\right)\pm\frac{\sqrt{2\beta_j}\omega_1}{\sqrt{\lambda_j+\gamma_j\sqrt{-\omega_1}}}\coth\left(\sqrt{-\omega_1}((x-\alpha t))\right)\right). \quad (55)$$

If $\omega_1 > 0$, then

$$p(x,t) = q(x,t) = e^{i(-\mathrm{kx}+\omega t)}\left(\mp\frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j+\gamma_j}}\sqrt{\omega_1}\cot\left(\sqrt{\omega_1}((x-\alpha t)\right)\pm\frac{\sqrt{2\beta_j}\omega_1}{\sqrt{\lambda_j+\gamma_j\sqrt{\omega_1}}}\tan\left(\sqrt{\omega_1}((x-\alpha t))\right)\right),$$
$$p(x,t) = q(x,t) = e^{i(-kx+\omega t)}\left(\pm\frac{\sqrt{2\beta_j}}{\sqrt{\lambda_j+\gamma_j}}\sqrt{\omega_1}\tan\left(\sqrt{\omega_1}((x-\alpha t)\right)\mp\frac{\sqrt{2\beta_j}\omega_1}{\sqrt{\lambda_j+\gamma_j\sqrt{\omega_1}}}\cot\left(\sqrt{\omega_1}((x-\alpha t))\right)\right). \quad (56)$$

Class 4:

$$A_1 = 0, B_1 = \pm\frac{\sqrt{2\beta_j}\omega_1}{\sqrt{\lambda_j+\gamma_j}}, A_0 = 0, a_j = \frac{-\alpha-3k^2\beta_j+2\beta_j\omega_1}{2k}. \quad (57)$$

If $\omega_1 < 0$, then

$$p(x,t) = q(x,t) = e^{i(-\mathrm{kx}+\omega t)}\left(\mp\frac{\sqrt{2\beta_j}\omega_1}{\sqrt{\lambda_j+\gamma_j\sqrt{-\omega_1}}}\tanh\left(\sqrt{-\omega_1}((x-\alpha t))\right)\right), \quad (58)$$

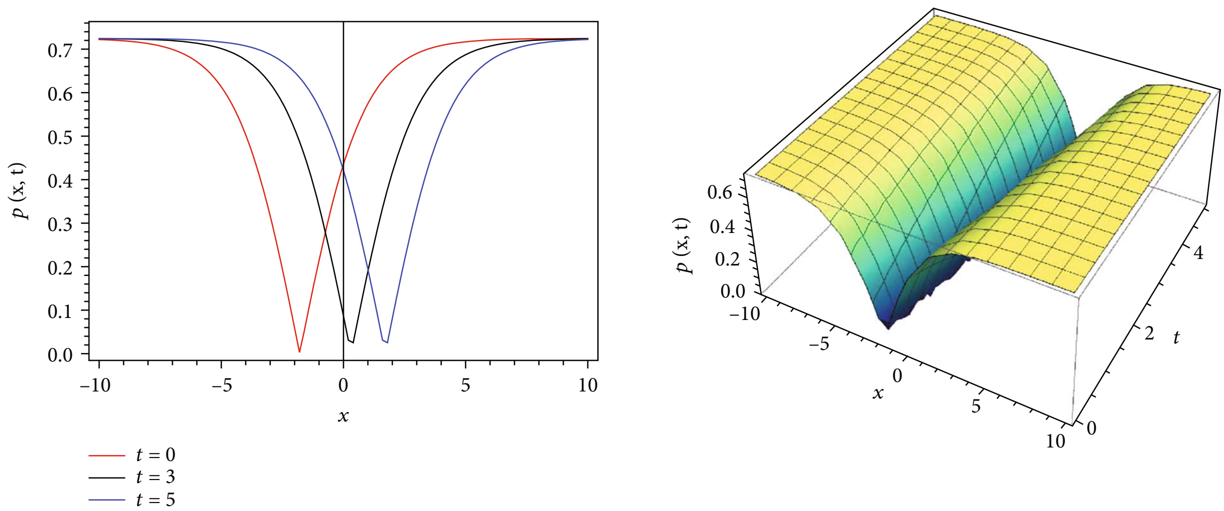


Figure 1: Graph of (31) using the $(G'/G)$-expansion method at $\lambda_j = 0.2, \beta_j = 0.7, \gamma_j = 0.2, k = 0.4, \alpha = 0.7, \lambda = 1, \mu = 0.1, h_1 = 0.5, h_2 = 0.3$.

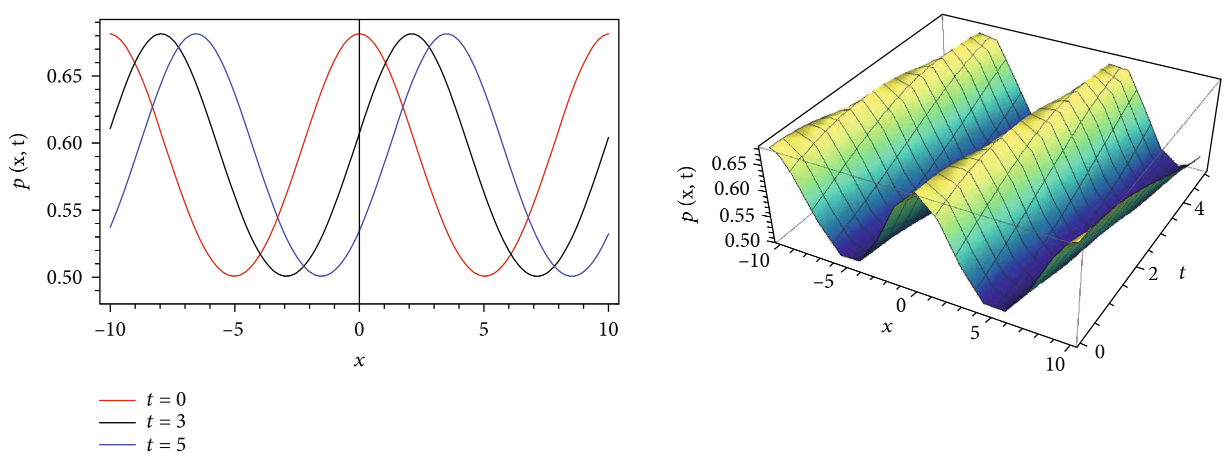


Figure 2: Graph of (32) using the $(G'/G)$-expansion method at $\lambda_j = 0.2, \beta_j = 0.7, \gamma_j = 0.2, k = 0.4, \alpha = 0.7, \lambda = 0.1, \mu = 0.1, h_1 = 0.6, h_2 = 0.7$.

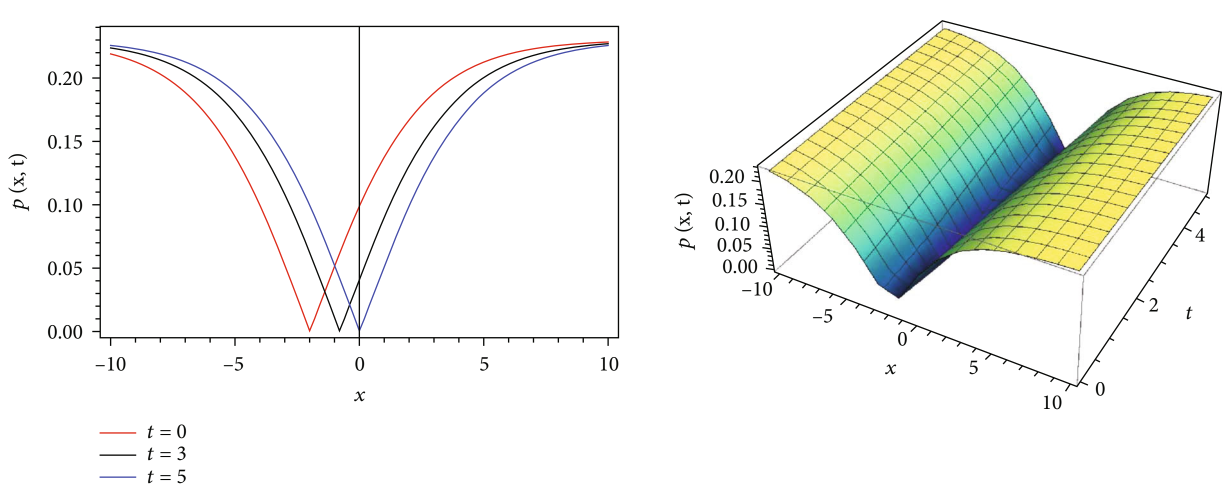


Figure 3: Graph of (41) using the new Kudryashov's method at $\lambda_j = 0.1, \beta_j = 0.1, \gamma_j = 0.1, k = 0.6, A_0 = 0.1, m = 0.01, d = 0.5, \rho = 0.4, \delta = 2$ , $\alpha = 0.4, b = 0.1$.

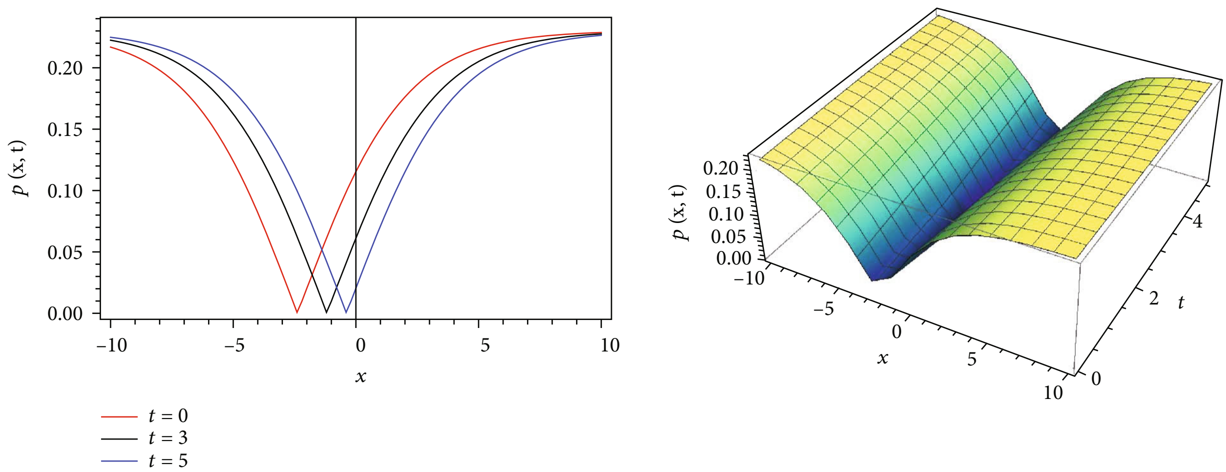


Figure 4: Graph of (42) using the new Kudryashov's method at $\lambda_j = 0.1$, $\beta_j = 0.1$, $\gamma_j = 0.1$, $k = 0.6$, $A_0 = 0.1$, $m = 0.01$, $d = 0.5$, $\rho = 0.4$, $\delta = 1.2$, $\alpha = 0.4$, $b = 0.1$.

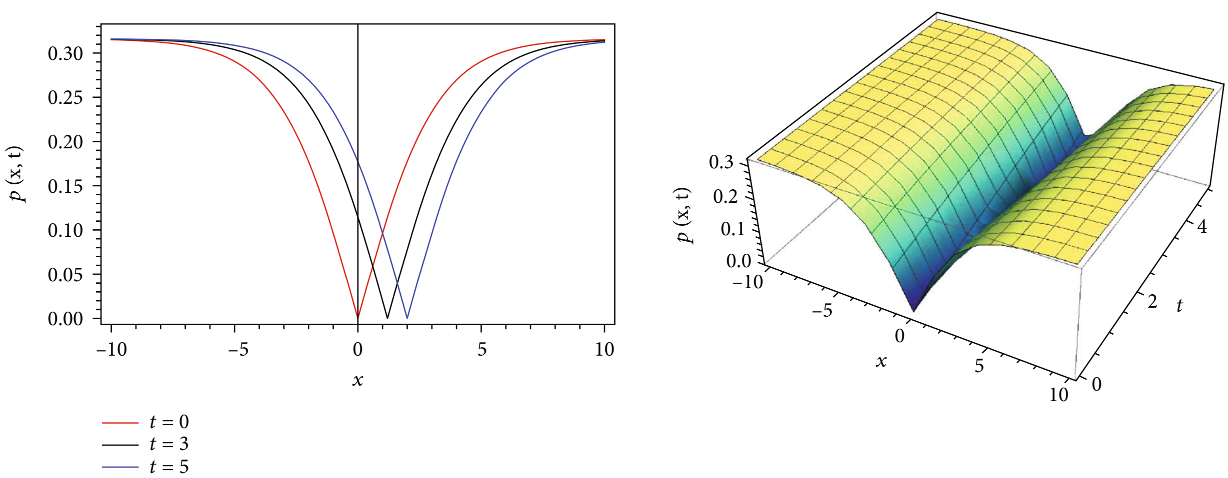


Figure 5: Graph of (47) using the extended tanh-function method at $\lambda_j = 0.1$, $\beta_j = 0.1$, $\gamma_j = 0.1$, $k = 0.5$, $\alpha = 0.4$, $\omega_1 = -0.1$.

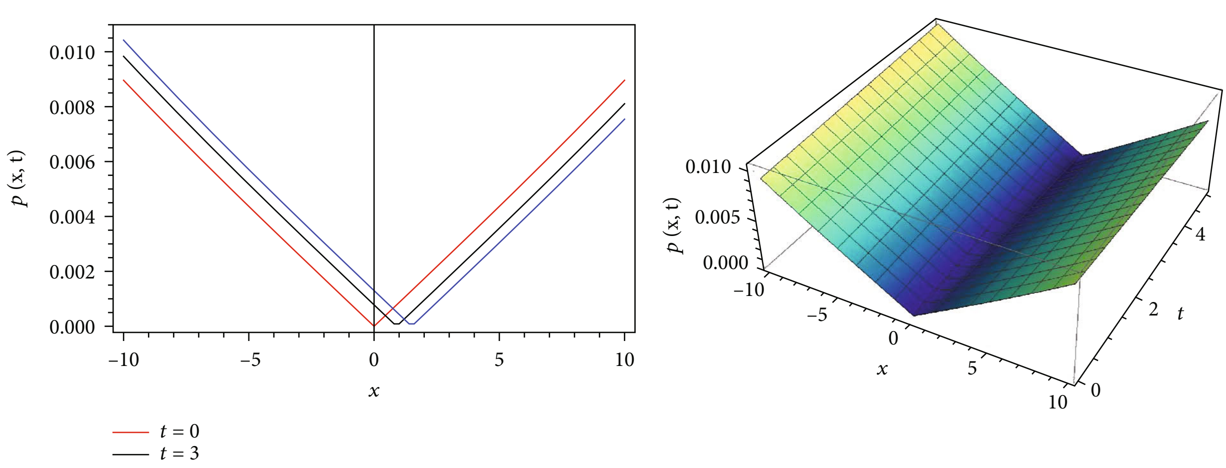


Figure 6: Graph of (50) using the extended tanh-function method at $\lambda_j = 0.4$, $\beta_j = 0.3$, $\gamma_j = 0.4$, $k = 0.2$, $\alpha = 0.3$, $\omega_1 = 0.001$.

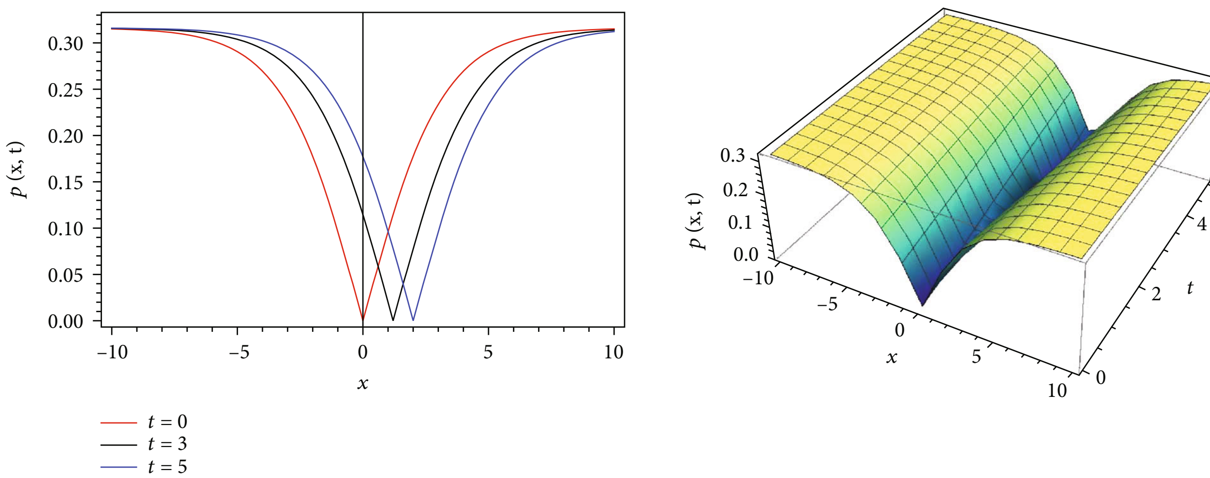


Figure 7: Graph of (58) using the extended tanh-function method at $\lambda_j = 0.1, \beta_j = 0.1, \gamma_j = 0.1, k = 0.5, \alpha = 0.4, \omega_1 = -0.1$.

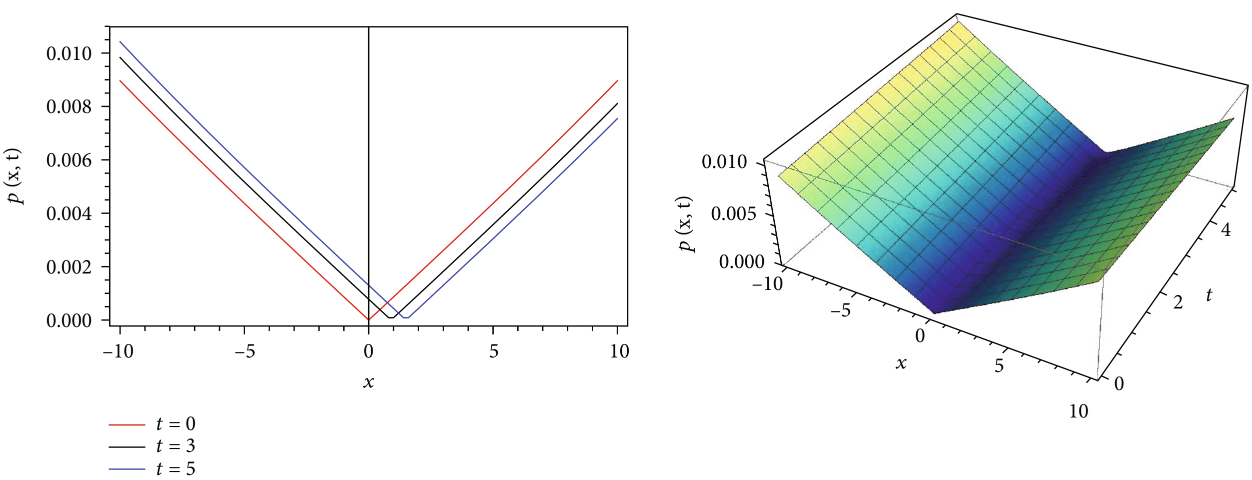


Figure 8: Graph of (47) using the extended tanh-function method at $\lambda_j = 0.4, \beta_j = 0.3, \gamma_j = 0.4, k = 0.2, \alpha = 0.3, \omega_1 = 0.001$.

$$p(x,t) = q(x,t) = e^{i(-\mathrm{kx}+\omega t)} \left( \mp \frac{\sqrt{2\beta_j}\omega_1}{\sqrt{\lambda_j + \gamma_j\sqrt{-\omega_1}}} \coth\left(\sqrt{-\omega_1}((x-\alpha t)\right) \right). \tag{59}$$

If $\omega_1 > 0$, then

$$p(x,t) = q(x,t) = e^{i(-\mathrm{kx}+\omega t)} \left( \mp \frac{\sqrt{2\beta_j}\omega_1}{\sqrt{\lambda_j + \gamma_j\sqrt{\omega_1}}} \tan\left(\sqrt{\omega_1}((x-\alpha t)\right) \right),$$
$$p(x,t) = q(x,t) = e^{i(-\mathrm{kx}+\omega t)} \left( \pm \frac{\sqrt{2\beta_j}\omega_1}{\sqrt{\lambda_j + \gamma_j\sqrt{\omega_1}}} \cot\left(\sqrt{\omega_1}((x-\alpha t)\right) \right). \tag{60}$$

## 4. Graphical Illustrations

Herein, we present some figures in the two-dimensional, and three-dimensional to clarify the solutions that we presented. Some of the analytical solutions are presented in Figures 1–8. In Figure 1, we introduce the graph of (31) using the $(G'/G)$-expansion method at $\lambda_j = 0.2, \beta_j = 0.7, \gamma_j = 0.2, k = 0.4, \alpha = 0.7, \lambda = 1, \mu = 0.1, h_1 = 0.5, h_2 = 0.3$.. Also, the graph of (32) using the $(G'/G)$-expansion method at $\theta_1 = 0, \theta_1 = 0, \lambda_j = 0.2, \beta_j = 0.7, \gamma_j = 0.2, k = 0.4, \alpha = 0.7, \lambda = 0.1, \mu = 0.1, h_1 = 0.6, h_2 = 0.7$, is presented in Figure 2. Moreover, we present the graph of (41) using the new Kudryashov's method at $\lambda_j = 0.1, \beta_j = 0.1, \gamma_j = 0.1, k = 0.6, A_0 = 0.1, m = 0.01, d = 0.5, \rho = 0.4, \delta = 2, \alpha = 0.4, b = 0.1$ in Figure 3. In Figure 4, we introduce the graph of (50) using the extended tanh-function method at $\lambda_j = 0.4, \beta_j = 0.3, \gamma_j = 0.4, k = 0.2, \alpha = 0.3, \omega_1 = 0.001$. The graph of (47) using the extended tanh-function method at $\lambda_j = 0.1, \beta_j = 0.1, \gamma_j = 0.1, k = 0.5, \alpha = 0.4, \omega_1 = -0.1$ is presented in Figure 5. Also, the graph of (58) using the extended tanh-function method at $\lambda_j = 0.1$, $\beta_j = 0.1, \gamma_j = 0.1, k = 0.5, \alpha = 0.4, \omega_1 = -0.1$ is presented in Figure 6. In Figure 7, we introduce the graph of (58) using the extended tanh-function method at $\lambda_j = 0.1, \beta_j = 0.1, \gamma_j = 0.1, k = 0.5, \alpha = 0.4, \omega_1 = -0.1$. Finally, in Figure 8, we introduce the graph of (47) using the extended tanh-function method at $\lambda_j = 0.4, \beta_j = 0.3, \gamma_j = 0.4, k = 0.2, \alpha = 0.3, \omega_1 = 0.001$.

## 5. Discussion

Graphs are effective visual tools because they present information quickly. In the previous section, we depict some of the obtained solutions with several values of the parameters. The 2D and 3D profile of the solution (31) are presented in Figure 1 exhibit the transfer of the wave to the right with time progress. In Figure 2, the soliton solution (32) present bell shape soliton, and we notice the movement of the wave towards right. As displayed in Figures 3 and 4, the waves travels to the right as time increasing. As we see in Figures 5 and 7, as time proceed, the wave also proceed towards right. Finally, the wave moves to the right in Figures 6 and 8 with progress of time.

## 6. Conclusion

Soliton radiation presented by the Radhakrishnan-Kundu-Lakshmanan equation, which is known also by dispersive optical solitons. The results are fascinating, and highly influential in the field of optical fiber and useful for improving the performance capacity of transmission networks in the telecommunications industry. In this article, we have efficaciously ascertained diverse soliton solutions to the Radhakrishnan-Kundu-Lakshmanan equation by implementing three dynamic techniques, for instance, the $(G'/G)$-expansion method, the generalized Kudryashov method, and the extended tanh method. We attain scores of solutions in different shapes, including rational, hyperbolic, and trigonometric functions. The attained solutions are inclusive and for definite values of the constrains well-known optical solitons are produced. To illustrate the context, the results have also been explained with the help of suitable 2D and 3D graphs. The gained solutions may be applied to explain the model simply and straightforwardly.

## Data Availability

The data used to support the findings of this study are included within the article.

## Ethical Approval

This article does not involve with any human or animal studies. We also confirm that, we have read and abided by the statement of ethical standards for the manuscript submission to this journal and that the manuscript has not been copyrighted, published, or submitted elsewhere.


## Conflicts of Interest

The authors declare that they have no competing interests.


## Authors' Contributions

Khalid K. Ali was responsible for the conceptualization, methodology, software, validation, resources, and writing-original draft. M. S. Mehanna worked on the formal analysis, investigation, visualization, and writing-review editing. M. Ali Akbar was assigned for the supervision, project administration, and funding acquisition. Prasun Chakrabarti managed the data curation, formal analysis, and writing-review editing. All the authors with the consultation of each other completed this research and drafted the manuscript together. All the authors have read and approved the final manuscript.

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