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Research Article

A Comparative Analysis of the Fractional-Order Coupled Korteweg–De Vries Equations with the Mittag–Leffler Law

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This article applies efficient methods, namely, modified decomposition method and new iterative transformation method, to analyze a nonlinear system of Korteweg–de Vries equations with the Atangana–Baleanu fractional derivative. The nonlinear fractional coupled systems investigated in this current analysis are the system of Korteweg–de Vries and the modified system of Korteweg–de Vries equations applied as a model in nonlinear physical phenomena arising in chemistry, biology, physics, and applied sciences. Approximate analytical results are represented in the form of a series with straightforward components, and some aspects showed an appropriate dependence on the values of the fractional-order derivatives. The convergence and uniqueness analysis is carried out. To comprehend the analytical procedure of both methods, three test examples are provided for the analytical results of the time-fractional KdV equation. Additionally, the efficiency of the mentioned procedures and the reduction in calculations provide broader applicability. It is also illustrated that the findings of the current methodology are in close harmony with the exact solutions. The series result achieved applying this technique is proved to be accurate and reliable with minimal calculations. The numerical simulations for obtained solutions are discussed for different values of the fractional order.

1. Introduction

Many researchers have been working on various aspects of fractional derivatives in recent years. Caputo and Fabrizio modified the existing Caputo derivative to develop the Caputo–Fabrizio fractional derivative [1–5] based on a nonsingular kernel. Because of its advantages, numerous researchers utilized this operator to investigate various types of fractional-order partial differential equations [6–9]. To address this issue, Atangana and Baleanu proposed a new fractional operator called the Atangana–Baleanu derivative, which combines Caputo and Riemann–Liouville derivatives. Because of the existence of the Mittag–Leffler kernel, which is a generalization of the exponential kernel, this new Atangana–Baleanu derivative has a long memory. Moreover, the Atangana–Baleanu operator outperforms other operators, and different scientific models have been successfully solved. Many advances have been made in fractional calculus over the last few years by borrowing ideas from classical calculus, but it does not remain easy. Scholars have the main concern to obtain a numerical solution; for this, numerous efficient methodologies have been constructed for fractional differential equations, such as the Adomian decomposition transform method [10], variational iteration transform method [11, 12], optimal homotopy asymptotic method [13], and homotopy perturbation method [14, 15].

Korteweg and de Vries introduced the Korteweg–de Vries equation in 1895 to model shallow water waves in a canal [16]. The suggested system Korteweg–de Vries equations play a crucial role in diverse engineering and applied sciences such as plasma physics, water waves, hydrodynamics, and theory of the quantum field. The Korteweg–de Vries equations are usually investigated in the analysis of nonlinear dispersive waves [17]. They define the interactions among two long waves with various dispersion relations. Many researchers have been interested in these schemes, and a lot of works have been done. For example, Ghoreishi et al. applied the homotopy analysis method to achieve numerical results of a modified system of Korteweg–de Vries equations [18]. Kaya and Inan in [19] achieved traveling wave results of the system of Korteweg–de Vries and modified system of Korteweg–de Vries equations. The fractional-order system of Korteweg–de Vries equations is defined as follows:

$$\begin{aligned}
\frac{\partial^{\gamma}\mathbb{U}}{\partial\mathfrak{I}^{\gamma}} &= -\rho\frac{\partial^{3}\mathbb{U}}{\partial\varphi^{3}} - 6\rho\mathbb{U}\frac{\partial\mathbb{U}}{\partial\varphi} + 6\mathbb{V}\frac{\partial\mathbb{V}}{\partial\varphi},\\
\frac{\partial^{\gamma}\mathbb{V}}{\partial\mathfrak{I}^{\gamma}} &= -\rho\frac{\partial^{3}\mathbb{V}}{\partial\varphi^{3}} - 3\vartheta\mathbb{U}\frac{\partial\mathbb{V}}{\partial\varphi}, \quad \mathfrak{I} > 0, 0 < \gamma \le 1,
\end{aligned} \tag{1}$$

where γ is the fractional-order derivative of $\mathbb{U}(\varphi,\mathfrak{I})$ and $\mathbb{V}(\varphi,\mathfrak{I})$, ϑ, and ρ are constants, respectively. The functions $\mathbb{U}(\varphi,\mathfrak{I})$ and $\mathbb{V}(\varphi,\mathfrak{I})$ are considered as important functions of time and space, disappearing for $\mathfrak{I}$ and φ, respectively. The other method eliminates to the conventional coupled Korteweg–de Vries equations since $\rho = \vartheta = 1$ is implemented.

A classic model in this hierarchy is the modified coupled Korteweg–de Vries system. The following nonlinear partial differential equations govern this model [20]:

$$\begin{aligned}
\frac{\partial^{\gamma}\mathbb{U}}{\partial\mathfrak{I}^{\gamma}} &= \frac{1}{2}\frac{\partial^{3}\mathbb{U}}{\partial\mathfrak{I}^{3}} - 3\mathbb{U}^{2}\frac{\partial\mathbb{U}}{\partial\varphi} + \frac{3}{2}\mathbb{W}\frac{\partial^{2}\mathbb{V}}{\partial\varphi^{2}} + 3\frac{\partial\mathbb{V}}{\partial\varphi}\frac{\partial\mathbb{W}}{\partial\varphi}\\
&\quad + \frac{3}{2}\mathbb{V}\frac{\partial^{2}\mathbb{W}}{\partial\varphi^{2}} + 3\text{yx}\frac{\partial\mathbb{U}}{\partial\varphi} + 3\text{zx}\frac{\partial\mathbb{V}}{\partial\varphi} + 3\text{zy}\frac{\partial\mathbb{W}}{\partial\varphi},\\
\frac{\partial^{\gamma}\mathbb{V}}{\partial\mathfrak{I}^{\gamma}} &= -\frac{\partial^{3}\mathbb{V}}{\partial\varphi^{3}} - 3\frac{\partial\mathbb{U}}{\partial\varphi}\frac{\partial\mathbb{V}}{\partial\varphi} - 3\mathbb{V}\frac{\partial^{2}\mathbb{U}}{\partial\varphi^{2}} - 3\mathbb{V}^{2}\frac{\partial\mathbb{W}}{\partial\varphi}\\
&\quad + 6\text{zy}\frac{\partial\mathbb{U}}{\partial\varphi} + 3\mathbb{U}^{2}\frac{\partial\mathbb{V}}{\partial\varphi},\\
\frac{\partial^{\gamma}\mathbb{W}}{\partial\mathfrak{I}^{\gamma}} &= -\frac{\partial^{3}\mathbb{W}}{\partial\varphi^{3}} - 3\frac{\partial\mathbb{U}}{\partial\varphi}\frac{\partial\mathbb{W}}{\partial\varphi} - 3\mathbb{W}\frac{\partial^{2}\mathbb{U}}{\partial\varphi^{2}} - 3\mathbb{W}^{2}\frac{\partial\mathbb{V}}{\partial\varphi}\\
&\quad + 6\text{zx}\frac{\partial\mathbb{U}}{\partial\varphi} + 3\mathbb{U}^{2}\frac{\partial\mathbb{W}}{\partial\varphi}, \quad \mathfrak{I} > 0, 0 < \gamma \le 1.
\end{aligned} \tag{2}$$

The modified Korteweg–de Vries equation in its standard type is simplified by the modified couple Korteweg–de Vries equation (2), with $\mathbb{V} = \mathbb{W} = 0$. Korteweg–de Vries models are a source of nonevolution equations with a wide range of implementations in science and engineering. The Korteweg–de Vries models, for instance, generate ion-acoustic result in fluid mechanics [21, 22]. Long waves characterise geophysical fluid dynamics in shallow and deep oceans [23, 24]. Various studies have suggested numerous systems to overcome the fractional-order Korteweg–de Vries equation employing various methodologies, such as the differential transform method [25], Adomian decomposition method [26], natural decomposition method [27], homotopy analysis method [28], Elzaki projected differential transform method [29], variational iteration method [30], new iterative method [31], modified tanh technique [32], and Lie symmetry analysis [33]. Analogously, same solutions for (2) have been suggested by Inc and Cavlak [34], Fan [35], Lin et al. [36], Inc et al. [37], and Ghoreishi et al. [18].

Daftardar-Gejji and Jafari [38] proposed an innovative iterative method of solving functional equations with approximation solutions. The new iterative approach is constructed on the justification of disappearing the nonlinear functions is identified as the iterative transformation technique [39]. This procedure is quick and accurate, and it avoids the utilization of complicated integrals, unconditioned matrix, and infinite series forms. This technique does not require any expressive parameters for the model. Numerous researchers have analyzed new iterative transformation methods to solve partial differential equations, such as the Fornberg–Whitham equation [40], KdV equation [31], and Klein–Gordon equation [41].

The Adomian decomposition method was firstly introduced by Adomian in 1980 and implemented by several investigators. In recent decades, numerous researchers have investigated the solutions of integral and differential equations by different techniques with the mixed Laplace transform. The Adomian decomposition method was modified with many integral transformations, such as Laplace, ρ-Laplace, Elzaki, Aboodh, and Mohand. Modification of Laplace Adomian decomposition method for solving nonlinear Volterra integral and integro-differential equations based on Newton Raphson formula [42] for solving nonlinear integrodifferential and Volterra integral equations based on the Newton–Raphson method, discrete Adomian decomposition technique [43] applied for investigating the fractional-order Navier–Stokes model, Laplace–Adomian decomposition method [44] study of implicit-impulsive differential equations involving Caputo-Fabrizio fractional derivative.

2. Basic Definitions

Definition 1. The fractional-order Caputo derivative is defined by

$$\begin{aligned}
{}^{\text{LC}}D_{\mathfrak{I}}^{\gamma}\{f(\mathfrak{I})\} &= \frac{1}{(n-\gamma)}\int_{0}^{\mathfrak{I}}(\mathfrak{I}-k)^{n-\gamma-1}f^{n}(k)\text{d}k,\\
&\text{where } n < \gamma \le n+1.
\end{aligned} \tag{3}$$

Definition 2. The Laplace transformation connected with fractional Caputo derivative ${}^{\text{LC}}D_{\mathfrak{I}}^{\gamma}\{f(\mathfrak{I})\}$ is expressed by

$$\mathbb{L}\left\{{}^{\mathrm{LC}}D_{\mathfrak{J}}^{\gamma}\{f(\mathfrak{J})\}\right\}(s) = \frac{1}{s^{n-\gamma}}\left[s^{n}\mathbb{L}\{f(x,\mathfrak{J})\}(s) - s^{n-1}f(x,0) - \cdots - f^{n-1}(x,0)\right]. \tag{4}$$

Definition 3. In the Caputo sense, the Atangana–Baleanu derivative is defined as

$${}^{ABC}D_{\mathfrak{J}}^{\gamma}\{f(\mathfrak{J})\} = \frac{A(\gamma)}{1-\gamma}\int_{a}^{\mathfrak{J}} f'(k)E_{\gamma}\left[-\frac{\gamma}{1-\gamma}(1-k)^{\gamma}\right]\mathrm{d}k, \tag{5}$$

where $A(\gamma)$ is a normalization function such that $A(0) = A(1) = 1, f \in H^{1}(a,b), b > a, \gamma \in [0,1]$, and E_{γ} represents the Mittag–Leffler function.

Definition 4. The Atangana–Baleanu derivative in the Riemann–Liouville sense is defined as

$${}^{ABC}D_{\mathfrak{J}}^{\gamma}\{f(\mathfrak{J})\} = \frac{A(\gamma)}{1-\gamma}\frac{\mathrm{d}}{\mathrm{d}\mathfrak{J}}\int_{a}^{\mathfrak{J}} f(k)E_{\gamma}\left[-\frac{\gamma}{1-\gamma}(1-k)^{\gamma}\right]\mathrm{d}k. \tag{6}$$

Definition 5. The Laplace transform connected with the Atangana–Baleanu operator is defined as

$${}^{AB}D_{\mathfrak{J}}^{\gamma}\{f(\mathfrak{J})\}(s) = \frac{A(\gamma)s^{\gamma}\mathbb{L}\{f(\mathfrak{J})\}(s) - s^{\gamma-1}f(0)}{(1-\gamma)\left(s^{\gamma} + (\gamma/(1-\gamma))\right)}. \tag{7}$$

Definition 6. Consider $0 < \gamma < 1$, and f is a function of γ; then, the fractional-order integral operator of γ is given as

$${}^{ABC}I_{\mathfrak{J}}^{\gamma}\{f(\mathfrak{J})\} = \frac{1-\gamma}{A(\gamma)}f(\mathfrak{J}) + \frac{\gamma}{A(\gamma)\Gamma(\gamma)}\int_{a}^{\mathfrak{J}} f(k)(\mathfrak{J}-k)^{\gamma-1}\mathrm{d}k. \tag{8}$$

3. The General Implementation of the Modified Decomposition Method

Suppose the nonlinear fractional partial differential equations

$$\mathscr{D}_{\mathfrak{J}}^{\gamma}\mathbb{U}(\varphi,\mathfrak{J}) + \mathscr{L}\mathbb{U}(\varphi,\mathfrak{J}) + \mathscr{N}\mathbb{U}(\varphi,\mathfrak{J}) = \mathscr{H}(\varphi,\mathfrak{J}), \quad \mathfrak{J} > 0, 0 < \gamma \leq 1, \tag{9}$$

with the condition

$$\mathbb{U}(\varphi,0) = \mathscr{G}(\varphi), \tag{10}$$

where $\mathscr{D}_{\mathfrak{J}}^{\gamma} = (\partial^{\gamma}\mathbb{U}(\varphi,\mathfrak{J})/\partial\mathfrak{J}^{\gamma})$ show the fractional-order Caputo derivative operator with $0 < \gamma \leq 1$, while $\mathscr{L}$ is linear, $\mathscr{N}$ are nonlinear functions, and $\mathscr{H}(\varphi,\mathfrak{J})$ defines the source term.

Applying the Laplace transformation to (9), we get

$$\mathbb{L}\left[\mathscr{D}_{\mathfrak{J}}^{\gamma}\mathbb{U}(\varphi,\mathfrak{J}) + \mathscr{L}\mathbb{U}(\varphi,\mathfrak{J}) + \mathscr{N}\mathbb{U}(\varphi,\mathfrak{J})\right] = \mathbb{L}[\mathscr{H}(\varphi,\mathfrak{J})]. \tag{11}$$

Taking the Laplace transformation differentiation, we find

$$\frac{\nu^{\gamma}}{(\nu^{\gamma}(1-\gamma)+\gamma)}\mathscr{U}(\nu,\omega) = \sum_{\kappa=0}^{j-1}\left(\frac{1}{\nu}\right)^{\gamma-\kappa-1}\mathbb{U}^{(\kappa)}(0) + \mathbb{L}[\mathscr{L}\mathbb{U}(\varphi,\mathfrak{J}) + \mathscr{N}\mathbb{U}(\varphi,\mathfrak{J})] + \mathbb{L}[\mathscr{H}(\varphi,\mathfrak{J})]. \tag{12}$$

The inverse Laplace transformation of (12) gives

$$\mathbb{U}(\varphi,\mathfrak{J}) = \mathbb{L}^{-1}\left[\sum_{\kappa=0}^{j-1}\left(\frac{1}{\nu}\right)^{\gamma-\kappa-1}\mathbb{U}^{(\kappa)}(0) + \frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}[\mathscr{H}(\varphi,\mathfrak{J})]\right] - \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}[\mathscr{L}\mathbb{U}(\varphi,\mathfrak{J}) + \mathscr{N}\mathbb{U}(\varphi,\mathfrak{J})]\right]. \tag{13}$$

The adomain decomposition method series form solution is defined as

$$\mathbb{U}(\varphi,\mathfrak{J}) = \sum_{j=0}^{\infty}\mathbb{U}_{j}(\varphi,\mathfrak{J}). \tag{14}$$

Thus, the nonlinear function $\mathscr{N}(\varphi,\mathfrak{J})$ can be calculated by the Adomian polynomials defined as

$$\mathscr{N}\mathbb{U}(\varphi,\mathfrak{J}) = \sum_{j=0}^{\infty}\widetilde{A}_{j}(\mathbb{U}_{0},\mathbb{U}_{1},\ldots), \quad j = 0,1,\ldots, \tag{15}$$

where

$$\widetilde{A}_{j}(\mathbb{U}_{0},\mathbb{U}_{1},\ldots) = \frac{1}{j!}\left[\frac{\mathrm{d}^{j}}{\mathrm{d}\lambda^{j}}\mathscr{N}\left(\sum_{J=0}^{\infty}\lambda^{J}\mathbb{U}_{J}\right)\right]_{\lambda=0}, \quad j > 0. \tag{16}$$

Putting (14) and (15) into (13), we have

$$\sum_{j=0}^{\infty}\mathbb{U}_{j}(\varphi,\mathfrak{J}) = \mathscr{G}(\varphi) + \widetilde{\mathscr{G}}(\varphi) - \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[\mathscr{L}\mathbb{U}(\varphi,\mathfrak{J}) + \sum_{j=0}^{\infty}\widetilde{A}_{j}\right]\right]. \tag{17}$$

Lastly, the iterative methodology for (17) is achieved as

$$\mathbb{U}_0(\varphi,\mathfrak{I}) = \mathscr{G}(\varphi) + \widetilde{\mathscr{G}}(\varphi), \quad j = 0,$$
$$\mathbb{U}_{j+1}(\varphi,\mathfrak{I}) = -\mathbb{L}^{-1}\left[\frac{(\nu^\gamma(1-\gamma)+\gamma)}{\nu^\gamma}\mathbb{L}\left[\mathscr{L}\mathbb{U}(\varphi,\mathfrak{I}) + \sum_{j=0}^{\infty}\widetilde{A}_j\right]\right], \quad j \geq 1. \tag{18}$$

4. The General Discussion of the New Iterative Transformation Method

Let us assume the following general fractional partial differential equation

$$\mathscr{D}_{\mathfrak{I}}^{\gamma}\mathbb{U}(\varphi,\mathfrak{I}) + \mathscr{L}\mathbb{U}(\varphi,\mathfrak{I}) + \mathscr{N}\mathbb{U}(\varphi,\mathfrak{I}) = \mathscr{H}(\varphi,\mathfrak{I}),$$
$$\mathfrak{I} > 0, j-1 < \gamma \leq j, j \in \mathbb{N}, \tag{19}$$

with the condition

$$\mathbb{U}^{(\kappa)}(\varphi,0) = \mathscr{G}_\kappa(\varphi), \quad \kappa = 0,1,2,\ldots,j-1, \tag{20}$$

where $\mathscr{L}$ and $\mathscr{N}$ are linear and nonlinear terms and $\mathscr{H}(\varphi,\mathfrak{I})$ shows the source term.

Using the Laplace transformation to (19), we get

$$\mathbb{L}\left[\mathscr{D}_{\mathfrak{I}}^{\gamma}\mathbb{U}(\varphi,\mathfrak{I}) + \mathscr{L}\mathbb{U}(\varphi,\mathfrak{I}) + \mathscr{N}\mathbb{U}(\varphi,\mathfrak{I})\right] = \mathbb{L}[\mathscr{H}(\varphi,\mathfrak{I})]. \tag{21}$$

Taking the Laplace transformation differentiation property, we get

$$\frac{\nu^\gamma}{(\nu^\gamma(1-\gamma)+\gamma)}\mathscr{U}(\nu,\omega) = \sum_{\kappa=0}^{j-1}\left(\frac{1}{\nu}\right)^{\gamma-\kappa-1}\mathbb{U}^{(\kappa)}(0)$$
$$+\mathbb{L}[\mathscr{L}\mathbb{U}(\varphi,\mathfrak{I}) + \mathscr{N}\mathbb{U}(\varphi,\mathfrak{I})]$$
$$+\mathbb{L}[\mathscr{H}(\varphi,\mathfrak{I})]. \tag{22}$$

The inverse Laplace transformation of (22) gives

$$\mathbb{U}(\varphi,\mathfrak{I}) = \mathbb{L}^{-1}\left[\sum_{\kappa=0}^{j-1}\left(\frac{1}{\nu}\right)^{\gamma-\kappa-1}\mathbb{U}^{(\kappa)}(0) + \frac{(\nu^\gamma(1-\gamma)+\gamma)}{\nu^\gamma}\mathbb{L}[\mathscr{H}(\varphi,\mathfrak{I})]\right]$$
$$-\mathbb{L}^{-1}\left[\frac{(\nu^\gamma(1-\gamma)+\gamma)}{\nu^\gamma}\mathbb{L}[\mathscr{L}\mathbb{U}(\varphi,\mathfrak{I}) + \mathscr{N}\mathbb{U}(\varphi,\mathfrak{I})]\right]. \tag{23}$$

From the iterative connection, we achieve

$$\mathbb{U}(\varphi,\mathfrak{I}) = \sum_{j=0}^{\infty}\mathbb{U}_j(\varphi,\mathfrak{I}). \tag{24}$$

Also, the linear operator is $\mathscr{L}$; therefore,

$$\mathscr{L}\left(\sum_{j=0}^{\infty}\mathbb{U}_j(\varphi,\mathfrak{I})\right) = \sum_{j=0}^{\infty}\mathscr{L}\left[\mathbb{U}_j(\varphi,\mathfrak{I})\right], \tag{25}$$

and $\mathscr{N}$ defines the nonlinear term as in [38].

$$\mathscr{N}\left(\sum_{j=0}^{\infty}\mathbb{U}_j(\varphi,\mathfrak{I})\right) = \mathscr{N}(\mathbb{U}_0(\varphi,\mathfrak{I}))$$
$$+\sum_{j=0}^{\infty}\left[\mathscr{N}\left(\sum_{k=0}^{\infty}\mathbb{U}_\kappa(\varphi,\mathfrak{I})\right) - \mathscr{N}\left(\sum_{k=1}^{\infty}\mathbb{U}_\kappa(\varphi,\mathfrak{I})\right)\right]$$
$$= \mathscr{N}(\mathbb{U}_0) + \sum_{k=1}^{\infty}D_j, \tag{26}$$

where $D_j = \mathscr{N}(\sum_{\kappa=0}^{j}\mathbb{U}_\kappa) - \mathscr{N}(\sum_{\kappa=0}^{j-1}\mathbb{U}_\kappa)$.

By putting (24), (25), and (??) into (23), we obtain

$$\sum_{j=0}^{\infty}\mathbb{U}_j(\varphi,\mathfrak{I})\mathbb{L}^{-1}\left[\sum_{\kappa=0}^{j-1}\left(\frac{1}{\nu}\right)^{\gamma-\kappa-1}\mathbb{U}^{(\kappa)}(0) + \frac{(\nu^\gamma(1-\gamma)+\gamma)}{\nu^\gamma}\mathbb{L}[\mathscr{H}(\varphi,\mathfrak{I})]\right]$$
$$-\mathbb{L}^{-1}\left\{\frac{(\nu^\gamma(1-\gamma)+\gamma)}{\nu^\gamma}\mathbb{L}\left[\mathscr{L}\left(\sum_{\kappa=0}^{\infty}\mathbb{U}_\kappa(\varphi,\mathfrak{I})\right) + \mathscr{N}(\mathbb{U}_0) + \sum_{\kappa=1}^{j}D_j\right]\right\}. \tag{27}$$

As a result, we determine the next iteration

$$
\begin{aligned}
\mathbb{U}_0(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\sum_{\kappa=0}^{j-1}\left(\frac{1}{\nu}\right)^{\gamma-\kappa-1}\mathbb{U}^{(\kappa)}(0)+\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}[\mathscr{H}(\varphi,\mathfrak{T})]\right],\\
\mathbb{U}_1(\varphi,\mathfrak{T}) &= -\mathbb{L}^{-1}\left\{\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}[\mathscr{L}(\mathbb{U}_0(\varphi,\mathfrak{T}))+\mathscr{N}(\mathbb{U}_0(\varphi,\mathfrak{T}))]\right\},\\
&\vdots\\
\mathbb{U}_{j+1}(\varphi,\mathfrak{T}) &= -\mathbb{L}^{-1}\left\{\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[\mathscr{L}\left(\mathbb{U}_j(\varphi,\mathfrak{T})\right)+D_j\right]\right\},\quad m\geq 1.
\end{aligned}
\tag{28}
$$

Finally, (19) and (20) yield the j-term result in the series form, defined as

$$
\begin{aligned}
\mathbb{U}(\varphi,\mathfrak{T}) &\approx \mathbb{U}_0(\varphi,\mathfrak{T})+\mathbb{U}_1(\varphi,\mathfrak{T})+\mathbb{U}_2(\varphi,\mathfrak{T})\\
&\quad+\cdots+\mathbb{U}_j(\varphi,\mathfrak{T}),\quad j\in\mathbb{N}.
\end{aligned}
\tag{29}
$$

5. Uniqueness and Existence Solutions for the Modified Decomposition Method

Theorem 1 (uniqueness theorem). *The unique result of equation (9) provide space whenever* $0<\varepsilon<1$*, where* $\varepsilon=(\breve{L}_1+\breve{L}_2+\breve{L}_3)((1-\gamma)+(\gamma\mathfrak{T}^{\gamma}/\Gamma(\gamma+1)))$.

Proof. Assume that $J=(\mathscr{C}[I],\|.\|)$ represents all continuous mappings on the Banach space, defined on $I=[0,\mathbb{T}]$ having the norm $\|.\|$. For this, we introduce a mapping $W\colon M\mapsto M$, and we have

$$
\begin{aligned}
\mathbb{U}_{n+1}(\varphi,\mathfrak{T}) &= \mathbb{U}(\varphi,\mathfrak{T})+\mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}[\mathscr{L}[\mathbb{U}_n(\varphi,\mathfrak{T})]\right.\\
&\quad\left.+\mathscr{R}[\mathbb{U}_n(\varphi,\mathfrak{T})]+\mathscr{N}[\mathbb{U}_n(\varphi,\mathfrak{T})]]\right],\quad n\geq 0,
\end{aligned}
\tag{30}
$$

when $\mathscr{L}[\mathbb{U}(\varphi,\mathfrak{T})]\equiv(\partial^3\mathbb{U}(\varphi,\mathfrak{T})/\partial\varphi^2)$ and $\mathscr{R}[\mathbb{U}(\varphi,\mathfrak{T})]\equiv(\partial\mathbb{U}(\varphi,\mathfrak{T})/\partial\varphi)$. Suppose that $\mathscr{L}[\mathbb{U}(\varphi,\mathfrak{T})]$ and $\mathscr{M}[\mathbb{U}(\varphi,\mathfrak{T})]$ are also Lipschitzian with $|\mathscr{R}z-\mathscr{R}\breve{\mathbb{U}}|<\breve{L}_1|\mathbb{U}-\breve{\mathbb{U}}|$ and $|\mathscr{L}\mathbb{U}-\mathscr{L}\breve{\mathbb{U}}|<\breve{L}_2|\mathbb{U}-\breve{\mathbb{U}}|$ where $\breve{L}_1$ and $\breve{L}_2$ are Lipschitz constants, respectively, and $\mathbb{U},\breve{\mathbb{U}}$ are various values of the mapping.

$$
\begin{aligned}
\|W\mathbb{U}-W\breve{\mathbb{U}}\| &= \max_{\mathfrak{T}\in I}\left|\begin{array}{l}\mathbb{L}^{-1}\left[\dfrac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}[\mathscr{L}[\mathbb{U}(\varphi,\mathfrak{T})]+\mathscr{R}[\mathbb{U}(\varphi,\mathfrak{T})]+\mathscr{N}[\mathbb{U}(\varphi,\mathfrak{T})]]\right]\\ -\mathbb{L}^{-1}\left[\dfrac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}[\mathscr{L}[\breve{\mathbb{U}}(\varphi,\mathfrak{T})]+\mathscr{R}[\breve{\mathbb{U}}(\varphi,\mathfrak{T})]+\mathscr{N}[\breve{\mathbb{U}}(\varphi,\mathfrak{T})]]\right]\end{array}\right|\\
&\leq \max_{\mathfrak{T}\in I}\left|\begin{array}{l}\mathbb{L}^{-1}\left[\dfrac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}[\mathscr{L}[\mathbb{U}(\varphi,\mathfrak{T})]-\mathscr{L}[\breve{\mathbb{U}}(\varphi,\mathfrak{T})]]\right]\\ +\mathbb{L}^{-1}\left[\dfrac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}[\mathscr{R}[\mathbb{U}(\varphi,\mathfrak{T})]-\mathscr{R}[\breve{\mathbb{U}}(\varphi,\mathfrak{T})]]\right]\\ +\mathbb{L}^{-1}\left[\dfrac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}[\mathscr{N}[\mathbb{U}(\varphi,\mathfrak{T})]-\mathscr{N}[\breve{\mathbb{U}}(\varphi,\mathfrak{T})]]\right]\end{array}\right|
\end{aligned}
$$

$$
\begin{aligned}
&\leq \max_{\mathfrak{I}\in I}\left[\begin{array}{l}\breve{L}_1\mathbb{L}^{-1}\left[\dfrac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}|\mathbb{U}(\varphi,\mathfrak{I})-\breve{\mathbb{U}}(\varphi,\mathfrak{I})|\right]\\ +\breve{L}_2\mathbb{L}^{-1}\left[\dfrac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}|\mathbb{U}(\varphi,\mathfrak{I})-\breve{\mathbb{U}}(\varphi,\mathfrak{I})|\right]\\ +\breve{L}_3\mathbb{L}^{-1}\left[\dfrac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}|\mathbb{U}(\varphi,\mathfrak{I})-\breve{\mathbb{U}}(\varphi,\mathfrak{I})|\right]\end{array}\right]\\
&\leq \max_{\mathfrak{I}\in I}\left(\breve{L}_1+\breve{L}_2+\breve{L}_3\right)\mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}|\mathbb{U}(\varphi,\mathfrak{I})-\breve{\mathbb{U}}(\varphi,\mathfrak{I})|\right]\\
&\leq \left(\breve{L}_1+\breve{L}_2+\breve{L}_3\right)\mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\|\mathbb{U}(\varphi,\mathfrak{I})-\breve{\mathbb{U}}(\varphi,\mathfrak{I})\|\right]\\
&= \left(\breve{L}_1+\breve{L}_2+\breve{L}_3\right)\left((1-\gamma)+\frac{\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right)\|\mathbb{U}(\varphi,\mathfrak{I})-\breve{\mathbb{U}}(\varphi,\mathfrak{I})\|.
\end{aligned}
\tag{31}
$$

The mapping is a contraction under the assumption $0<\varepsilon<1$. As a result of the Banach contraction fixed point theorem, there is a unique solution to (9). As a result, the proof is complete. □

Theorem 2 (convergence analysis). *The solution general form of (9) will be convergent.*

Proof. Suppose $\widehat{S}_n$ is the nth partial sum; that is, $\widehat{W}_n=\sum_{j=0}^{n}\mathbb{U}_j(\varphi,\mathfrak{I})$. Firstly, we define that $\{\widehat{W}_n\}$ is a Banach space Cauchy sequence in M. Using into consideration of Adomian polynomials, we achieve

$$
\begin{aligned}
\overline{R}\left(\widehat{W}_n\right) &= \breve{H}_n+\sum_{p=0}^{n-1}\breve{H}_p,\\
\overline{N}\left(\widehat{W}_n\right) &= \breve{H}_n+\sum_{c=0}^{n-1}\breve{H}_c.
\end{aligned}
\tag{32}
$$

Now,

$$
\left\|\widehat{W}_n-\widehat{W}_q\right\| = \max_{\mathfrak{I}\in I}\left|\widehat{W}_n-\widehat{W}_q\right|
$$

$$
= \max_{\mathfrak{I}\in I}\left|\sum_{j=q+1}^{n}\breve{\mathbb{U}}(\varphi,\mathfrak{I})\right|,\ (j=1,2,3,\ldots)
$$

$$
\leq \max_{\mathfrak{I}\in I}\left|\begin{array}{l}\mathbb{L}^{-1}\left[\dfrac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[\displaystyle\sum_{j=q+1}^{n}\mathscr{L}[\mathbb{U}_{n-1}(\varphi,\mathfrak{I})]\right]\right]\\ +\mathbb{L}^{-1}\left[\dfrac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[\displaystyle\sum_{j=q+1}^{n}\mathscr{R}[\mathbb{U}_{n-1}(\varphi,\mathfrak{I})]\right]\right]\\ +\mathbb{L}^{-1}\left[\dfrac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[\displaystyle\sum_{j=q+1}^{n}\breve{H}_{n-1}(\varphi,\mathfrak{I})\right]\right]\end{array}\right|
$$

$$
= \max_{\mathfrak{I}\in I}\left|\begin{array}{l}\mathbb{L}^{-1}\left[\dfrac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[\displaystyle\sum_{j=q}^{n-1}\mathscr{L}[\mathbb{U}_{n}(\varphi,\mathfrak{I})]\right]\right]\\ +\mathbb{L}^{-1}\left[\dfrac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[\displaystyle\sum_{j=q}^{n-1}\mathscr{R}[\mathbb{U}_{n}(\varphi,\mathfrak{I})]\right]\right]\\ +\mathbb{L}^{-1}\left[\dfrac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[\displaystyle\sum_{j=q}^{n-1}\breve{H}_{n}(\varphi,\mathfrak{I})\right]\right]\end{array}\right|
$$

$$
\leq \max_{\mathfrak{I}\in I}\left|\begin{array}{l}\mathbb{L}^{-1}\left[\dfrac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[\displaystyle\sum_{j=q}^{n-1}\mathscr{L}\left(\widehat{W}_{n-1}\right)-\mathscr{L}\left(\widehat{W}_{q-1}\right)\right]\right]\\ +\mathbb{L}^{-1}\left[\dfrac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[\displaystyle\sum_{j=q}^{n-1}\mathscr{R}\left(\widehat{W}_{n-1}\right)-\mathscr{R}\left(\widehat{W}_{q-1}\right)\right]\right]\\ +\mathbb{L}^{-1}\left[\dfrac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[\displaystyle\sum_{j=q}^{n-1}\mathscr{N}\left(\widehat{W}_{n-1}\right)-\mathscr{N}\left(\widehat{W}_{q-1}\right)\right]\right]\end{array}\right|
$$

$$
\leq \max_{\mathfrak{I}\in I}\left|\begin{array}{l}\mathbb{L}^{-1}\left[\dfrac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[\mathscr{L}\left(\widehat{W}_{n-1}\right)-\mathscr{L}\left(\widehat{W}_{q-1}\right)\right]\right]\\ +\mathbb{L}^{-1}\left[\dfrac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[\mathscr{R}\left(\widehat{W}_{n-1}\right)-\mathscr{R}\left(\widehat{W}_{q-1}\right)\right]\right]\\ +\mathbb{L}^{-1}\left[\dfrac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[\mathscr{N}\left(\widehat{W}_{n-1}\right)-\mathscr{N}\left(\widehat{W}_{q-1}\right)\right]\right]\end{array}\right|
$$

$$
\leq \breve{L}_1\max_{\mathfrak{I}\in I}\mathbb{L}^{-1}\left|\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[\left(\widehat{W}_{n-1}\right)-\left(\widehat{W}_{q-1}\right)\right]\right]\right|
$$

$$
+\breve{L}_2\max_{\mathfrak{I}\in I}\left|\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[\left(\widehat{W}_{n-1}\right)-\left(\widehat{W}_{q-1}\right)\right]\right]\right|
$$

$$+\breve{L}_3 \max_{\mathfrak{I}\in I}\left\|\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[\left(\widehat{W}_{n-1}\right)-\left(\widehat{W}_{q-1}\right)\right]\right]\right\|$$
$$=\left(\breve{L}_1+\breve{L}_2+\breve{L}_3\right)\left((1-\gamma)+\frac{\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right)\left\|\widehat{W}_{n-1}-\widehat{W}_{q-1}\right\|. \tag{33}$$

Consider $n = q + 1$; then,

$$\left\|\widehat{W}_{q+1}-\widehat{W}_q\right\| \leq \varepsilon\left\|\widehat{W}_q-\widehat{W}_{q-1}\right\| \leq \varepsilon^2\left\|\widehat{W}_{q-1}-\widehat{W}_{q-2}\right\| \leq \cdots \leq \varepsilon^q\left\|\widehat{W}_1-\widehat{W}_0\right\|, \tag{34}$$

where $((\breve{L}_1+\breve{L}_2+\breve{L}_3)\mathfrak{I}^{(\gamma-1)}/\gamma!)$. Similarly, we have the triangular inequality

$$\begin{aligned}\left\|\widehat{W}_n-\widehat{W}_q\right\| &\leq \left\|\widehat{W}_{q+1}-\widehat{W}_q\right\|+\left\|\widehat{W}_{q+2}-\widehat{W}_{q+1}\right\| \\ &\quad+\cdots+\left\|\widehat{W}_n-\widehat{W}_{n-1}\right\| \\ &\leq \left[\varepsilon^q+\varepsilon^{q+1}+\cdots+\varepsilon^{n-1}\right]\left\|\widehat{W}_1-\widehat{W}_0\right\| \\ &\leq \varepsilon^q\left(\frac{1-\varepsilon^{n-q}}{\varepsilon}\right)\left\|\mathbb{U}_1\right\|,\end{aligned} \tag{35}$$

and since $0<\varepsilon<1$, we get $(1-\varepsilon^{n-q})<1$; then,

$$\left\|\widehat{W}_n-\widehat{W}_q\right\| \leq \frac{\varepsilon^q}{1-\varepsilon}\max_{\mathfrak{I}\in I}\left\|\mathbb{U}_1\right\|. \tag{36}$$

However, $|\mathbb{U}_1|<\infty$ (since $\mathbb{U}(\varphi,\mathfrak{I})$ is bounded). Thus, as $q\mapsto\infty$, $\|\widehat{W}_n-\widehat{W}_q\|\mapsto 0$. Hence, $\{\widehat{W}_1\}$ is a Cauchy sequence in K. As a solution, the series $\sum_{n=0}^{\infty}\mathbb{U}_n$ converges, and this completes the proof. □

Theorem 3 (error estimate). *The maximum absolute truncation error of series solution (9) to (??) is computed as*

$$\max_{\mathfrak{I}\in I}\left|\mathbb{U}(\varphi,\mathfrak{I})\sum_{n=1}^{q}\mathbb{U}_n(\varphi,\mathfrak{I})\right| \leq \frac{\varepsilon^q}{1-\varepsilon}\max_{\mathfrak{I}\in I}\left\|\mathbb{U}_1\right\|. \tag{37}$$

6. Numerical Results

This section describes several test examples by applying two novel techniques, modified decomposition technique and new iterative transformation technique, via the Atangana–Baleanu derivative operator. Also, the stability and convergence of the technique are discussed.

Example 1 (see [31]). Consider the fractional-order nonlinear system of Korteweg–de Vries equation (1) with $\vartheta = \rho = 1$, with the initial conditions

$$\begin{aligned}\mathbb{U}(\varphi,0) &= \varrho^2\sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right), \\ \mathbb{V}(\varphi,0) &= \sqrt{\frac{\rho}{2}}\varrho^2\sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right).\end{aligned} \tag{38}$$

Case I: first, we apply the modified decomposition technique for Example 1.

Applying the Laplace transform to (1), we get

$$\begin{aligned}&\frac{\nu^{\gamma}}{(\nu^{\gamma}(1-\gamma)+\gamma)}\left\{\mathcal{U}(\varphi,\nu)-\sum_{\kappa=0}^{j-1}\left(\frac{1}{\nu}\right)^{\gamma-\kappa-1}\mathbb{U}^{(\kappa)}(0)\right\} \\ &\quad=\mathbb{L}\left[-\rho\frac{\partial^3\mathbb{U}}{\partial\varphi^3}-6\rho\mathbb{U}\frac{\partial\mathbb{U}}{\partial\varphi}+6\mathbb{V}\frac{\partial\mathbb{V}}{\partial\varphi}\right], \\ &\frac{\nu^{\gamma}}{(\nu^{\gamma}(1-\gamma)+\gamma)}\left\{\mathcal{V}(\varphi,\nu)-\sum_{\kappa=0}^{j-1}\left(\frac{1}{\nu}\right)^{\gamma-\kappa-1}\mathbb{V}^{(\kappa)}(0)\right\} \\ &\quad=\mathbb{L}\left[-\rho\frac{\partial^3\mathbb{V}}{\partial\varphi^3}-3\rho\mathbb{U}\frac{\partial\mathbb{V}}{\partial\varphi}\right].\end{aligned} \tag{39}$$

In view of (38) and analytical method procedure as follows:

$$\begin{aligned}\mathcal{U}(\varphi,\nu) &= \frac{1}{\nu}\mathbb{U}^{(0)}(\varphi,0)+\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L} \\ &\quad\cdot\left[-\rho\frac{\partial^3\mathbb{U}}{\partial\varphi^3}-6\rho\mathbb{U}\frac{\partial\mathbb{U}}{\partial\varphi}+6\mathbb{V}\frac{\partial\mathbb{V}}{\partial\varphi}\right], \\ \mathcal{V}(\varphi,\nu) &= \frac{1}{\nu}\mathbb{V}^{(0)}(\varphi,0)+\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L} \\ &\quad\cdot\left[-\rho\frac{\partial^3\mathbb{V}}{\partial\varphi^3}-3\rho\mathbb{U}\frac{\partial\mathbb{V}}{\partial\varphi}\right].\end{aligned} \tag{40}$$

Using the inverse Laplace transformation, we get

$$\begin{aligned}\mathbb{U}(\varphi,\mathfrak{I}) &= \mathbb{L}^{-1}\left[\frac{1}{\nu}\mathbb{U}(\varphi,0)\right]+\mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\right. \\ &\quad\left.\cdot\left[-\rho\frac{\partial^3\mathbb{U}}{\partial\varphi^3}-6\rho\mathbb{U}\frac{\partial\mathbb{U}}{\partial\varphi}+6\mathbb{V}\frac{\partial\mathbb{V}}{\partial\varphi}\right]\right], \\ \mathbb{V}(\varphi,\mathfrak{I}) &= \mathbb{L}^{-1}\left[\frac{1}{\nu}\mathbb{V}(\varphi,0)\right]+\mathbb{L}^{-1} \\ &\quad\cdot\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\rho\frac{\partial^3\mathbb{V}}{\partial\varphi^3}-3\rho\mathbb{U}\frac{\partial\mathbb{V}}{\partial\varphi}\right]\right].\end{aligned} \tag{41}$$

By morality of the modified decomposition technique, we get

$$\begin{aligned}
\mathbb{U}_0(\varphi,\mathfrak{I}) &= \mathbb{L}^{-1}\left[\frac{\omega}{\nu}\mathbb{U}(\varphi,0)\right] \\
&= \mathbb{L}^{-1}\left[\frac{1}{\nu}\varrho^2 \sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\right] \\
&= \varrho^2 \sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right), \\
\mathbb{V}_0(\varphi,\mathfrak{I}) &= \mathbb{L}^{-1}\left[\frac{\omega}{\nu}\mathbb{V}(\varphi,0)\right] \\
&= \sqrt{\frac{\rho}{2}}\varrho^2 \sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right), \\
\sum_{j=0}^{\infty}\mathbb{U}_{j+1}(\varphi,\mathfrak{I}) &= \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\right. \\
&\quad \left.\cdot\left[-\rho\sum_{j=0}^{\infty}\left(\mathbb{U}_{\varphi\varphi\varphi}\right)_j - 6\rho\sum_{j=0}^{\infty}\mathscr{A}_j + 6\sum_{j=0}^{\infty}\mathscr{B}_j\right]\right], \\
\sum_{j=0}^{\infty}\mathbb{V}_{j+1}(\varphi,\mathfrak{I}) &= \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\right. \\
&\quad \left.\cdot\left[-\rho\sum_{j=0}^{\infty}\left(\mathbb{V}_{\varphi\varphi\varphi}\right)_j - 3\rho\sum_{j=0}^{\infty}\mathscr{C}_j\right]\right], \\
&\quad j = 0,1,2,\ldots.
\end{aligned} \tag{42}$$

The Adomian polynomials' some terms are defined as follows:

$$\begin{aligned}
\mathscr{A}_0\left(\mathbb{U}\mathbb{U}_{\varphi}\right) &= \mathbb{U}_0\mathbb{U}_{0\varphi}, \\
\mathscr{A}_1\left(\mathbb{U}\mathbb{U}_{\varphi}\right) &= \mathbb{U}_0\mathbb{U}_{1\varphi} + \mathbb{U}_1\mathbb{U}_{0\varphi}, \\
\mathscr{A}_2\left(\mathbb{U}\mathbb{U}_{\varphi}\right) &= \mathbb{U}_1\mathbb{U}_{2\varphi} + \mathbb{U}_1\mathbb{U}_{1\varphi} + \mathbb{U}_2\mathbb{U}_{0\varphi}, \\
\mathscr{B}_0\left(\mathbb{V}\mathbb{V}_{\varphi}\right) &= \mathbb{V}_0\mathbb{V}_{0\varphi}, \\
\mathscr{B}_1\left(\mathbb{V}\mathbb{V}_{\varphi}\right) &= \mathbb{V}_0\mathbb{V}_{1\varphi} + \mathbb{V}_1\mathbb{V}_{0\varphi}, \\
\mathscr{B}_2\left(\mathbb{V}\mathbb{V}_{\varphi}\right) &= \mathbb{V}_1\mathbb{V}_{2\varphi} + \mathbb{V}_1\mathbb{V}_{1\varphi} + \mathbb{V}_2\mathbb{V}_{0\varphi}, \\
\mathscr{C}_0\left(\mathbb{U}\mathbb{V}_{\varphi}\right) &= \mathbb{U}_0\mathbb{V}_{0\varphi}, \\
\mathscr{C}_1\left(\mathbb{U}\mathbb{V}_{\varphi}\right) &= \mathbb{U}_0\mathbb{V}_{1\varphi} + \mathbb{U}_1\mathbb{V}_{0\varphi}, \\
\mathscr{C}_2\left(\mathbb{U}\mathbb{V}_{\varphi}\right) &= \mathbb{U}_1\mathbb{V}_{2\varphi} + \mathbb{U}_1\mathbb{V}_{1\varphi} + \mathbb{U}_2\mathbb{V}_{0\varphi}.
\end{aligned} \tag{43}$$

For $j = 0,1,2,3,\ldots,$

$$\begin{aligned}
\mathbb{U}_1(\varphi,\mathfrak{I}) &= \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\rho\left(\mathbb{U}_{\varphi\varphi\varphi}\right)_0 - 6\rho\mathscr{A}_0 + 6\mathscr{B}_0\right]\right] \\
&= \mathbb{L}^{-1}\left[\frac{\omega^{\gamma+2}}{\nu^{\gamma+2}}\varrho^5\rho\tanh\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\right] \\
&= \varrho^5\rho\tanh\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\left((1-\gamma)+\frac{\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right) \\
\mathbb{V}_1(\varphi,\mathfrak{I}) &= \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\rho\left(\mathbb{V}_{\varphi\varphi\varphi}\right)_0 - 3\rho\mathscr{C}_0\right]\right] \\
&= \frac{\varrho^5\rho^{3/2}}{\sqrt{2}}\tanh\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\left((1-\gamma)+\frac{\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right) \\
\mathbb{U}_2(\varphi,\mathfrak{I}) &= \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\rho\left(\mathbb{U}_{\varphi\varphi\varphi}\right)_1 - 6\rho\mathscr{A}_1 + 6\mathscr{B}_1\right]\right]
\end{aligned}$$

$$\begin{aligned}
&= \mathbb{L}^{-1}\left[\frac{\omega^{2\gamma+2}}{\nu^{2\gamma+2}}\frac{\varrho^8\rho^2}{2}\left[2\cosh^2\left(\frac{\rho}{2}+\frac{\varrho\varphi}{2}\right)-3\right]\sec h^4\left(\frac{\rho}{2}+\frac{\varrho\varphi}{2}\right)\right]\\
&= \frac{\varrho^8\rho^2}{2}\left[2\cosh^2\left(\frac{\rho}{2}+\frac{\varrho\varphi}{2}\right)-3\right]\sec h^4\left(\frac{\rho}{2}+\frac{\varrho\varphi}{2}\right)\left((1-\gamma)^2+\frac{\gamma^2\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{2(1-\gamma)\gamma\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
\mathbb{V}_2(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\rho\left(\mathbb{V}_{\varphi\varphi\varphi}\right)_1-3\rho\mathscr{C}_1\right]\right]\\
&= \frac{\varrho^5\rho^{5/2}}{2\sqrt{2}}\left[2\cosh^2\left(\frac{\rho}{2}+\frac{\varrho\varphi}{2}\right)-3\right]\sec h^4\left(\frac{\rho}{2}+\frac{\varrho\varphi}{2}\right)\left((1-\gamma)^2+\frac{\gamma^2\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{2(1-\gamma)\gamma\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
&\vdots\,.
\end{aligned}\tag{44}$$

The modified decomposition technique result for Example 1 is shown as

$$\begin{aligned}
\mathbb{U}(\varphi,\mathfrak{T}) &= \mathbb{U}_0(\varphi,\mathfrak{T})+\mathbb{U}_1(\varphi,\mathfrak{T})+\mathbb{U}_2(\varphi,\mathfrak{T})+\mathbb{U}_3(\varphi,\mathfrak{T})+\cdots\\
&= \varrho^2\sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)+\varrho^5\rho\tanh\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\left((1-\gamma)+\frac{\gamma\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
&\quad+\frac{\varrho^8\rho^2}{2}\left[2\cosh^2\left(\frac{\rho}{2}+\frac{\varrho\varphi}{2}\right)-3\right]\sec h^4\left(\frac{\rho}{2}+\frac{\varrho\varphi}{2}\right)\left((1-\gamma)^2+\frac{\gamma^2\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{2(1-\gamma)\gamma\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right)+\cdots.
\end{aligned}\tag{45}$$

Similarly, we get

$$\begin{aligned}
\mathbb{V}(\varphi,\mathfrak{T}) = \sqrt{\frac{\rho}{2}}\varrho^2\sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)+\frac{\varrho^5\rho^{3/2}}{\sqrt{2}}\tanh\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\left((1-\gamma)+\frac{\gamma\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
+\frac{\varrho^5\rho^{5/2}}{2\sqrt{2}}\left[2\cosh^2\left(\frac{\rho}{2}+\frac{\varrho\varphi}{2}\right)-3\right]\sec h^4\left(\frac{\rho}{2}+\frac{\varrho\varphi}{2}\right)\left((1-\gamma)^2+\frac{\gamma^2\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{2(1-\gamma)\gamma\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right)+\cdots.
\end{aligned}\tag{46}$$

By putting $\gamma = 1$, we achieve the exact result of the system of Korteweg–de Vries equation (1):

$$\begin{aligned}
\mathbb{U}(\varphi,\mathfrak{T}) &= \varrho^2\sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}-\frac{\rho\varrho^3\mathfrak{T}}{2}\right),\\
\mathbb{V}(\varphi,\mathfrak{T}) &= \sqrt{\frac{\rho}{2}}\varrho^2\sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}-\frac{\rho\varrho^3\mathfrak{T}}{2}\right).
\end{aligned}\tag{47}$$

Case II: now, we apply the new iterative transformation technique on Example 1.

Using the suggested analytical method, we have

$$
\begin{aligned}
\mathbb{U}_0(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{1}{\nu}\mathbb{U}(\varphi,0)\right] = \mathbb{L}^{-1}\left[\frac{1}{\nu}\varrho^2 \sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\right]\\
&= \varrho^2 \sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\\
\mathbb{V}_0(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{1}{\nu}\mathbb{V}(\varphi,0)\right]\\
\mathbb{U}_1(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\rho\frac{\partial^3\mathbb{U}_0}{\partial\varphi^3}-6\rho\mathbb{U}_0\frac{\partial\mathbb{U}_0}{\partial\varphi}+6\mathbb{V}_0\frac{\partial\mathbb{V}_0}{\partial\varphi}\right]\right]\\
&= \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\varrho^5\rho \tanh\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\right]\\
&= \varrho^5\rho \tanh\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\left((1-\gamma)+\frac{\gamma\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
\mathbb{V}_1(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\rho\frac{\partial^3\mathbb{V}_0}{\partial\varphi^3}-3\rho\mathbb{U}_0\frac{\partial\mathbb{V}_0}{\partial\varphi}\right]\right]\\
&= \frac{\varrho^5\rho^{3/2}}{\sqrt{2}}\tanh\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\left((1-\gamma)+\frac{\gamma\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
\mathbb{U}_2(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\rho\frac{\partial^3\mathbb{U}_1}{\partial\varphi^3}-6\rho\mathbb{U}_1\frac{\partial\mathbb{U}_1}{\partial\varphi}+6\mathbb{V}_1\frac{\partial\mathbb{V}_1}{\partial\varphi}\right]\right]\\
&= \mathbb{L}^{-1}\left[\frac{\omega^{2\gamma+2}}{\nu^{2\gamma+2}}\frac{\varrho^8\rho^2}{2}\left[2\cosh^2\left(\frac{\rho}{2}+\frac{\varrho\varphi}{2}\right)-3\right]\sec h^4\left(\frac{\rho}{2}+\frac{\varrho\varphi}{2}\right)\right]\\
&= \frac{\varrho^8\rho^2}{2}\left[2\cosh^2\left(\frac{\rho}{2}+\frac{\varrho\varphi}{2}\right)-3\right]\sec h^4\left(\frac{\rho}{2}+\frac{\varrho\varphi}{2}\right)\left((1-\gamma)^2+\frac{\gamma^2\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{2(1-\gamma)\gamma\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
\mathbb{V}_2(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\rho\frac{\partial^3\mathbb{V}_1}{\partial\varphi^3}-3\rho\mathbb{U}_1\frac{\partial\mathbb{V}_1}{\partial\varphi}\right]\right]\\
&= \frac{\varrho^5\rho^{5/2}}{2\sqrt{2}}\left[2\cosh^2\left(\frac{\rho}{2}+\frac{\varrho\varphi}{2}\right)-3\right]\sec h^4\left(\frac{\rho}{2}+\frac{\varrho\varphi}{2}\right)\left((1-\gamma)^2+\frac{\gamma^2\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{2(1-\gamma)\gamma\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
&\vdots\\
\mathbb{U}_n(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\rho\frac{\partial^3\mathbb{U}_{j-1}}{\partial\varphi^3}-6\rho\mathbb{U}_{j-1}\frac{\partial\mathbb{U}_{j-1}}{\partial\varphi}+6\mathbb{V}_{j-1}\frac{\partial\mathbb{V}_{j-1}}{\partial\varphi}\right]\right]\\
\mathbb{V}_j(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\rho\frac{\partial^3\mathbb{V}_{j-1}}{\partial\varphi^3}-3\rho\mathbb{U}_{j-1}\frac{\partial\mathbb{V}_{j-1}}{\partial\varphi}\right]\right].
\end{aligned}
\tag{48}
$$

The series of solutions for Example 1 is expressed as

$$\begin{aligned}\mathbb{U}(\varphi,\mathfrak{I}) &= \mathbb{U}_0(\varphi,\mathfrak{I}) + \mathbb{U}_1(\varphi,\mathfrak{I}) + \mathbb{U}_2(\varphi,\mathfrak{I}) \\ &\quad + \mathbb{U}_3(\varphi,\mathfrak{I}) + \cdots \mathbb{U}_j(\varphi,\mathfrak{I}), \\ \mathbb{V}(\varphi,\mathfrak{I}) &= \mathbb{V}_0(\varphi,\mathfrak{I}) + \mathbb{V}_1(\varphi,\mathfrak{I}) + \mathbb{V}_2(\varphi,\mathfrak{I}) \\ &\quad + \mathbb{V}_3(\varphi,\mathfrak{I}) + \cdots \mathbb{V}_j(\varphi,\mathfrak{I}).\end{aligned} \tag{49}$$

Consequently, we have

$$\begin{aligned}\mathbb{U}(\varphi,\mathfrak{I}) &= \varrho^2 \sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right) + \varrho^5\rho \tanh\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\left((1-\gamma)+\frac{\gamma\mathfrak{I}^\gamma}{\Gamma(\gamma+1)}\right) \\ &\quad + \frac{\varrho^8\rho^2}{2}\left[2\cosh^2\left(\frac{\rho}{2}+\frac{\varrho\varphi}{2}\right)-3\right]\sec h^4\left(\frac{\rho}{2}+\frac{\varrho\varphi}{2}\right)\left((1-\gamma)^2+\frac{\gamma^2\mathfrak{I}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{2(1-\gamma)\gamma\mathfrak{I}^\gamma}{\Gamma(\gamma+1)}\right)+\cdots, \\ \mathbb{V}(\varphi,\mathfrak{I}) &= \sqrt{\frac{\rho}{2}}\varrho^2 \sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right) + \frac{\varrho^5\rho^{3/2}}{\sqrt{2}} \tanh\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}\right)\left((1-\gamma)+\frac{\gamma\mathfrak{I}^\gamma}{\Gamma(\gamma+1)}\right) \\ &\quad + \frac{\varrho^5\rho^{5/2}}{2\sqrt{2}}\left[2\cosh^2\left(\frac{\rho}{2}+\frac{\varrho\varphi}{2}\right)-3\right]\sec h^4\left(\frac{\rho}{2}+\frac{\varrho\varphi}{2}\right)\left((1-\gamma)^2+\frac{\gamma^2\mathfrak{I}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{2(1-\gamma)\gamma\mathfrak{I}^\gamma}{\Gamma(\gamma+1)}\right)+\cdots.\end{aligned} \tag{50}$$

By putting $\gamma = 1$, we get the exact result of the system of Korteweg–de Vries equation (1):

$$\begin{aligned}\mathbb{U}(\varphi,\mathfrak{I}) &= \varrho^2 \sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}-\frac{\rho\varrho^3\mathfrak{I}}{2}\right), \\ \mathbb{V}(\varphi,\mathfrak{I}) &= \sqrt{\frac{\rho}{2}}\varrho^2 \sec h^2\left(\frac{\delta}{2}+\frac{\varrho\varphi}{2}-\frac{\rho\varrho^3\mathfrak{I}}{2}\right).\end{aligned} \tag{51}$$

In Figures 1 and 2, the actual and analytical solutions of $\mathbb{U}(\varphi,\mathfrak{I})$ and $\mathbb{V}(\varphi,\mathfrak{I})$ are proved at $\delta = 2$, $\rho = 0.5$, and $\varrho = 1$. In Figures 3 and 4, the surface and two-dimensional figure for $\mathbb{U}(\varphi,\mathfrak{I})$ and $\mathbb{V}(\varphi,\mathfrak{I})$ for numerous fractional orders are described which demonstrate that the modified decomposition technique and new iterative transformation technique obtained series form solutions are in close contact with the analytical and the exact results. This comparison shows a strong connection among the modified decomposition method and actual solutions. Consequently, the modified decomposition technique and new iterative transformation technique are accurate innovative techniques which need less calculation time and are very simple and more flexible than the homotopy analysis technique and homotopy perturbation technique.

Example 2 (see [31]). Consider the fractional-order nonlinear system of Korteweg–de Vries equation given as

$$\begin{aligned}\frac{\partial^\gamma\mathbb{U}}{\partial\mathfrak{I}^\gamma} &= -\frac{\partial\mathbb{V}}{\partial\varphi}-\frac{1}{2}\frac{\partial\mathbb{U}^2}{\partial\varphi}, \\ \frac{\partial^\gamma\mathbb{V}}{\partial\mathfrak{I}^\gamma} &= -\frac{\partial\mathbb{U}}{\partial\varphi}-\frac{\partial^3\mathbb{U}}{\partial\varphi^3}-\frac{\partial zy}{\partial\varphi}, \quad \mathfrak{I}>0, 0<\gamma\le 1,\end{aligned} \tag{52}$$

with the conditions

$$\begin{aligned}\mathbb{U}(\varphi,0) &= \rho\left[\tanh\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)+1\right], \\ \mathbb{V}(\varphi,0) &= \frac{\rho^2}{2}\sec h^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)-1.\end{aligned} \tag{53}$$

Case I: first, we apply the modified decomposition technique for Example 2.

Applying the Laplace transform to (52), we find

$$\begin{aligned}\frac{\nu^\gamma}{(\nu^\gamma(1-\gamma)+\gamma)}\mathscr{U}(\varphi,\nu)-\sum_{\kappa=0}^{j-1}\left(\frac{1}{\nu}\right)^{\gamma-\kappa-1}\mathbb{U}^{(\kappa)}(0) &= \mathbb{L}\left[-\frac{\partial\mathbb{V}}{\partial\varphi}-\frac{1}{2}\frac{\partial\mathbb{U}^2}{\partial\varphi}\right], \\ \frac{\nu^\gamma}{(\nu^\gamma(1-\gamma)+\gamma)}\mathscr{V}(\varphi,\nu)-\sum_{\kappa=0}^{j-1}\left(\frac{1}{\nu}\right)^{\gamma-\kappa-1}\mathbb{V}^{(\kappa)}(0) &= \mathbb{L}\left[-\frac{\partial\mathbb{U}}{\partial\varphi}-\frac{\partial^3\mathbb{U}}{\partial\varphi^3}-\frac{\partial zy}{\partial\varphi}\right].\end{aligned} \tag{54}$$

In view of (29) and straightforward approximate achieve

$$\mathscr{U}(\varphi,\nu)=\frac{1}{\nu}\mathbb{U}^{(0)}(\varphi,0)+\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\frac{\partial\mathbb{V}}{\partial\varphi}-\frac{1}{2}\frac{\partial\mathbb{U}^{2}}{\partial\varphi}\right],$$
$$\mathscr{V}(\varphi,\nu)=\frac{1}{\nu}\mathbb{V}^{(0)}(\varphi,0)+\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\cdot\left[-\frac{\partial\mathbb{U}}{\partial\varphi}-\frac{\partial^{3}\mathbb{U}}{\partial\varphi^{3}}-\frac{\partial \mathrm{zy}}{\partial\varphi}\right]. \tag{55}$$

Using the inverse Laplace transformation, we get

$$\mathbb{U}(\varphi,\mathfrak{I})=\mathbb{L}^{-1}\left[\frac{1}{\nu}\mathbb{U}(\varphi,0)\right]+\mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\cdot\left[-\frac{\partial\mathbb{V}}{\partial\varphi}-\frac{1}{2}\frac{\partial\mathbb{U}^{2}}{\partial\varphi}\right]\right],$$
$$\mathbb{V}(\varphi,\mathfrak{I})=\mathbb{L}^{-1}\left[\frac{1}{\nu}\mathbb{V}(\varphi,0)\right]+\mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\cdot\left[-\frac{\partial\mathbb{U}}{\partial\varphi}-\frac{\partial^{3}\mathbb{U}}{\partial\varphi^{3}}-\frac{\partial \mathrm{zy}}{\partial\varphi}\right]\right]. \tag{56}$$

By the consequence of the modified decomposition technique, we get

$$\begin{aligned}\mathbb{U}_0(\varphi,\mathfrak{I})&=\mathbb{L}^{-1}\left[\frac{1}{\nu}\mathbb{U}(\varphi,0)\right]\\&=\mathbb{L}^{-1}\left[\frac{1}{\nu}\rho\left(\tanh\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)+1\right)\right]\\&=\rho\left(\tanh\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)+1\right),\end{aligned} \tag{57}$$
$$\mathbb{V}_0(\varphi,\mathfrak{I})=\mathbb{L}^{-1}\left[\frac{1}{\nu}\mathbb{V}(\varphi,0)\right]$$

It follows that

$$\sum_{j=0}^{\infty}\mathbb{U}_{j+1}(\varphi,\mathfrak{I})=\mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\cdot\left[-\rho\sum_{j=0}^{\infty}\left(\mathbb{V}_{\varphi}\right)_j-\frac{1}{2}\sum_{j=0}^{\infty}\mathscr{D}_j\right]\right],$$
$$\sum_{j=0}^{\infty}\mathbb{V}_{j+1}(\varphi,\mathfrak{I})=\mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\cdot\left[-\sum_{j=0}^{\infty}\left(\mathbb{U}_{\varphi}\right)_j-\sum_{j=0}^{\infty}\left(\mathbb{V}_{\varphi\varphi\varphi}\right)_j-\sum_{j=0}^{\infty}\left(\mathbb{U}\mathbb{V}_{\varphi}\right)_j\right]\right],$$
$$j=0,1,2,\ldots. \tag{58}$$

The Adomian polynomials' some terms are expressed as

$$\begin{aligned}\mathscr{D}_0\left(\mathbb{U}^2\right)&=\mathbb{U}_0^2,\\\mathscr{D}_1\left(\mathbb{U}^2\right)&=2\mathbb{U}_0\mathbb{U}_1,\\\mathscr{D}_2\left(\mathbb{U}^2\right)&=2\mathbb{U}_0\mathbb{U}_2+\mathbb{U}_1^2.\end{aligned} \tag{59}$$

For $j=0,1,2,\ldots$,

$$\begin{aligned}\mathbb{U}_1(\varphi,\mathfrak{I})&=\mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\rho\left(\mathbb{V}_{\varphi}\right)_0-\frac{1}{2}\mathscr{D}_0\right]\right]\\&=-\frac{\rho^2}{2}\mathbb{L}^{-1}\left[\frac{\omega^{\gamma+2}}{\nu^{\gamma+2}}\sec h^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\right]\\&=-\frac{\rho^2}{2}\sec h^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\left((1-\gamma)+\frac{\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right)\end{aligned}$$
$$\mathbb{V}_1(\varphi,\mathfrak{I})=\mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\left(\mathbb{U}_{\varphi}\right)_0-\left(\mathbb{V}_{\varphi\varphi\varphi}\right)_0-\left((\mathrm{zy})_{\varphi}\right)_0\right]\right]$$

$$
\begin{aligned}
&= \frac{\rho^3}{2}\sinh\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\sec h^3\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\left((1-\gamma)+\frac{\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
\mathbb{U}_2(\varphi,\mathfrak{I}) &= \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\rho\left(\mathbb{V}_{\varphi}\right)_1-\frac{1}{2}\mathcal{D}_1\right]\right]\\
&= \mathbb{L}^{-1}\left[-\frac{\rho^5}{4}\sec h^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)+\frac{3\rho^5}{4}\frac{\omega^{2\gamma+2}}{\nu^{2\gamma+2}}\sinh^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\sec h^4\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\right]\\
&\quad+\frac{\rho^7}{4}\mathbb{L}^{-1}\left[\frac{\Gamma(2\gamma+1)}{\Gamma^2(\gamma+1)}\frac{\omega^{3\gamma+2}}{\nu^{3\gamma+2}}\sinh\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\sec h^5\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\right]\\
&= \left[-\frac{\rho^5}{4}\sec h^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)+\frac{3\rho^5}{4}\sinh^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\sec h^4\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\right]\left((1-\gamma)+\frac{\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
&\quad+\frac{2(1-\gamma)\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}+\frac{\rho^7}{4}\sinh\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\sec h^5\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\frac{\Gamma(2\gamma+1)\mathfrak{I}^{3\gamma}}{\Gamma^2(\gamma+1)\Gamma(3\gamma+1)}\\
\mathbb{V}_2(\varphi,\mathfrak{I}) &= \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\left[-\left(\mathbb{U}_{\varphi}\right)_1-\left(\mathbb{V}_{\varphi\varphi\varphi}\right)_1-\left((\mathrm{zy})_{\varphi}\right)_1\right]\right]\\
&= \frac{\rho^6}{4}\left[2\cosh^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)-3\right]\sec h^4\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\left((1-\gamma)^2+\frac{\gamma^2\mathfrak{I}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{2(1-\gamma)\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
&\ \vdots\,.
\end{aligned}
\tag{60}
$$

The modified decomposition technique result for Example 2 is represented as

$$
\begin{aligned}
\mathbb{U}(\varphi,\mathfrak{I}) &= \mathbb{U}_0(\varphi,\mathfrak{I})+\mathbb{U}_1(\varphi,\mathfrak{I})+\mathbb{U}_2(\varphi,\mathfrak{I})+\cdots,\\
&= \rho\left(\tanh\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)+1\right)-\frac{\rho^2}{2}\sec h^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\left((1-\gamma)+\frac{\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
&\quad+\left[-\frac{\rho^5}{4}\sec h^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)+\frac{3\rho^5}{4}\sinh^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\sec h^4\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\right]\left((1-\gamma)+\frac{\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
&\quad+\frac{2(1-\gamma)\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}+\frac{\rho^7}{4}\sinh\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\sec h^5\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\frac{\Gamma(2\gamma+1)\mathfrak{I}^{3\gamma}}{\Gamma^2(\gamma+1)\Gamma(3\gamma+1)}+\cdots.
\end{aligned}
\tag{61}
$$

Consequently, we get

$$
\begin{aligned}
\mathbb{V}(\varphi,\mathfrak{I}) &= -1+\frac{\rho^2}{2}\sec h^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)+\frac{\rho^3}{2}\sinh\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\sec h^3\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\left((1-\gamma)+\frac{\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
&\quad+\frac{\rho^6}{4}\left[2\cosh^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)-3\right]\sec h^4\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\left((1-\gamma)^2+\frac{\gamma^2\mathfrak{I}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{2(1-\gamma)\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right)+\cdots.
\end{aligned}
\tag{62}
$$

By putting $\gamma = 1$, we achieve the exact result of the system of Korteweg–de Vries equation:

$$\mathbb{U}(\varphi,\mathfrak{I}) = \rho\left(\tanh\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}-\frac{\rho^2\mathfrak{I}}{2}\right)+1\right),$$
$$\mathbb{V}(\varphi,\mathfrak{I}) = \frac{\rho^2}{2}\sec h^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}-\frac{\rho^2\mathfrak{I}}{2}\right)-1. \tag{63}$$

Case II: now, we implement the new iterative transformation technique on Example 2.

By using the suggested analytical technique, we get

$$\begin{aligned}
\mathbb{U}_0(\varphi,\mathfrak{I}) &= \mathbb{L}^{-1}\left[\frac{1}{\nu}\mathbb{U}(\varphi,0)\right] = \mathbb{L}^{-1}\left[\frac{1}{\nu}\rho\left(\tanh\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)+1\right)\right]\\
&= \rho\left(\tanh\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)+1\right)\\
\mathbb{V}_0(\varphi,\mathfrak{I}) &= \mathbb{L}^{-1}\left[\frac{1}{\nu}\mathbb{V}(\varphi,0)\right]\\
\mathbb{U}_1(\varphi,\mathfrak{I}) &= \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\frac{\partial\mathbb{V}_0}{\partial\varphi}-\frac{1}{2}\frac{\partial\mathbb{U}_0^2}{\partial\varphi}\right]\right]\\
&= -\frac{\rho^2}{2}\mathbb{L}^{-1}\left[\frac{\omega^{\gamma+2}}{\nu^{\gamma+2}}\sec h^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\right]\\
&= -\frac{\rho^2}{2}\sec h^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\left((1-\gamma)+\frac{\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
\mathbb{V}_1(\varphi,\mathfrak{I}) &= \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\frac{\partial\mathbb{U}_0}{\partial\varphi}-\frac{\partial^3\mathbb{U}_0}{\partial\varphi^3}-\frac{\partial\mathbb{U}_0\mathbb{V}_0}{\partial\varphi}\right]\right]\\
&= \frac{\rho^3}{2}\sinh\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\sec h^3\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\left((1-\gamma)+\frac{\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
\mathbb{U}_2(\varphi,\mathfrak{I}) &= \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\frac{\partial\mathbb{V}_1}{\partial\varphi}-\frac{1}{2}\frac{\partial\mathbb{U}_1^2}{\partial\varphi}\right]\right]\\
&= \mathbb{L}^{-1}\left[-\frac{\rho^5}{4}\sec h^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)+\frac{3\rho^5}{4}\frac{\omega^{2\gamma+2}}{\nu^{2\gamma+2}}\sinh^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\sec h^4\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\right]\\
&\quad+\frac{\rho^7}{4}\mathbb{L}^{-1}\left[\frac{\Gamma(2\gamma+1)}{\Gamma^2(\gamma+1)}\frac{\omega^{3\gamma+2}}{\nu^{3\gamma+2}}\sinh\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\sec h^5\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\right]\\
&= \left[-\frac{\rho^5}{4}\sec h^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)+\frac{3\rho^5}{4}\sinh^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\sec h^4\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\right]\left((1-\gamma)+\frac{\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
&\quad+\frac{2(1-\gamma)\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}+\frac{\rho^7}{4}\sinh\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\sec h^5\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\frac{\Gamma(2\gamma+1)\mathfrak{I}^{3\gamma}}{\Gamma^2(\gamma+1)\Gamma(3\gamma+1)}\\
\mathbb{V}_2(\varphi,\mathfrak{I}) &= \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\left[-\frac{\partial\mathbb{U}_1}{\partial\varphi}-\frac{\partial^3\mathbb{U}_1}{\partial\varphi^3}-\frac{\partial\mathbb{U}_1\mathbb{V}_1}{\partial\varphi}\right]\right]\\
&= \frac{\rho^6}{4}\left[2\cosh^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)-3\right]\sec h^4\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\left((1-\gamma)^2+\frac{\gamma^2\mathfrak{I}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{2(1-\gamma)\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
&\vdots\\
\mathbb{U}_j(\varphi,\mathfrak{I}) &= \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\frac{\partial\mathbb{V}_{j-1}}{\partial\varphi}-\frac{1}{2}\frac{\partial\mathbb{U}_{j-1}^2}{\partial\varphi}\right]\right]\\
\mathbb{V}_j(\varphi,\mathfrak{I}) &= \mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\left[-\frac{\partial\mathbb{U}_{j-1}}{\partial\varphi}-\frac{\partial^3\mathbb{U}_{j-1}}{\partial\varphi^3}-\frac{\partial\mathbb{U}_{j-1}\mathbb{V}_{j-1}}{\partial\varphi}\right]\right].
\end{aligned} \tag{64}$$

The series of results for Example 2 is expressed as

$$\mathbb{U}(\varphi,\mathfrak{T}) = \mathbb{U}_0(\varphi,\mathfrak{T}) + \mathbb{U}_1(\varphi,\mathfrak{T}) + \mathbb{U}_2(\varphi,\mathfrak{T}) + \cdots \mathbb{U}_j(\varphi,\mathfrak{T}),$$
$$\mathbb{V}(\varphi,\mathfrak{T}) = \mathbb{V}_0(\varphi,\mathfrak{T}) + \mathbb{V}_1(\varphi,\mathfrak{T}) + \mathbb{V}_2(\varphi,\mathfrak{T}) + \cdots \mathbb{V}_j(\varphi,\mathfrak{T}). \tag{65}$$

Consequently, we have

$$\begin{aligned}\mathbb{U}(\varphi,\mathfrak{T}) &= \rho\left(\tanh\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)+1\right) - \frac{\rho^2}{2}\sec h^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\left((1-\gamma)+\frac{\gamma\mathfrak{T}^\gamma}{\Gamma(\gamma+1)}\right)\\
&+\left[-\frac{\rho^5}{4}\sec h^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)+\frac{3\rho^5}{4}\sinh^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\sec h^4\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\right]\left((1-\gamma)+\frac{\gamma\mathfrak{T}^\gamma}{\Gamma(\gamma+1)}\right)\\
&+\frac{2(1-\gamma)\gamma\mathfrak{T}^\gamma}{\Gamma(\gamma+1)}+\frac{\rho^7}{4}\sinh\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\sec h^5\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\frac{\Gamma(2\gamma+1)\mathfrak{T}^{3\gamma}}{\Gamma^2(\gamma+1)\Gamma(3\gamma+1)}+\cdots\\
\mathbb{V}(\varphi,\mathfrak{T}) &= -1+\frac{\rho^2}{2}\sec h^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)+\frac{\rho^3}{2}\sinh\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\sec h^3\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\left((1-\gamma)+\frac{\gamma\mathfrak{T}^\gamma}{\Gamma(\gamma+1)}\right)\\
&\frac{\rho^6}{4}\left[2\cosh^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)-3\right]\sec h^4\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}\right)\left((1-\gamma)^2+\frac{\gamma^2\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{2(1-\gamma)\gamma\mathfrak{T}^\gamma}{\Gamma(\gamma+1)}\right)+\cdots.\end{aligned} \tag{66}$$

By putting $\gamma = 1$, we obtain the actual result of the system of Korteweg–de Vries equation (??):

$$\mathbb{U}(\varphi,\mathfrak{T}) = \rho\left(\tanh\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}-\frac{\rho^2\mathfrak{T}}{2}\right)+1\right),$$
$$\mathbb{V}(\varphi,\mathfrak{T}) = \frac{\rho^2}{2}\sec h^2\left(\frac{\varrho}{2}+\frac{\rho\varphi}{2}-\frac{\rho^2\mathfrak{T}}{2}\right)-1. \tag{67}$$

In Figures 5 and 6, the actual and analytical solutions of $\mathbb{U}(\varphi,\mathfrak{T})$ and $\mathbb{V}(\varphi,\mathfrak{T})$ are proved at $\delta = 2$, $\rho = 0.5$, and $\varrho = 1$. In Figures 7 and 8, the surface and two-dimensional figure for $\mathbb{U}(\varphi,\mathfrak{T})$ and $\mathbb{V}(\varphi,\mathfrak{T})$ for numerous fractional orders are described which demonstrate that the modified decomposition technique and new iterative transformation technique approximated obtained results are in close contact with the analytical and the exact results. This comparison shows a strong connection among the modified decomposition method and actual solutions. Consequently, the modified decomposition technique and new iterative transformation technique are accurate innovative techniques which need less calculation time and are very simple and more flexible than the homotopy analysis technique and homotopy perturbation technique.

Example 3. (see [31]). Consider the fractional-order nonlinear system of modified Korteweg–de Vries equations given as (2) with the conditions

$$\mathbb{U}(\varphi,0) = \frac{2+\tanh\varphi}{2},$$
$$\mathbb{V}(\varphi,0) = \frac{2-\tanh\varphi}{4}, \tag{68}$$
$$\mathbb{W}(\varphi,0) = 2-\tanh\varphi.$$

Case I: first, we apply the modified decomposition technique for Example 3.

Using the Laplace transformation to (2), we have

$$\begin{aligned}\frac{\nu^\gamma}{(\nu^\gamma(1-\gamma)+\gamma)}\mathcal{U}(\varphi,\nu) - \sum_{\kappa=0}^{j-1}\left(\frac{1}{\nu}\right)^{\gamma-\kappa-1}\mathbb{U}^{(\kappa)}(0) = \mathbb{L}\Bigg[&\frac{1}{2}\frac{\partial^3\mathbb{U}}{\partial\mathfrak{T}^3} - 3\mathbb{U}^2\frac{\partial\mathbb{U}}{\partial\varphi}+\frac{3}{2}\mathbb{W}\frac{\partial^2\mathbb{V}}{\partial\varphi^2}+3\frac{\partial\mathbb{V}}{\partial\varphi}\frac{\partial\mathbb{W}}{\partial\varphi}+\frac{3}{2}\mathbb{V}\frac{\partial^2\mathbb{W}}{\partial\varphi^2}\\
&+3yx\frac{\partial\mathbb{U}}{\partial\varphi}+3zx\frac{\partial\mathbb{V}}{\partial\varphi}+3zy\frac{\partial\mathbb{W}}{\partial\varphi}\Bigg],\end{aligned}$$

$$\frac{\nu^{\gamma}}{(\nu^{\gamma}(1-\gamma)+\gamma)}\mathscr{V}(\varphi,\nu)-\sum_{\kappa=0}^{j-1}\left(\frac{1}{\nu}\right)^{\gamma-\kappa-1}\mathbb{V}^{(\kappa)}(0)=\mathbb{L}\left[-\frac{\partial^3\mathbb{V}}{\partial\varphi^3}-3\frac{\partial\mathbb{U}}{\partial\varphi}\frac{\partial\mathbb{V}}{\partial\varphi}-3\mathbb{V}\frac{\partial^2\mathbb{U}}{\partial\varphi^2}-3\mathbb{V}^2\frac{\partial\mathbb{W}}{\partial\varphi}+6zy\frac{\partial\mathbb{U}}{\partial\varphi}+3\mathbb{U}^2\frac{\partial\mathbb{V}}{\partial\varphi}\right],$$
$$\frac{\nu^{\gamma}}{(\nu^{\gamma}(1-\gamma)+\gamma)}\mathscr{W}(\varphi,\nu)-\sum_{\kappa=0}^{j-1}\left(\frac{1}{\nu}\right)^{\gamma-\kappa-1}\mathbb{W}^{(\kappa)}(0)=\mathbb{L}\left[-\frac{\partial^3\mathbb{W}}{\partial\varphi^3}-3\frac{\partial\mathbb{U}}{\partial\varphi}\frac{\partial\mathbb{W}}{\partial\varphi}-3\mathbb{W}\frac{\partial^2\mathbb{U}}{\partial\varphi^2}-3\mathbb{V}^2\frac{\partial\mathbb{W}}{\partial\varphi}+6zy\frac{\partial\mathbb{U}}{\partial\varphi}+3\mathbb{U}^2\frac{\partial\mathbb{V}}{\partial\varphi}\right]. \tag{69}$$

In view of (68) and straightforward calculations,

$$\mathscr{U}(\varphi,\nu)=\frac{1}{\nu}\mathbb{U}^{(0)}(\varphi,0)+\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[\frac{1}{2}\frac{\partial^3\mathbb{U}}{\partial\mathfrak{J}^3}-3\mathbb{U}^2\frac{\partial\mathbb{U}}{\partial\varphi}+\frac{3}{2}\mathbb{W}\frac{\partial^2\mathbb{V}}{\partial\varphi^2}+3\frac{\partial\mathbb{V}}{\partial\varphi}\frac{\partial\mathbb{W}}{\partial\varphi}+\frac{3}{2}\mathbb{V}\frac{\partial^2\mathbb{W}}{\partial\varphi^2}\right.$$
$$\left.+3yx\frac{\partial\mathbb{U}}{\partial\varphi}+3zx\frac{\partial\mathbb{V}}{\partial\varphi}+3zy\frac{\partial\mathbb{W}}{\partial\varphi}\right],$$
$$\mathscr{V}(\varphi,\nu)=\frac{1}{\nu}\mathbb{V}^{(0)}(\varphi,0)+\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\frac{\partial^3\mathbb{V}}{\partial\varphi^3}-3\frac{\partial\mathbb{U}}{\partial\varphi}\frac{\partial\mathbb{V}}{\partial\varphi}-3\mathbb{V}\frac{\partial^2\mathbb{U}}{\partial\varphi^2}-3\mathbb{V}^2\frac{\partial\mathbb{W}}{\partial\varphi}+6zy\frac{\partial\mathbb{U}}{\partial\varphi}+3\mathbb{U}^2\frac{\partial\mathbb{V}}{\partial\varphi}\right],$$
$$\mathscr{W}(\varphi,\nu)=\frac{1}{\nu}\mathbb{W}^{(0)}(\varphi,0)+\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\frac{\partial^3\mathbb{W}}{\partial\varphi^3}-3\frac{\partial\mathbb{U}}{\partial\varphi}\frac{\partial\mathbb{W}}{\partial\varphi}-3\mathbb{W}\frac{\partial^2\mathbb{U}}{\partial\varphi^2}-3\mathbb{W}^2\frac{\partial\mathbb{V}}{\partial\varphi}+6zx\frac{\partial\mathbb{U}}{\partial\varphi}+3\mathbb{U}^2\frac{\partial\mathbb{W}}{\partial\varphi}\right]. \tag{70}$$

Applying the Laplace transform, we have

$$\mathbb{U}(\varphi,\mathfrak{J})=\mathbb{L}^{-1}\left[\frac{1}{\nu}\mathbb{U}(\varphi,0)\right]+\mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[\frac{1}{2}\frac{\partial^3\mathbb{U}}{\partial\mathfrak{J}^3}-3\mathbb{U}^2\frac{\partial\mathbb{U}}{\partial\varphi}+\frac{3}{2}\mathbb{W}\frac{\partial^2\mathbb{V}}{\partial\varphi^2}+3\frac{\partial\mathbb{V}}{\partial\varphi}\frac{\partial\mathbb{W}}{\partial\varphi}+\frac{3}{2}\mathbb{V}\frac{\partial^2\mathbb{W}}{\partial\varphi^2}+3yx\frac{\partial\mathbb{U}}{\partial\varphi}\right.\right.$$
$$\left.\left.+3zx\frac{\partial\mathbb{V}}{\partial\varphi}+3zy\frac{\partial\mathbb{W}}{\partial\varphi}\right]\right],$$
$$\mathbb{V}(\varphi,\mathfrak{J})=\mathbb{L}^{-1}\left[\frac{1}{\nu}\mathbb{V}(\varphi,0)\right]+\mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\frac{\partial^3\mathbb{V}}{\partial\varphi^3}-3\frac{\partial\mathbb{U}}{\partial\varphi}\frac{\partial\mathbb{V}}{\partial\varphi}-3\mathbb{V}\frac{\partial^2\mathbb{U}}{\partial\varphi^2}-3\mathbb{V}^2\frac{\partial\mathbb{W}}{\partial\varphi}+6zy\frac{\partial\mathbb{U}}{\partial\varphi}+3\mathbb{U}^2\frac{\partial\mathbb{V}}{\partial\varphi}\right]\right],$$
$$\mathbb{W}(\varphi,\mathfrak{J})=\mathbb{L}^{-1}\left[\frac{1}{\nu}\mathbb{W}(\varphi,0)\right]+\mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\frac{\partial^3\mathbb{W}}{\partial\varphi^3}-3\frac{\partial\mathbb{U}}{\partial\varphi}\frac{\partial\mathbb{W}}{\partial\varphi}-3\mathbb{W}\frac{\partial^2\mathbb{U}}{\partial\varphi^2}-3\mathbb{W}^2\frac{\partial\mathbb{V}}{\partial\varphi}+6zx\frac{\partial\mathbb{U}}{\partial\varphi}+3\mathbb{U}^2\frac{\partial\mathbb{W}}{\partial\varphi}\right]\right]. \tag{71}$$

By the consequence of the modified decomposition technique, we get

$$\mathbb{U}_0(\varphi,\mathfrak{J})=\mathbb{L}^{-1}\left[\frac{1}{\nu}\mathbb{U}(\varphi,0)\right]=\frac{1}{2}\mathbb{L}^{-1}\left[\frac{1}{\nu}(2+\tanh\varphi)\right]$$
$$\mathbb{V}_0(\varphi,\mathfrak{J})=\mathbb{L}^{-1}\left[\frac{1}{\nu}\mathbb{V}(\varphi,0)\right]=\frac{1}{4}(2-\tanh\varphi),$$
$$\mathbb{W}_0(\varphi,\mathfrak{J})=\mathbb{L}^{-1}\left[\frac{1}{\nu}\mathbb{W}(\varphi,0)\right]=(2-\tanh\varphi). \tag{72}$$

It follows that

$$\sum_{j=0}^{\infty}\mathbb{U}_{j+1}(\varphi,\mathfrak{T})=\mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[\frac{1}{2}\sum_{j=0}^{\infty}\left(\mathbb{U}_{\varphi\varphi\varphi}\right)_{j}-3\sum_{j=0}^{\infty}\mathscr{E}_{j}+\frac{3}{2}\sum_{j=0}^{\infty}\mathscr{F}_{j}+3\sum_{j=0}^{\infty}\mathscr{G}_{j}+\frac{3}{2}\sum_{j=0}^{\infty}\mathscr{H}_{j}\right.\right.$$
$$\left.\left.+3\sum_{j=0}^{\infty}I_{j}+3\sum_{j=0}^{\infty}\mathscr{J}_{j}+3\sum_{j=0}^{\infty}\mathscr{K}_{j}\right]\right],\quad j$$
$$\sum_{j=0}^{\infty}\mathbb{V}_{j+1}(\varphi,\mathfrak{T})=\mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\sum_{j=0}^{\infty}\left(\mathbb{V}_{\varphi\varphi\varphi}\right)_{j}-3\sum_{j=0}^{\infty}\mathscr{M}_{j}-3\sum_{j=0}^{\infty}\mathscr{N}_{j}-3\sum_{j=0}^{\infty}\mathscr{O}_{j}+6\sum_{j=0}^{\infty}\mathrm{x}_{j}+3\sum_{j=0}^{\infty}\mathscr{Q}_{j}\right]\right],\tag{73}$$
$$\sum_{j=0}^{\infty}\mathbb{W}_{j+1}(\varphi,\mathfrak{T})=\mathbb{L}^{-1}\left[\frac{(\nu^{\gamma}(1-\gamma)+\gamma)}{\nu^{\gamma}}\mathbb{L}\left[-\sum_{j=0}^{\infty}\left(\mathbb{W}_{\varphi\varphi\varphi}\right)_{j}-3\sum_{j=0}^{\infty}\mathscr{R}_{j}-3\sum_{j=0}^{\infty}W_{j}-3\sum_{j=0}^{\infty}\mathscr{T}_{j}+6\sum_{j=0}^{\infty}\mathscr{X}_{j}+3\sum_{j=0}^{\infty}\mathscr{Y}_{j}\right]\right].$$

The Adomian polynomials' some terms are defined as

$$\mathscr{E}_{J}\left(\mathbb{U}^{2}\mathbb{U}_{\varphi}\right)=\begin{cases}\mathbb{U}_{0}^{2}\mathbb{U}_{0\varphi}, & \text{for } J=0,\\ \left(2\mathbb{U}_{0}\mathbb{U}_{1}\right)\mathbb{U}_{0\varphi}+\mathbb{U}_{0}^{2}\mathbb{U}_{1\varphi}, & \text{for } J=1,\\ \left(2\mathbb{U}_{0}\mathbb{U}_{2}+\mathbb{U}_{1}^{2}\right)\mathbb{U}_{0\varphi}+\left(2\mathbb{U}_{0}\mathbb{U}_{1}\right)\mathbb{U}_{1\varphi}+\mathbb{U}_{0}^{2}\mathbb{U}_{2\varphi}, & \text{for } J=2,\end{cases}$$

$$\mathscr{F}_{J}\left(\mathbb{W}\mathbb{V}_{\varphi\varphi}\right)=\begin{cases}\mathbb{W}_{0}\mathbb{V}_{0\varphi\varphi}, & \text{for } J=0,\\ \mathbb{W}_{1}\mathbb{V}_{0\varphi\varphi}+\mathbb{W}_{0}\mathbb{V}_{1\varphi\varphi}, & \text{for } J=1,\\ \mathbb{W}_{2}\mathbb{V}_{0\varphi\varphi}+\mathbb{W}_{1}\mathbb{V}_{1\varphi\varphi}+\mathbb{W}_{0}\mathbb{V}_{2\varphi\varphi}, & \text{for } J=2,\end{cases}$$

$$\mathscr{G}_{J}\left(\mathbb{V}_{\varphi}\mathbb{W}_{\varphi}\right)=\begin{cases}\mathbb{V}_{0\varphi}\mathbb{W}_{0\varphi}, & \text{for } J=0,\\ \mathbb{V}_{0\varphi}\mathbb{W}_{1\varphi}+\mathbb{V}_{1\varphi}\mathbb{W}_{0\varphi}, & \text{for } J=1,\\ \mathbb{V}_{2\varphi}\mathbb{W}_{0\varphi}+\mathbb{V}_{1\varphi}\mathbb{W}_{1\varphi}+\mathbb{V}_{0\varphi}\mathbb{W}_{2\varphi}, & \text{for } J=2,\end{cases}$$

$$\mathscr{H}_{J}\left(\mathbb{V}_{\varphi}\mathbb{W}_{\varphi\varphi}\right)=\begin{cases}\mathbb{V}_{0\varphi}\mathbb{W}_{0\varphi\varphi}, & \text{for } J=0,\\ \mathbb{V}_{0\varphi}\mathbb{W}_{1\varphi\varphi}+\mathbb{V}_{1\varphi}\mathbb{W}_{0\varphi\varphi}, & \text{for } J=1,\\ \mathbb{V}_{2\varphi}\mathbb{W}_{0\varphi\varphi}+\mathbb{V}_{1\varphi}\mathbb{W}_{1\varphi\varphi}+\mathbb{V}_{0\varphi}\mathbb{W}_{2\varphi\varphi}, & \text{for } J=2,\end{cases}$$

$$I_{J}\left(\mathbb{V}z\mathbb{U}_{\varphi}\right)=\begin{cases}(\mathrm{yx})_{0}\mathbb{U}_{0\varphi}, & \text{for } J=0,\\ (\mathrm{yx})_{0}\mathbb{U}_{1\varphi}+(\mathrm{yx})_{1}\mathbb{U}_{0\varphi}, & \text{for } J=1,\\ (\mathrm{yx})_{0}\mathbb{U}_{2\varphi}+(\mathrm{yx})_{1}\mathbb{U}_{1\varphi}+(\mathrm{yx})_{2}\mathbb{U}_{0\varphi}, & \text{for } J=2,\end{cases}$$

$$\mathscr{J}_{J}\left(\mathbb{U}z\mathbb{V}_{\varphi}\right)=\begin{cases}(\mathrm{zx})_{0}\mathbb{U}_{0\varphi}, & \text{for } J=0,\\ (\mathrm{zx})_{0}\mathbb{U}_{1\varphi}+(\mathrm{zx})_{1}\mathbb{U}_{0\varphi}, & \text{for } J=1,\\ (\mathrm{zx})_{0}\mathbb{U}_{2\varphi}+(\mathrm{zx})_{1}\mathbb{U}_{1\varphi}+(\mathrm{zx})_{2}\mathbb{U}_{0\varphi}, & \text{for } J=2,\end{cases}$$

$$\mathscr{K}_{J}\left(\mathrm{zy}\mathbb{W}_{\varphi}\right)=\begin{cases}(\mathrm{zy})_{0}\mathbb{U}_{0\varphi}, & \text{for } J=0,\\ (\mathrm{zy})_{0}\mathbb{U}_{1\varphi}+(\mathrm{zy})_{1}\mathbb{U}_{0\varphi}, & \text{for } J=1,\\ (\mathrm{zy})_{0}\mathbb{U}_{2\varphi}+(\mathrm{zy})_{1}\mathbb{U}_{1\varphi}+(\mathrm{zy})_{2}\mathbb{U}_{0\varphi}, & \text{for } J=2,\end{cases}$$

$$\mathscr{M}_{J}\left(\mathbb{U}_{\varphi}\mathbb{V}_{\varphi}\right)=\begin{cases}\mathbb{U}_{0\varphi}\mathbb{V}_{0\varphi}, & \text{for } J=0,\\ \mathbb{U}_{0\varphi}\mathbb{V}_{1\varphi}+\mathbb{U}_{\varphi}\mathbb{V}_{0\varphi}, & \text{for } J=1,\\ \mathbb{U}_{2\varphi}\mathbb{V}_{0\varphi}+\mathbb{U}_{1\varphi}\mathbb{V}_{1\varphi}+\mathbb{U}_{0\varphi}\mathbb{V}_{1\varphi}, & \text{for } J=2,\end{cases}$$

$$\mathcal{N}_J\left(\mathbb{V}\mathbb{U}_{\varphi\varphi}\right)=\begin{cases}\mathbb{V}_0\mathbb{U}_{0\varphi\varphi}, & \text{for } J=0,\\ \mathbb{V}_0\mathbb{U}_{1\varphi\varphi}+\mathbb{V}_1\mathbb{U}_{0\varphi\varphi}, & \text{for } J=1,\\ \mathbb{V}_2\mathbb{U}_{0\varphi\varphi}+\mathbb{V}_1\mathbb{U}_{1\varphi\varphi}+\mathbb{V}_0\mathbb{U}_{2\varphi\varphi}, & \text{for } J=2,\end{cases}$$

$$\mathcal{O}_J\left(\mathbb{V}^2\mathbb{W}_\varphi\right)=\begin{cases}\mathbb{V}_0^2\mathbb{W}_{0\varphi}, & \text{for } J=0,\\ (2\mathbb{V}_0\mathbb{V}_1)\mathbb{W}_{0\varphi}+\mathbb{V}_0^2\mathbb{W}_{1\varphi}, & \text{for } J=1,\\ \left(2\mathbb{V}_0\mathbb{V}_2+\mathbb{V}_1^2\right)\mathbb{W}_{0\varphi}+(2\mathbb{V}_0\mathbb{V}_1)\mathbb{W}_{1\varphi}+\mathbb{V}_0^2\mathbb{W}_{2\varphi}, & \text{for } J=2,\end{cases}$$

$$\mathrm{x}_J\left(\mathrm{zy}\mathbb{W}_\varphi\right)=\begin{cases}(\mathrm{zy})_0\mathbb{U}_{0\varphi}, & \text{for } J=0,\\ (\mathrm{zy})_0\mathbb{U}_{1\varphi}+(\mathrm{zy})_0\mathbb{U}_{1\varphi}, & \text{for } J=1,\\ (\mathrm{zy})_0\mathbb{U}_{2\varphi}+(\mathrm{zy})_1\mathbb{U}_{1\varphi}+(\mathrm{zy})_2\mathbb{U}_{0\varphi}, & \text{for } J=2,\end{cases}$$

$$\mathcal{Q}_J\left(\mathbb{U}^2\mathbb{V}_\varphi\right)=\begin{cases}\mathbb{U}_0^2\mathbb{V}_{0\varphi}, & \text{for } J=0,\\ (2\mathbb{U}_0\mathbb{U}_1)\mathbb{V}_{0\varphi}+\mathbb{U}_0^2\mathbb{V}_{1\varphi}, & \text{for } J=1,\\ \left(2\mathbb{U}_0\mathbb{U}_2+\mathbb{U}_1^2\right)\mathbb{V}_{0\varphi}+(2\mathbb{U}_0\mathbb{U}_1)\mathbb{V}_{1\varphi}+\mathbb{U}_0^2\mathbb{V}_{2\varphi} & \text{for } J=2,\end{cases}$$

$$\mathcal{R}_0\left(\mathbb{U}_\varphi\mathbb{V}_\varphi\right)=\begin{cases}\mathbb{U}_{0\varphi}\mathbb{V}_{0\varphi}, & \text{for } J=0,\\ \mathbb{U}_{0\varphi}\mathbb{V}_{1\varphi}+\mathbb{U}_{1\varphi}\mathbb{V}_{0\varphi}, & \text{for } J=1,\\ \mathbb{U}_{2\varphi}\mathbb{V}_{0\varphi}+\mathbb{U}_{1\varphi}\mathbb{V}_{1\varphi}+\mathbb{U}_{0\varphi}\mathbb{V}_{2\varphi}, & \text{for } J=2,\end{cases} \quad (74)$$

$$S_J\left(\mathbb{W}\mathbb{U}_{\varphi\varphi}\right)=\begin{cases}\mathbb{W}_0\mathbb{U}_{0\varphi\varphi}, & \text{for } J=0,\\ \mathbb{W}_0\mathbb{U}_{1\varphi\varphi}+\mathbb{W}_1\mathbb{U}_{0\varphi\varphi}, & \text{for } J=1,\\ \mathbb{W}_2\mathbb{U}_{0\varphi\varphi}+\mathbb{W}_1\mathbb{U}_{1\varphi\varphi}+\mathbb{W}_0\mathbb{U}_{2\varphi\varphi}, & \text{for } J=2,\end{cases}$$

$$\mathcal{T}_J\left(\mathbb{W}^2\mathbb{V}_\varphi\right)=\begin{cases}\mathbb{W}_0^2\mathbb{V}_{0\varphi}, & \text{for } J=0,\\ (2\mathbb{W}_0\mathbb{W}_1)\mathbb{V}_{0\varphi}+\mathbb{W}_0^2\mathbb{V}_{1\varphi}, & \text{for } J=1,\\ \left(2\mathbb{W}_0\mathbb{W}_2+\mathbb{W}_1^2\right)\mathbb{V}_{0\varphi}+(2\mathbb{W}_0\mathbb{W}_1)\mathbb{V}_{1\varphi}+\mathbb{W}_0^2\mathbb{V}_{2\varphi}, & \text{for } J=2,\end{cases}$$

$$\mathcal{X}_J\left(\mathbb{U}\mathbb{W}_1\mathbb{U}_\varphi\right)=\begin{cases}(\mathrm{zx})_0\mathbb{U}_{0\varphi}, & \text{for } J=0,\\ (\mathrm{zx})_0\mathbb{U}_{1\varphi}+(\mathrm{zx})_1\mathbb{U}_{0\varphi}, & \text{for } J=1,\\ (\mathrm{zx})_2\mathbb{U}_{0\varphi}+(\mathrm{zx})_1\mathbb{U}_{1\varphi}+(\mathrm{zx})_2\mathbb{U}_{0\varphi}, & \text{for } J=2,\end{cases}$$

$$\mathcal{Y}_J\left(\mathbb{U}^2\mathbb{W}_\varphi\right)=\begin{cases}\mathbb{U}_0^2\mathbb{W}_{0\varphi}, & \text{for } J=0,\\ (2\mathbb{U}_0\mathbb{U}_1)\mathbb{W}_{0\varphi}+\mathbb{U}_0^2\mathbb{W}_{1\varphi}, & \text{for } J=1,\\ \left(2\mathbb{U}_0\mathbb{U}_2+\mathbb{U}_1^2\right)\mathbb{W}_{0\varphi}+(2\mathbb{U}_0\mathbb{U}_1)\mathbb{W}_{1\varphi}+\mathbb{U}_0^2\mathbb{W}_{2\varphi}, & \text{for } J=2.\end{cases}$$

For $j=0,1,2,3,\ldots,$

$$\begin{aligned}\mathbb{U}_1(\varphi,\mathfrak{T})&=\mathbb{L}^{-1}\left[\frac{\left(\nu^\gamma(1-\gamma)+\gamma\right)}{\nu^\gamma}\mathbb{L}\left[\frac{1}{2}\left(\mathbb{U}_{\varphi\varphi\varphi}\right)_0-3\mathcal{E}_0+\frac{3}{2}\mathcal{F}_0+3\mathcal{G}_0+\frac{3}{2}\mathcal{H}_0+3I_0+3\mathcal{J}_0+3\mathcal{K}_0\right]\right]\\ &=\frac{11}{2}\sec h^2(\varphi)\left((1-\gamma)+\frac{\gamma\mathfrak{T}^\gamma}{\Gamma(\gamma+1)}\right)\\ \mathbb{V}_1(\varphi,\mathfrak{T})&=\mathbb{L}^{-1}\left[\frac{\left(\nu^\gamma(1-\gamma)+\gamma\right)}{\nu^\gamma}\mathbb{L}\left[\left(\mathbb{V}_{\varphi\varphi\varphi}\right)_0-3\mathcal{M}_0-3\mathcal{N}_0-3\mathcal{O}_0+6\mathrm{x}_0+3\mathcal{Q}_0\right]\right]\\ &=-\frac{11}{8}\sec h^2(\varphi)\left((1-\gamma)+\frac{\gamma\mathfrak{T}^\gamma}{\Gamma(\gamma+1)}\right)\end{aligned}$$

$$
\begin{aligned}
\mathbb{W}_1(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{(v^{\gamma}(1-\gamma)+\gamma)}{v^{\gamma}}\mathbb{L}\left[\left(\mathbb{W}_{\varphi\varphi\varphi}\right)_0 - 3\mathscr{R}_0 - 3\widehat{\mathbb{S}}_0 - 3\mathscr{T}_0 + 6\mathscr{X}_0 + 3\mathscr{Y}_0\right]\right] \\
&= -\frac{11}{2}\sec h^2(\varphi)\left((1-\gamma)+\frac{\gamma\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right) \\
\mathbb{U}_2(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{(v^{\gamma}(1-\gamma)+\gamma)}{v^{\gamma}}\mathbb{L}\left[\frac{1}{2}\left(\mathbb{U}_{\varphi\varphi\varphi}\right)_1 - 3\mathscr{E}_1 + \frac{3}{2}\mathscr{F}_1 + 3\mathscr{G}_1 + \frac{3}{2}\mathscr{H}_1 + 3I_1 + 3\mathscr{J}_1 + 3\mathscr{K}_1\right]\right] \\
&= \frac{-121}{8}\tanh(\varphi)\sec h^2(\varphi)\left((1-\gamma)^2+\frac{\gamma^2\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{2(1-\gamma)\gamma\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right) \\
\mathbb{V}_2(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{(v^{\gamma}(1-\gamma)+\gamma)}{v^{\gamma}}\mathbb{L}\left[\left(\mathbb{V}_{\varphi\varphi\varphi}\right)_1 - 3\mathscr{M}_1 - 3\mathscr{N}_1 - 3\mathscr{O}_1 + 6\mathrm{x}_1 + 3\mathscr{Q}_1\right]\right] \\
&= \frac{121}{8}\tanh(\varphi)\sec h^2(\varphi)\left((1-\gamma)^2+\frac{\gamma^2\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{2(1-\gamma)\gamma\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right) \\
\mathbb{W}_2(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{(v^{\gamma}(1-\gamma)+\gamma)}{v^{\gamma}}\mathbb{L}\left[\left(\mathbb{W}_{\varphi\varphi\varphi}\right)_1 - 3\mathscr{R}_1 - 3\widehat{\mathbb{S}}_1 - 3\mathscr{T}_1 + 6\mathscr{X}_1 + 3\mathscr{Y}_1\right]\right] \\
&= \frac{242}{8}anh(\varphi)\sec h^2(\varphi)\left((1-\gamma)^2+\frac{\gamma^2\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{2(1-\gamma)\gamma\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right) \\
\mathbb{U}_3(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{(v^{\gamma}(1-\gamma)+\gamma)}{v^{\gamma}}\mathbb{L}\left[\frac{1}{2}\left(\mathbb{U}_{\varphi\varphi\varphi}\right)_2 - 3\mathscr{E}_2 + \frac{3}{2}\mathscr{F}_2 + 3\mathscr{G}_2 + \frac{3}{2}\mathscr{H}_2 + 3I_2 + 3\mathscr{J}_2 + 3\mathscr{K}_2\right]\right] \\
&= \frac{1331}{48}\sec h^4(\varphi)[\cosh(2\varphi)-2]\left\{(1-\gamma)^3+\gamma(1-\gamma)\left(1+\gamma+2\gamma^2\right)\frac{\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}+\frac{3\gamma^2(1-\gamma)\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{\gamma^3\Gamma(2\gamma+1)\mathfrak{T}^{3\gamma}}{\Gamma(3\gamma+1)}\right\} \\
\mathbb{V}_3(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{(v^{\gamma}(1-\gamma)+\gamma)}{v^{\gamma}}\mathbb{L}\left[\left(\mathbb{V}_{\varphi\varphi\varphi}\right)_2 - 3\mathscr{M}_2 - 3\mathscr{N}_2 - 3\mathscr{O}_2 + 6\mathrm{x}_2 + 3\mathscr{Q}_2\right]\right] \\
&= \frac{2662}{96}\sec h^4(\varphi)[\cosh(2\varphi)-2]\left\{(1-\gamma)^3+\gamma(1-\gamma)\left(1+\gamma+2\gamma^2\right)\frac{\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}+\frac{3\gamma^2(1-\gamma)\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{\gamma^3\Gamma(2\gamma+1)\mathfrak{T}^{3\gamma}}{\Gamma(3\gamma+1)}\right\} \\
\mathbb{W}_3(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{(v^{\gamma}(1-\gamma)+\gamma)}{v^{\gamma}}\mathbb{L}\left[\left(\mathbb{W}_{\varphi\varphi\varphi}\right)_2 - 3\mathscr{R}_2 - 3\widehat{\mathbb{S}}_2 - 3\mathscr{T}_2 + 6\mathscr{X}_2 + 3\mathscr{Y}_2\right]\right] \\
&= \frac{-2662}{48}\sec h^4(\varphi)[\cosh(2\varphi)-2]\left\{(1-\gamma)^3+\gamma(1-\gamma)\left(1+\gamma+2\gamma^2\right)\frac{\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}+\frac{3\gamma^2(1-\gamma)\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{\gamma^3\Gamma(2\gamma+1)\mathfrak{T}^{3\gamma}}{\Gamma(3\gamma+1)}\right\} \\
&\vdots .
\end{aligned}
\tag{75}
$$

The modified decomposition technique result for Example 3 is given as

$$
\begin{aligned}
\mathbb{U}(\varphi,\mathfrak{T}) &= \mathbb{U}_0(\varphi,\mathfrak{T}) + \mathbb{U}_1(\varphi,\mathfrak{T}) + \mathbb{U}_2(\varphi,\mathfrak{T}) + \mathbb{U}_3(\varphi,\mathfrak{T})\ldots, \\
&= \frac{1}{2}(2+\tanh\varphi) + \frac{11}{2}\sec h^2(\varphi)\left((1-\gamma)+\frac{\gamma\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right) \\
&\quad - \frac{121}{8}\tanh(\varphi)\sec h^2(\varphi)\left((1-\gamma)^2+\frac{\gamma^2\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{2(1-\gamma)\gamma\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right) \\
&\quad + \frac{1331}{48}\sec h^4(\varphi)[\cosh(2\varphi)-2]\left\{(1-\gamma)^3+\gamma(1-\gamma)\left(1+\gamma+2\gamma^2\right)\frac{\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}+\frac{3\gamma^2(1-\gamma)\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{\gamma^3\Gamma(2\gamma+1)\mathfrak{T}^{3\gamma}}{\Gamma(3\gamma+1)}\right\} \\
&\quad + \cdots .
\end{aligned}
\tag{76}
$$

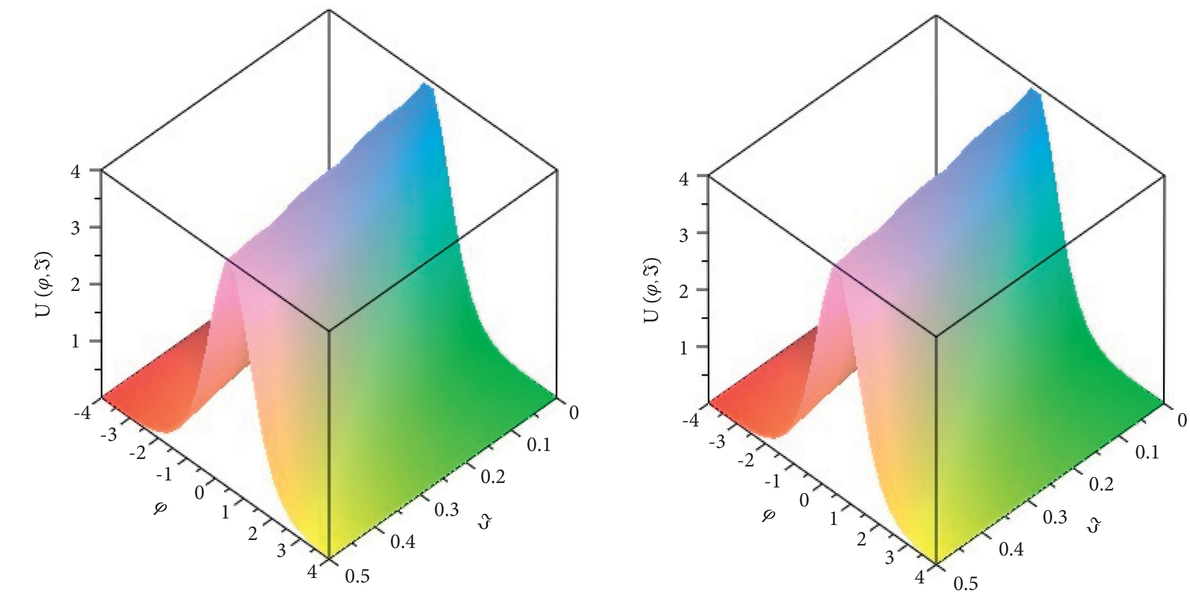


Figure 1: The actual and analytical (MDM/NITM) result figure at $\mathbb{U}(\varphi, \mathfrak{T})$ of Example 1 for $\varrho = 1$, $\rho = 0.5$, and $\delta = 2$.

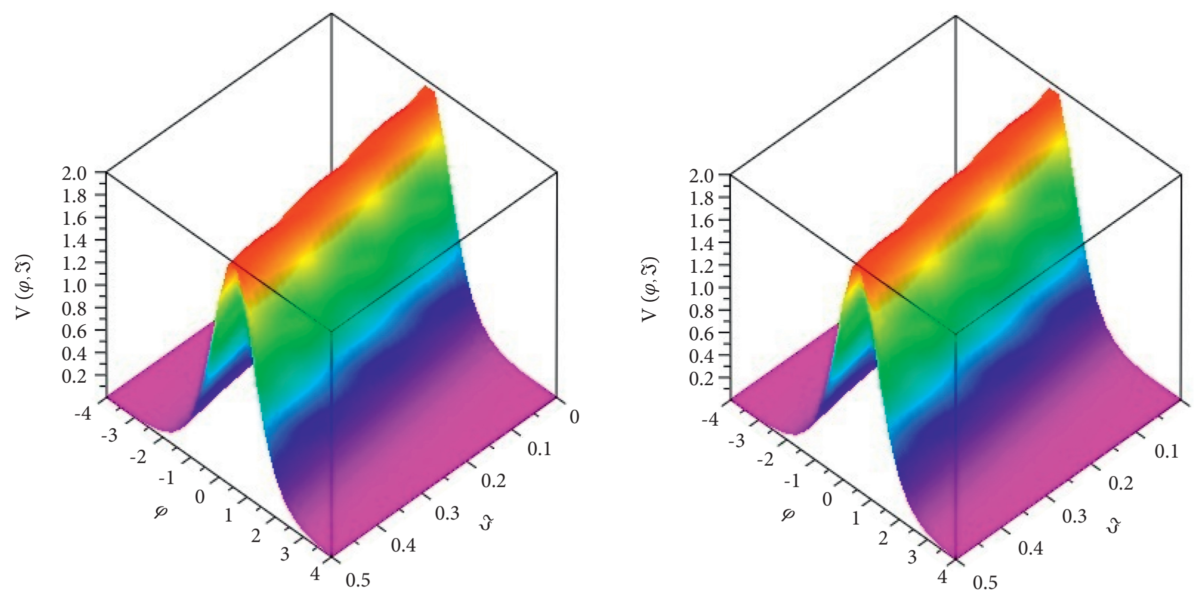


Figure 2: The actual and analytical (MDM/NITM) result figure at $\mathbb{V}(\varphi, \mathfrak{T})$ of Example 1 for $\varrho = 1$, $\rho = 0.5$, and $\delta = 2$.

Consequently, we get

$$
\begin{aligned}
\mathbb{V}(\varphi, \mathfrak{T}) &= \frac{1}{4}(2 - \tanh\varphi) - \frac{11}{8}\sec h^2(\varphi)\left((1-\gamma) + \frac{\gamma \mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right) + \frac{121}{8}\tanh(\varphi)\sec h^2(\varphi)\left((1-\gamma)^2 + \frac{\gamma^2 \mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)} + \frac{2(1-\gamma)\gamma \mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right) \\
&\quad - \frac{1331}{48}\sec h^4(\varphi)[\cosh(2\varphi) - 2]\left\{(1-\gamma)^3 + \gamma(1-\gamma)\left(1+\gamma+2\gamma^2\right)\frac{\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)} + \frac{3\gamma^2(1-\gamma)\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)} + \frac{\gamma^3\Gamma(2\gamma+1)\mathfrak{T}^{3\gamma}}{\Gamma(3\gamma+1)}\right\} + \cdots \\
\mathbb{W}(\varphi, \mathfrak{T}) &= (2 - \tanh\varphi) - \frac{11}{2}\sec h^2(\varphi)\left((1-\gamma) + \frac{\gamma \mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right) + \frac{121}{8}\tanh(\varphi)\sec h^2(\varphi)\left((1-\gamma)^2 + \frac{\gamma^2 \mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)} + \frac{2(1-\gamma)\gamma \mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right) \\
&\quad - \frac{2662}{96}\sec h^4(\varphi)[\cosh(2\varphi) - 2]\left\{(1-\gamma)^3 + \gamma(1-\gamma)\left(1+\gamma+2\gamma^2\right)\frac{\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)} + \frac{3\gamma^2(1-\gamma)\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)} + \frac{\gamma^3\Gamma(2\gamma+1)\mathfrak{T}^{3\gamma}}{\Gamma(3\gamma+1)}\right\} + \cdots.
\end{aligned}
\tag{77}
$$

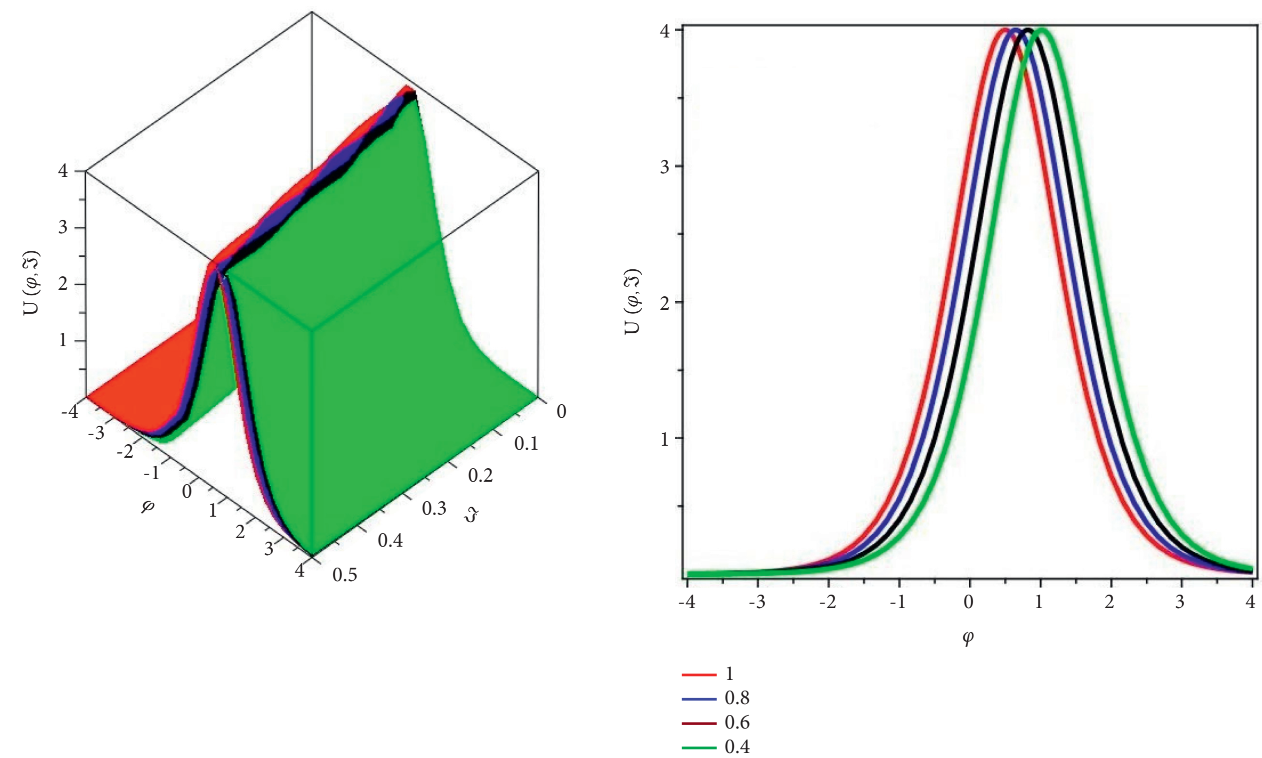


FIGURE 3: Analytical investigation of figure $\mathbb{U}(\varphi, \mathfrak{J})$ for Example 1 for different fractional orders $\gamma = 1.0, 0.8, 0.6, 0.4$, $\rho = 0.5$, $\varrho = 1$, and $\delta = 2$.

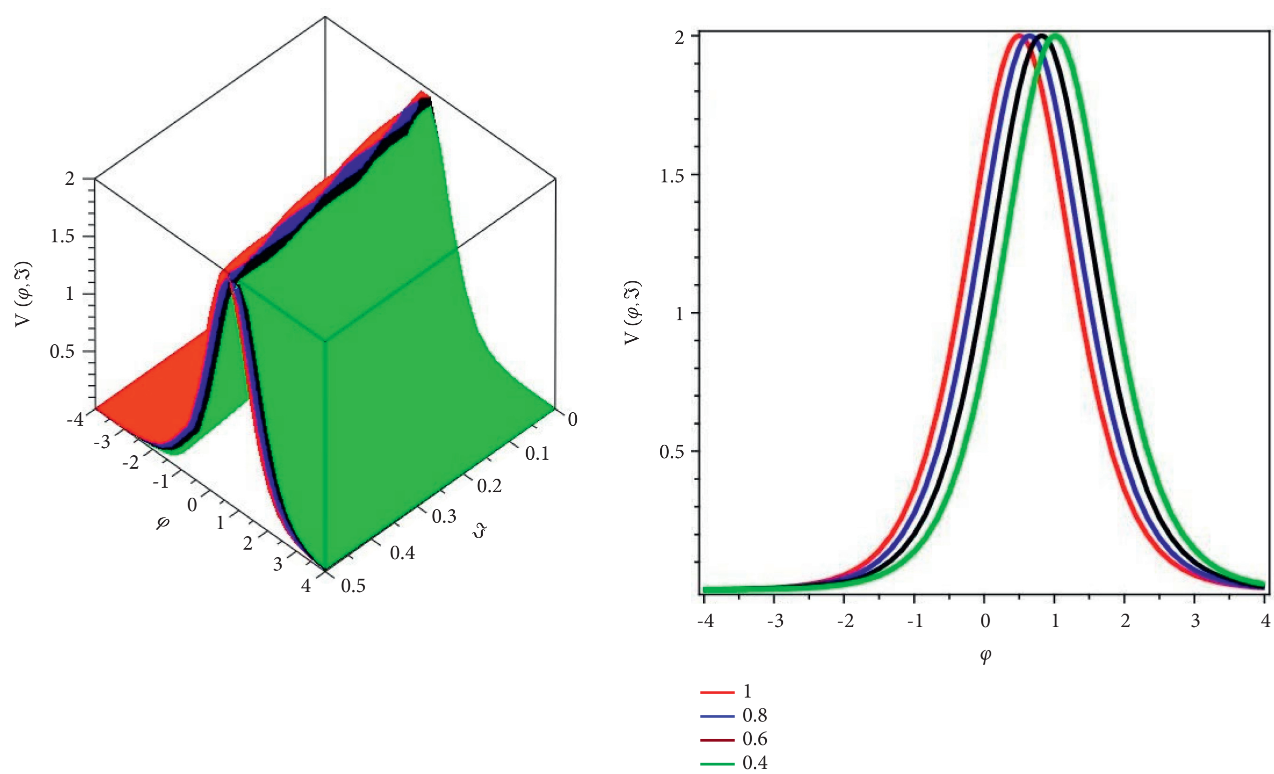


FIGURE 4: Analytical investigation of figure $\mathbb{V}(\varphi, \mathfrak{J})$ for Example 1 for different fractional orders $\gamma = 1.0, 0.8, 0.6, 0.4$, $\rho = 0.5$, $\varrho = 1$, and $\delta = 2$.

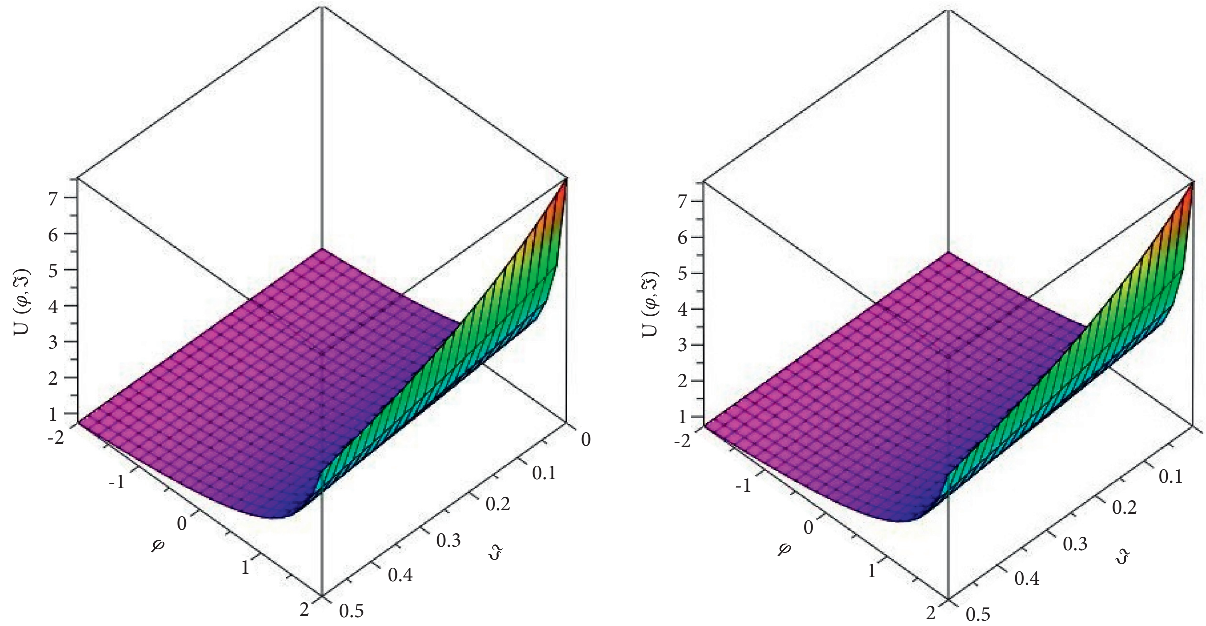


Figure 5: The analytical and exact (MDM/NITM) solution plot at $\mathbb{U}(\varphi, \mathfrak{J})$ of Example 2 for $\rho = 0.5$, $\varrho = 1$, and $\delta = 2$.

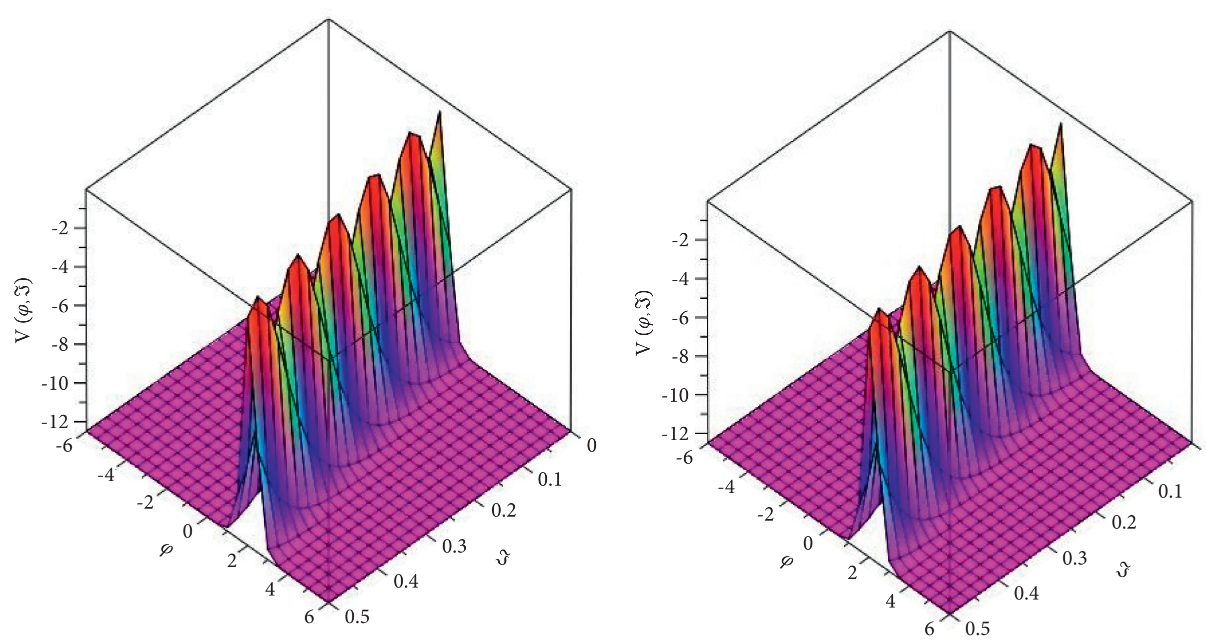


Figure 6: The analytical and exact (MDM/NITM) solution plot at $\mathbb{V}(\varphi, \mathfrak{J})$ of Example 2 for $\rho = 0.5$, $\varrho = 1$, and $\delta = 2$.

By putting $\gamma = 1$, we obtain the exact result of the system of Korteweg–de Vries equation (2):

$$\begin{aligned}
\mathbb{U}(\varphi, \mathfrak{J}) &= \frac{1}{2}\left(2 + \tanh\left(\varphi - \frac{11\mathfrak{J}}{2}\right)\right),\\
\mathbb{V}(\varphi, \mathfrak{J}) &= \frac{1}{4}\left(2 - \tanh\left(\varphi - \frac{11\mathfrak{J}}{2}\right)\right),\\
\mathbb{W}(\varphi, \mathfrak{J}) &= \left(2 - \tanh\left(\varphi - \frac{11\mathfrak{J}}{2}\right)\right).
\end{aligned} \tag{78}$$

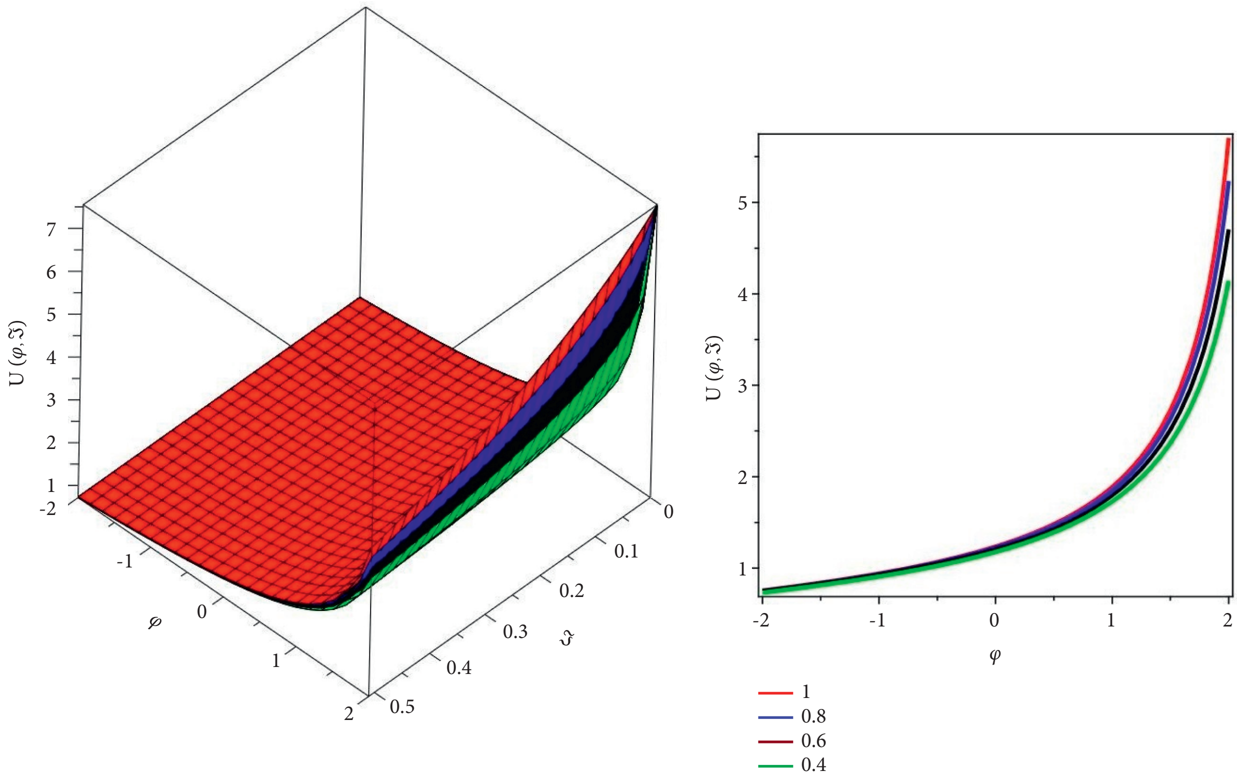


Figure 7: Mathematical analysis of the plot of $\mathbb{V}(\varphi, \mathfrak{J})$ for Example 2 for different fractional orders $\gamma = 1.0, 0.8, 0.6, 0.4$, $\rho = 0.5$, $\varrho = 1$, and $\delta = 2$.

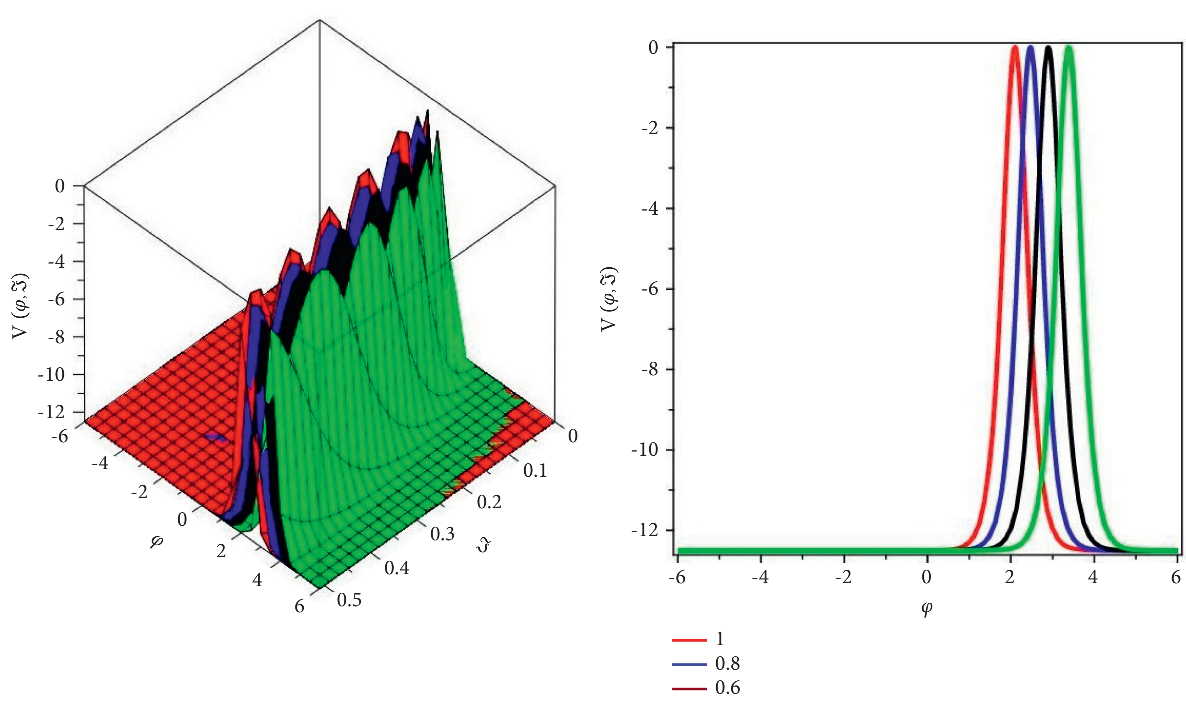


Figure 8: Mathematical analysis of the plot of $\mathbb{V}(\varphi, \mathfrak{J})$ for Example 2 for different fractional orders $\gamma = 1.0, 0.8, 0.6, 0.4 \rho = 0.5$, $\varrho = 1$, and $\delta = 2$.

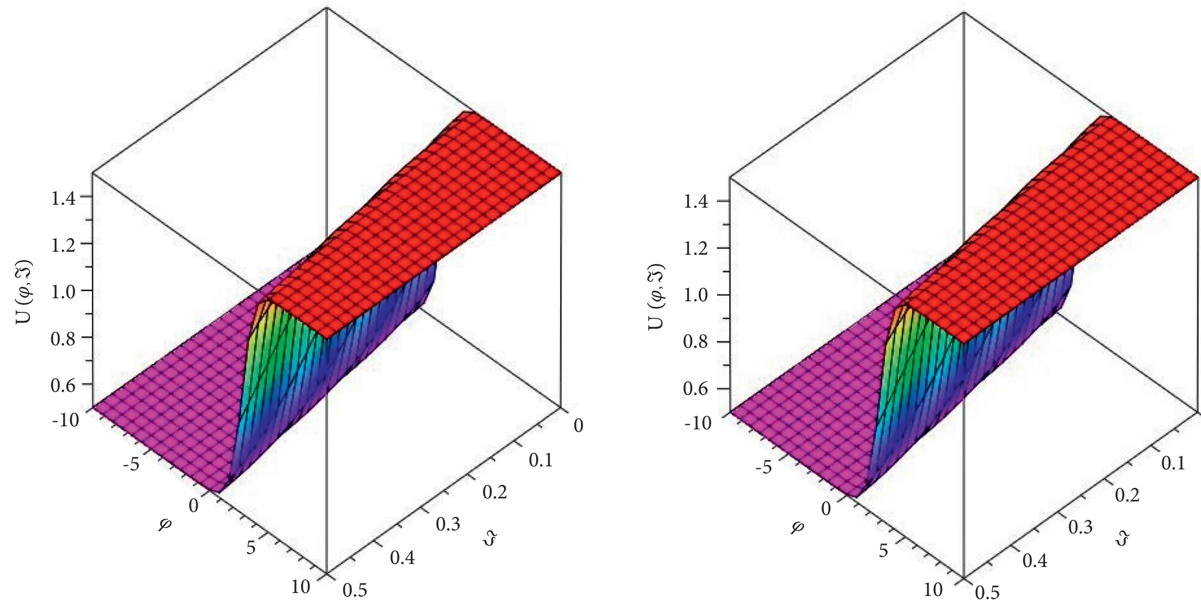


Figure 9: The analytical and exact result plot at $\mathbb{U}(\varphi, \mathfrak{I})$ of Example 3 for $\rho = 0.5$, $\varrho = 1$, and $\delta = 2$.

Case II: now, we apply the new iterative transformation technique for Example 3.

By using the suggested analytical method, we get

$$\mathbb{U}_0(\varphi, \mathfrak{I}) = \frac{1}{2}(2 + \tanh\varphi)$$

$$\mathbb{V}_0(\varphi, \mathfrak{I}) = \frac{1}{4}(2 - \tanh\varphi)$$

$$\mathbb{W}_0(\varphi, \mathfrak{I}) = (2 - \tanh\varphi)$$

$$\mathbb{U}_1(\varphi, \mathfrak{I}) = \mathbb{L}^{-1}\left[\frac{(v^{\gamma}(1-\gamma)+\gamma)}{v^{\gamma}}\mathbb{L}\left[\frac{1}{2}\frac{\partial^3 \mathbb{U}_0}{\partial \mathfrak{I}^3} - 3\mathbb{U}_0^2\frac{\partial \mathbb{U}_0}{\partial \varphi} + \frac{3}{2}\mathbb{W}_0\frac{\partial^2 \mathbb{V}_0}{\partial \varphi^2} + 3\frac{\partial \mathbb{V}_0}{\partial \varphi}\frac{\partial \mathbb{W}_0}{\partial \varphi} + \frac{3}{2}\mathbb{V}_0\frac{\partial^2 \mathbb{W}_0}{\partial \varphi^2} + 3\mathbb{V}_0\mathbb{W}_0\frac{\partial \mathbb{U}_0}{\partial \varphi} + 3\mathbb{U}_0\mathbb{W}_0\frac{\partial \mathbb{V}_0}{\partial \varphi} + 3\mathbb{U}_0\mathbb{V}_0\frac{\partial \mathbb{W}_0}{\partial \varphi}\right]\right]$$

$$= \frac{11}{2}\sec h^2(\varphi)\mathbb{L}^{-1}\left[\frac{\omega^{\gamma+2}}{v^{\gamma+2}}\right]$$

$$\mathbb{V}_1(\varphi, \mathfrak{I}) = \mathbb{L}^{-1}\left[\frac{(v^{\gamma}(1-\gamma)+\gamma)}{v^{\gamma}}\mathbb{L}\left[-\frac{\partial^3 \mathbb{V}_0}{\partial \varphi^3} - 3\frac{\partial \mathbb{U}_0}{\partial \varphi}\frac{\partial \mathbb{V}_0}{\partial \varphi} - 3\mathbb{V}_0\frac{\partial^2 \mathbb{U}_0}{\partial \varphi^2} - 3\mathbb{V}_0^2\frac{\partial \mathbb{W}_0}{\partial \varphi} + 6\mathbb{U}_0\mathbb{V}_0\frac{\partial \mathbb{U}_0}{\partial \varphi} + 3\mathbb{U}_0^2\frac{\partial \mathbb{V}_0}{\partial \varphi}\right]\right]$$

$$= -\frac{11}{8}\sec h^2(\varphi)\left((1-\gamma) + \frac{\gamma \mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right)$$

$$\mathbb{W}_1(\varphi, \mathfrak{I}) = \mathbb{L}^{-1}\left[\frac{(v^{\gamma}(1-\gamma)+\gamma)}{v^{\gamma}}\mathbb{L}\left[-\frac{\partial^3 \mathbb{W}_0}{\partial \varphi^3} - 3\frac{\partial \mathbb{U}_0}{\partial \varphi}\frac{\partial \mathbb{W}_0}{\partial \varphi} - 3\mathbb{W}_0\frac{\partial^2 \mathbb{U}_0}{\partial \varphi^2} - 3\mathbb{W}_0^2\frac{\partial \mathbb{V}_0}{\partial \varphi} + 6\mathbb{U}_0\mathbb{W}_0\frac{\partial \mathbb{U}_0}{\partial \varphi} + 3\mathbb{U}_0^2\frac{\partial \mathbb{W}_0}{\partial \varphi}\right]\right]$$

$$= -\frac{11}{2}\sec h^2(\varphi)\left((1-\gamma) + \frac{\gamma \mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right)$$

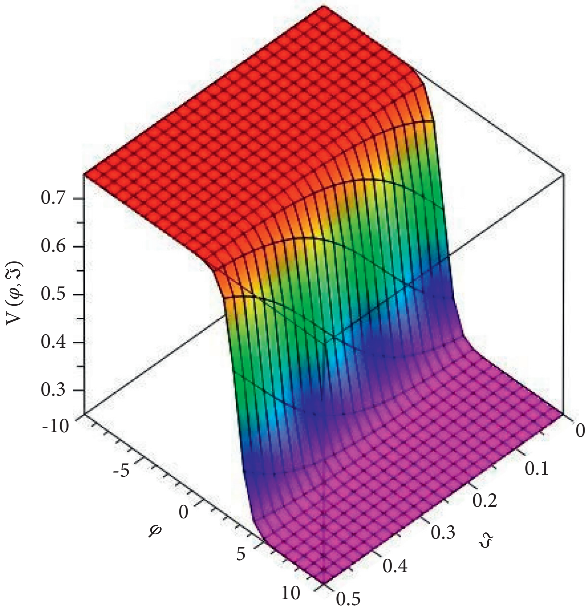
$$\mathbb{U}_2(\varphi, \mathfrak{I}) = \mathbb{L}^{-1}\left[\frac{(v^{\gamma}(1-\gamma)+\gamma)}{v^{\gamma}}\mathbb{L}\left[\frac{1}{2}\frac{\partial^3 \mathbb{U}_1}{\partial \mathfrak{I}^3} - 3\mathbb{U}_1^2\frac{\partial \mathbb{U}_1}{\partial \varphi} + \frac{3}{2}\mathbb{W}_1\frac{\partial^2 \mathbb{V}_1}{\partial \varphi^2} + 3\frac{\partial \mathbb{V}_1}{\partial \varphi}\frac{\partial \mathbb{W}_1}{\partial \varphi} + \frac{3}{2}\mathbb{V}_1\frac{\partial^2 \mathbb{W}_1}{\partial \varphi^2} + 3\mathbb{V}_1\mathbb{W}_1\frac{\partial \mathbb{U}_1}{\partial \varphi} + 3\mathbb{U}_1\mathbb{W}_1\frac{\partial \mathbb{V}_1}{\partial \varphi} + 3\mathbb{U}_1\mathbb{V}_1\frac{\partial \mathbb{W}_1}{\partial \varphi}\right]\right]$$

$$= \frac{-121}{8}\tanh(\varphi)\sec h^2(\varphi)\left((1-\gamma)^2 + \frac{\gamma^2 \mathfrak{I}^{2\gamma}}{\Gamma(2\gamma+1)} + \frac{2(1-\gamma)\gamma \mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right)$$

$$\begin{aligned}
\mathbb{V}_2(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{(v^{\gamma}(1-\gamma)+\gamma)}{v^{\gamma}}\mathbb{L}\left[-\frac{\partial^3\mathbb{V}_1}{\partial\varphi^3}-3\frac{\partial\mathbb{U}_1}{\partial\varphi}\frac{\partial\mathbb{V}_1}{\partial\varphi}-3\mathbb{V}_1\frac{\partial^2\mathbb{U}_1}{\partial\varphi^2}-3\mathbb{V}_1^2\frac{\partial\mathbb{W}_1}{\partial\varphi}+6\mathbb{U}_1\mathbb{V}_1\frac{\partial\mathbb{U}_1}{\partial\varphi}+3\mathbb{U}_1^2\frac{\partial\mathbb{V}_1}{\partial\varphi}\right]\right]\\
&= \frac{121}{8}\tanh(\varphi)\sec h^2(\varphi)\left((1-\gamma)^2+\frac{\gamma^2\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{2(1-\gamma)\gamma\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
\mathbb{W}_2(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{(v^{\gamma}(1-\gamma)+\gamma)}{v^{\gamma}}\mathbb{L}\left[-\frac{\partial^3\mathbb{W}_1}{\partial\varphi^3}-3\frac{\partial\mathbb{U}_1}{\partial\varphi}\frac{\partial\mathbb{W}_1}{\partial\varphi}-3\mathbb{W}_1\frac{\partial^2\mathbb{U}_1}{\partial\varphi^2}-3\mathbb{W}_1^2\frac{\partial\mathbb{V}_1}{\partial\varphi}+6\mathbb{U}_1\mathbb{W}_1\frac{\partial\mathbb{U}_1}{\partial\varphi}+3\mathbb{U}_1^2\frac{\partial\mathbb{W}_1}{\partial\varphi}\right]\right]\\
&= \frac{242}{8}\tanh(\varphi)\sec h^2(\varphi)\left((1-\gamma)^2+\frac{\gamma^2\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{2(1-\gamma)\gamma\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
\mathbb{U}_3(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{(v^{\gamma}(1-\gamma)+\gamma)}{v^{\gamma}}\mathbb{L}\left[\frac{1}{2}\frac{\partial^3\mathbb{U}_2}{\partial\mathfrak{T}^3}-3\mathbb{U}_2^2\frac{\partial\mathbb{U}_2}{\partial\varphi}+\frac{3}{2}\mathbb{W}_2\frac{\partial^2\mathbb{V}_2}{\partial\varphi^2}+3\frac{\partial\mathbb{V}_2}{\partial\varphi}\frac{\partial\mathbb{W}_2}{\partial\varphi}+\frac{3}{2}\mathbb{V}_2\frac{\partial^2\mathbb{W}_2}{\partial\varphi^2}+3\mathbb{V}_2\mathbb{W}_2\frac{\partial\mathbb{U}_2}{\partial\varphi}+3\mathbb{U}_2\mathbb{W}_2\frac{\partial\mathbb{V}_2}{\partial\varphi}+3\mathbb{U}_2\mathbb{V}_2\frac{\partial\mathbb{W}_2}{\partial\varphi}\right]\right]\\
&= \frac{1331}{48}\sec h^4(\varphi)[\cosh(2\varphi)-2]\left\{(1-\gamma)^3+\gamma(1-\gamma)\left(1+\gamma+2\gamma^2\right)\frac{\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}+\frac{3\gamma^2(1-\gamma)\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{\gamma^3\Gamma(2\gamma+1)\mathfrak{T}^{3\gamma}}{\Gamma(3\gamma+1)}\right\}\\
\mathbb{V}_3(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{(v^{\gamma}(1-\gamma)+\gamma)}{v^{\gamma}}\mathbb{L}\left[-\frac{\partial^3\mathbb{V}_2}{\partial\varphi^3}-3\frac{\partial\mathbb{U}_2}{\partial\varphi}\frac{\partial\mathbb{V}_2}{\partial\varphi}-3\mathbb{V}_2\frac{\partial^2\mathbb{U}_2}{\partial\varphi^2}-3\mathbb{V}_2^2\frac{\partial\mathbb{W}_2}{\partial\varphi}+6\mathbb{U}_2\mathbb{V}_2\frac{\partial\mathbb{U}_2}{\partial\varphi}+3\mathbb{U}_2^2\frac{\partial\mathbb{V}_2}{\partial\varphi}\right]\right]\\
&= \frac{2662}{96}\sec h^4(\varphi)[\cosh(2\varphi)-2]\left\{(1-\gamma)^3+\gamma(1-\gamma)\left(1+\gamma+2\gamma^2\right)\frac{\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}+\frac{3\gamma^2(1-\gamma)\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{\gamma^3\Gamma(2\gamma+1)\mathfrak{T}^{3\gamma}}{\Gamma(3\gamma+1)}\right\}\\
\mathbb{W}_3(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{(v^{\gamma}(1-\gamma)+\gamma)}{v^{\gamma}}\mathbb{L}\left[-\frac{\partial^3\mathbb{W}_2}{\partial\varphi^3}-3\frac{\partial\mathbb{U}_2}{\partial\varphi}\frac{\partial\mathbb{W}_2}{\partial\varphi}-3\mathbb{W}_2\frac{\partial^2\mathbb{U}_2}{\partial\varphi^2}-3\mathbb{W}_2^2\frac{\partial\mathbb{V}_2}{\partial\varphi}+6\mathbb{U}_2\mathbb{W}_2\frac{\partial\mathbb{U}_2}{\partial\varphi}+3\mathbb{U}_2^2\frac{\partial\mathbb{W}_2}{\partial\varphi}\right]\right]\\
&= \frac{-2662}{48}\sec h^4(\varphi)[\cosh(2\varphi)-2]\left\{(1-\gamma)^3+\gamma(1-\gamma)\left(1+\gamma+2\gamma^2\right)\frac{\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}+\frac{3\gamma^2(1-\gamma)\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{\gamma^3\Gamma(2\gamma+1)\mathfrak{T}^{3\gamma}}{\Gamma(3\gamma+1)}\right\}\\
&\vdots\\
\mathbb{U}_j(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{v^{\gamma}(1-\gamma)+\gamma}{v^{\gamma}}\mathbb{L}\left[\frac{1}{2}\frac{\partial^3\mathbb{U}_{j-1}}{\partial\mathfrak{T}^3}-3\mathbb{U}_{j-1}^2\frac{\partial\mathbb{U}_{j-1}}{\partial\varphi}+\frac{3}{2}\mathbb{W}_{j-1}\frac{\partial^2\mathbb{V}_{j-1}}{\partial\varphi^2}+3\frac{\partial\mathbb{V}_{j-1}}{\partial\varphi}\frac{\partial\mathbb{W}_{j-1}}{\partial\varphi}+\frac{3}{2}\mathbb{V}_{j-1}\frac{\partial^2\mathbb{W}_{j-1}}{\partial\varphi^2}\right.\right.\\
&\quad\left.\left.+3\mathbb{V}_{j-1}\mathbb{W}_{j-1}\frac{\partial\mathbb{U}_{j-1}}{\partial\varphi}+3\mathbb{U}_{j-1}\mathbb{W}_{j-1}\frac{\partial\mathbb{V}_{j-1}}{\partial\varphi}+3\mathbb{U}_{j-1}\mathbb{V}_{j-1}\frac{\partial\mathbb{W}_{j-1}}{\partial\varphi}\right]\right]\\
\mathbb{V}_j(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{v^{\gamma}(1-\gamma)+\gamma}{v^{\gamma}}\mathbb{L}\left[-\frac{\partial^3\mathbb{V}_{j-1}}{\partial\varphi^3}-3\frac{\partial\mathbb{U}_{j-1}}{\partial\varphi}\frac{\partial\mathbb{V}_{j-1}}{\partial\varphi}-3\mathbb{V}_{j-1}\frac{\partial^2\mathbb{U}_{j-1}}{\partial\varphi^2}-3\mathbb{V}_{j-1}^2\frac{\partial\mathbb{W}_{j-1}}{\partial\varphi}+6\mathbb{U}_{j-1}\mathbb{V}_{j-1}\frac{\partial\mathbb{U}_{j-1}}{\partial\varphi}+3\mathbb{U}_{j-1}^2\frac{\partial\mathbb{V}_{j-1}}{\partial\varphi}\right]\right]\\
\mathbb{W}_j(\varphi,\mathfrak{T}) &= \mathbb{L}^{-1}\left[\frac{v^{\gamma}(1-\gamma)+\gamma}{v^{\gamma}}\mathbb{L}\left[-\frac{\partial^3\mathbb{W}_{j-1}}{\partial\varphi^3}-3\frac{\partial\mathbb{U}_{j-1}}{\partial\varphi}\frac{\partial\mathbb{W}_{j-1}}{\partial\varphi}-3\mathbb{W}_{j-1}\frac{\partial^2\mathbb{U}_{j-1}}{\partial\varphi^2}-3\mathbb{W}_{j-1}^2\frac{\partial\mathbb{V}_{j-1}}{\partial\varphi}+6\mathbb{U}_{j-1}\mathbb{W}_{j-1}\frac{\partial\mathbb{U}_{j-1}}{\partial\varphi}+3\mathbb{U}_{j-1}^2\frac{\partial\mathbb{W}_{j-1}}{\partial\varphi}\right]\right].
\end{aligned} \tag{79}$$

The series of results for Example 3 is given as

$$\begin{aligned}
\mathbb{U}(\varphi,\mathfrak{T}) &= \mathbb{U}_0(\varphi,\mathfrak{T})+\mathbb{U}_1(\varphi,\mathfrak{T})+\mathbb{U}_2(\varphi,\mathfrak{T})+\mathbb{U}_3(\varphi,\mathfrak{T})+\cdots\mathbb{U}_j(\varphi,\mathfrak{T})\\
&= \frac{1}{2}(2+\tanh\varphi)+\frac{11}{2}\sec h^2(\varphi)\left((1-\gamma)+\frac{\gamma\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
&\quad-\frac{121}{8}\tanh(\varphi)\sec h^2(\varphi)\left((1-\gamma)^2+\frac{\gamma^2\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{2(1-\gamma)\gamma\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
&\quad+\frac{1331}{48}\sec h^4(\varphi)[\cosh(2\varphi)-2]\left\{(1-\gamma)^3+\gamma(1-\gamma)\left(1+\gamma+2\gamma^2\right)\frac{\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}+\frac{3\gamma^2(1-\gamma)\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{\gamma^3\Gamma(2\gamma+1)\mathfrak{T}^{3\gamma}}{\Gamma(3\gamma+1)}\right\}+\cdots.
\end{aligned} \tag{80}$$



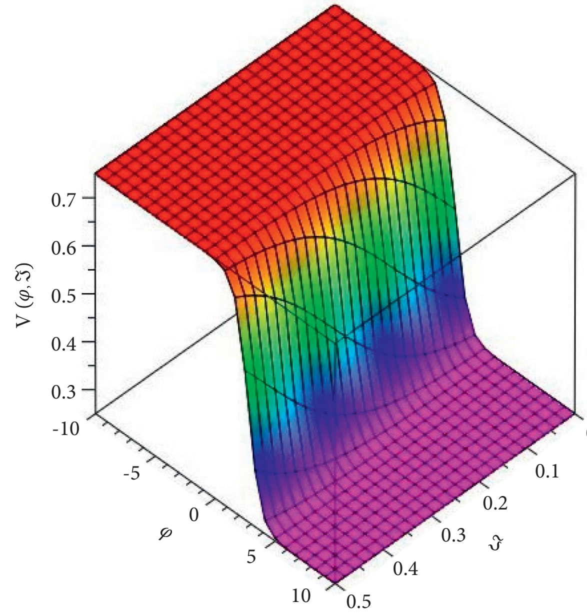


FIGURE 10: The analytical and exact result plot at $\mathbb{V}(\varphi, \mathfrak{I})$ of Example 3 for $\rho = 0.5$, $\varrho = 1$, and $\delta = 2$.

Consequently, we get

$$\begin{aligned}
\mathbb{V}(\varphi,\mathfrak{I}) &= \frac{1}{4}(2-\tanh\varphi) - \frac{11}{8}\sec h^2(\varphi)\left((1-\gamma)+\frac{\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right) + \frac{121}{8}\tanh(\varphi)\sec h^2(\varphi)\left((1-\gamma)^2+\frac{\gamma^2\mathfrak{I}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{2(1-\gamma)\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
&\quad - \frac{1331}{48}\sec h^4(\varphi)[\cosh(2\varphi)-2]\left\{(1-\gamma)^3+\gamma(1-\gamma)\left(1+\gamma+2\gamma^2\right)\frac{\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}+\frac{3\gamma^2(1-\gamma)\mathfrak{I}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{\gamma^3\Gamma(2\gamma+1)\mathfrak{I}^{3\gamma}}{\Gamma(3\gamma+1)}\right\}+\cdots,\\
\mathbb{W}(\varphi,\mathfrak{I}) &= (2-\tanh\varphi) - \frac{11}{2}\sec h^2(\varphi)\left((1-\gamma)+\frac{\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right) + \frac{121}{4}\tanh(\varphi)\sec h^2(\varphi)\left((1-\gamma)^2+\frac{\gamma^2\mathfrak{I}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{2(1-\gamma)\gamma\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}\right)\\
&\quad - \frac{2662}{48}\sec h^4(\varphi)[\cosh(2\varphi)-2]\left\{(1-\gamma)^3+\gamma(1-\gamma)\left(1+\gamma+2\gamma^2\right)\frac{\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}+\frac{3\gamma^2(1-\gamma)\mathfrak{I}^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{\gamma^3\Gamma(2\gamma+1)\mathfrak{I}^{3\gamma}}{\Gamma(3\gamma+1)}\right\}+\cdots.
\end{aligned} \tag{81}$$

By putting $\gamma = 1$, we obtain the exact result of modified couple Korteweg–de Vries equation (2):

$$\begin{aligned}
\mathbb{U}(\varphi,\mathfrak{I}) &= \frac{1}{2}\left(2+\tanh\left(\varphi-\frac{11\mathfrak{I}}{2}\right)\right),\\
\mathbb{V}(\varphi,\mathfrak{I}) &= \frac{1}{4}\left(2-\tanh\left(\varphi-\frac{11\mathfrak{I}}{2}\right)\right),\\
\mathbb{W}(\varphi,\mathfrak{I}) &= \left(2-\tanh\left(\varphi-\frac{11\mathfrak{I}}{2}\right)\right).
\end{aligned} \tag{82}$$

In Figures 9–11, the actual and analytical solutions of $\mathbb{U}(\varphi,\mathfrak{I}), \mathbb{V}(\varphi,\mathfrak{I})$, and $\mathbb{W}(\varphi,\mathfrak{I})$ are proved at $\delta = 2$, $\rho = 0.5$, and $\varrho = 1$. In Figures 12–14, the surface and two-dimensional figure for $\mathbb{U}(\varphi,\mathfrak{I}), \mathbb{V}(\varphi,\mathfrak{I})$, and $\mathbb{W}(\varphi,\mathfrak{I})$ for numerous fractional orders are described which demonstrate that the modified decomposition technique and

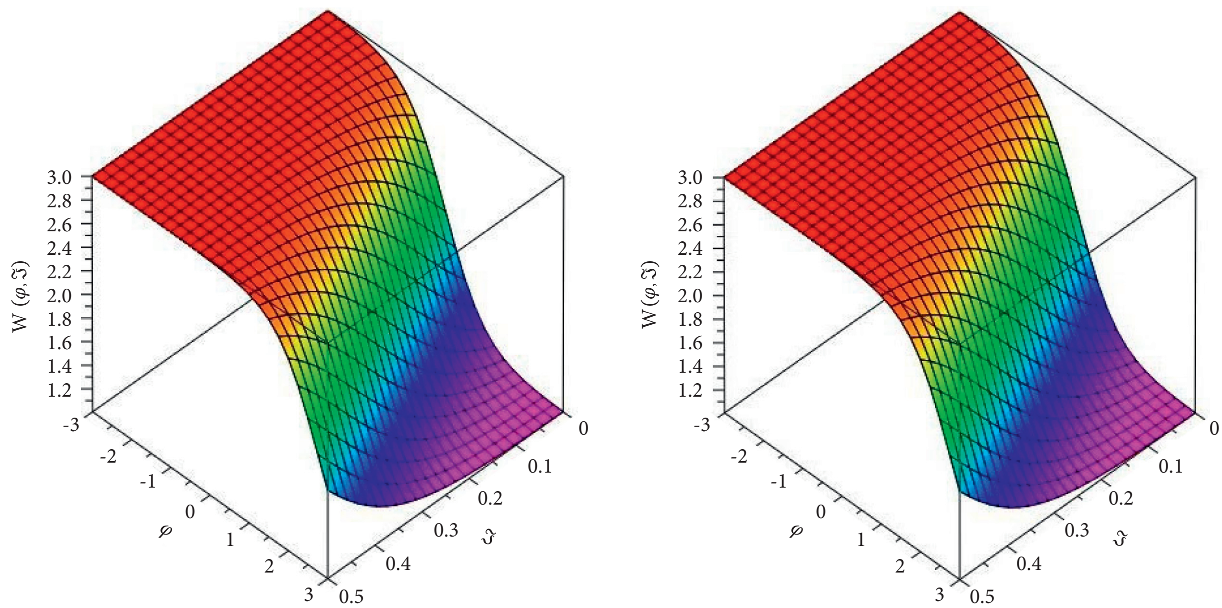


FIGURE 11: The analytical and exact result plot at $\mathbb{W}(\varphi, \mathfrak{I})$ of Example 3 for $\rho = 0.5$, $\varrho = 1$, and $\delta = 2$.

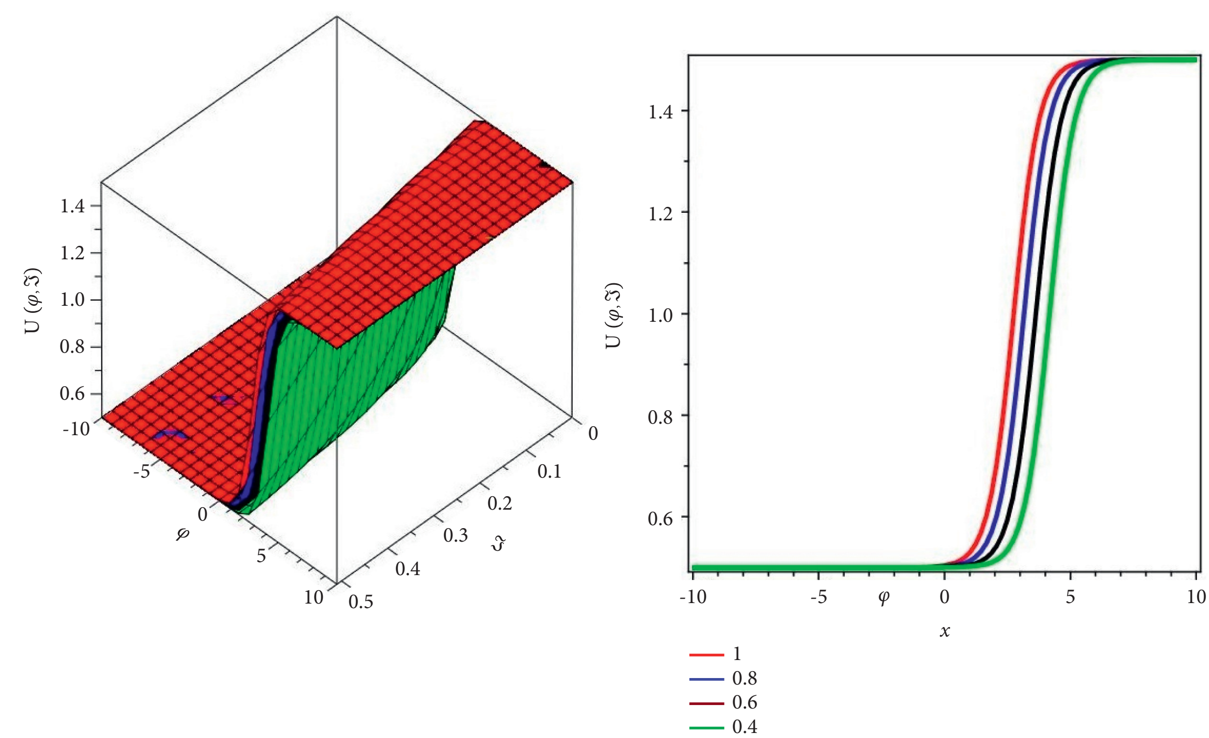


FIGURE 12: The three and two dimensional different fractional order of Example 3 with respect to U (φ, $\mathfrak{I}$).

new iterative transformation technique approximated obtained results are in close contact with the analytical and the exact results. This comparison shows a strong connection among the modified decomposition method and actual solutions. Consequently, the modified decomposition technique and new iterative transformation technique are accurate innovative techniques which need less calculation time and are very simple and more flexible than the homotopy analysis technique and homotopy perturbation technique.

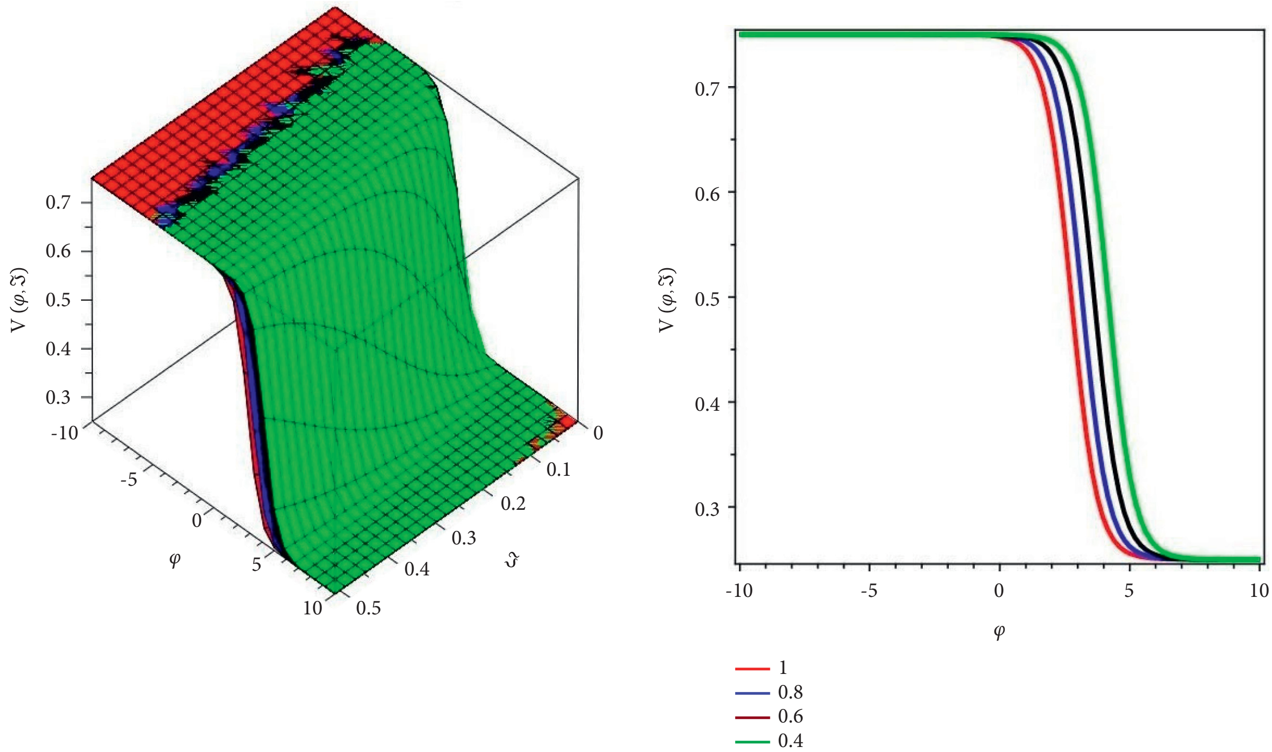


Figure 13: The three and two dimensional different fractional order of Example 3 with respect to V (φ, ℑ).

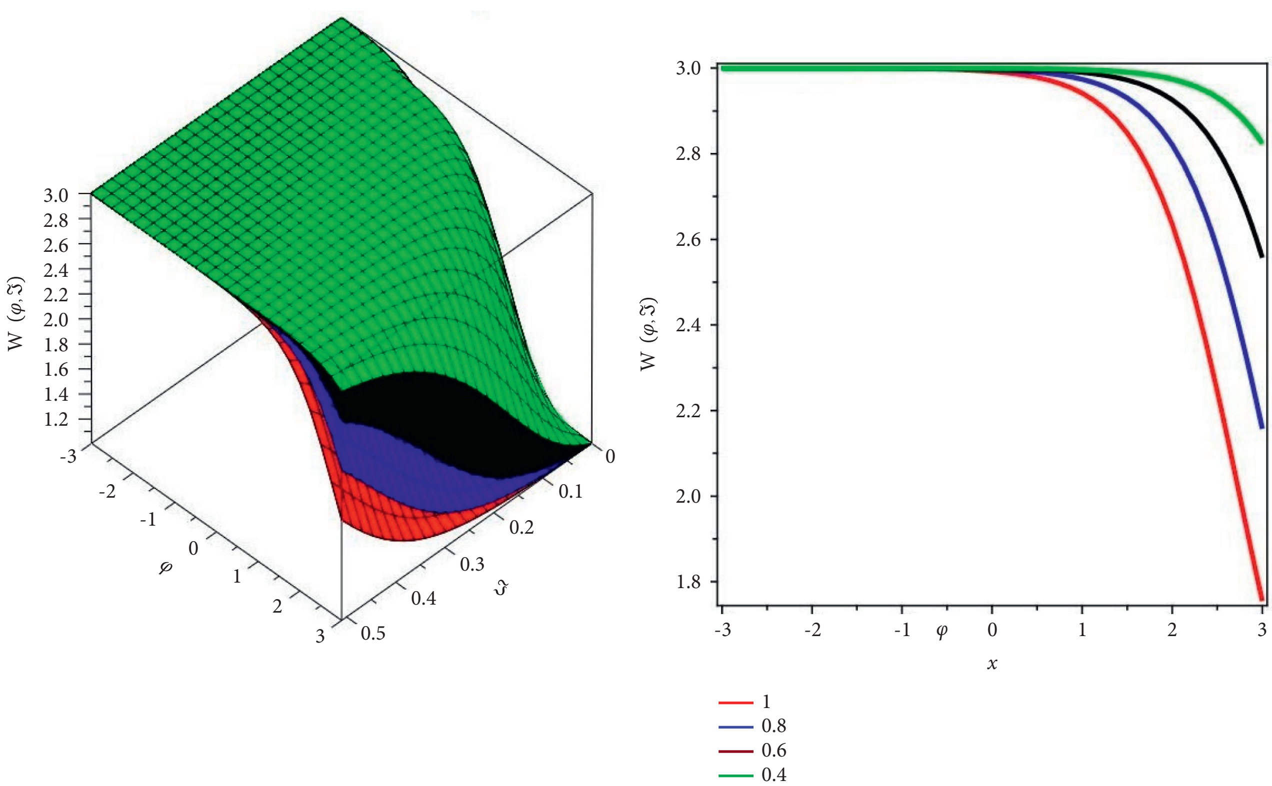


Figure 14: The three and two dimensional different fractional order of Example 3 with respect to W (φ, ℑ).

7. Conclusion

In this article, we have considered the nonlinear fractional-order Korteweg–de Vries equations in the sense of the Atangana–Baleanu derivative which is able to perform more extensive analysis due to the nonsingular kernel in its structure. The mathematical solutions are obtained with the help of the modified decomposition method and new iterative transformation method associated with the Atangana–Baleanu derivative. The present analysis illuminates the effectiveness of the considered derivative operator. We can conclude from the analytical results that these are very reliable, simple, and powerful methods for finding approximate results of many fractional physical models which arise in applied sciences. In this approach, we do not need the Lagrange multiplier, correction functional, and stationary conditions or to calculate heavy integrals because the results established are noise free, which overcomes the shortcomings of existing methods. It is remarkable that the projected approaches are well-organized analytical methods for finding approximate analytical solutions to complex nonlinear partial differential equations. Finally, we conclude that this scheme, in future, will be taken into account in order to cope with other complex nonlinear fractional-order systems of equations.

Data Availability

The numerical data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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