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Research Article

The Role of Soft θ-Topological Operators in Characterizing Various Soft Separation Axioms

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This manuscript begins with an introduction to a soft θ-kernel operator. Then, the main properties and connections of this soft topological operator with other known soft topological operators are examined. We show that soft θ-kernel operator is weaker than soft kernel operator but stronger than soft θ-closure. Both soft θ-closure and soft θ-kernel operators are equivalent on soft compact sets. Furthermore, the stated operators are utilized to obtain several new characterizations of soft R_i-topologies and soft T_j-topologies, for $i = 0, 1$ and $j = 0, 1, 2$.

1. Introduction

The majority of real-world problems in engineering, medicine, economics, the environment, and other professions are fraught with uncertainty. Molodtsov [1] presented the soft set theory in 1999 as a mathematical model for reducing uncertainty. This is freed of the drawbacks of prior theories such as fuzzy set theory [2], rough set theory [3], and so on. The nature of parameter sets, particularly those connected to soft sets, provides a consistent foundation for modeling uncertain data. This leads to the rapid growth of soft set theory in a short amount of time, as well as a wide range of real-world applications of soft sets.

Influenced by the standard postulates of ordinary topological space, Shabir and Naz [4], and Çağman et al. [5], separately, established another branch of topology known as "soft topology," which is a mixture of soft set theory and topology. It focuses on the development of the system of all soft sets. The study in [4, 5], in particular, was essential in building the subject of soft topology. Following these works, researchers have been discussed the topological concepts via soft topological spaces such as soft bases [6] and soft compactness [7]. In [8], the authors applied some soft operators to generate soft topologies.

The separation axioms are simply axioms in the sense that these criteria could be added as additional hypotheses to the definition of topological space to create a more restricted description of what a topological space is. These axioms have a great role in developing (classical) topology. Correspondingly, soft separation axioms are a significant aspect in the late development of soft topology; see for example [9, 10] for soft T_j-separation axioms and [11] for soft R_i-separation axioms. Despite the fact that intensive studies have been conducted on these axioms, however, significant contributions can indeed be made. Hence, we characterize both soft T_j and soft T_i-separation axioms in terms of the discussed soft topological operators. It should be noted that some amendments for a number of properties of separation axioms in soft settings were given in [12, 13].

The following is how the paper's body is organized: we provide an overview of the literature on soft set theory and soft topology in Section 2. The essential points of a soft θ-kernel operator and its link to the associated soft topological operators are discussed in Section 3. Sections 4 and 5 use soft operators provided in Section 3 to characterize soft R_i and soft T_j topologies for $i = 0, 1$ and $j = 0, 1, 2$, respectively. A brief summary and conclusions conclude Section 6 of our paper.

2. Preliminaries

Let X be a domain set, 2^X be the set of all subsets of X, and E be a set of parameters. A pair $(F, E) = \{(e, F(e)): e \in E\}$ is said to be a soft set over X, where $F: E \longrightarrow 2^X$ is a set-valued mapping. The set of all soft sets over X parameterized by E is identified by $S_E(X)$. We call a soft set (F, E) over X a soft point [14, 15], denoted by x_e, if $F(e) = \{x\}$ and $F(e') = \varnothing$ for every $e' \in E$ with $e' \neq e$, where $e \in E$ and $x \in X$. An argument $x_e \in (F, E)$ means that $x \in F(e)$. The set of all soft points over X is denoted by $P_E(X)$. A soft set $(X, E) - (F, E)$ (or simply $(F, E)^c$) is the complement of (F, E), where $F^c: E \longrightarrow 2^X$ is given by $F^c(e) = X - F(e)$ for every $e \in E$. If $(F, E) \in S_E(X)$, it is denoted by Φ if $F(e) = \varnothing$ for every $e \in E$ and is denoted by $\widetilde{X}$ if $F(e) = X$ for every $e \in E$. Evidently, $\widetilde{X}^c = \Phi$ and $\Phi^c = \widetilde{X}$. A soft set (F, E) is called degenerate if $(F, E) = \{x_e\}$ or $(F, E) = \Phi$. It is said that (A, E_1) is a soft subset of (B, E_2) (written by $(A, E_1) \widetilde{\subseteq} (B, E_2)$, [16]) if $E_1 \subseteq E_2$ and $A(e) \subseteq B(e)$ for every $e \in E_1$, and $(A, E_1) = (B, E_2)$ if $(A, E_1) \widetilde{\subseteq} (B, E_2)$ and $(B, E_2) \widetilde{\subseteq} (A, E_1)$. The union of soft sets $(A, E), (B, E)$ is represented by $(F, E) = (A, E) \widetilde{\cup} (B, E)$, where $F(e) = A(e) \cup B(e)$ for every $e \in E$, and intersection of soft sets $(A, E), (B, E)$ is given by $(F, E) = (A, E) \widetilde{\cap} (B, E)$, where $F(e) = A(e) \cap B(e)$ for every $e \in E$ (see, [17]).

Definition 1 (see [4, 5]). A collection $\mathcal{T}$ of $S_E(X)$ is said to be a soft topology on X if it satisfies the following axioms:

(T.1) $\Phi, \widetilde{X} \in \mathcal{T}$.

(T.2) If $(F_1, E), (F_2, E) \in \mathcal{T}$, then $(F_1, E) \widetilde{\cap} (F_2, E) \in \mathcal{T}$.

(T.3) If $\{(F_i, E): i \in I\} \widetilde{\subseteq} \mathcal{T}$, then $\widetilde{\cup}_{i \in I} (F_i, E) \in \mathcal{T}$.

Terminologically, we call $(X, \mathcal{T}, E)$ a soft topological space on X. The elements of $\mathcal{T}$ are called soft open sets. The complements of every soft open or elements of $\mathcal{T}^c$ are called soft closed sets. The lattice of all soft topologies on X is referred to $T_E(X)$ (see, [18]).

Definition 2 (see [4, 19]). Let $(B, E) \in S_E(X)$ and $\mathcal{T} \in T_E(X)$.

(1) The soft closure of (B, E) is $cl(B, E) := \widetilde{\cap}\{(F, E): (B, E) \widetilde{\subseteq} (F, E), (F, E) \in \mathcal{T}^c\}$.

(2) The soft interior of (B, E) is $\operatorname{int}(B, E) := \widetilde{\cup}\{(F, E): (F, E) \widetilde{\subseteq} (B, E), (F, E) \in \mathcal{T}\}$.

(3) The soft kernel of (B, E) is $\ker(B, E) := \widetilde{\cap}\{(G, E): (B, E) \widetilde{\subseteq} (G, E), (G, E) \in \mathcal{T}\}$.

Definition 3. [5] Let $(B, E) \in S_E(X)$ and $\mathcal{T} \in T_E(X)$. A point $x_e \in P_E(X)$ is called a soft limit point of (B, E) if $(G, E) \widetilde{\cap} (B, E) - \{x_e\} \neq \Phi$ for all $(G, E) \in \mathcal{T}$ with $x_e \in (G, E)$. The set of all soft limit points is symbolized by $\operatorname{der}(B, E)$. Then, $\operatorname{cl}(F, E) = (F, E) \widetilde{\cup} \operatorname{der}(F, E)$ (see, Theorem 5 in [5])

Definition 4 (see [20]). Let $\mathcal{T} \in T_E(X)$. A set $(A, E) \in S_E(X)$ is called soft locally closed if there exist $(G, E) \in \mathcal{T}$ and $(F, E) \in \mathcal{T}^c$ such that $(A, E) = (G, E) \widetilde{\cap} (F, E)$. The family of all soft locally closed sets over X is referred to $LC(X)$.

Definition 5. [21] Let $\mathcal{T} \in T_E(X)$. A set $(A, E) \in S_E(X)$ is called soft θ-open if for every $x_e \in (A, E)$, there exists $(G, E) \in \mathcal{T}$ such that $x_e \in (G, E) \widetilde{\subseteq} \operatorname{cl}(G, E) \widetilde{\subseteq} (A, E)$. The set of all soft θ-open sets forms a soft topology on X and denoted by $\mathcal{T}_\theta$. The complement of soft θ-open sets are soft θ-closed and their family is denoted by $\mathcal{T}^c_\theta$.

Remark 1. One can easily check that $\mathcal{T}_\theta \widetilde{\subseteq} \mathcal{T}$.

Definition 6 (see [21]). Let $(B, E) \in S_E(X)$ and $\mathcal{T} \in T_E(X)$.

(1) The soft θ-interior of (B, E) is $\operatorname{int}_\theta(B, E) := \widetilde{\cup}\{(F, E): (F, E) \widetilde{\subseteq} (B, E), (F, E) \in \mathcal{T}_\theta\}$.

(2) The soft θ-closure of (B, E) is $\operatorname{cl}_\theta(B, E) := \widetilde{\cap}\{(F, E): (B, E) \widetilde{\subseteq} (F, E), (F, E) \in \mathcal{T}^c_\theta\}$.

Lemma 1 (see [19, 21]). *Let $(B, E) \in S_E(X)$ and $\mathcal{T} \in T_E(X)$. Then,*

(1) $(B, E) \in \mathcal{T}^c_\theta$ whenever $cl_\theta(B, E) = (B, E)$.

(2) $cl(B, E) \widetilde{\subseteq} cl_\theta(B, E)$.

(3) $cl_\theta(B, E) \in \mathcal{T}^c$.

(4) $cl(B, E) = cl_\theta(B, E)$ whenever $(B, E) \in \mathcal{T}$.

(5) $ker(B, E) = \{x_e \in P_E(X): cl(\{x_e\}) \widetilde{\cap} (B, E) \neq \Phi\}$

Definition 7 (see [19]). For $x_e \in P_E(X)$ and $\mathcal{T} \in T_E(X)$, we define

(1) the soft derived set of x_e as $\operatorname{der}(\{x_e\}) := \operatorname{cl}(\{x_e\}) - \{x_e\}$.

(2) the soft shell of x_e as $\operatorname{shel}(\{x_e\}) := \ker(\{x_e\}) - \{x_e\}$.

(3) the soft set $\langle x_e \rangle := \operatorname{cl}(\{x_e\}) \widetilde{\cap} \ker(\{x_e\})$.

Lemma 2 (see [19]). *The following properties are valid for every $x_e, y_{e'} \in P_E(X)$ and $\mathcal{T} \in T_E(X)$:*

(1) $y_{e'} \in ker(\{x_e\}) \Leftrightarrow x_e \in cl(\{y_{e'}\})$.

(2) $y_{e'} \in shel(\{x_e\}) \Leftrightarrow x_e \in der(\{y_{e'}\})$.

(3) $y_{e'} \in cl(\{x_e\}) \Rightarrow cl(\{y_{e'}\}) \widetilde{\subseteq} cl(\{x_e\})$.

(4) $y_{e'} \in ker(\{x_e\}) \Rightarrow ker(\{y_{e'}\}) \widetilde{\subseteq} ker(\{x_e\})$.

(5) $shel(\{x_e\})$ is degenerate iff for every $y_{e'} \in P_E(X)$ with $y_{e'} \neq x_e$, $der(\{x_e\}) \widetilde{\cap} der(\{y_{e'}\}) = \Phi$.

(6) $der(\{x_e\})$ is degenerate iff for every $y_{e'} \in P_E(X)$ with $y_{e'} \neq x_e$, $shel(\{x_e\}) \widetilde{\cap} shel(\{y_{e'}\}) = \Phi$.

(7) *If* $y_{e'} \in \langle x_e \rangle$, *then* $\langle y_{e'} \rangle = \langle x_e \rangle$.

(8) *Either* $\langle y_{e'} \rangle = \langle x_e \rangle$ *or* $\langle y_{e'} \rangle \widetilde{\cap} \langle x_e \rangle = \Phi$.

Definition 8 (see [11, 22]). A soft space $(X, E, \mathcal{T})$ (or simply soft topology $\mathcal{T} \in T_E(X)$) is called

(1) soft T_0 if for every $x_e, y_{e'} \in P_E(X)$ with $x_e \neq y_{e'}$, there exist $(U, E), (V, E) \in \mathcal{T}$ such that $x_e \in (U, E)$, $y_{e'} \notin (U, E)$ or $y_{e'} \in (V, E)$, $x_e \notin (V, E)$.

(2) soft T_1 if for every $x_e, y_{e'} \in P_E(X)$ with $x_e \neq y_{e'}$, there exist $(U, E), (V, E) \in \mathcal{T}$ such that $x_e \in (U, E)$, $y_{e'} \notin (U, E)$ and $y_{e'} \in (V, E)$, $x_e \notin (V, E)$.

(3) soft T_2 if for every $x_e, y_{e'} \in P_E(X)$ with $x_e \neq y_{e'}$, there exist $(U, E), (V, E) \in \mathcal{T}$ such that $x_e \in (U, E)$, $y_{e'} \in (V, E)$, and $(U, E) \widetilde{\cap} (V, E) = \Phi$.

(4) soft R_0 if for every $x_e \in P_E(X)$ and every $(U, E) \in \mathcal{T}$ with $x_e \in (U, E)$, we have $\mathrm{cl}(\{x_e\}) \widetilde{\subseteq} (U, E)$.

(5) soft R_1 if for every $x_e, y_{e'} \in P_E(X)$ with $\mathrm{cl}(\{x_e\}) \neq \mathrm{cl}(\{y_{e'}\})$, there exist $(U, E), (V, E) \in \mathcal{T}$ such that $\mathrm{cl}(\{x_e\}) \in (U, E)$, $\mathrm{cl}(\{y_{e'}\}) \in (V, E)$ and $(U, E) \widetilde{\cap} (V, E) = \Phi$.

Lemma 3 (see [22], Theorem 4.1). *Let* $\mathcal{T} \in T_E(X)$. *Then,* $\mathcal{T}$ *is soft* T_1 *iff* $cl(\{x_e\}) = \{x_e\}$ *for every* $x_e \in P_E(X)$.

Theorem 1 (see [11], Theorem 3.5). *Let* $\mathcal{T} \in T_E(X)$. *The following properties are equivalent:*

(1) $\mathcal{T}$ *is soft* R_0.

(2) *Either* $cl(\{x_e\}) = cl(\{y_{e'}\})$ *or* $cl(\{x_e\}) \widetilde{\cap} cl\ (\{y_{e'}\}) = \Phi$ *for every* $x_e, y_{e'} \in P_E(X)$ *with* $x_e \neq y_{e'}$.

(3) *For every* $x_e \in P_E(X)$ *and every* $(F, E) \in \mathcal{T}^c$ *with* $x_e \notin (F, E)$, $cl(\{x_e\}) \widetilde{\cap} (F, E) = \Phi$.

(4) *For every* $x_e \in P_E(X)$ *and every* $(F, E) \in \mathcal{T}^c$ *with* $x_e \notin (F, E)$, *there is* $(G, E) \in \mathcal{T}$ *such that* $(F, E) \widetilde{\subseteq} (G, E)$ *and* $x_e \notin (G, E)$.

Theorem 2 (see [11], Theorem 3.13). *Let* $\mathcal{T} \in T_E(X)$. *The following properties are equivalent:*

(1) $\mathcal{T}$ *is soft* R_0.

(2) *If* $(F, E) \in \mathcal{T}^c$, *then* $ker(F, E) = (F, E)$.

(3) *If* $x_e \in (F, E) \in \mathcal{T}^c$, *then* $ker(\{x_e\}) \widetilde{\subseteq} (F, E)$.

(4) $ker(\{x_e\}) \widetilde{\subseteq} cl(\{y_{e'}\})$ *for every* $x_e \in P_E(X)$.

Lemma 4 (see [11], Proposition 3.18). *Let* $\mathcal{T} \in T_E(X)$. *Then,* $\mathcal{T}$ *is soft* T_1 *iff it soft* T_0 *and soft* R_0.

3. Some Soft Topological Operators

In this section, we define "soft θ-kernel" and "soft θ-derived set" as soft topological operators. Then, the connections between soft θ-kernel, soft kernel, soft closure, soft θ-derived set, and soft derived set operators are obtained. The results of the present part will be used to characterize several soft separation axioms.

Definition 9. Let $(F, E) \in S_E(X)$ and let $\mathcal{T} \in T_E(X)$. The soft θ-kernel of (F, E) is defined by

$$\ker_\theta(F, E) := \widetilde{\cap}\{(G, E) : (G, E) \in \mathcal{T}_\theta, (F, E) \widetilde{\subseteq} (G, E)\}. \quad (1)$$

Definition 10. For $x_e \in P_E(X)$ and $\mathcal{T} \in T_E(X)$, we define the soft θ-derived set of x_e as $\mathrm{der}_\theta(\{x_e\}) := \mathrm{cl}_\theta(\{x_e\}) - \{x_e\}$.

Lemma 5. *Let* $(F, E), (G, E) \in S_E(X)$ *and* $\mathcal{T} \in T_E(X)$. *The following properties are valid:*

(1) $(F, E) \widetilde{\subseteq} ker_\theta(F, E)$.

(2) $ker_\theta(F, E) \widetilde{\subseteq} ker_\theta(ker_\theta(F, E))$.

(3) $(F, E) \widetilde{\subseteq} (G, E) \Rightarrow ker_\theta(F, E) \widetilde{\subseteq} ker_\theta(G, E)$.

(4) $ker_\theta[(F, E) \widetilde{\cap} (G, E)] \widetilde{\subseteq} ker_\theta(F, E) \widetilde{\cap} ker_\theta(G, E)$.

(5) $ker_\theta[(F, E) \widetilde{\cup} (G, E)] = ker_\theta(F, E) \widetilde{\cup} ker_\theta(G, E)$.

Proof. Standard.

Recall that a soft space $(X, \mathcal{T}, E)$ is called soft compact [23] if every soft open cover of $\widetilde{X}$ possesses a finite subcover. □

Lemma 6. *The following properties are valid for every* $(F, E) \in P_E(X)$ *and* $\mathcal{T} \in T_E(X)$:

(1) $ker_\theta(F, E) = \{x_e \in P_E(X) : cl_\theta(\{x_e\}) \widetilde{\cap} (F, E) \neq \Phi\}$.

(2) $ker(F, E) \widetilde{\subseteq} ker_\theta(F, E) \widetilde{\subseteq} cl_\theta(F, E)$.

(3) *If* (F, E) *is soft compact, then* $cl_\theta(F, E) = ker_\theta(F, E)$.

Proof

(1) Let $x_e \in \ker_\theta(F, E)$. If $\mathrm{cl}_\theta(\{x_e\}) \widetilde{\cap} (F, E) = \Phi$, then one can find $(G, E) \in \mathcal{T}_\theta$ such that it contains (F, E) but not x_e, a contradiction.

Conversely, if $x_e \notin \ker_\theta(F, E)$ but $\mathrm{cl}_\theta(\{x_e\}) \widetilde{\cap} (F, E) \neq \Phi$, then there is $(G, E) \in \mathcal{T}_\theta$ such that $(F, E) \widetilde{\subseteq} (G, E)$ but $x_e \notin (G, E)$ and $y_{e'} \in \mathrm{cl}_\theta(\{x_e\}) \widetilde{\cap} (F, E)$. Therefore, $\widetilde{X} - (G, E) \in \mathcal{T}_\theta^c$ including x_e but not $y_{e'}$. However, this contradicts to $y_{e'} \in \mathrm{cl}_\theta(\{x_e\}) \widetilde{\cap} (F, E)$. Thus, $x_e \in \ker_\theta(F, E)$.

(2) It follows from the fact that $\mathcal{T}_\theta \widetilde{\subseteq} \mathcal{T}$ and $\mathrm{cl}_\theta(F, E) = \{x_e \in P_E(X) :$ $\mathrm{cl}(G, E) \widetilde{\cap} (F, E) \neq \Phi, (G, E) \in \mathcal{T}, x_e \in (G, E)\}$. That is, $\mathrm{cl}_\theta(F, E)$ can be seen as the intersection of the soft closure of every soft open set (G, E) that includes (F, E). Equivalently, it is a soft closed set including $\ker(F, E)$.

(3) From (2), it suffices to prove that $\mathrm{cl}_\theta(F, E) \widetilde{\subseteq} \ker_\theta(F, E)$. Suppose (F, E) is a soft compact set. If $x_e \notin \ker_\theta(F, E)$, then $\mathrm{cl}_\theta(\{x_e\}) \widetilde{\cap} (F, E) = \Phi$. Therefore, there exist $(G_{y_{e'}}, E)$, $(H_{y_{e'}}, E) \in \mathcal{T}$ such that $x_e \in (G_{y_{e'}}, E)$, $y_{e'} \in (H_{y_{e'}}, E)$, and $(G_{y_{e'}}, E) \widetilde{\cap} (H_{y_{e'}}, E) = \Phi$ for every $y_{e'} \in (F, E)$. Thus, $\mathcal{H} = \{(H_{y_{e'}}, E) : y_{e'} \in (F, E)\}$

forms a soft open cover of (F,E). Then, there is a finite subclass $\{y_{e'}^1, y_{e'}^2, \ldots, y_{e'}^n\}$ of $\mathscr{H}$ such that $(F,E)\widetilde{\subseteq}\widetilde{\cup}_{i=1}^n (H_{y_{e'}^i},E)$. Set $(A,E) = \widetilde{\cap}_{i=1}^n (G_{y_{e'}^i},E)$ and $(B,E) = \widetilde{\cup}_{i=1}^n (H_{y_{e'}^i},E)$. Therefore, (A,E), $(B,E) \in \mathscr{T}$ such that $x_e \widetilde{\in} (A,E)$, $(F,E)\widetilde{\subseteq}(B,E)$, and $(A,E)\widetilde{\cap}(B,E) = \Phi$. This means that $x_e \notin \mathrm{cl}_\theta(F,E)$. We are done. □

Lemma 7. *The following properties are valid for every $x_e, y_{e'} \in P_E(X)$ and $\mathscr{T} \in T_E(X)$:*

(1) $\langle x_e\rangle = \langle y_{e'}\rangle \Leftrightarrow \ker(\{x_e\}) = \ker(\{y_{e'}\}) \Leftrightarrow cl(\{x_e\}) = cl(\{y_{e'}\})$.

(2) $x_e \in cl_\theta(\{y_{e'}\}) \Leftrightarrow y_{e'} \in cl_\theta(\{x_e\})$.

(3) $\ker_\theta(\{x_e\}) = cl_\theta(\{x_e\})$.

(4) $cl(\langle x_e\rangle) = cl(\{x_e\})$.

(5) $cl_\theta(\langle x_e\rangle) = cl_\theta(\{x_e\})$.

(6) $ker(\langle x_e\rangle) = ker(\{x_e\})$.

(7) If $(F,E) \in \mathscr{T}\widetilde{\cup}\mathscr{T}^c$ and $x_e \in (F,E)$, then $\langle x_e\rangle \widetilde{\subseteq} (F,E)$.

Proof

(1) It is enough to show that $\ker(\{x_e\}) = \ker(\{y_{e'}\}) \Leftrightarrow \mathrm{cl}(\{x_e\}) = \mathrm{cl}(\{y_{e'}\})$. If $\ker(\{x_e\}) \neq \ker(\{y_{e'}\})$, then one can find $z_{e^*} \in \ker(\{x_e\})$ but $z_{e^*} \notin \ker(\{y_{e'}\})$. From $z_{e^*} \in \ker(\{x_e\})$, we get $x_e \in \mathrm{cl}(\{z_{e^*}\})$ and then $\mathrm{cl}(\{x_e\})\widetilde{\subseteq}\mathrm{cl}(\{z_{e^*}\})$. Since $z_{e^*} \notin \ker(\{y_{e'}\})$, by Lemma 2. (1), $\mathrm{cl}(\{z_{e^*}\}) \widetilde{\cap} y_{e'} = \Phi$. Therefore, $\mathrm{cl}(\{z_{e^*}\})\widetilde{\cap} y_{e'} = \Phi$ implies $y_{e'} \notin \mathrm{cl}(\{x_e\})$. Hence, $\mathrm{cl}(\{y_{e'}\}) \neq \mathrm{cl}(\{x_e\})$.

(2) If $x_e \notin \mathrm{cl}_\theta(\{y_{e'}\})$, then there are $(G,E), (H,E) \in \mathscr{T}$, respectively, containing x_e, $y_{e'}$ such that $(G,E)\widetilde{\cap}(H,E) = \Phi$. This implies that $y_{e'} \notin \mathrm{cl}_\theta(\{x_e\})$.

(3) It follows from Lemma 6 (3) as every $\{x_e\}$ is soft compact.

(4) Since $x_e \in \langle x_e\rangle$, so $\mathrm{cl}(\langle x_e\rangle)\widetilde{\supseteq}\mathrm{cl}(\{x_e\})$. On the other hand, $\mathrm{cl}(\langle x_e\rangle) = \mathrm{cl}(\mathrm{cl}(\{x_e\})) \widetilde{\cap}\mathrm{cl}(\ker(\{x_e\}))\widetilde{\subseteq}\mathrm{cl}(\{x_e\})$. Hence, $\mathrm{cl}(\langle x_e\rangle) = \mathrm{cl}(\{x_e\})$.

Other parts are similar or simple. □

4. Characterizations of Soft R_i-Spaces, $i \in \{0,1\}$

In this part, we obtain some characterizations of soft R_0 and soft R_1-spaces via certain soft topological operators.

Theorem 3. *Let $\mathscr{T} \in T_E(X)$. Then, $\mathscr{T}$ is soft R_0 iff $cl_\theta(\{x_e\}) - cl(\{x_e\})$ is a union of soft closed sets for every $x_e \in P_E(X)$.*

Proof. Given $x_e \in P_E(X)$. W.l.o.g, we let $\mathrm{cl}_\theta(\{x_e\}) - \mathrm{cl}(\{x_e\}) \neq \Phi$, otherwise, the conclusion is trivially true. Suppose $y_{e'} \in \mathrm{cl}_\theta(\{x_e\}) - \mathrm{cl}(\{x_e\})$. Then, $\mathrm{cl}(\{y_{e'}\})\widetilde{\subseteq}\mathrm{cl}_\theta(\{x_e\})$. Since $\mathscr{T}$ is soft R_0, by Theorem 1, $\mathrm{cl}(\{y_{e'}\})\widetilde{\cap}\mathrm{cl}(\{x_e\}) = \Phi$. Thus, $\mathrm{cl}(\{y_{e'}\})\widetilde{\subseteq}\mathrm{cl}_\theta(\{x_e\}) - \mathrm{cl}(\{x_e\})$ and so $\mathrm{cl}_\theta(\{x_e\}) - \mathrm{cl}(\{x_e\})$ is a union of soft closed sets.

Conversely, let $x_e, y_{e'} \in P_E(X)$. Suppose $\mathrm{cl}_\theta(\{x_e\}) - \mathrm{cl}(\{x_e\}) = \widetilde{\cup}\{(F,E)\colon (F,E) \in \mathscr{T}^c\}$. In order to prove that $\mathscr{T}$ is soft R_0, we study the following cases:

(i) Assume $y_{e'} \in \mathrm{cl}_\theta(\{x_e\}) - \mathrm{cl}(\{x_e\})$. Then, there is $(F,E) \in \mathscr{T}^c$ such that $(F,E)\widetilde{\subseteq}\mathrm{cl}_\theta(\{x_e\}) - \mathrm{cl}(\{x_e\})$ and $y_{e'} \in (F,E)$. Therefore, $\mathrm{cl}(\{y_{e'}\})\widetilde{\subseteq}(F,E)$. This implies that $\mathrm{cl}(\{y_{e'}\})\widetilde{\cap}\mathrm{cl}(\{x_e\}) = \Phi$.

(ii) Assume $y_{e'} \in \mathrm{cl}(\{x_e\})$. Clearly, $y_{e'} \in \mathrm{cl}_\theta(\{x_e\})$ and $x_e \in \mathrm{cl}_\theta(\{y_{e'}\})$. If $x_e \notin \mathrm{cl}(\{y_{e'}\})$, then $x_e \in \mathrm{cl}_\theta(\{y_{e'}\}) - \mathrm{cl}(\{y_{e'}\})$. By (i), $\mathrm{cl}(\{y_{e'}\})\widetilde{\cap}\mathrm{cl}(\{x_e\}) = \Phi$, which is a contradiction. Therefore, we must have $x_e \in \mathrm{cl}(\{y_{e'}\})$ and so $\mathrm{cl}(\{x_e\}) = \mathrm{cl}(\{y_{e'}\})$.

(iii) Assume $y_{e'} \notin \mathrm{cl}_\theta(\{x_e\})$. Suppose if possible $\mathrm{cl}(\{x_e\})\widetilde{\cap}\mathrm{cl}(\{y_{e'}\}) \neq \Phi$, then there is $z_{e^*} \in \mathrm{cl}(\{x_e\})$ and $z_{e^*} \in \mathrm{cl}(\{y_{e'}\})$. By (ii), $\mathrm{cl}(\{x_e\}) = \mathrm{cl}(\{y_{e'}\}) = \mathrm{cl}(\{z_{e^*}\})$. Therefore, $y_{e'} \in \mathrm{cl}(\{x_e\})$, a contradiction. Hence, $\mathrm{cl}(\{x_e\})\widetilde{\cap}\mathrm{cl}(\{y_{e'}\}) = \Phi$.

In conclusion, we have shown that for every $x_e, y_{e'} \in P_E(X)$, either $\mathrm{cl}(\{x_e\}) = \mathrm{cl}(\{y_{e'}\})$ or $\mathrm{cl}(\{x_e\})\widetilde{\cap}\mathrm{cl}(\{y_{e'}\}) = \Phi$. Thus, $\mathscr{T}$ is soft R_0. □

Proposition 1. *Let $\mathscr{T} \in T_E(X)$. Then, $\mathscr{T}$ is soft R_1 iff $\langle x_e\rangle = cl_\theta(\{x_e\})$ for every $x_e \in P_E(X)$.*

Proof. Suppose $\mathscr{T}$ is soft R_1. The first direction is simple. That is $\langle x_e\rangle = \mathrm{cl}(\{x_e\})\widetilde{\cap}\ker(\{x_e\})\widetilde{\subseteq}\mathrm{cl}(\{x_e\})\widetilde{\subseteq}\mathrm{cl}_\theta(\{x_e\})$, see Lemma 2 (2). For the reverse, let $y_{e'} \notin \langle x_e\rangle$. By Lemma 2 (7), $\langle x_e\rangle \neq \langle y_{e'}\rangle$. By Lemma 7 (1), $\mathrm{cl}(\{x_e\}) \neq \mathrm{cl}(\{y_{e'}\})$. Since $\mathscr{T}$ is soft R_1, there are $(G,E), (H,E) \in \mathscr{T}$ such that $x_e \in (G,E)$, $y_{e'} \in (H,E)$, and $(G,E)\widetilde{\cap}(H,E) = \Phi$. This implies $y_{e'} \notin \mathrm{cl}_\theta(\{x_e\})$, and hence, $\mathrm{cl}_\theta(\{x_e\})\widetilde{\subseteq}\langle x_e\rangle$. Thus, $\langle x_e\rangle = cl_\theta(\{x_e\})$.

The converse can be proved similarly. □

Proposition 2. *Let $\mathscr{T} \in T_E(X)$. Then, $\mathscr{T}$ is soft R_1 iff $\langle x_e\rangle \in \mathscr{T}_\theta^c$ for every $x_e \in P_E(X)$.*

Proof. Suppose $\mathscr{T}$ is soft R_1. It suffices to show that $\mathrm{cl}_\theta(\langle x_e\rangle)\widetilde{\subseteq}\langle x_e\rangle$. Let $y_{e'} \in \mathrm{cl}_\theta(\langle x_e\rangle)$. By Lemma 7 (5), $\mathrm{cl}_\theta(\langle x_e\rangle) = \mathrm{cl}_\theta(\{x_e\})$, and so $y_{e'} \in \mathrm{cl}_\theta(\{x_e\})$. This means that for all $(G,E), (H,E) \in \mathscr{T}$ containing $x_e, y_{e'}$, respectively, $(G,E)\widetilde{\cap}(H,E) = \Phi$. We must have $\mathrm{cl}(\{x_e\}) = \mathrm{cl}(\{y_{e'}\})$, otherwise, we get a contradiction as $\mathrm{cl}(\{x_e\})$ and $\mathrm{cl}(\{y_{e'}\})$ can be separated by two disjoint soft open. Hence, $y_{e'} \in \langle x_e\rangle$.

Assume $\langle x_e\rangle \in \mathscr{T}_\theta^c$ for every $x_e \in P_E(X)$. That is, $\mathrm{cl}_\theta(\langle x_e\rangle) = \langle x_e\rangle$. Since $\langle x_e\rangle\widetilde{\subseteq}\ker(\{x_e\})$ and $\mathrm{cl}(\{x_e\})\widetilde{\subseteq}\mathrm{cl}_\theta(\{x_e\}) = \mathrm{cl}_\theta(\langle x_e\rangle)$, then we obtain that $\langle x_e\rangle = \mathrm{cl}(\{x_e\}) = \mathrm{cl}_\theta(\{x_e\}) = \ker(\{x_e\})$. For $x_e, y_{e'} \in P_E(X)$, if $\mathrm{cl}(\{x_e\}) \neq \mathrm{cl}(\{y_{e'}\})$, then $y_{e'} \notin \langle x_e\rangle$. Therefore, there are $(G,E), (H,E) \in \mathscr{T}$ containing $x_e, y_{e'}$, respectively, $(G,E)\widetilde{\cap}(H,E) = \Phi$. Obviously, $\mathrm{cl}(\{x_e\})\widetilde{\subseteq}(G,E)$ and $\mathrm{cl}(\{y_{e'}\})\widetilde{\subseteq}(H,E)$ as $\mathrm{cl}(\{x_e\}) = \ker(\{x_e\})$, $\mathrm{cl}(\{y_{e'}\}) = \ker(\{y_{e'}\})$. Hence, $\mathscr{T}$ is soft R_1. □

Proposition 3. *Let $\mathscr{T} \in T_E(X)$. The following properties are equivalent:*

(1) $\mathscr{T}$ is soft R_1.

(2) $cl_\theta(\{x_e\}) = cl(\{x_e\})$ for every $x_e \in P_E(X)$.

(3) $cl_\theta(\{x_e\}) = ker(\{x_e\})$ for every $x_e \in P_E(X)$.

(4) $cl(\{x_e\}) \in \mathscr{T}_\theta^c$ for every $x_e \in P_E(X)$.

(5) $ker(\{x_e\}) \in \mathscr{T}_\theta^c$ for every $x_e \in P_E(X)$.

Proof. If $\mathscr{T}$ is soft R_1, by Proposition 2, $\mathrm{cl}_\theta(\langle x_e\rangle) = \langle x_e\rangle$. Since $\langle x_e\rangle \tilde{\subseteq} \ker(\{x_e\})$ and $\mathrm{cl}(\{x_e\}) \tilde{\subseteq} \mathrm{cl}_\theta(\{x_e\}) = \mathrm{cl}_\theta(\langle x_e\rangle)$, then we obtain that $\langle x_e\rangle = \mathrm{cl}(\{x_e\}) = \ker(\{x_e\}) = \mathrm{cl}_\theta(\{x_e\})$. The equivalence of these statements can be easily concluded. □

Proposition 4. *Let $\mathscr{T} \in T_E(X)$. The following properties are equivalent:*

(1) $\mathscr{T}$ is soft R_1.

(2) If $(F, E) \in \mathscr{T}^c$ and $x_e \in (F, E)$, then $cl_\theta(\{x_e\}) \tilde{\subseteq} (F, E)$.

(3) If $(G, E) \in \mathscr{T}$ and $x_e \in (G, E)$, then $cl_\theta(\{x_e\}) \tilde{\subseteq} (G, E)$.

Proof. It follows from Lemma 7 (7) and Proposition 1. □

Proposition 5. *Let $\mathscr{T} \in T_E(X)$. Then, $\mathscr{T}$ is soft R_1 iff either $\langle x_e\rangle = \langle y_{e'}\rangle$ or $cl_\theta(\{x_e\}) \tilde{\cap} cl_\theta(\{y_{e'}\}) = \Phi$, for every $x_e, y_{e'} \in P_E(X)$.*

Proof. Assume $\mathscr{T}$ is soft R_1. Given $x_e, y_{e'} \in P_E(X)$, then either $\mathrm{cl}(\{x_e\}) = \mathrm{cl}(\{y_{e'}\})$ or $\mathrm{cl}(\{x_e\}) \neq \mathrm{cl}(\{y_{e'}\})$. If $\mathrm{cl}(\{x_e\}) = \mathrm{cl}(\{y_{e'}\})$, then by Lemma 7 (1), $\langle x_e\rangle = \langle y_{e'}\rangle$. If $\mathrm{cl}(\{x_e\}) \neq \mathrm{cl}(\{y_{e'}\})$, since $\mathscr{T}$ is soft R_1, then there exist disjoint $(U, E), (V, E) \in \mathscr{T}$ such that $\mathrm{cl}(\{x_e\}) \tilde{\subseteq} (U, E)$ and $\mathrm{cl}(\{y_{e'}\}) \tilde{\subseteq} (V, E)$. By Proposition 3 (2), $\mathrm{cl}_\theta(\{x_e\}) \tilde{\cap} \mathrm{cl}_\theta(\{y_{e'}\}) = \Phi$.

Conversely, let $x_e, y_{e'} \in P_E(X)$ such that $\mathrm{cl}(\{x_e\}) \neq \mathrm{cl}(\{y_{e'}\})$. Since $\mathrm{cl}(\{x_e\}) \tilde{\subseteq} \mathrm{cl}_\theta(\{x_e\})$, by assumption, $\mathrm{cl}(\{x_e\}) \tilde{\cap} \mathrm{cl}(\{y_{e'}\}) = \Phi$. Set $(U, E) = \tilde{X} - \mathrm{cl}(\{y_{e'}\})$ and $(V, E) = \tilde{X} - \mathrm{cl}(\{x_e\})$. Therefore, $(U, E), (V, E) \in \mathscr{T}$ such that $\mathrm{cl}(\{x_e\}) \tilde{\subseteq} (U, E)$ and $\mathrm{cl}(\{y_{e'}\}) \tilde{\subseteq} (V, E)$. Hence, $\mathscr{T}$ is soft R_1. □

Proposition 6. *Let $\mathscr{T} \in T_E(X)$. The following properties are equivalent:*

(1) $\mathscr{T}$ is soft R_1.

(2) For every $x_e, y_{e'} \in P_E(X)$, either there exists $(G, E) \in \mathscr{T}$ such that $x_e \in (G, E)$ iff $y_{e'} \in (G, E)$ or there exist disjoint sets $(U, E), (V, E) \in \mathscr{T}$ containing them.

(3) For every $x_e, y_{e'} \in P_E(X)$ with $cl(\{x_e\}) \neq cl(\{y_{e'}\})$, there exist $(F, E), (D, E) \in \mathscr{T}^c$ such that $x_e \in (F, E)$, $y_{e'} \in (D, E)$, and $\tilde{X} = (F, E) \tilde{\cup} (D, E)$.

Proof. It follows from the definition of a soft R_1-space and Proposition 5.

When all of the preceding propositions are added together, the following result arises: □

Theorem 4. *Let $\mathscr{T} \in T_E(X)$. The following properties are equivalent:*

(1) $\mathscr{T}$ is soft R_1.

(2) For every $x_e \in P_E(X)$, $\langle x_e\rangle = cl_\theta(\{x_e\})$.

(3) For every $x_e \in P_E(X)$, $\langle x_e\rangle = cl_\theta(\langle x_e\rangle)$.

(4) For every $x_e \in P_E(X)$, $cl_\theta(\{x_e\}) = cl(\{x_e\})$.

(5) For every $x_e \in P_E(X)$, $cl_\theta(\{x_e\}) = ker(\{x_e\})$.

(6) For every $x_e \in P_E(X)$, $cl(\{x_e\}) \in \mathscr{T}_\theta^c$.

(7) For every $x_e \in P_E(X)$, $ker(\{x_e\}) \in \mathscr{T}_\theta^c$.

(8) If $(F, E) \in \mathscr{T}^c$ and $x_e \in (F, E)$, then $cl_\theta(\{x_e\}) \tilde{\subseteq} (F, E)$.

(9) If $(G, E) \in \mathscr{T}$ and $x_e \in (G, E)$, then $cl_\theta(\{x_e\}) \tilde{\subseteq} (G, E)$.

(10) For every $x_e, y_{e'} \in P_E(X)$, either $\langle x_e\rangle = \langle y_{e'}\rangle$ or $cl_\theta(\{x_e\}) \tilde{\cap} cl_\theta(\{y_{e'}\}) = \Phi$.

(11) For every $x_e, y_{e'} \in P_E(X)$ with $cl(\{x_e\}) \neq cl(\{y_{e'}\})$, there exist $(F, E), (D, E) \in \mathscr{T}^c$ such that $x_e \in (F, E)$, $y_{e'} \in (D, E)$, and $\tilde{X} = (F, E) \tilde{\cup} (D, E)$.

Theorem 5. *For $\mathscr{T} \in T_E(X)$, the following properties are equivalent:*

(1) $\mathscr{T}$ is soft R_1.

(2) $\ker_\theta(F, E) = \ker(F, E)$ *for every $(F, E) \in S_E(X)$.*

(3) $\ker_\theta(F, E) = cl(F, E)$ *for every soft compact $(F, E) \in S_E(X)$.*

(4) $cl_\theta(F, E) = cl(F, E)$ for every soft compact $(F, E) \in S_E(X)$.

Proof

(1) ⇒ (2) Suppose $\mathscr{T}$ is soft R_1. By Theorem 4 (4), we have $\ker_\theta(F, E) = \{x_e \in P_E(X): \mathrm{cl}_\theta(\{x_e\}) \tilde{\cap} (F, E) \neq \Phi\} = \{x_e \in P_E(X): \mathrm{cl}(\{x_e\}) \tilde{\cap} (F, E) \neq \Phi\} = \ker(F, E)$.

(2) ⇒ (3) Given a soft compact set (F, E), by (2) and Lemma 6 (3), $\mathrm{cl}_\theta(F, E) = \ker_\theta(F, E) = \ker(F, E)$. Since $\mathrm{cl}(F, E) \tilde{\subseteq} \mathrm{cl}_\theta(F, E)$, so $\mathrm{cl}(F, E) \tilde{\subseteq} \ker_\theta(F, E)$. By Theorem 2, $\ker(A) \tilde{\subseteq} \mathrm{cl}(F, E)$. Therefore, $\ker_\theta(A) \tilde{\subseteq} \mathrm{cl}(F, E)$. Hence, (3).

(3) ⇒ (4) It derives from Lemma 6 (3).

(4) ⇒ (1) It concludes from Theorem 4 (4). □

5. Characterizations of Soft T_i-Spaces, $i \in \{0, 1, 2\}$

In this section, we give characterizations of soft T_0, soft T_1, and soft T_2-spaces via the soft topological operators mentioned in Section 3.

Theorem 6 (see [19]). *Let $\mathcal{T} \in T_E(X)$. Then, $\mathcal{T}$ is soft T_0 iff $der(\{x_e\})$ is a union of soft closed sets for every $x_e \in P_E(X)$.*

Using the soft θ-derived set operator, a conclusion similar to the above can be established for soft T_1 topologies.

Theorem 7. *Let $\mathcal{T} \in T_E(X)$. Then, $\mathcal{T}$ is soft T_1 iff $der_\theta(\{x_e\})$ is a union of soft closed sets for every $x_e \in P_E(X)$.*

Proof. Suppose $\mathcal{T}$ is soft T_1. By Lemma 4, $\mathcal{T}$ is soft T_0 and soft R_0. By Theorems 3 and 6, we can easily conclude that $\mathrm{der}_\theta(\{x_e\})$ is a union of soft closed sets for every $x_e \in P_E(X)$.

Conversely, given $x_e \in P_E(X)$. If $y_{e'} \in \mathrm{der}(\{x_e\})$, then $y_{e'} \in \mathrm{der}_\theta(\{x_e\})$ and $x_e \in \mathrm{der}_\theta(\{y_{e'}\})$. Therefore, there exists $(F,E) \in \mathcal{T}^c$ with $(F,E) \tilde{\subseteq} \mathrm{der}_\theta(\{y_{e'}\})$ such that $x_e \in (F,E)$. Thus, $\mathrm{cl}(\{x_e\}) \tilde{\subseteq} \mathrm{der}_\theta(\{y_{e'}\})$. This implies that $y_{e'} \notin \mathrm{cl}(\{x_e\})$ and hence $\mathrm{der}(\{x_e\}) = \mathrm{cl}(\{x_e\}) - \{x_e\} = \Phi$. This proves that $\mathcal{T}$ is soft T_1. □

Theorem 8. *Let $\mathcal{T} \in T_E(X)$. Then, $\mathcal{T}$ is soft T_2 iff $cl_\theta(\{x_e\}) = \{x_e\}$ for every $x_e \in P_E(X)$.*

Proof. Suppose $\mathcal{T}$ is soft T_2. Given $x_e \in P_E(X)$. Then, for every $y_{e'} \in P_E(X)$ with $y_{e'} \neq x_e$, there are $(G,E), (H,E) \in \mathcal{T}$ such that $x_e \in (G,E)$, $y_{e'} \in (H,E)$, and $(G,E) \tilde{\cap} (H,E) = \Phi$. Therefore, $(G,E) \tilde{\cap} \mathrm{cl}(H,E) = \Phi$. This means that $y_{e'} \notin \mathrm{cl}_\theta(\{x_e\})$. Hence, $\mathrm{cl}_\theta(\{x_e\}) = \{x_e\}$.

Conversely, suppose $\mathrm{cl}_\theta(\{x_e\}) = \{x_e\}$ for every $x_e \in P_E(X)$. For any $y_{e'} \in P_E(X)$ with $y_{e'} \neq x_e$, $y_{e'} \notin \mathrm{cl}_\theta(\{x_e\})$. This implies that there are disjoint $(G,E), (H,E) \in \mathcal{T}$ such that $x_e \in (G,E)$, $y_{e'} \in (H,E)$. Thus, $\mathcal{T}$ is soft T_2.

The next result is an immediate consequence of the above theorem: □

Corollary 1. *Let $\mathcal{T} \in T_E(X)$. Then, $\mathcal{T}$ is soft T_2 iff $\mathcal{T}_\theta$ is soft T_1 iff $\mathcal{T}_\theta$ is soft T_0.*

Theorem 9. *For $\mathcal{T} \in T_E(X)$, the following properties are equivalent:*

(1) $\mathcal{T}$ is soft T_2.

(2) $cl_\theta(\{x_e\}) \tilde{\cap} cl_\theta(\{y_{e'}\}) = \Phi$ for every $x_e, y_{e'} \in P_E(X)$ with $x_e \neq y_{e'}$.

(3) $\ker_\theta(F,E) = (F,E)$ for every $(F,E) \in S_E(X)$.

(4) $\ker_\theta(F,E) = (F,E)$ for every soft compact $(F,E) \in S_E(X)$.

(5) $cl_\theta(F,E) = (F,E)$ for every soft compact $(F,E) \in S_E(X)$.

Proof

(1) ⇒ (2) Let $x_e, y_{e'} \in P_E(X)$ with $x_e \neq y_{e'}$. By (1) and Theorem 8, there exist disjoint $(G,E), (H,E) \in \mathcal{T}$ such that $\mathrm{cl}_\theta(\{x_e\}) \tilde{\subseteq} (G,E)$ and $\mathrm{cl}_\theta(\{y_{e'}\}) \tilde{\subseteq} (H,E)$. Hence, $\mathrm{cl}_\theta(\{x_e\}) \tilde{\cap} \mathrm{cl}_\theta(\{y_{e'}\}) = \Phi$.

(2) ⇒ (3) By Lemma 6 (1), $\ker_\theta(F,E) = \{x_e: \mathrm{cl}_\theta(\{x_e\}) \tilde{\cap} (F,E) \neq \Phi\}$. Since for every $x_e \in (F,E)$ and for every $y_{e'} \neq x_e$, $\mathrm{cl}_\theta(\{x_e\}) \tilde{\cap} \mathrm{cl}_\theta(\{y_{e'}\}) = \Phi$ and $x_e \in \mathrm{cl}_\theta(\{x_e\})$. We must have that $\ker_\theta(F,E) = (F,E)$.

(3) ⇒ (4) Evident.

(4) ⇒ (1) Since each $\{x_e\}$ is soft compact, by Theorem 8, $\mathcal{T}$ is soft T_2. □

6. Conclusion

Soft separation axioms are collections of conditions for classifying a system of soft topological spaces according to particular soft topological properties. These axioms are usually described in terms of soft open or soft closed sets in a topological space. In this paper, we propose soft θ-kernel and soft θ-derived set operators and make find their relationships with other soft topological operators. Then, the mentioned operators are used to characterize various soft separation axioms. We see that soft θ-kernel, soft θ-closure, and soft θ-derived set operators behave better than their corresponding soft operators for characterizing soft T_i and soft R_j-spaces, where $i = 0, 1, 2$ and $j = 0, 1$ (*c.f.* [19]).

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no competing interests.

References

[1] D. Molodtsov, "Soft set theory—first results," *Computers & Mathematics with Applications*, vol. 37, no. 4-5, pp. 19–31, 1999.

[2] L. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[3] Z. Pawlak, "Rough sets," *International Journal of Computer & Information Sciences*, vol. 11, no. 5, pp. 341–356, 1982.

[4] M. Shabir and M. Naz, "On soft topological spaces," *Computers & Mathematics with Applications*, vol. 61, no. 7, pp. 1786–1799, 2011.

[5] N. Çağman, S. Karataş, and S. Enginoglu, "Soft topology," *Computers & Mathematics with Applications*, vol. 62, no. 1, pp. 351–358, 2011.

[6] J. C. R. Alcantud, "Soft open bases and a novel construction of soft topologies from bases for topologies," *Mathematics*, vol. 8, no. 5, p. 672, 2020.

[7] T. Hida, "A comprasion of two formulations of soft compactness," *Annals of Fuzzy Mathematics and Informatics*, vol. 8, no. 4, pp. 511–524, 2014.

[8] A. A. Azzam, Z. A. Ameen, T. M. Al-shami, and M. E. El-Shafei, "Generating soft topologies via soft set operators," *Symmetry*, vol. 14, no. 5, p. 914, 2022.

[9] T. M. Al-shami and A. Mhemdi, "Two families of separation axioms on infra soft topological spaces," *Filomat*, vol. 36, no. 4, pp. 1143–1157, 2022.

[10] W. K. Min, "A note on soft topological spaces," *Computers & Mathematics with Applications*, vol. 62, no. 9, pp. 3524–3528, 2011.

[11] A. B. Khalaf, N. K. Ahmed, and Q. H. Hamko, “Soft separation axioms and functions with soft closed graphs,” *Proyecciones (Antofagasta)*, vol. 41, no. 1, pp. 177–195, 2022.
[12] T. M. Al-shami, “Comments on some results related to soft separation axioms,” *Afrika Matematika*, vol. 31, no. 7-8, pp. 1105–1119, 2020.
[13] A. Singh and N. S. Noorie, “Remarks on soft axioms,” *Annals of Fuzzy Mathematics and Informatics*, vol. 14, no. 5, pp. 503–513, 2017.
[14] A. Allam, T. H. Ismail, and R. Muhammed, “A new approach to soft belonging,” *Journal of Annals of Fuzzy Mathematics and Informatics*, vol. 13, no. 1, pp. 145–152, 2017.
[15] S. Nazmul and S. Samanta, “Neighbourhood properties of soft topological spaces,” *Annals of Fuzzy Mathematics and Informatics*, vol. 6, no. 1, pp. 1–15, 2013.
[16] P. K. Maji, R. Biswas, and A. R. Roy, “Soft set theory,” *Computers & Mathematics with Applications*, vol. 45, no. 4-5, pp. 555–562, 2003.
[17] M. I. Ali, F. Feng, X. Liu, W. K. Min, and M. Shabir, “On some new operations in soft set theory,” *Computers & Mathematics with Applications*, vol. 57, no. 9, pp. 1547–1553, 2009.
[18] S. Al Ghour and Z. A. Ameen, “Maximal soft compact and maximal soft connected topologies,” *Applied Computational Intelligence and Soft Computing*, vol. 2022, Article ID 9860015, 7 pages, 2022.
[19] T.M. Al-shami, Z. A. Ameen, A. Azzam, and M. E. El-Shafei, Soft separation axioms via soft topological operators, submitted, vol. 7, no. 8, pp. 15107–15119, 2022.
[20] A. H. Kocaman and N. Tozlu, “Soft locally closed sets and decompositions of soft continuity,” *Annals of Fuzzy Mathematics and Informatics*, vol. 11, no. 2, pp. 173–181, 2016.
[21] D. Georgiou, A. Megaritis, and V. Petropoulos, “On soft topological spaces,” *Applied Mathematics & Information Sciences*, vol. 7, no. 5, pp. 1889–1901, 2013.
[22] S. Bayramov and C. Gunduz, “A new approach to separability and compactness in soft topological spaces,” *TWMS Journal of Pure and Applied Mathematics*, vol. 9, no. 21, pp. 82–93, 2018.
[23] A. Aygünoğlu and H. Aygün, “Some notes on soft topological spaces,” *Neural Computing & Applications*, vol. 21, no. S1, pp. 113–119, 2012.