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Research Article

Further on Inequalities for $(\alpha, h-m)$-Convex Functions via k-Fractional Integral Operators

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The purpose of this article is to demonstrate new generalized k-fractional Hadamard and Fejér–Hadamard integral inequalities for $(\alpha, h-m)$-convex functions. To prove these inequalities, k-fractional integral operators including the generalization of the Mittag–Leffler function are used. The results presented in this article can be considered an important advancement of previously published inequalities.

1. Introduction

Theory of convexity offers an effective and charming area of research and is also a theory that featured prominently and surprisingly in distinct disciplines such as mathematical analysis, optimization, economics, finance, engineering and game theory. Convexity theory is very closely related with the theory of inequalities. Many inequalities well known in the literature are direct applications of the properties of convex functions. The usage of fractional integral operators for getting the generalized types of classic inequalities has become an important method in advanced mathematical studies of inequalities.

One of the convexity theory studies in the literature belongs to Gao et al. [1]. They presented a new type of functions called n-polynomial harmonically exponential type convex, and specified some of their algebraic features. Mehrez and Agarwal [2] established new type of integral inequalities for convex functions and indicated new inequalities for some special and q-special functions. Tariq [3] defined the concept of p–harmonic exponential type convex functions. Also, they investigated some integral inequalities in the form of applications for some means. Another study on convexity theory and inequalities was presented by Butt et al. [4]. They presented the notion of m–polynomial p-harmonic exponential type convex functions and demonstrated various new integral inequalities. Srivastava et al. [5] obtained a new class of the bi-close-to-convex functions described in the open unit disk by using the Borel distribution series of the Mittag–Leffler type. Also, the authors demonstrated the Fekete–Szego type inequalities via the bi-close-to-convex function class.

Fractional calculus, which is the study of integrals and derivatives of fractional order, has expanded significantly over the late nineteenth century. It ranges from chemical, viscoelasticity, and statistical physics to electrical and mechanical engineering. The fundamental working doctrine of fractional analysis is to present new fractional derivative and integral operators, and to analyze the benefits of these operators through the instrument of modeling studies, and collations. Integral operators, which form a significant part of fractional calculus, are now resources of many fields such as inequality theory, engineering, statistics, mathematical biology, and modeling, which take advantage of fractional analysis. Many inequalities have been generalized through the instrument of fractional integral operators and provide construction of new approximations.

One of the fractional calculus studies in the literature belongs to Abdeljawad et al. [6]. They obtained generalized Hermite–Hadamard type inequalities and generalized Simpson type inequalities for (s, m)-convex functions with the help of local fractional integration. Akdemir et al. [7] used generalized fractional integral operators. By using these operators, they proved new and general variants of Chebyshev's inequality. Butt et al. [8] established a general integral identity to acquire new integral inequalities of several Hadamard types. For this purpose, they used a new version of the Atangana–Baleanu integral operator. Khan et al. [9] explored two fractional integral operators related to Fox H-function owing to Saxena and Kumbhat. They proved series expansion of the images of the M-series with the help of these fractional operators. Another study to k-fractional integrals was presented by Qi et al. [10]. They constructed some generalized fractional integral inequalities of the Hermite–Hadamard type via (α, m)-convex functions. Also, they demonstrated that one can get and expand some Riemann–Liouville fractional integral inequalities and classical integral inequalities of Hermite–Hadamard's type. Tunc et al. [11] presented the generalized k-fractional integrals of a function with respect to the another function that generalizes many several types of fractional integrals. Also, they studied trapezoid inequalities for the functions whose derivatives in absolute value are convex. Önalan et al. [12] proved many Hermite–Hadamard type integral inequalities for functions whose absolute values of the second derivatives are s-convex and s-concave using fractional integral operators with the Mittag–Leffler kernel. Zhu et al. [13] explored a weighted integral identity of Simpson-like type. Relying on this identity, they obtained some estimation-type results connected with the weighted Simpson-like type integral inequalities for the first order differentiable functions. Srivastava et al. [14] established the homogeneous q-shift operator and the homogeneous q-difference operator. Based on these operators, they searched generalized Cauchy and Hahn polynomials.

2. Preliminaries

Now let us define some important functions.

Definition 1 (see [15]). A function $\varphi\colon [a, b] \longrightarrow \mathbb{R}$ *is called a convex function, if*

$$\varphi(\eta\iota + (1-\eta)\kappa) \leq \eta\varphi(\iota) + (1-\eta)\varphi(\kappa) \tag{1}$$

holds for all $\iota, \kappa \in [a, b]$ and $\eta \in [0, 1]$.

Definition 2 (see [16]). The function $\varphi\colon [0, b] \longrightarrow \mathbb{R}$, $b > 0$, *is called the* (α, m)*-convex function, if*

$$\varphi(\eta\iota + m(1-\eta)\kappa) \leq \eta^{\alpha}\varphi(\iota) + m(1-\eta^{\alpha})\varphi(\kappa) \tag{2}$$

holds for all $\iota, \kappa \in [0, b]$, $\eta \in [0, 1]$ and $(\alpha, m) \in [0, 1]^2$.

Definition 3 (see [17]). *A function* $\varphi\colon [0, b] \longrightarrow \mathbb{R}$ *is said to be* (s, m)*-convex, if*

$$\varphi(\eta\iota + m(1-\eta)\kappa) \leq \eta^{s}\varphi(\iota) + m(1-\eta^{s})\varphi(\kappa) \tag{3}$$

holds for all $\iota, \kappa \in [0, b]$, $\eta \in [0, 1]$ and $(s, m) \in (0, 1]$.

Definition 4 (see [18]). A function $\varphi\colon [0, b] \longrightarrow \mathbb{R}$ *is said to be* (s, m)*-convex in the second sense, if*

$$\varphi(\eta\iota + m(1-\eta)\kappa) \leq \eta^{s}\varphi(\iota) + m(1-\eta)^{s}\varphi(\kappa) \tag{4}$$

holds for all $\iota, \kappa \in [0, b]$, $\eta \in [0, 1]$ and $(s, m) \in (0, 1]^2$.

Definition 5 (see [19]). *Let* $J \subseteq \mathbb{R}$ *be an interval including* $(0, 1)$ *and let* $h\colon J \longrightarrow \mathbb{R}$ *be a nonnegative function. Then the function* $\varphi\colon [0, b] \longrightarrow \mathbb{R}$ *is called the* $(h - m)$*-convex function, if*

$$\varphi(\eta\iota + m(1-\eta)\kappa) \leq h(\eta)\varphi(\iota) + mh(1-\eta)\varphi(\kappa) \tag{5}$$

holds for all $\iota, \kappa \in [0, b]$, $\eta \in [0, 1]$ and $m \in [0, 1]$.

Definition 6 (see [20]). *Let* $J \subseteq \mathbb{R}$ *be an interval including* $(0, 1)$ *and let* $h\colon J \longrightarrow \mathbb{R}$ *be a nonnegative function. Then the function* $\varphi\colon [0, b] \longrightarrow \mathbb{R}$ *is called the* $(\alpha, h - m)$*-convex function, if*

$$\varphi(\eta\iota + m(1-\eta)\kappa) \leq h(\eta^{\alpha})\varphi(\iota) + mh(1-\eta^{\alpha})\varphi(\kappa) \tag{6}$$

holds for all $\iota, \kappa \in [0, b]$, $\eta \in [0, 1]$ and $(\alpha, m) \in [0, 1]^2$.

Remark 1

(i) *By taking* $m = \alpha = 1$ *and* $h(\eta) = \eta$ *in* (6), *we obtain the definition of convex function* (1).

(ii) *By taking* $h(\eta) = \eta$ *in* (6), *we obtain the definition of* (α, m)*-convex function* (2).

(iii) *By taking* $h(\eta) = \eta$ *and* $\alpha = s$ *in* (6), *we obtain the definition of* (s, m)*-convex function* (3).

(iv) *By taking* $h(\eta) = \eta^{s}$ *and* $\alpha = 1$ *in* (6), *we obtain the definition of* (s, m)*-convex function in the second sense* (4).

(v) *By taking* $\alpha = 1$ *in*(6), *we obtain the definition of* (h, m)*-convex function* (5).

(vi) *By taking* $\alpha = m = h(\eta) = 1$ *in* (6), *we obtain the definition of p-function described by Dragomir* et al. *in* [21].

Now let us represent some definitions of fractional integral operators that will form the basis for this article.

Definition 7 (see [22]). Let $\gamma, c, w, \alpha, l, \in \mathbb{C}, \mathfrak{R}(l), \mathfrak{R}(\alpha) > 0$, $\mathfrak{R}(c) > \mathfrak{R}(\gamma) > 0$ *with* $\mu, \delta > 0$, $\tilde{p} \geq 0$, *and* $0 < \nu \leq \delta + \mu$. *Let* $\varphi \in L_1[a, b]$, $\iota \in [a, b]$. *In that case, the generalized fractional operators are defined by*

$$\begin{aligned}
\left(F^{\gamma,\delta,\nu,c}_{\mu,\alpha,l,w,a+}\varphi\right)(\iota;\tilde{p}) &= \int_a^{\iota} (\iota-\eta)^{\alpha-1} E^{\gamma,\delta,\nu,c}_{\mu,\alpha,l}\left(w(\iota-\eta)^{\mu};\tilde{p}\right)\varphi(\eta)\mathrm{d}\eta,\\
\left(F^{\gamma,\delta,\nu,c}_{\mu,\alpha,l,w,b-}\varphi\right)(\iota;\tilde{p}) &= \int_{\iota}^{b} (\eta-\iota)^{\alpha-1} E^{\gamma,\delta,\nu,c}_{\mu,\alpha,l}\left(w(\eta-\iota)^{\mu};\tilde{p}\right)\varphi(\eta)\mathrm{d}\eta,
\end{aligned} \tag{7}$$

where

$$E_{\mu,\alpha,l}^{\gamma,\delta,\nu,c}(\eta;\tilde{p}) = \sum_{n=0}^{\infty}\frac{\beta_{\tilde{p}}(\gamma+n\nu, c-\gamma)}{\beta(\gamma, c-\gamma)}\frac{(c)_{n\nu}}{\Gamma(\mu n+\alpha)}\frac{\eta^n}{(l)_{n\delta}} \quad (8)$$

is generalized extended Mittag–Leffler function, and $\beta_{\tilde{p}}$ is the expansion of beta function described as below:

$$\beta_{\tilde{p}}(\iota,\kappa) = \int_0^1 \eta^{\iota-1}(1-\eta)^{\kappa-1}e^{-(\tilde{p}/\eta(1-\eta))}\mathrm{d}\eta, \quad (9)$$

where $\mathfrak{R}(\iota), \mathfrak{R}(\kappa), \mathfrak{R}(\tilde{p}) > 0$.

Definition 8 (see [23]). Let φ, ψ: $[a,b] \longrightarrow \mathbb{R}$ with $0<a<b$, be the functions, φ be positive, $\varphi \in L_1[a,b]$ and ψ be differentiable and strictly increasing. Let (ϕ/ι) be an increasing on $[a,\infty)$, $\gamma, c, w, \alpha, l \in \mathbb{C}$, $\mathfrak{R}(l), \mathfrak{R}(\alpha) > 0$, $\mathfrak{R}(c) > \mathfrak{R}(\gamma) > 0$ with $\mu, \delta > 0$, $\tilde{p} \geq 0$, and $0 < \nu \leq \mu + \delta$. In that case, for $\iota \in [a,b]$, the fractional operators are described by

$$\begin{aligned}\left({}_{\psi}F_{\mu,\alpha,l,w,a+}^{\phi,\gamma,\delta,\nu,c}\varphi\right)(\iota;\tilde{p}) &= \int_a^{\iota}\frac{\phi(\psi(\iota)-\psi(\eta))}{\psi(\iota)-\psi(\eta)}E_{\mu,\alpha,l}^{\gamma,\delta,\nu,c}\left(w(\psi(\iota)-\psi(\eta))^{\mu};\tilde{p}\right)\psi\prime(\eta)\varphi(\eta)\mathrm{d}\eta,\\ \left({}_{\psi}F_{\mu,\alpha,l,w,b-}^{\phi,\gamma,\delta,\nu,c}\varphi\right)(\iota;\tilde{p}) &= \int_{\iota}^{b}\frac{\phi(\psi(\eta)-\psi(\iota))}{\psi(\eta)-\psi(\iota)}E_{\mu,\alpha,l}^{\gamma,\delta,\nu,c}\left(w(\psi(\eta)-\psi(\iota))^{\mu};\tilde{p}\right)\psi\prime(\eta)\varphi(\eta)\mathrm{d}\eta.\end{aligned} \quad (10)$$

Definition 9 (see [23]). Let φ, ψ: $[a,b] \longrightarrow \mathbb{R}$ with $0<a<b$, be the functions such that φ be positive and $\varphi \in L_1[a,b]$ and ψ be differentiable and strictly increasing. Let $\gamma, c, w, \alpha, l, \in \mathbb{C}$, $\mathfrak{R}(l), \mathfrak{R}(\alpha) > 0$, $\mathfrak{R}(c) > \mathfrak{R}(\gamma) > 0$, $\mu, \delta > 0$, $\tilde{p} \geq 0$, and $0 < \nu \leq \mu + \delta$. In that case, for $\iota \in [a,b]$, the united operators are described by

$$\begin{aligned}\left({}_{\psi}F_{\mu,\alpha,l,w,a+}^{\gamma,\delta,\nu,c}\varphi\right)(\iota;\tilde{p}) &= \int_a^{\iota}(\psi(\iota)-\psi(\eta))^{\alpha-1}E_{\mu,\alpha,l}^{\gamma,\delta,\nu,c}\left(w(\psi(\iota)-\psi(\eta))^{\mu};\tilde{p}\right)\psi'(\eta)\varphi(\eta)\mathrm{d}\eta,\\ \left({}_{\psi}F_{\mu,\alpha,l,w,b-}^{\phi,\gamma,\delta,\nu,c}\varphi\right)(\iota;\tilde{p}) &= \int_{\iota}^{b}(\psi(\eta)-\psi(\iota))^{\alpha-1}E_{\mu,\alpha,l}^{\gamma,\delta,\nu,c}\left(w(\psi(\eta)-\psi(\iota))^{\mu};\tilde{p}\right)\psi'(\eta)\varphi(\eta)\mathrm{d}\eta.\end{aligned} \quad (11)$$

Recently, Yue et al. defined generalized k-fractional operators including a further extension of Mittag–Leffler function in [24] as noted below:

Definition 10. Let φ, ψ: $[a,b] \longrightarrow \mathbb{R}$ with $0<a<b$; be the functions such that φ be positive and $\varphi \in L_1[a,b]$ and ψ be differentiable and strictly increasing. Let $\gamma, c, w, \alpha, l, \in \mathbb{R}$ and $\alpha > k$, $l, \alpha > 0$, $c > \gamma > 0$ with $0 < \nu \leq \delta + \mu$, $\tilde{p} \geq 0$ and $\mu, \delta > 0$. In that case, for $\iota \in [a,b]$, the right-left generalized k-fractional operators $({}_{\psi}^{k}F_{\mu,\alpha,l,w,a+}^{\gamma,\delta,\nu,c}\varphi)$ and $({}_{\psi}^{k}F_{\mu,\alpha,l,w,b-}^{\gamma,\delta,\nu,c}\varphi)$ are defined by

$$\left({}_{\psi}^{k}F_{\mu,\alpha,l,w,a+}^{\gamma,\delta,\nu,c}\varphi\right)(\iota;\tilde{p}) = \int_a^{\iota}(\psi(\iota)-\psi(\eta))^{(\alpha/k)-1}E_{\mu,\alpha,l}^{\gamma,\delta,\nu,c}\left(w(\psi(\iota)-\psi(\eta))^{\mu};\tilde{p}\right)\psi\prime(\eta)\varphi(\eta)\mathrm{d}\eta, \quad (12)$$

$$\left(g_k F_{\mu,\alpha,l,w,b-}^{\gamma,\delta,\nu,c}\varphi\right)(\iota;\tilde{p}) = \int_{\iota}^{b}(\psi(\eta)-\psi(\iota))^{(\alpha/k)-1}E_{\mu,\alpha,l}^{\gamma,\delta,\nu,c}\left(w(\psi(\eta)-\psi(\iota))^{\mu};\tilde{p}\right)\psi\prime(\eta)\varphi(\eta)\mathrm{d}\eta. \quad (13)$$

The following inequality is the admitted Hadamard inequality.

Theorem 1. *Let φ: $[a,b] \longrightarrow \mathbb{R}$ with $a<b$, be a convex function. In that case, the below inequality occurs:*

$$\varphi\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b \varphi(\iota)d\iota \leq \frac{\varphi(a)+\varphi(b)}{2}. \quad (14)$$

Theorem 2. *Let φ: $[a,b] \longrightarrow \mathbb{R}$ be a convex and ψ: $[a,b] \longrightarrow \mathbb{R}$ be nonnegative and symmetric in respect of $((a+b)/2)$ and integrable. In that case, the below inequality occurs:*

$$\varphi\left(\frac{a+b}{2}\right)\int_a^b \psi(\iota)d\iota \leq \int_a^b \varphi(\iota)\psi(\iota)\mathrm{d}\iota \leq \frac{\varphi(a)+\varphi(b)}{2}\int_a^b \psi(\iota)\mathrm{d}\iota. \quad (15)$$

This inequality in [25] presented by Fejér is known as a weighted type of Hadamard's inequality.

Many authors have been established several refinements and extensions of the Hadamard and the Fejér–Hadamard inequalities for various fractional integral operators (for details see, [2, 7, 11, 16, 17, 19–21, 26–34] and references therein). This article aims to derive the Hadamard and Fejér–Hadamard inequalities about generalized k-fractional integrals involving Mittag–Leffler functions via $(\alpha, h-m)$-convex functions. In the upcoming section, we will utilize k-fractional integral operators and $(\alpha, h-m)$-convexity to prove the two versions of the Hadamard inequality and the Fejér–Hadamard inequality.

3. The k-Fractional Inequalities of Hadamard and Fejér–Hadamard Type

In this section, we first describe the below generalized k-fractional Hadamard's inequality.

Theorem 3. *Let $h: J \longrightarrow \mathbb{R}$ is nonnegative, nonzero and integrable function and $\varphi, \psi: [a,b] \longrightarrow \mathbb{R}$, $0 \le a < mb$, be the functions such that $\varphi \in L_1[a,b]$ and φ be positive and ψ be differentiable and strictly increasing. If φ is $(\alpha, h-m)$-convex, the below inequalities for k-fractional operators (12) and (13) occur:*

$$\begin{aligned}
&\varphi\left(\frac{\psi(a)+m\psi(b)}{2}\right)\left({}_{\psi}^{k}F^{\gamma,\delta,\nu,c}_{\mu,\tau,l,\overline{w},a+}1\right)(m\psi(b);\widetilde{p})\\
&\le h\left(\frac{1}{2^{\alpha}}\right)\left({}_{\psi}^{k}F^{\gamma,\delta,\nu,c}_{\mu,\tau,l,\overline{w},a+}\varphi^{\circ}\psi\right)(m\psi(b);\widetilde{p}) + m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\left({}_{\psi}^{k}F^{\gamma,\delta,\nu,c}_{\mu,\tau,l,\overline{w}m^{\mu},b-}\varphi^{\circ}\psi\right)\left(\frac{\psi(a)}{m};\widetilde{p}\right)\\
&\le \left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(a)) + m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(\psi(b))\right]\int_0^1 \eta^{(\tau/k)-1}E^{\gamma,\delta,\nu,c}_{\mu,\tau,l}(w\eta^{\mu};\widetilde{p})h(\eta^{\alpha})\mathrm{d}\eta\\
&+ m\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(b)) + m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi\left(\frac{\psi(a)}{m^2}\right)\right]\int_0^1 \eta^{(\tau/k)-1}E^{\gamma,\delta,\nu,c}_{\mu,\tau,l}(w\eta^{\mu};\widetilde{p})h(1-\eta^{\alpha})\mathrm{d}\eta,
\end{aligned} \tag{16}$$

where $\overline{w} = (w/(m\psi(b)-\psi(a))^{\mu})$ for all $\eta \in [a,b]$.

Proof. Since φ is $(\alpha, h-m)$-convex on $[a,b]$, for all $\iota, \kappa \in [a,b]$, we have

$$\varphi\left(\frac{\psi(\iota)+m\psi(\kappa)}{2}\right) \le h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(\iota)) + mh\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(\psi(\kappa)). \tag{17}$$

Setting $\psi(\iota) = \eta\psi(a) + m(1-\eta)\psi(b)$ and $\psi(\kappa) = (\psi(a)/m)(1-\eta) + \eta\psi(b)$ in above inequality, we have

$$\begin{aligned}
\varphi\left(\frac{\psi(a)+m\psi(b)}{2}\right) &\le h\left(\frac{1}{2^{\alpha}}\right)\varphi(\eta\psi(a) + m(1-\eta)\psi(b))\\
&+ mh\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi\left(\frac{\psi(a)}{m}(1-\eta) + \eta g(b)\right).
\end{aligned} \tag{18}$$

Multiplying both sides of (18) by $\eta^{(\tau/k)-1}E^{\gamma,\delta,\nu,c}_{\mu,\tau,l}(w\eta^{\mu};\widetilde{p})$, then integrating over $[0,1]$, we have

$$\begin{aligned}
&\varphi\left(\frac{\psi(a)+m\psi(b)}{2}\right)\int_0^1 \eta^{(\tau/k)-1}E^{\gamma,\delta,\nu,c}_{\mu,\tau,l}(w\eta^{\mu};\widetilde{p})d\eta\\
&\le h\left(\frac{1}{2^{\alpha}}\right)\int_0^1 \eta^{(\tau/k)-1}E^{\gamma,\delta,\nu,c}_{\mu,\tau,l}(w\eta^{\mu};\widetilde{p})\varphi(\eta\psi(a) + m(1-\eta)\psi(b))\mathrm{d}\eta\\
&+ mh\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\int_0^1 \eta^{(\tau/k)-1}E^{\gamma,\delta,\nu,c}_{\mu,\tau,l}(w\eta^{\mu};\widetilde{p})\varphi\left(\frac{\psi(a)}{m}(1-\eta) + \eta\psi(b)\right)\mathrm{d}\eta.
\end{aligned} \tag{19}$$

By specifying $\psi(\iota) = \eta\psi(a) + m(1-\eta)\psi(b)$ and $\psi(\kappa) = (\psi(a)/m)(1-\eta) + \eta\psi(b)$ in (19), we have

$$\begin{aligned}
&\varphi\left(\frac{\psi(a)+m\psi(b)}{2}\right)\int_a^{\psi^{-1}(m\psi(b))}(m\psi(b)-\psi(\iota))^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}(m\psi(b)-\psi(\iota))^{\mu};\widetilde{p}\right)\psi\prime(\iota)d\iota\\
&\leq h\left(\frac{1}{2^{\alpha}}\right)\int_a^{\psi^{-1}(m\psi(b))}(m\psi(b)-\psi(\iota))^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}(m\psi(b)-\psi(\iota))^{\mu};\widetilde{p}\right)\varphi(\psi(\iota))\psi\prime(\iota)d\iota\\
&\quad+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\int_{\psi^{-1}(\psi(a)/m)}^{b}\left(\psi(\kappa)-\frac{\psi(a)}{m}\right)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}m^{\mu}\left(\psi(\kappa)-\frac{\psi(a)}{m}\right)^{\mu};\widetilde{p}\right)\varphi(\psi(\kappa))\psi\prime(\kappa)d\kappa.
\end{aligned}\tag{20}$$

By usage k-fractional operators (12) and (13), the first side of (16) is achieved.

To evidence the second side of (16), once again $(\alpha, h-m)$-convexity of φ over $[a,b]$, for $\eta \in [0,1]$, we achieve

$$\begin{aligned}
&h\left(\frac{1}{2^{\alpha}}\right)\varphi(\eta\psi(a)+m(1-\eta)\psi(b))+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi\left((1-\eta)\frac{\psi(a)}{m}+\eta\psi(b)\right)\\
&\leq h(\eta^{\alpha})\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(a))+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(\psi(b))\right]\\
&\quad+mh(1-\eta^{\alpha})\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(b))+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi\left(\frac{\psi(a)}{m^{2}}\right)\right].
\end{aligned}\tag{21}$$

Multiplying both sides of (21) by $\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\widetilde{p})$, next integrating over $[0,1]$, we achieve

$$\begin{aligned}
&h\left(\frac{1}{2^{\alpha}}\right)\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\widetilde{p})\varphi(\eta\psi(a)+m(1-\eta)\psi(b))d\eta\\
&\quad+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\widetilde{p})\varphi\left((1-\eta)\frac{\psi(a)}{m}+\eta\psi(b)\right)d\eta\\
&\leq\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(a))+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(\psi(b))\right]\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\widetilde{p})h(\eta^{\alpha})d\eta\\
&\quad+m\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(b))+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi\left(\frac{\psi(a)}{m^{2}}\right)\right]\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\widetilde{p})h(1-\eta^{\alpha})d\eta.
\end{aligned}\tag{22}$$

Setting $\psi(\iota) = \eta\psi(a) + m(1-\eta)\psi(b)$ and $\psi(\kappa) = (1-\eta)(\psi(a)/m) + \eta\psi(b)$ in (22), in that case by utilizing k-fractional operators (12) and (13), the second side of (16) is achieved. □

Corollary 1. *By usage (16), anymore k-fractional inequalities are offered as noted below:*

(i) By choosing $\psi = I$ and $\widetilde{p} = w = 0$, we obtain

$$\begin{aligned}
&\varphi\left(\frac{a+mb}{2}\right)\int_a^{mb}(mb-\iota)^{(\tau/k)-1}\mathrm{d}\iota\\
&\leq h\left(\frac{1}{2^\alpha}\right)\int_a^{mb}(mb-\iota)^{(\tau/k)-1}\varphi(\iota)\mathrm{d}\iota+m^{(\tau/k)+1}h\left(\frac{2^\alpha-1}{2^\alpha}\right)\int_{\frac{a}{m}}^{b}\left(\kappa-\frac{a}{m}\right)^{(\tau/k)-1}\varphi(\kappa)\mathrm{d}\kappa\\
&\leq\left[h\left(\frac{1}{2^\alpha}\right)\varphi(a)+m^{(\tau/k)+1}h\left(\frac{2^\alpha-1}{2^\alpha}\right)\varphi(b)\right]\int_0^1\eta^{(\tau/k)-1}h(\eta^\alpha)\mathrm{d}\eta\\
&\quad+m\left[h\left(\frac{1}{2^\alpha}\right)\varphi(b)+m^{(\tau/k)+1}h\left(\frac{2^\alpha-1}{2^\alpha}\right)\varphi\left(\frac{a}{m^2}\right)\right]\int_0^1\eta^{(\tau/k)-1}h(1-\eta^\alpha)\mathrm{d}\eta.
\end{aligned}\tag{23}$$

(ii) By choosing $\psi = I$ and $\widetilde{p} = 0$, we obtain

$$\begin{aligned}
&\varphi\left(\frac{a+mb}{2}\right)\int_a^{mb}(mb-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}(mb-\iota)^\mu\right)\mathrm{d}\iota\\
&\leq h\left(\frac{1}{2^\alpha}\right)\int_a^{mb}(mb-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}(mb-\iota)^\mu\right)\varphi(\iota)\mathrm{d}\iota\\
&\quad+m^{(\tau/k)+1}h\left(\frac{2^\alpha-1}{2^\alpha}\right)\int_{(a/m)}^{b}\left(\kappa-\frac{a}{m}\right)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}m^\mu\left(\kappa-\frac{a}{m}\right)^\mu\right)\varphi(\kappa)\mathrm{d}\kappa\\
&\leq\left[h\left(\frac{1}{2^\alpha}\right)\varphi(a)+m^{(\tau/k)+1}h\left(\frac{2^\alpha-1}{2^\alpha}\right)\varphi(b)\right]\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^\mu)h(\eta^\alpha)\mathrm{d}\eta\\
&\quad+m\left[h\left(\frac{1}{2^\alpha}\right)\varphi(b)+m^{(\tau/k)+1}h\left(\frac{2^\alpha-1}{2^\alpha}\right)\varphi\left(\frac{a}{m^2}\right)\right]\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^\mu)h(1-\eta^\alpha)\mathrm{d}\eta.
\end{aligned}\tag{24}$$

(iii) By setting $m = 1$ and $\psi = I$, we obtain

$$\begin{aligned}
&\varphi\left(\frac{a+b}{2}\right)\int_a^{b}(b-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}(b-\iota)^\mu;\widetilde{p}\right)\mathrm{d}\iota\\
&\leq h\left(\frac{1}{2^\alpha}\right)\int_a^{b}(b-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}(b-\iota)^\mu;\widetilde{p}\right)\varphi(\iota)\mathrm{d}\iota\\
&\quad+h\left(\frac{2^\alpha-1}{2^\alpha}\right)\int_a^{b}(\kappa-a)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}(\kappa-a)^\mu;\widetilde{p}\right)\varphi(\kappa)\mathrm{d}\kappa\\
&\leq\left[h\left(\frac{1}{2^\alpha}\right)f(a)+h\left(\frac{2^\alpha-1}{2^\alpha}\right)\varphi(b)\right]\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^\mu;\widetilde{p})h(\eta^\alpha)\mathrm{d}\eta\\
&\quad+\left[h\left(\frac{1}{2^\alpha}\right)\varphi(b)+h\left(\frac{2^\alpha-1}{2^\alpha}\right)\varphi(a)\right]\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^\mu;\widetilde{p})h(1-\eta^\alpha)\mathrm{d}\eta.
\end{aligned}\tag{25}$$

(iv) By choosing $h(\eta) = \eta$ and $\widetilde{p} = w = 0$, we obtain

$$\begin{aligned}
&\varphi\left(\frac{\psi(a)+m\psi(b)}{2}\right)\int_a^{\psi^{-1}(m\psi(b))}(m\psi(b)-\psi(\iota))^{(\tau/k)-1}\psi\prime(\iota)\mathrm{d}\iota \\
&\leq\left(\frac{1}{2^\alpha}\right)\int_a^{\psi^{-1}(m\psi(b))}(m\psi(b)-\psi(\iota))^{(\tau/k)-1}\varphi(\psi(\iota))\psi\prime(\iota)\mathrm{d}\iota \\
&\quad+m^{(\tau/k)+1}\left(\frac{2^\alpha-1}{2^\alpha}\right)\int_{\psi^{-1}(\psi(a)/m)}^{b}(\psi(\kappa)-\frac{\psi(a)}{m})^{(\tau/k)-1}\varphi(\psi(\kappa))\psi\prime(\kappa)\mathrm{d}\kappa \\
&\leq\left[\left(\frac{1}{2^\alpha}\right)\varphi(\psi(a))+m^{(\tau/k)+1}\left(\frac{2^\alpha-1}{2^\alpha}\right)\varphi(\psi(b))\right]\left(\frac{k}{\tau+\alpha k}\right) \\
&\quad+m\left[\left(\frac{1}{2^\alpha}\right)\varphi(\psi(b))+m^{(\tau/k)+1}\left(\frac{2^\alpha-1}{2^\alpha}\right)\varphi\left(\frac{\psi(a)}{m^2}\right)\right]\left(\frac{\alpha k^2}{\tau(\tau+\alpha k)}\right).
\end{aligned} \tag{26}$$

(v) By setting $\alpha = 1$ and $\psi = I$, we get

$$\begin{aligned}
&\varphi\left(\frac{a+mb}{2}\right)\int_a^{mb}(mb-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(\overline{w}(mb-\iota)^\mu;\widetilde{p})\mathrm{d}\iota \\
&\leq h\left(\frac{1}{2}\right)\int_a^{mb}(mb-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(\overline{w}(mb-\iota)^\mu;\widetilde{p})\varphi(\iota)\mathrm{d}\iota \\
&\quad+m^{(\tau/k)+1}h\left(\frac{1}{2}\right)\int_{(a/m)}^{b}\left(\kappa-\frac{a}{m}\right)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}m^\mu\left(\kappa-\frac{a}{m}\right)^\mu;\widetilde{p}\right)\varphi(\kappa)\mathrm{d}\kappa \\
&\leq\left[h\left(\frac{1}{2}\right)\varphi(a)+m^{(\tau/k)+1}h\left(\frac{1}{2}\right)\varphi(b)\right]\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^\mu;\widetilde{p})h(\eta)\mathrm{d}\eta \\
&\quad+m\left[h\left(\frac{1}{2}\right)\varphi(b)+m^{(\tau/k)+1}h\left(\frac{1}{2}\right)\varphi\left(\frac{a}{m^2}\right)\right]\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^\mu;\widetilde{p})h(1-\eta)\mathrm{d}\eta.
\end{aligned} \tag{27}$$

(vi) By setting $\alpha = m = 1$, $h(\eta) = \eta$ and $\psi = I$, we get

$$\begin{aligned}
&\varphi\left(\frac{a+b}{2}\right)\int_a^{b}(b-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(\overline{w}(b-\iota)^\mu;\widetilde{p})\mathrm{d}\iota \\
&\leq\frac{1}{2}\left[\int_a^{b}(b-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(\overline{w}(b-\iota)^\mu;\widetilde{p})\varphi(\iota)\mathrm{d}\iota+\int_a^{b}(\kappa-a)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(\overline{w}(\kappa-a)^\mu;\widetilde{p})\varphi(\kappa)\mathrm{d}\kappa\right] \\
&\leq\left(\frac{\varphi(a)+\varphi(b)}{2}\right)\left[\int_0^1\eta^{(\tau/k)}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^\mu;\widetilde{p})\mathrm{d}\eta+\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^\mu;\widetilde{p})(1-\eta)\mathrm{d}\eta\right].
\end{aligned} \tag{28}$$

Remark 2. The above k-fractional inequalities are farther in line with already known conclusions as noted below: (i) By choosing $k = 1$ in Corollary 1 *(v), an inequality for extended generalized fractional integrals is acquired. (ii) By choosing $k = 1$ and $\widetilde{p} = 0$ in* Corollary 1 *(v), Theorem 2.1 of* [28] *is acquired. (iii) By choosing $m = 1$, and $h(\eta) = \eta$ in* Corollary 1 *(v), Theorem 2.1 of* [27] *is acquired. (iv) By choosing $\widetilde{p} = w = 0$ in* Corollary 1 *(v), Theorem 2.1 of* [20] *is acquired.*

Remark 3. (i) By choosing $k = 1$ and $\widetilde{p} = 0$ in Remark 1 *(iii), an inequality for extended generalized fractional integrals is acquired. (ii) By choosing $k = 1$ and $\widetilde{p} = w = 0$ in* Remark 1 *(iii),* Theorem 2 *of* [29] *is acquired. (iii) By choosing $k = 1$ in* Remark 1 *(iv), Corollary 2.2 of* [20] *is acquired.*

The below lemma is beneficial to offer the Fejér–Hadamard's inequality for generalized k-fractional integrals.

Lemma 1. *Let $\varphi, \psi\colon [a,b] \longrightarrow \mathbb{R}$ with $0 \le a < mb$, be the functions such that $\varphi \in L_1[a,b]$ and φ positive and ψ be differentiable and strictly increasing. If $\varphi(\psi(\iota)) = \varphi(\psi(a) + m\psi(b) - \psi(\iota))$, in that case for generalized k-fractional operators (11) and (12), we get*

$$\begin{aligned}\left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{w},a+}^{\gamma,\delta,\nu,c}\varphi^{\circ}\psi\right)(m\psi(b);\widetilde{p}) &= \left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{w}m^{\mu},b-}^{\gamma,\delta,\nu,c}\varphi^{\circ}\psi\right)\left(\frac{\psi(a)}{m};\widetilde{p}\right)\\ &= \frac{1}{2}\left[\left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{w},a+}^{\gamma,\delta,\nu,c}\varphi^{\circ}g\right)(m\psi(b);\widetilde{p}) + \left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{w}m^{\mu},b-}^{\gamma,\delta,\nu,c}\varphi^{\circ}\psi\right)\left(\frac{\psi(a)}{m};\widetilde{p}\right)\right],\end{aligned} \tag{29}$$

for all $\eta \in [a,b]$.

Proof. By description of generalized k-fractional operators (12) and (13), we get

$$\begin{aligned}&\left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{w},a+}^{\gamma,\delta,\nu,c}\varphi^{\circ}\psi\right)(m\psi(b);\widetilde{p})\\ &= \int_{a}^{\psi^{-1}(m\psi(b))}(m\psi(b)-\psi(\iota))^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}(m\psi(b)-\psi(\iota))^{\mu};\widetilde{p}\right)(\varphi^{\circ}\psi)(\iota)\psi\prime(\iota)\mathrm{d}\iota\\ &= \int_{a}^{\psi^{-1}(m\psi(b))}(m\psi(b)-\psi(\iota))^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}(m\psi(b)-\psi(\iota))^{\mu};\widetilde{p}\right)\varphi(\psi(\iota))\psi\prime(\iota)\mathrm{d}\iota\\ &= \int_{a}^{g^{-1}(m\psi(b))}(m\psi(b)-\psi(\iota))^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}(m\psi(b)-\psi(\iota))^{\mu};\widetilde{p}\right)\varphi(\psi(a)+m\psi(b)-\psi(\iota))\psi\prime(\iota)\mathrm{d}\iota.\end{aligned} \tag{30}$$

Setting $\psi(\eta) = \psi(a) + m\psi(b) - \psi(\iota)$ in the above equation and using $\varphi(\psi(\iota)) = \varphi(\psi(a) + m\psi(b) - \psi(\iota))$, we have

$$\begin{aligned}&\left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{w},a+}^{\gamma,\delta,\nu,c}\varphi^{\circ}\psi\right)(m\psi(b);\widetilde{p})\\ &= \int_{\psi^{-1}(\psi(a)/m)}^{b}(m\psi(\eta)-\psi(a))^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}(m\psi(\eta)-\psi(a))^{\mu};\widetilde{p}\right)\varphi(\psi(\eta))\psi\prime(\eta)\mathrm{d}\eta\\ &= \int_{\psi^{-1}(\psi(a)/m)}^{b}(m\psi(\eta)-\psi(a))^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}(m\psi(\eta)-\psi(a))^{\mu};\widetilde{p}\right)(\varphi^{\circ}\psi)(\eta)\psi\prime(\eta)\mathrm{d}\eta.\end{aligned} \tag{31}$$

This implies

$$\left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{w},a+}^{\gamma,\delta,\nu,c}\varphi^{\circ}\psi\right)(m\psi(b);\widetilde{p}) = \left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{w}m^{\mu},b-}^{\gamma,\delta,\nu,c}\varphi^{\circ}\psi\right)\left(\frac{\psi(a)}{m};\widetilde{p}\right). \tag{32}$$

By adding $\left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{w},a+}^{\gamma,\delta,\nu,c}\varphi^{\circ}\psi\right)(m\psi(b);\widetilde{p})$ on both sides of (32), we have

$$\begin{aligned}2\left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{w},a+}^{\gamma,\delta,\nu,c}\varphi^{\circ}\psi\right)(m\psi(b);\widetilde{p}) &= \left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{w}m^{\mu},b-}^{\gamma,\delta,\nu,c}\varphi^{\circ}\psi\right)\left(\frac{\psi(a)}{m};\widetilde{p}\right)\\ &\quad + \left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{w},a+}^{\gamma,\delta,\nu,c}\varphi^{\circ}\psi\right)(m\psi(b);\widetilde{p}).\end{aligned} \tag{33}$$

From equations (32) and (33), the result can be obtained. □

The first type of Fejér–Hadamard inequality is endued through generalized k-fractional integrals as noted below:

Theorem 4. *Let h: $J \longrightarrow \mathbb{R}$ be nonnegative, nonzero, and integrable function and φ, ψ: $[a, b] \longrightarrow \mathbb{R}$, $0 \le a < mb$, be the functions such that $\varphi \in L_1[a, b]$ and φ be positive and ψ be differentiable and strictly increasing, r is a nonnegative and integrable function. If φ is $(\alpha, h - m)$-convex and $\varphi(\psi(\iota)) = \varphi(\psi(a) + m\psi(b) - \psi(\iota))$, in that case the below inequalities for generalized k-fractional operators (12) and (13) occur:*

$$\begin{aligned}
&\varphi\left(\frac{\psi(a)+m\psi(b)}{2}\right)\left[\left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{w},a+}^{\gamma,\delta,\nu,c}r^{\circ}\psi\right)(m\psi(b);\widetilde{p})+\left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{w}m^{\mu},b-}^{\gamma,\delta,\nu,c}r^{\circ}\psi\right)\left(\frac{\psi(a)}{m};\widetilde{p}\right)\right]\\
&\le 2h\left(\frac{1}{2^{\alpha}}\right)\left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{w},a+}^{\gamma,\delta,\nu,c}\varphi^{\circ}r^{\circ}\psi\right)(m\psi(b);\widetilde{p})\\
&+2m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{w}m^{\mu},b-}^{\gamma,\delta,\nu,c}\varphi^{\circ}r^{\circ}\psi\right)\left(\frac{\psi(a)}{m};\widetilde{p}\right)\\
&\le 2\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(a))+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(\psi(b))\right]\\
&\times\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\widetilde{p})r(\eta\psi(a)+m(1-\eta)\psi(b))h(\eta^{\alpha})d\eta\\
&+2m\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(b))+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi\left(\frac{\psi(a)}{m^{2}}\right)\right]\\
&\times\int_0^1 t^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\widetilde{p})r(\eta\psi(a)+m(1-\eta)\psi(b))h(1-\eta^{\alpha})d\eta\,,
\end{aligned}\tag{34}$$

where $\overline{w} = (w/(m\psi(b) - \psi(a))^{\mu})$ for all $\eta \in [a, b]$.

Proof. We demonstrate the claim as follows:

Multiplying both sides of (18) by $\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\widetilde{p})r(\eta\psi(a)+m(1-\eta)\psi(b))$ and then integrating over $[0, 1]$, we have

$$\begin{aligned}
&\varphi\left(\frac{\psi(a)+m\psi(b)}{2}\right)\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\widetilde{p})r(\eta\psi(a)+m(1-\eta)\psi(b))\mathrm{d}\eta\\
&\le h\left(\frac{1}{2^{\alpha}}\right)\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\widetilde{p})\varphi(\eta\psi(a)+m(1-\eta)\psi(b))r(\eta\psi(a)+m(1-\eta)\psi(b))\mathrm{d}\eta\\
&+mh\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\widetilde{p})\varphi\left((1-\eta)\frac{\psi(a)}{m}+\eta\psi(b)\right)r(\eta\psi(a)+m(1-\eta)\psi(b))\mathrm{d}\eta.
\end{aligned}\tag{35}$$

By specifying $\psi(\iota) = \eta\psi(a) + m(1-\eta)\psi(b)$ and $\psi(\kappa) = (1-\eta)(\psi(a)/m) + \eta\psi(b)$, that is $\psi(a) + m\psi(b) - \psi(\iota) = (1-\eta)\psi(a) + m\eta\psi(b)$, in (35), then using $\varphi(\psi(\iota)) = \varphi(\psi(a) + m\psi(b) - \psi(\iota))$, we have

$$\begin{aligned}
&\varphi\left(\frac{\psi(a)+m\psi(b)}{2}\right)\int_a^{\psi^{-1}(m\psi(b))}(m\psi(b)-\psi(\iota))^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}(m\psi(b)-\psi(\iota))^{\mu};\widetilde{p}\right)(r^{\circ}\psi)(\iota)\psi\prime(\iota)\mathrm{d}\iota\\
&\le h\left(\frac{1}{2^{\alpha}}\right)\int_a^{\psi^{-1}(m\psi(b))}(m\psi(b)-\psi(\iota))^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}(m\psi(b)-\psi(\iota))^{\mu};\widetilde{p}\right)(\varphi^{\circ}\psi)(\iota)(r^{\circ}\psi)(\iota)\psi\prime(\iota)\mathrm{d}\iota\\
&+mh\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\int_{\psi^{-1}(\psi(a)/m)}^{b}(\psi(\iota)-\frac{\psi(a)}{m})^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}m^{\mu}\left(\psi(\iota)-\frac{\psi(a)}{m}\right)^{\mu};\widetilde{p}\right)(\varphi^{\circ}\psi)(\iota)(r^{\circ}\psi)(\iota)\psi\prime(\iota)\mathrm{d}\iota.
\end{aligned}\tag{36}$$

This implies

$$\varphi\left(\frac{\psi(a)+m\psi(b)}{2}\right)\left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{w},a+}^{\gamma,\delta,\nu,c}r^{\circ}\psi\right)(m\psi(b);\widetilde{p})\leq h\left(\frac{1}{2^{\alpha}}\right)\left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{w},a+}^{\gamma,\delta,\nu,c}\varphi^{\circ}r^{\circ}\psi\right)(m\psi(b);\widetilde{p}) + m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{w}m^{\mu},b-}^{\gamma,\delta,\nu,c}\varphi^{\circ}r^{\circ}\psi\right)\left(\frac{\psi(a)}{m};\widetilde{p}\right). \tag{37}$$

Using Lemma 1 in the above inequality, we have the first side of (34).

To demonstrate second side of (34), multiplying both parts of (21) by $2\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\widetilde{p})r(\psi(a)+m(1-\eta)\psi(b))$ and then integrating over $[0,1]$, we have

$$\begin{aligned}
&2h\left(\frac{1}{2^{\alpha}}\right)\int_{0}^{1}\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\widetilde{p})r(\eta\psi(a)+m(1-\eta)\psi(b))\varphi(\eta\psi(a)+m(1-\eta)\psi(b))d\eta\\
&\quad+2m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\int_{0}^{1}\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\widetilde{p})r(\eta\psi(a)+m(1-\eta)\psi(b))\varphi\left((1-\eta)\frac{\psi(a)}{m}+\psi(b)\right)\mathrm{d}\eta\\
&\leq 2\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(a))+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(\psi(b))\right]\\
&\quad\times\int_{0}^{1}\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\widetilde{p})r(\eta\psi(a)+m(1-\eta)\psi(b))h(\eta^{\alpha})\mathrm{d}\eta\\
&\quad+2m\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(b))+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi\left(\frac{\psi(a)}{m^{2}}\right)\right]\\
&\quad\times\int_{0}^{1}\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\widetilde{p})r(\eta\psi(a)+m(1-\eta)\psi(b))h(1-\eta^{\alpha})\mathrm{d}\eta.
\end{aligned} \tag{38}$$

Setting $\psi(\iota)=\eta\psi(a)+m(1-\eta)\psi(b)$ and $\psi(\kappa)=(1-\eta)(\psi(a)/m)+\eta\psi(b)$, then using $\varphi(\psi(\iota))=\varphi(\psi(a)+m\psi(b)-\psi(\iota))$ in (38), we have

$$\begin{aligned}
&2h\left(\frac{1}{2^{\alpha}}\right)\left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{w},a+}^{\gamma,\delta,\nu,c}\varphi^{\circ}r^{\circ}\psi\right)(m\psi(b);\widetilde{p})\\
&\quad+2m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{w}m^{\mu},b-}^{\gamma,\delta,\nu,c}\varphi^{\circ}r^{\circ}\psi\right)\left(\frac{\psi(a)}{m};\widetilde{p}\right)\\
&\leq 2\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(a))+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(\psi(b))\right]\\
&\quad\times\int_{0}^{1}\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\widetilde{p})r(\eta\psi(a)+m(1-\eta)\psi(b))h(\eta^{\alpha})\mathrm{d}\eta\\
&\quad+2m\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(b))+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi\left(\frac{\psi(a)}{m^{2}}\right)\right]\\
&\quad\times\int_{0}^{1}\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\widetilde{p})r(\eta\psi(a)+m(1-\eta)\psi(b))h(1-\eta^{\alpha})\mathrm{d}\eta.
\end{aligned} \tag{39}$$

By usage Lemma 1 in the above inequality, we have the second side of (34). ☐

Corollary 2. *By using (34), some more k-fractional inequalities are offered as noted below:*

(i) By choosing $\psi = I$ and $\widetilde{p} = w = 0$, we obtain

$$\begin{aligned}
&\varphi\left(\frac{a+mb}{2}\right)\int_a^{mb}(mb-\iota)^{(\tau/k)-1}r(\iota)\mathrm{d}\iota \\
&\leq 2h\left(\frac{1}{2^\alpha}\right)\int_a^{mb}(mb-\iota)^{(\tau/k)-1}(\varphi\circ r)(\iota)\mathrm{d}\iota + 2m^{(\tau/k)+1}h\left(\frac{2^\alpha-1}{2^\alpha}\right)\int_{(a/m)}^{b}\left(\kappa-\frac{a}{m}\right)^{(\tau/k)-1}(\varphi\circ r)(\kappa)\mathrm{d}\kappa \\
&\leq 2\left[h\left(\frac{1}{2^\alpha}\right)\varphi(a) + m^{(\tau/k)+1}h\left(\frac{2^\alpha-1}{2^\alpha}\right)\varphi(b)\right]\int_0^1 \eta^{(\tau/k)-1}r(\eta a+m(1-\eta)b)h(\eta^\alpha)\mathrm{d}\eta \\
&\quad + 2m\left[h\left(\frac{1}{2^\alpha}\right)\varphi(b) + m^{(\tau/k)+1}h\left(\frac{2^\alpha-1}{2^\alpha}\right)\varphi\left(\frac{a}{m^2}\right)\right]\int_0^1 \eta^{(\tau/k)-1}r(\eta a+m(1-\eta)b)h(1-\eta^\alpha)\mathrm{d}\eta.
\end{aligned} \tag{40}$$

(ii) By choosing $\widetilde{p} = 0$ and $\psi = I$, we obtain

$$\begin{aligned}
&\varphi\left(\frac{a+mb}{2}\right)\int_a^{mb}(mb-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}(mb-\iota)^\mu\right)r(\iota)\mathrm{d}\iota \\
&\leq 2h\left(\frac{1}{2^\alpha}\right)\int_a^{mb}(mb-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}(mb-\iota)^\mu\right)(\varphi\circ r)(\iota)\mathrm{d}\iota \\
&\quad + 2m^{(\tau/k)+1}h\left(\frac{2^\alpha-1}{2^\alpha}\right)\int_{(a/m)}^{b}\left(\kappa-\frac{a}{m}\right)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}m^\mu\left(\kappa-\frac{a}{m}\right)^\mu\right)(\varphi\circ r)(\kappa)\mathrm{d}\kappa \\
&\leq 2\left[h\left(\frac{1}{2^\alpha}\right)\varphi(a) + m^{(\tau/k)+1}h\left(\frac{2^\alpha-1}{2^\alpha}\right)\varphi(b)\right]\int_0^1 \eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^\mu)r(\eta a+m(1-\eta)b)h(\eta^\alpha)\mathrm{d}\eta \\
&\quad + 2m\left[h\left(\frac{1}{2^\alpha}\right)\varphi(b) + m^{(\tau/k)+1}h\left(\frac{2^\alpha-1}{2^\alpha}\right)\varphi\left(\frac{a}{m^2}\right)\right]\int_0^1 \eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^\mu)r(\eta a+m(1-\eta)b)h(1-\eta^\alpha)\mathrm{d}\eta.
\end{aligned} \tag{41}$$

(iii) By choosing $m = 1$ and $\psi = I$, we obtain

$$\begin{aligned}
&\varphi\left(\frac{a+b}{2}\right)\int_a^{b}(b-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}(b-\iota)^\mu;\widetilde{p}\right)r(\iota)\mathrm{d}\iota \\
&\leq 2h\left(\frac{1}{2^\alpha}\right)\int_a^{b}(b-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}(b-\iota)^\mu;\widetilde{p}\right)(\varphi\circ r)(\iota)\mathrm{d}\iota \\
&\quad + 2h\left(\frac{2^\alpha-1}{2^\alpha}\right)\int_a^{b}(\kappa-a)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}(\kappa-a)^\mu;\widetilde{p}\right)(\varphi\circ r)(\kappa)\mathrm{d}\kappa \\
&\leq 2\left[h\left(\frac{1}{2^\alpha}\right)\varphi(a) + h\left(\frac{2^\alpha-1}{2^\alpha}\right)\varphi(b)\right]\int_0^1 \eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^\mu;\widetilde{p})r(\eta a+(1-\eta)b)h(\eta^\alpha)\mathrm{d}\eta \\
&\quad + 2\left[h\left(\frac{1}{2^\alpha}\right)\varphi(b) + h\left(\frac{2^\alpha-1}{2^\alpha}\right)\varphi(a)\right]\int_0^1 \eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^\mu;\widetilde{p})r(\eta a+(1-\eta)b)h(1-\eta^\alpha)\mathrm{d}\eta.
\end{aligned} \tag{42}$$

(iv) By choosing $\widetilde{p} = w = 0$ and $h(\eta) = \eta$, we obtain

$$\begin{aligned}
&\varphi\left(\frac{\psi(a)+m\psi(b)}{2}\right)\int_a^{\psi^{-1}(m\psi(b))}(m\psi(b)-\psi(\iota))^{(\tau/k)-1}(r\circ\psi)(\iota)\psi\prime(\iota)\mathrm{d}\iota\\
&\leq\left(\frac{1}{2^{\alpha-1}}\right)\int_a^{\psi^{-1}(m\psi(b))}(m\psi(b)-\psi(\iota))^{(\tau/k)-1}(\varphi\circ r\circ\psi)(\iota)\psi\prime(\iota)\mathrm{d}\iota\\
&+m^{(\tau/k)+1}\left(\frac{2^\alpha-1}{2^{\alpha-1}}\right)\int_{\psi^{-1}\left(\frac{\psi(a)}{m}\right)}^{b}(\psi(\kappa)-\frac{\psi(a)}{m})^{(\tau/k)-1}(\varphi\circ r\circ\psi)(\kappa)\psi\prime(\kappa)\mathrm{d}\kappa\\
&\leq 2\left[\left(\frac{1}{2^\alpha}\right)\varphi(\psi(a))+m^{(\tau/k)+1}\left(\frac{2^\alpha-1}{2^\alpha}\right)\varphi(\psi(b))\right]\int_0^1\eta^{(\tau/k)-1}r(\eta\psi(a)+m(1-\eta)\psi(b))(\eta^\alpha)\mathrm{d}\eta\\
&+2m\left[\left(\frac{1}{2^\alpha}\right)\varphi(\psi(b))+m^{(\tau/k)+1}\left(\frac{2^\alpha-1}{2^\alpha}\right)\varphi\left(\frac{\psi(a)}{m^2}\right)\right]\int_0^1\eta^{(\tau/k)-1}r(\eta\psi(a)+m(1-\eta)\psi(b))(1-\eta^\alpha)\mathrm{d}\eta.
\end{aligned}\tag{43}$$

(v) By choosing $\alpha = 1$ and $\psi = I$, we obtain

$$\begin{aligned}
&\varphi\left(\frac{a+mb}{2}\right)\int_a^{mb}(mb-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(\overline{w}(mb-\iota)^\mu;\widetilde{p})r(\iota)\mathrm{d}\iota\\
&\leq 2h\left(\frac{1}{2}\right)\int_a^{mb}(mb-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(\overline{w}(mb-\iota)^\mu;\widetilde{p})(\varphi\circ r)(\iota)\mathrm{d}\iota\\
&+2m^{(\tau/k)+1}h\left(\frac{1}{2}\right)\int_{(a/m)}^{b}\left(\kappa-\frac{a}{m}\right)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}m^\mu\left(\kappa-\frac{a}{m}\right)^\mu;\widetilde{p}\right)(\varphi\circ r)(\kappa)\mathrm{d}\kappa\\
&\leq 2\left[h\left(\frac{1}{2}\right)\varphi(a)+m^{(\tau/k)+1}h\left(\frac{1}{2}\right)\varphi(b)\right]\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^\mu;\widetilde{p})r(\eta a+m(1-\eta)b)h(\eta)\mathrm{d}\eta\\
&+2m\left[h\left(\frac{1}{2}\right)\varphi(b)+m^{(\tau/k)+1}h\left(\frac{1}{2}\right)\varphi\left(\frac{a}{m^2}\right)\right]\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^\mu;\widetilde{p})r(\eta a+m(1-\eta)b)h(1-\eta)\mathrm{d}\eta.
\end{aligned}\tag{44}$$

(vi) By choosing $\alpha = m = 1$, $h(\eta) = \eta$ and, we obtain

$$\begin{aligned}
&\varphi\left(\frac{a+b}{2}\right)\int_a^{b}(b-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(\overline{w}(b-\iota)^\mu;\widetilde{p})r(\iota)\mathrm{d}\iota\\
&\leq\left[\int_a^{b}(b-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(\overline{w}(b-\iota)^\mu;\widetilde{p})(\varphi\circ r)(\iota)\mathrm{d}\iota\right.\\
&\left.+\int_a^{b}(\kappa-a)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(\overline{w}(\kappa-a)^\mu;\widetilde{p})(\varphi\circ r)(\kappa)\mathrm{d}\kappa\right]\\
&\leq(\varphi(a)+\varphi(b))[\int_0^1\eta^{(\tau/k)}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^\mu;\widetilde{p})r(\eta a+(1-\eta)b)\mathrm{d}\eta\\
&+\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^\mu;\widetilde{p})r(\eta a+(1-\eta)b)(1-\eta)\mathrm{d}\eta].
\end{aligned}\tag{45}$$

(vii) By choosing $\alpha = k = 1$ and $\psi = I$, we obtain

$$\begin{aligned}
&\varphi\left(\frac{a+mb}{2}\right)\int_a^{mb}(mb-\iota)^{\tau-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}(mb-\iota)^{\mu};\tilde{p}\right)r(\iota)\mathrm{d}\iota\\
&\leq 2h\left(\frac{1}{2}\right)\int_a^{mb}(mb-\iota)^{\tau-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}(mb-\iota)^{\mu};\tilde{p}\right)(\varphi\circ r)(\iota)\mathrm{d}\iota\\
&\quad+2m^{\tau+1}h\left(\frac{1}{2}\right)\int_{(a/m)}^{b}\left(\kappa-\frac{a}{m}\right)^{\tau-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{w}m^{\mu}\left(\kappa-\frac{a}{m}\right)^{\mu};\tilde{p}\right)(\varphi\circ r)(\kappa)\mathrm{d}\kappa\\
&\leq 2\left[h\left(\frac{1}{2}\right)\varphi(a)+m^{\tau+1}h\left(\frac{1}{2}\right)\varphi(b)\right]\int_0^1\eta^{\tau-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(w\eta^{\mu};\tilde{p}\right)r(\eta a+m(1-\eta)b)h(\eta)\mathrm{d}\eta\\
&\quad+2m\left[h\left(\frac{1}{2}\right)\varphi(b)+m^{\tau+1}h\left(\frac{1}{2}\right)\varphi\left(\frac{a}{m^2}\right)\right]\int_0^1\eta^{\tau-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(w\eta^{\mu};\tilde{p}\right)r(\eta a+m(1-\eta)b)h(1-\eta)\mathrm{d}\eta.
\end{aligned}\tag{46}$$

Remark 4. The above k-fractional inequalities are farther in line with foreknown conclusions as noted below: (i) By choosing $k = 1$ in Corollary 2 *(vi), Theorem 2.2 of* [27] *is acquired. (ii) By choosing $\tilde{p} = 0$ in in* Corollary 2 *(vii), Theorem 2.5 of* [28] *is acquired. (iii) By choosing $k = I$, $\tilde{p} = w = 0$ and $h(\eta) = \eta$ in* Corollary 2 *(v), an inequality for m-convex functions via Riemann–Liouville integrals is acquired. (iv) By choosing $k = 1$ and $\tilde{p} = 0$ in in* Corollary 2 *(vi), an inequality for extended generalized fractional integrals is acquired. (v) By choosing $k = 1$ and $\tilde{p} = w = 0$ in in* Corollary 2 *(vi), Theorem 4* of [26] *is acquired. (vi) By choosing $h(\eta) = \eta$ in in Corollary* 3.2 *(vii), Theorem 3.1* of [27] *is acquired.*

In the subsequent theorem, we offer another type of Hadamard's inequality.

Theorem 5. *Let $h\colon J \longrightarrow \mathbb{R}$ is nonnegative, nonzero and integrable function and $\varphi, \psi\colon [a,b] \longrightarrow \mathbb{R}$, $0 \leq a < mb$, be the functions such that $\varphi \in L_1[a,b]$ and φ be positive and ψ be differentiable and strictly increasing. If φ is $(\alpha, h-m)$-convex, in that case for generalized k-fractional operators (12) and (13), we acquire*

$$\begin{aligned}
&\varphi\left(\frac{\psi(a)+m\psi(b)}{2}\right)\left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{\overline{w}},\psi^{-1}(m\psi(b)+\psi(a)/2)+}^{\gamma,\delta,\nu,c}1\right)(m\psi(b);\tilde{p})\\
&\leq h\left(\frac{1}{2^{\alpha}}\right)\left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{\overline{w}},\psi^{-1}(m\psi(b)+\psi(a)/2)+}^{\gamma,\delta,\nu,c}\varphi^{\circ}\psi\right)(m\psi(b);\tilde{p})\\
&\quad+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{\overline{w}}m^{\mu},\psi^{-1}(m\psi(b)+\psi(a)/2m)-}^{\gamma,\delta,\nu,c}\varphi^{\circ}\psi\right)\left(\frac{\psi(a)}{m};\tilde{p}\right)\\
&\leq\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(a))+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(\psi(b))\right]\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(w\eta^{\mu};\tilde{p}\right)h\left(\frac{\eta^{\alpha}}{2^{\alpha}}\right)d\eta\\
&\quad+m\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(b))+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi\left(\frac{\psi(a)}{m^2}\right)\right]\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(w\eta^{\mu};\tilde{p}\right)h\left(\frac{2^{\alpha}-\eta^{\alpha}}{2^{\alpha}}\right)\mathrm{d}\eta,
\end{aligned}\tag{47}$$

where $\overline{\overline{w}} = (2^{\mu}w/(m\psi(b) - \psi(a))^{\mu})$ for all $\eta \in [a,b]$.

Proof. Setting $\psi(\iota) = (\eta/2)\psi(a) + m(2-\eta/2)\psi(b)$ and $\psi(\kappa) = (2-\eta/2)(\psi(a)/m) + \eta/2\psi(b)$ in (3.2), we have

$$\begin{aligned}
\varphi\left(\frac{\psi(a)+m\psi(b)}{2}\right)&\leq h\left(\frac{1}{2^{\alpha}}\right)\varphi\left(\frac{\eta}{2}\psi(a)+m\left(\frac{2-\eta}{2}\right)\psi(b)\right)\\
&\quad+mh\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi\left(\left(\frac{2-\eta}{2}\right)\frac{\psi(a)}{m}+\frac{\eta}{2}\psi(b)\right).
\end{aligned}\tag{48}$$

Multiplying both parts of (48) by $\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\tilde{p})$ and then integrating over $[0,1]$, we have

$$\begin{aligned}
&\varphi\left(\frac{\psi(a)+m\psi(b)}{2}\right)\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\tilde{p})\mathrm{d}\eta\\
&\le h\left(\frac{1}{2^{\alpha}}\right)\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\tilde{p})\varphi\left(\frac{\eta}{2}\psi(a)+m\left(\frac{2-\eta}{2}\right)\psi(b)\right)\mathrm{d}\eta\\
&\quad+mh\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\tilde{p})\varphi\left(\left(\frac{2-\eta}{2}\right)\frac{\psi(a)}{m}+\frac{\eta}{2}\psi(b)\right)\mathrm{d}\eta.
\end{aligned}\tag{49}$$

By taking $\psi(\iota)=(\eta/2)\psi(a)+m(2-\eta/2)\psi(b)$ and $\psi(\kappa)=(2-\eta/2)(\psi(a)/m)+\eta/2\psi(b)$ in (49), in that case by usage k-fractional operators (2.12) and (2.13), the first side of (47) is acquired.

To demonstrate the second side of (47), once again $(\alpha,h-m)$-convexity of φ over $[a,b]$, for $\eta\in[0,1]$, we get

$$\begin{aligned}
&h\left(\frac{1}{2^{\alpha}}\right)\varphi\left(\frac{\eta}{2}\psi(a)+m\left(\frac{2-\eta}{2}\right)\psi(b)\right)+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi\left(\left(\frac{2-\eta}{2}\right)\frac{\psi(a)}{m}+\frac{\eta}{2}\psi(b)\right)\\
&\le h\left(\frac{\eta^{\alpha}}{2^{\alpha}}\right)\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(a))+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(\psi(b))\right]\\
&\quad+mh\left(\frac{2^{\alpha}-\eta^{\alpha}}{2^{\alpha}}\right)\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(b))+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi\left(\frac{\psi(a)}{m^2}\right)\right].
\end{aligned}\tag{50}$$

Multiplying both sides of (50) by $\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\tilde{p})$, then integrating over $[0,1]$, we acquire

$$\begin{aligned}
&h\left(\frac{1}{2^{\alpha}}\right)\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\tilde{p})\varphi\left(\frac{\eta}{2}\psi(a)+m\left(\frac{2-\eta}{2}\right)\psi(b)\right)\mathrm{d}\eta\\
&\quad+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\tilde{p})\varphi\left(\left(\frac{2-\eta}{2}\right)\frac{\psi(a)}{m}+\frac{\eta}{2}\psi(b)\right)\mathrm{d}\eta\\
&\le\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(a))+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(\psi(b))\right]\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\tilde{p})h\left(\frac{\eta^{\alpha}}{2^{\alpha}}\right)\mathrm{d}\eta\\
&\quad+m\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(b))+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi\left(\frac{\psi(a)}{m^2}\right)\right]\int_0^1\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\tilde{p})h\left(\frac{2^{\alpha}-\eta^{\alpha}}{2^{\alpha}}\right)\mathrm{d}\eta.
\end{aligned}\tag{51}$$

Choosing $\psi(\iota)=(\eta/2)\psi(a)+m(2-\eta/2)\psi(b)$ and $\psi(\kappa)=(2-\eta/2)(\psi(a)/m)+\eta/2\psi(b)$ in (51), in that case by usage k-fractional operators (12) and (13), the second side of (47) is acquired. □

Corollary 3. *By using (47), anymore k-fractional inequalities are offered as noted below:*

(i) By choosing $\psi=I$ and $\tilde{p}=w=0$, we have

$$\begin{aligned}
&\varphi\left(\frac{a+mb}{2}\right)\int_{a+mb/2}^{mb}(mb-\iota)^{\frac{\tau}{k}-1}d\iota \\
&\leq h\left(\frac{1}{2^{\alpha}}\right)\int_{a+mb/2}^{mb}(mb-\iota)^{\frac{\tau}{k}-1}\varphi(\iota)d\iota+m^{\frac{\tau}{k}+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\int_{a/m}^{a+mb/2}\left(\kappa-\frac{a}{m}\right)^{\frac{\tau}{k}-1}\varphi(\kappa)d\kappa \\
&\leq\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(a)+m^{\frac{\tau}{k}+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(b)\right]\int_{0}^{1}\eta^{\frac{\tau}{k}-1}h\left(\frac{\eta^{\alpha}}{2^{\alpha}}\right)d\eta \\
&+m\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(b)+m^{\frac{\tau}{k}+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi\left(\frac{a}{m^{2}}\right)\right]\int_{0}^{1}\eta^{\frac{\tau}{k}-1}h\left(\frac{2^{\alpha}-\eta^{\alpha}}{2^{\alpha}}\right)d\eta .
\end{aligned} \tag{52}$$

(ii) By choosing $\widetilde{p}=0$ *and* $\psi=I$, *we have*

$$\begin{aligned}
&\varphi\left(\frac{a+mb}{2}\right)\int_{a+mb/2}^{mb}(mb-\iota)^{\frac{\tau}{k}-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{\overline{w}}(mb-\iota)^{\mu}\right)d\iota \\
&\leq h\left(\frac{1}{2^{\alpha}}\right)\int_{a+mb/2}^{mb}(mb-\iota)^{\frac{\tau}{k}-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{\overline{w}}(mb-\iota)^{\mu}\right)\varphi(\iota)d\iota \\
&+m^{\frac{\tau}{k}+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\int_{\frac{a}{m}}^{\frac{a+mb}{2}}\left(\kappa-\frac{a}{m}\right)^{\frac{\tau}{k}-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{\overline{w}}m^{\mu}\left(\kappa-\frac{a}{m}\right)^{\mu}\right)\varphi(\kappa)d\kappa \\
&\leq\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(a)+m^{\frac{\tau}{k}+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(b)\right]\int_{0}^{1}\eta^{\frac{\tau}{k}-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(w\eta^{\mu}\right)h\left(\frac{\eta^{\alpha}}{2^{\alpha}}\right)d\eta \\
&+m\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(b)+m^{\frac{\tau}{k}+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi\left(\frac{a}{m^{2}}\right)\right]\int_{0}^{1}\eta^{\frac{\tau}{k}-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(w\eta^{\mu}\right)h\left(\frac{2^{\alpha}-\eta^{\alpha}}{2^{\alpha}}\right)d\eta .
\end{aligned} \tag{53}$$

(iii) By choosing $m=1$ *and* $\psi=I$, *we acquire*

$$\begin{aligned}
&\varphi\left(\frac{a+b}{2}\right)\int_{(a+b/2)}^{b}(b-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{\overline{w}}(b-\iota)^{\mu};\widetilde{p}\right)\mathrm{d}\iota \\
&\leq h\left(\frac{1}{2^{\alpha}}\right)\int_{(a+b/2)}^{b}(b-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{\overline{w}}(b-\iota)^{\mu};\widetilde{p}\right)\varphi(\iota)\mathrm{d}\iota \\
&+h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\int_{a}^{b}(\kappa-a)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{\overline{w}}(\kappa-a)^{\mu};\widetilde{p}\right)\varphi(\kappa)\mathrm{d}\kappa \\
&\leq\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(a)+h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(b)\right]\int_{0}^{1}\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(w\eta^{\mu};\widetilde{p}\right)h\left(\frac{\eta^{\alpha}}{2^{\alpha}}\right)\mathrm{d}\eta \\
&+\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(b)+h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(a)\right]\int_{0}^{1}\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(w\eta^{\mu};\widetilde{p}\right)h\left(\frac{2^{\alpha}-\eta^{\alpha}}{2^{\alpha}}\right)d\eta.
\end{aligned} \tag{54}$$

(iv) By choosing $\widetilde{p} = w = 0$ and $h(\eta) = \eta$, we have

$$\begin{aligned}
&\varphi\left(\frac{\psi(a) + m\psi(b)}{2}\right) \int_{\psi^{-1}(\psi(a)+m\psi(b)/2)}^{\psi^{-1}(m\psi(b))} (m\psi(b) - \psi(\iota))^{(\tau/k)-1} \psi\prime(\iota)\mathrm{d}\iota \\
&\leq \left(\frac{1}{2^{\alpha}}\right) \int_{\psi^{-1}(\psi(a)+m\psi(b)/2)}^{\psi^{-1}(m\psi(b))} (m\psi(b) - \psi(\iota))^{(\tau/k)-1} \varphi(\psi(\iota))\psi\prime(\iota)\mathrm{d}\iota \\
&\quad + m^{(\tau/k)+1}\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right) \int_{\psi^{-1}(\psi(a)/m)}^{\psi^{-1}(\psi(a)+m\psi(b)/2m)} \left(\psi(\kappa) - \frac{\psi(a)}{m}\right)^{(\tau/k)-1} \varphi(\psi(\kappa))\psi\prime(\kappa)\mathrm{d}\kappa \\
&\leq \left[\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(a)) + m^{(\tau/k)+1}\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(\psi(b))\right]\left(\frac{k}{2^{\alpha}(\tau+\alpha k)}\right) \\
&\quad + m\left[\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(b)) + m^{(\tau/k)+1}\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi\left(\frac{\psi(a)}{m^2}\right)\right]\left(\frac{k}{\tau} - \frac{k}{2^{\alpha}(\tau+\alpha k)}\right).
\end{aligned} \tag{55}$$

(v) By choosing $\alpha = 1$ and $\psi = I$, we have

$$\begin{aligned}
&\varphi\left(\frac{a+mb}{2}\right) \int_{a+mb/2}^{mb} (mb-\iota)^{(\tau/k)-1} E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{\overline{w}}(mb-\iota)^{\mu}; \widetilde{p}\right)\mathrm{d}\iota \\
&\leq h\left(\frac{1}{2}\right) \int_{a+mb/2}^{mb} (mb-\iota)^{(\tau/k)-1} E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{\overline{w}}(mb-\iota)^{\mu}; \widetilde{p}\right)\varphi(\iota)\mathrm{d}\iota \\
&\quad + m^{(\tau/k)+1} h\left(\frac{1}{2}\right) \int_{(a/m)}^{(a+mb)/2} \left(\kappa - \frac{a}{m}\right)^{(\tau/k)-1} E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{\overline{w}}m^{\mu}\left(\kappa - \frac{a}{m}\right)^{\mu}; \widetilde{p}\right)\varphi(\kappa)\mathrm{d}\kappa \\
&\leq h\left(\frac{1}{2}\right)\left[\varphi(a) + m^{(\tau/k)+1} f(b)\right] \int_0^1 \eta^{(\tau/k)-1} E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(w\eta^{\mu}; \widetilde{p}\right) h\left(\frac{\eta}{2}\right)\mathrm{d}\eta \\
&\quad + mh\left(\frac{1}{2}\right)\left[\varphi(b) + m^{(\tau/k)+1}\varphi\left(\frac{a}{m^2}\right)\right] \int_0^1 \eta^{(\tau/k)-1} E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(w\eta^{\mu}; \widetilde{p}\right) h\left(\frac{2-\eta}{2}\right)\mathrm{d}\eta.
\end{aligned} \tag{56}$$

(vi) By choosing $\alpha = m = 1$, $h(\eta) = \eta$ and $\psi = I$, we have

$$\begin{aligned}
&\varphi\left(\frac{a+b}{2}\right) \int_{(a+b/2)}^{b} (b-\iota)^{(\tau/k)-1} E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{\overline{w}}(b-\iota)^{\mu}; \widetilde{p}\right)\mathrm{d}\iota \\
&\leq \frac{1}{2}\Big[\int_{(a+b/2)}^{b} (b-\iota)^{(\tau/k)-1} E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{\overline{w}}(b-\iota)^{\mu}; \widetilde{p}\right)\varphi(\iota)\mathrm{d}\iota \\
&\quad + \int_a^{(a+b/2)} (\kappa-a)^{(\tau/k)-1} E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{\overline{w}}(\kappa-a)^{\mu}; \widetilde{p}\right)\varphi(\kappa)\mathrm{d}\kappa\Big] \\
&\leq \left(\frac{\varphi(a)+\varphi(b)}{2}\right) \int_0^1 \eta^{(\tau/k)-1} E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(w\eta^{\mu}; \widetilde{p}\right)\mathrm{d}\eta.
\end{aligned} \tag{57}$$

Remark 5. The above k-fractional inequalities are farther in line with foreknown conclusions as noted below: (i) By choosing $k = 1$ in Corollary 3 *(v), an inequality for extended generalized fractional integrals is acquired. (ii) By choosing $k = 1$ and $\widetilde{p} = 0$ in* Corollary 3 *(v), Theorem 2.2 of* [28] *is acquired.*

The second type of the Fejér–Hadamard's inequality for generalized k-fractional integrals is dedicated as noted below:

Theorem 6. *Let $h\colon J \longrightarrow \mathbb{R}$ is nonnegative, nonzero and integrable function and $\varphi, \psi\colon [a,b] \longrightarrow \mathbb{R}$, $0 \leq a < mb$, be the functions such that $\varphi \in L_1[a,b]$ and φ be positive and ψ be differentiable and strictly increasing, r is a nonnegative and integrable function. If φ is $(\alpha, h-m)$-convex and $\varphi(\psi(\iota)) = \varphi(\psi(a) + m\psi(b) - \psi(\iota))$, in that case the below inequalities for generalized k-fractional operators (12) and (13) occur:*

$$
\begin{aligned}
&\varphi\left(\frac{\psi(a)+m\psi(b)}{2}\right)\left({}_{\psi}^{k}F^{\gamma,\delta,\nu,c}_{\mu,\tau,l,\overline{\overline{w}},\psi^{-1}((m\psi(b)+\psi(a))/2)+}r^{\circ}\psi\right)(m\psi(b);\widetilde{p})\\
&\leq h\left(\frac{1}{2^{\alpha}}\right)\left({}_{\psi}^{k}F_{\mu,\tau,l,\overline{\overline{w}}},\psi^{-1}((m\psi(b)+\psi(a))/2)+^{\gamma,\delta,\nu,c}\varphi^{\circ}r^{\circ}\psi\right)(m\psi(b);\widetilde{p})\\
&\quad + m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\left({}_{\psi}^{k}F^{\gamma,\delta,\nu,c}_{\mu,\tau,l,\overline{\overline{w}}m^{\mu},\psi^{-1}((m\psi(b)+\psi(a))/2m)-}\varphi^{\circ}r^{\circ}\psi\right)\left(\frac{\psi(a)}{m};\widetilde{p}\right)\\
&\leq\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(a))+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(\psi(b))\right]\\
&\quad\times\int_0^1\eta^{(\tau/k)-1}E^{\gamma,\delta,\nu,c}_{\mu,\tau,l}(w\eta^{\mu};\widetilde{p})r\left(\frac{\eta}{2}\psi(a)+m\left(\frac{2-\eta}{2}\right)\psi(b)\right)h\left(\frac{\eta^{\alpha}}{2^{\alpha}}\right)d\eta\\
&\quad+m\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(b))+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi\left(\frac{\psi(a)}{m^{2}}\right)\right]\\
&\quad\times\int_0^1\eta^{(\tau/k)-1}E^{\gamma,\delta,\nu,c}_{\mu,\tau,l}(w\eta^{\mu};\widetilde{p})r\left(\frac{\eta}{2}\psi(a)+m\left(\frac{2-\eta}{2}\right)\psi(b)\right)h\left(\frac{2^{\alpha}-\eta^{\alpha}}{2^{\alpha}}\right)d\eta,
\end{aligned}
\tag{58}
$$

where $\overline{\overline{w}} = (2^{\mu}w/(m\psi(b) - \psi(a))^{\mu})$ for all $\eta \in [a^{p}, b^{p}]$.

Proof. We demonstrate the claim as follows:

Multiplying (48) by $\eta^{(\tau/k)-1}E^{\gamma,\delta,\nu,c}_{\mu,\tau,l}(w\eta^{\mu};\widetilde{p})r((\eta/2)\psi(a) + m(2-\eta/2)\psi(b))$ and then integrating over $[0,1]$, we have

$$
\begin{aligned}
&\varphi\left(\frac{\psi(a)+m\psi(b)}{2}\right)\int_0^1\eta^{(\tau/k)-1}E^{\gamma,\delta,\nu,c}_{\mu,\tau,l}(w\eta^{\mu};\widetilde{p})r\left(\frac{\eta}{2}\psi(a)+m\left(\frac{2-\eta}{2}\right)\psi(b)\right)d\eta\\
&\leq h\left(\frac{1}{2^{\alpha}}\right)\int_0^1\eta^{(\tau/k)-1}E^{\gamma,\delta,\nu,c}_{\mu,\tau,l}(w\eta^{\mu};\widetilde{p})\varphi\left(\frac{\eta}{2}\psi(a)+m\left(\frac{2-\eta}{2}\right)\psi(b)\right)r\left(\frac{\eta}{2}\psi(a)+m\left(\frac{2-\eta}{2}\right)\psi(b)\right)d\eta\\
&\quad+mh\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\int_0^1\eta^{(\tau/k)-1}E^{\gamma,\delta,\nu,c}_{\mu,\tau,l}(w\eta^{\mu};\widetilde{p})\varphi\left(\left(\frac{2-\eta}{2}\right)\frac{\psi(a)}{m}+\frac{\eta}{2}\psi(b)\right)r\left(\frac{\eta}{2}\psi(a)+m\left(\frac{2-\eta}{2}\right)\psi(b)\right)d\eta.
\end{aligned}
\tag{59}
$$

By setting $\psi(\iota) = (\eta/2)\psi(a) + m(2-\eta/2)\psi(b)$ and $\psi(\kappa) = (2-\eta/2)(\psi(a)/m) + \eta/2\psi(b)$, that is, $\psi(a) + m\psi(b) - \psi(\iota) = (2-\eta/2)\psi(a) + m(\eta/2)\psi(b)$, in (59), in that case by usage $\varphi(\psi(\iota)) = \varphi(\psi(a) + m\psi(b) - \psi(\iota))$ and k-fractional integral operators (12) and (13), the first side of (58) is acquired.

To demonstrate the second side of (58), multiplying both parts of (50) by

$$
\eta^{(\tau/k)-1}E^{\gamma,\delta,\nu,c}_{\mu,\tau,l}(w\eta^{\mu};\widetilde{p})r\left(\frac{\eta}{2}\psi(a)+m\left(\frac{2-\eta}{2}\right)\psi(b)\right),
\tag{60}
$$

and then integrating over $[0,1]$, we have

$$
\begin{aligned}
&h\left(\frac{1}{2^{\alpha}}\right)\int_{0}^{1}\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\tilde{p})\varphi\left(\frac{\eta}{2}\psi(a)+m\left(\frac{2-\eta}{2}\right)\psi(b)\right)r\left(\frac{\eta}{2}\psi(a)+m\left(\frac{2-\eta}{2}\right)\psi(b)\right)\mathrm{d}\eta\\
&\quad+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\int_{0}^{1}\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\tilde{p})\varphi\left(\left(\frac{2-\eta}{2}\right)\frac{\psi(a)}{m}+\frac{\eta}{2}\psi(b)\right)r\left(\frac{\eta}{2}g(a)+m\left(\frac{2-\eta}{2}\right)\psi(b)\right)\mathrm{d}\eta\\
&\leq\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(a))+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(\psi(b))\right]\\
&\quad\times\int_{0}^{1}\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\tilde{p})r\left(\frac{\eta}{2}\psi(a)+m\left(\frac{2-\eta}{2}\right)\psi(b)\right)h\left(\frac{\eta^{\alpha}}{2^{\alpha}}\right)\mathrm{d}\eta\\
&\quad+m\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(b))+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi\left(\frac{\psi(a)}{m^{2}}\right)\right]\\
&\quad\times\int_{0}^{1}\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\tilde{p})r\left(\frac{\eta}{2}\psi(a)+\left(\frac{2-\eta}{2}\right)\psi(b)\right)h\left(\frac{2^{\alpha}-\eta^{\alpha}}{2^{\alpha}}\right)\mathrm{d}\eta.
\end{aligned}
\tag{61}
$$

Setting $\psi(\iota)=(\eta/2)\psi(a)+m(2-\eta/2)\psi(b)$ and $\psi(\kappa)=(2-\eta/2)(\psi(a)/m)+\eta/2\psi(b)$ in (59), then by using $\varphi(\psi(\iota))=\varphi(\psi(a)+m\psi(b)-\psi(\iota))$ and k-fractional integral operators (12) and (13), the second inequality of (58) is obtained. □

Corollary 4. *By using (58), some more k-fractional inequalities are offered as noted below:*

(i) By choosing $\psi=I$ and $\tilde{p}=w=0$, we obtain

$$
\begin{aligned}
&\varphi\left(\frac{a+mb}{2}\right)\int_{((a+mb)/2)}^{mb}(mb-\iota)^{(\tau/k)-1}r(\iota)\mathrm{d}\iota\\
&\leq h\left(\frac{1}{2^{\alpha}}\right)\int_{((a+mb)/2)}^{mb}(mb-\iota)^{(\tau/k)-1}(\varphi^{\circ}r)(\iota)\mathrm{d}\iota\\
&\quad+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\int_{(a/m)}^{((a+mb)/2)}\left(\kappa-\frac{a}{m}\right)^{(\tau/k)-1}(\varphi^{\circ}r)(\kappa)\mathrm{d}\kappa\\
&\leq\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(a)+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(b)\right]\\
&\quad\times\int_{0}^{1}\eta^{(\tau/k)-1}r\left(\frac{\eta}{2}a+m\left(\frac{2-\eta}{2}\right)b\right)h\left(\frac{\eta^{\alpha}}{2^{\alpha}}\right)\mathrm{d}\eta\\
&\quad+m\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(b)+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi\left(\frac{a}{m^{2}}\right)\right]\\
&\quad\times\int_{0}^{1}\eta^{(\tau/k)-1}r\left(\frac{\eta}{2}a+m\left(\frac{2-\eta}{2}\right)b\right)h\left(\frac{2^{\alpha}-\eta^{\alpha}}{2^{\alpha}}\right)\mathrm{d}\eta.
\end{aligned}
\tag{62}
$$

(ii) By choosing $\psi=I$ and $\tilde{p}=0$, we obtain

$$\begin{aligned}
&\varphi\left(\frac{a+mb}{2}\right)\int_{((a+mb)/2)}^{mb}(mb-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{\overline{w}}(mb-\iota)^{\mu}\right)r(\iota)\mathrm{d}\iota\\
&\leq h\left(\frac{1}{2^{\alpha}}\right)\int_{((a+mb)/2)}^{mb}(mb-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{\overline{w}}(mb-\iota)^{\mu}\right)(\varphi\circ r)(\iota)\mathrm{d}\iota\\
&\quad+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\int_{(a/m)}^{((a+mb)/2)}\left(\kappa-\frac{a}{m}\right)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{\overline{w}}m^{\mu}\left(\kappa-\frac{a}{m}\right)^{\mu}\right)(\varphi\circ r)(\kappa)\mathrm{d}\kappa\\
&\leq\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(a)+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(b)\right]\\
&\quad\times\int_{0}^{1}\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(w\eta^{\mu}\right)r\left(\frac{\eta}{2}a+m\left(\frac{2-\eta}{2}\right)b\right)h\left(\frac{\eta^{\alpha}}{2^{\alpha}}\right)\mathrm{d}\eta\\
&\quad+m\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(b)+m^{(\tau/k)+1}h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi\left(\frac{a}{m^{2}}\right)\right]\\
&\quad\times\int_{0}^{1}\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(w\eta^{\mu}\right)r\left(\frac{\eta}{2}a+m\left(\frac{2-\eta}{2}\right)b\right)h\left(\frac{2^{\alpha}-\eta^{\alpha}}{2^{\alpha}}\right)\mathrm{d}\eta.
\end{aligned}\tag{63}$$

(iii) By choosing $m=1$ and $\psi=I$, we obtain

$$\begin{aligned}
&\varphi\left(\frac{a+b}{2}\right)\int_{(a+b/2)}^{b}(b-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{\overline{w}}(b-\iota)^{\mu};\widetilde{p}\right)r(\iota)\mathrm{d}\iota\\
&\leq h\left(\frac{1}{2^{\alpha}}\right)\int_{(a+b/2)}^{b}(b-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{\overline{w}}(b-\iota)^{\mu};\widetilde{p}\right)(\varphi\circ r)(\iota)\mathrm{d}\iota\\
&\quad+h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\int_{a}^{b}(\kappa-a)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{\overline{w}}(\kappa-a)^{\mu};\widetilde{p}\right)(\varphi\circ r)(\kappa)\mathrm{d}\kappa\\
&\leq\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(a)+h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(b)\right]\int_{0}^{1}\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(w\eta^{\mu};\widetilde{p}\right)r\left(\frac{\eta}{2}a+\left(\frac{2-\eta}{2}\right)b\right)h\left(\frac{\eta^{\alpha}}{2^{\alpha}}\right)\mathrm{d}\eta\\
&\quad+\left[h\left(\frac{1}{2^{\alpha}}\right)\varphi(b)+h\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(a)\right]\int_{0}^{1}\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(w\eta^{\mu};\widetilde{p}\right)r\left(\frac{\eta}{2}a+\left(\frac{2-\eta}{2}\right)b\right)h\left(\frac{2^{\alpha}-\eta^{\alpha}}{2^{\alpha}}\right)\mathrm{d}\eta.
\end{aligned}\tag{64}$$

(iv) By choosing $\widetilde{p}=w=0$ and $h(\eta)=\eta$, we obtain

$$\begin{aligned}
&\varphi\left(\frac{\psi(a)+m\psi(b)}{2}\right)\int_{\psi^{-1}(\psi(a)+m\psi(b)/2)}^{\psi^{-1}(m\psi(b))}(m\psi(b)-\psi(\iota))^{(\tau/k)-1}r(\psi(\iota))\psi\prime(\iota)\mathrm{d}\iota\\
&\leq\left(\frac{1}{2^{\alpha}}\right)\int_{\psi^{-1}(\psi(a)+m\psi(b)/2)}^{\psi^{-1}(m\psi(b))}(m\psi(b)-\psi(\iota))^{(\tau/k)-1}(\varphi\circ r\circ\psi)(\iota)\psi\prime(\iota)\mathrm{d}\iota\\
&\quad+m^{(\tau/k)+1}\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\int_{\psi^{-1}(\psi(a)/2)}^{\psi^{-1}(\psi(a)+m\psi(b)/2m)}(\psi(\kappa)-\frac{\psi(a)}{m})^{(\tau/k)-1}(\varphi\circ r\circ\psi)(\kappa)\psi\prime(\kappa)\mathrm{d}\kappa\\
&\leq\left[\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(a))+m^{(\tau/k)+1}\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi(\psi(b))\right]\\
&\quad+m\left[\left(\frac{1}{2^{\alpha}}\right)\varphi(\psi(b))+m^{(\tau/k)+1}\left(\frac{2^{\alpha}-1}{2^{\alpha}}\right)\varphi\left(\frac{\psi(a)}{m^{2}}\right)\right]\\
&\quad\times\int_{0}^{1}\eta^{(\tau/k)-1}r\left(\frac{\eta}{2}\psi(a)+m\left(\frac{2-\eta}{2}\right)\psi(b)\right)h\left(\frac{2^{\alpha}-\eta^{\alpha}}{2^{\alpha}}\right)\mathrm{d}\eta.
\end{aligned}\tag{65}$$

(v) By choosing $\alpha=1$ and $\psi=I$, we get

$$\begin{aligned}
&\varphi\left(\frac{a+mb}{2}\right)\int_{(a+mb/2)}^{mb}(mb-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(\overline{\overline{w}}(mb-\iota)^{\mu};\widetilde{p})r(\iota)\mathrm{d}\iota\\
&\leq h\left(\frac{1}{2}\right)\int_{(a+mb/2)}^{mb}(mb-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(\overline{\overline{w}}(mb-\iota)^{\mu};\widetilde{p})(\varphi\circ r)(\iota)\mathrm{d}\iota\\
&\quad+m^{(\tau/k)+1}h\left(\frac{1}{2}\right)\int_{(a/m)}^{(a+mb/2)}\left(\kappa-\frac{a}{m}\right)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}\left(\overline{\overline{w}}m^{\mu}\left(\kappa-\frac{a}{m}\right)^{\mu};\widetilde{p}\right)(\varphi\circ r)(\kappa)\mathrm{d}\kappa\\
&\leq h\left(\frac{1}{2}\right)\left[\varphi(a)+m^{(\tau/k)+1}\varphi(b)\right]\int_{0}^{1}\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\widetilde{p})r\left(\frac{\eta}{2}a+m\left(\frac{2-\eta}{2}\right)b\right)h\left(\frac{\eta}{2}\right)\mathrm{d}\eta\\
&\quad+mh\left(\frac{1}{2}\right)\left[f(b)+m^{(\tau/k)+1}\varphi\left(\frac{a}{m^{2}}\right)\right]\int_{0}^{1}\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\widetilde{p})r\left(\frac{\eta}{2}a+m\left(\frac{2-\eta}{2}\right)b\right)h\left(\frac{2-\eta}{2}\right)\mathrm{d}\eta.
\end{aligned}\tag{66}$$

(vi) By choosing $\alpha=m=1, h(\eta)=\eta$ and $\psi=I$, we get

$$\begin{aligned}
&\varphi\left(\frac{a+b}{2}\right)\int_{(a+b/2)}^{b}(b-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(\overline{\overline{w}}(b-\iota)^{\mu};\widetilde{p})r(\iota)d\iota\\
&\leq\frac{1}{2}\Big[\int_{(a+b/2)}^{b}(b-\iota)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(\overline{\overline{w}}(b-\iota)^{\mu};\widetilde{p})(\varphi\circ r)(\iota)\mathrm{d}\iota\\
&\quad+\int_{a}^{(a+b/2)}(\kappa-a)^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(\overline{\overline{w}}(\kappa-a)^{\mu};\widetilde{p})(\varphi\circ r)(\kappa)\mathrm{d}\kappa\Big]\\
&\leq\left(\frac{\varphi(a)+\varphi(b)}{2}\right)\int_{0}^{1}\eta^{(\tau/k)-1}E_{\mu,\tau,l}^{\gamma,\delta,\nu,c}(w\eta^{\mu};\widetilde{p})r\left(\frac{\eta}{2}a+\left(\frac{2-\eta}{2}\right)b\right)\mathrm{d}\eta.
\end{aligned}\tag{67}$$

Remark 6. Those as mentioned above k-fractional inequalities are farther in line with foreknown conclusions as by choosing $k=1$ in Corollary 4 *(v), an inequality for extended generalized fractional integrals is obtained.*

Data Availability

There are no data required for this paper

Conflicts of Interest

The authors declare no conflicts of interest.

Authors' Contributions

All the authors made equal contributions.

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