Hindawi
Journal of Mathematics
Volume 2022, Article ID 9148581, 7 pages
https://doi.org/10.1155/2022/9148581

Research Article

Multiplicative Attributes Derived from Graph Invariants for *$Saztec_4$* Diamond

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Received 14 July 2022; Accepted 23 August 2022; Published 22 September 2022

Academic Editor: Hassan Raza

This study consists of developing some closed and updated formulas derived from multiplicative graph invariants such as general Randic index (GRI)$R_{\lambda^0}(\varkappa)$ for $\lambda^0 = \{\pm 1, \pm 1/2\}$, ordinary general geometric-arithmetic (OGA), general version of harmonic index (GHI), sum connectivity index (SI), general sum connectivity index (GSI), 1st and 2nd Gourava and hyper-Gourava indices, (ABC) index, Shegehalli and Kanabur indices, 1st generalised version of Zagreb index (GZI), and forgotten index (FI) for the subdivided Aztec diamond network. Aztec diamond is constructed based on the squares boxes. These square boxes are placed at the centre point and nourish the condition $|s - (1/2)| + |r - (1/2)| \leq n$. Furthermore, we put a new vertex of degree-2 at each edge of the small boxes, squares in shapes. A new structure is obtained that has the same properties as its parental graph and is called a subdivided Aztec diamond and symbolised as $Saztec_n$. Subsequently, we compute the multiplicative topological attributes to get some new formulas. For this purpose, a simple, connected, and the finite graph is considered by supposing it Y as the graph of the $Saztec_n$. The order and size have also been discussed in this study and found three different kinds of edges (2, 2), (2, 3), and (2, 4) for computing. The discussion on the networks mentioned above provides us with essential results that can be used in the determination of bio and physio activities and can be interspersed with the molecular compounds and their graphical structures better to understand their physical as well as biological properties.

1. Introduction

The branch of mathematics concerned with graphs, their application, and their correlation with chemical compounds is known as chemical graph theory. In molecular modelling, we use this tool of mathematics. First, the chemical compound's structure is drawn and compared with its mathematical graphical structure. This theory needs to be used as a mathematical tool to recognise a particular molecular web's physical and biological features.

Vukicevi'c and Furtula' developed the 1st GA [1] index in 2009 and formulated as

$$\mathrm{GA}(\Upsilon) = \prod_{ro \in E(\Upsilon)} \frac{2\sqrt{d_r \times d_o}}{d_r + d_o}. \tag{1}$$

An OGA invariant [2] was determined in 2011 and symbolised as given in the following $\forall$ real w:

$$\mathrm{OGA}_w(\Upsilon) = \prod_{ef\in E(\Upsilon)} \left[\frac{\sqrt{4d_e d_f}}{d_e + d_f}\right]^w. \quad (2)$$

In 2017, V.R. Kulli developed 1st and 2nd Gourava and hyper-Gourava indices [3, 4] are computed by

$$\begin{aligned}
\mathrm{GO}_1(\Upsilon) &= \prod_{ef\in E(\Upsilon)} \left[\left(d_e + d_f\right) + \left(d_e d_f\right)\right],\\
\mathrm{GO}_2(\Upsilon) &= \prod_{ef\in E(\Upsilon)} \left[\left(d_e + d_f\right)\left(d_e d_f\right)\right],\\
\mathrm{HGO}_1(\Upsilon) &= \prod_{ef\in E(\Upsilon)} \left[\left(d_e + d_f\right) + \left(d_e d_f\right)\right]^2,\\
\mathrm{HGO}_2(\Upsilon) &= \prod_{ef\in E(\Upsilon)} \left[\left(d_e + d_f\right)\left(d_e d_f\right)\right]^2.
\end{aligned} \quad (3)$$

Randic' index was developed [5] by Milan Randic' in 1975 and calculated by

$$R_{1/2}(\Upsilon) = \prod_{ef\in E(\Upsilon)} \frac{1}{\sqrt{\left(d_e d_f\right)}}. \quad (4)$$

Then, Erdos and Bollobas' invented its general version, familiarised with the general Randic' index for λ^o, where $\lambda^o \in R$ [6], and evaluated by

$$R_{\lambda^o}(\Upsilon) = \prod_{ef\in E(\Upsilon)} \left(d_e d_f\right)^{\lambda^o}, \quad \text{for } \lambda^o = \left\{-1, 1, -\frac{1}{2}, \frac{1}{2}\right\}. \quad (5)$$

Zhong [7], in 2012, gave the idea of the harmonic index and familiarised by

$$\mathrm{HI}(\Upsilon) = \prod_{ef\in E(\Upsilon)} \frac{2}{d_e + d_f}. \quad (6)$$

Yan determined its generalised form [8] in 2015 and symbolised as

$$H_w I(\Upsilon) = \prod_{ef\in E(\Upsilon)} \left[\frac{2}{d_e + d_f}\right]. \quad (7)$$

ABC invariant was introduced in 1998 by Estrada et al. [9]; that is,

$$\mathrm{ABC}(\Upsilon) = \prod_{ef\in E(\Upsilon)} \sqrt{\frac{d_e + d_f - 2}{d_e d_f}}. \quad (8)$$

SK, SK_1, and SK_2 invariants [10] were introduced by Shegehalli & Kanabur that are

$$\begin{aligned}
\mathrm{SK}(\Upsilon) &= \prod_{ef\in E(\Upsilon)} \frac{d_e + d_f}{2},\\
\mathrm{SK}_1(\Upsilon) &= \prod_{ef\in E(\Upsilon)} \frac{d_e d_f}{2},\\
\mathrm{SK}_2(\Upsilon) &= \prod_{ef\in E(\Upsilon)} \left[\frac{d_e + d_f}{2}\right]^2.
\end{aligned} \quad (9)$$

In 2009, Lucic' described (SI) [11] and computed it as

$$G_{-(1/2)}(\Upsilon) = \prod_{ef\in E(\Upsilon)} \left[d_e + d_f\right]^{-(1/2)}. \quad (10)$$

Then, in 2010, Zhou and Trinajstic generalised it [8, 12] as

$$G_k(\Upsilon) = \prod_{ef\in E(\Upsilon)} \left[d_e + d_f\right]^k. \quad (11)$$

Zheng, in 2005, developed the general version of the 1st Zagreb index [13]

$${}^sM_1(\Upsilon) = \prod_{ef\in E(\Upsilon)} \left[d_e^{s-1} + d_f^{s-1}\right], \quad s \in R, s \neq 0 \text{ and } s \neq 1. \quad (12)$$

Furtula' and Gutman [14] formulated an invariant known as F-index

$$F(\Upsilon) = \prod_{ef\in E(\Upsilon)} \left[d_e^2 + d_f^2\right] \quad (13)$$

2. Material and Methods

Aztec diamond is created based on the square lattices. These square lattices are kept centred at (s, r), satisfying $|s - (1/2)| + |r - (1/2)| \leq n$. In addition, we place a new node having 2 as the degree at every edge of the small squares. In this way, we get a new structure known as a subdivided Aztec diamond $Saztec_n$. Next, we evaluate the multiplicative topological attributes in order to obtain new formulas. Let us suppose Y is the graph of the $Saztec_n$. The cardinality of $Saztec_n$ with respect to vertices is $|V(\Upsilon)| = 6n^2 + 14n + 1$, and with respect to edges, is $|E(\Upsilon)| = 8n^2 + 16n$. There are three different kinds of edges (2, 2), (2, 3), and (2, 4).

3. Results and Discussion

We have implemented various multiplicative graph invariants [15] over the given molecular structures. Figures 1–4 have been depicted for better understanding. We describe two essential components, nodes and edges. Following, some theorems have been constructed with the help of these particular graphical invariants. Let $e = |E(\Upsilon)|$ be the cardinality of Y with respect to edge set. The $Saztec_n$ is established at the terminal nodes of every edge.

Let's choose Y as a graph of the subdivided Aztec diamond network $Saztec_n$, defining the terms d_e and d_f as the degrees of nodes e and f. We have developed the following theorems.

Theorem 1. *For $Saztec_n$, its OGA can be developed as*

$$\ln\left[\mathrm{OGA}_w(\Upsilon)\right] = (12w)\ln\left[\frac{\sqrt{24}}{5}\right] + w\left(8n^2 + 8n - 12\right)\ln\left[\frac{\sqrt{32}}{6}\right]. \quad (14)$$

Proof. With the help of Table 1, we infer

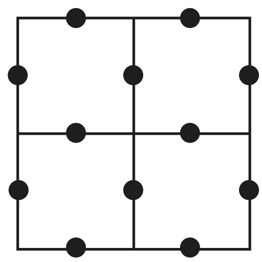

Figure 1: *Saztec* (1).

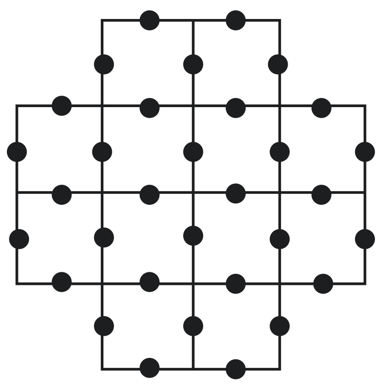

Figure 2: *Saztec* (2).

$$\begin{aligned}
\mathrm{OGA}_w(\Upsilon) &= \prod_{ef\in E(\Upsilon)} \left[\frac{\sqrt{4d_e\times d_f}}{d_e+d_f}\right]^w,\\
\mathrm{OGA}_w(\Upsilon) &= \left[\frac{\sqrt{16}}{2+2}\right]^{8nw} \times \left[\frac{\sqrt{24}}{5}\right]^{12w} \times \left[\frac{\sqrt{32}}{6}\right]^{w\left(8n^2+8n-12\right)},\\
\mathrm{OGA}_w(\Upsilon) &= \left[\frac{\sqrt{24}}{5}\right]^{12w} \times \left[\frac{\sqrt{32}}{6}\right]^{w\left(8n^2+8n-12\right)}.
\end{aligned} \tag{15}$$

By making some computations, we get

$$\ln\left[\mathrm{OGA}_w(\Upsilon)\right] = (12w)\ln\left[\frac{\sqrt{24}}{5}\right] + w\left(8n^2+8n-12\right)\ln\left[\frac{\sqrt{32}}{6}\right]. \tag{16}$$

□

Theorem 2. *For Saztec$_n$, 1st and 2nd Gourava descriptors can be developed as*

$$\begin{aligned}
\ln\left[\mathrm{GO}_1(\Upsilon)\right] &= \ln[0.05535] + (8n)\ln[112] + \left(8n^2\right)\ln[14],\\
\ln\left[GO_2(\Upsilon)\right] &= \ln\left[3.552\times 10^{-3}\right] + (8n)\ln[768] + \left(8n^2\right)\ln[48].
\end{aligned} \tag{17}$$

Proof. With the help of Table 1, we infer

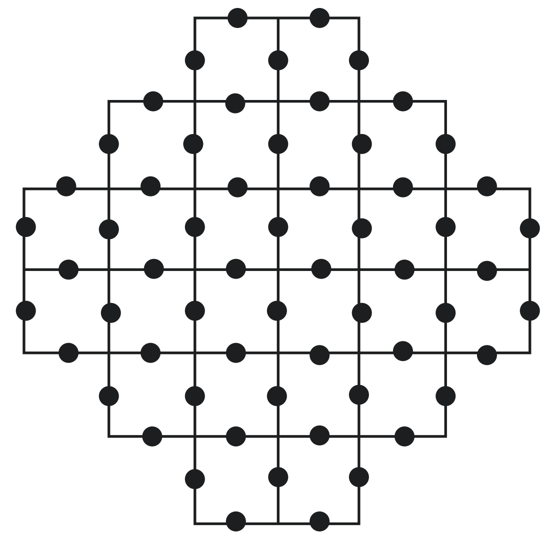

Figure 3: *Saztec* (3).

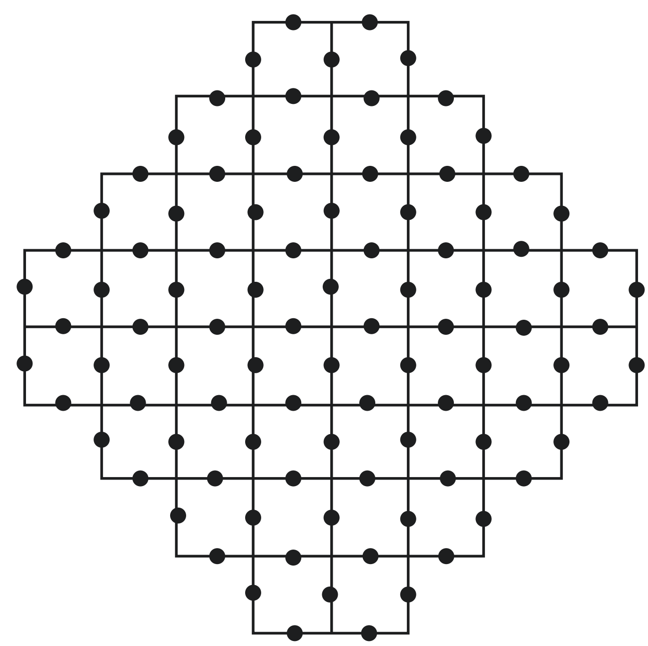

Figure 4: *Saztec* (4).

Table 1: Describes the partition of edges for graph Y.

(d_e, d_f) for $ef \in E(\Upsilon)$	Number of $E(\Upsilon)$
(2, 2)	$8n$
(2, 3)	12
(2, 4)	$8n^2 + 8n - 12$

$$\begin{aligned}
\mathrm{GO}_1(\Upsilon) &= \prod_{ef\in E(\Upsilon)} \left[\left(d_e+d_f\right)+\left(d_e d_f\right)\right],\\
\mathrm{GO}_2(\Upsilon) &= \prod_{ef\in E(\Upsilon)} \left[\left(d_e+d_f\right)\left(d_e d_f\right)\right].
\end{aligned} \tag{18}$$

After some calculations, we have

$$\begin{aligned}
\mathrm{GO}_1(\Upsilon) &= 8^{8n} \times 11^{12} \times 14^{(8n^2+8n-12)} \\
&= 0.05535\left(112^{8n} \times 14^{8n^2}\right), \\
\ln[\mathrm{GO}_1(\Upsilon)] &= \ln[0.05535] + (8n)\ln[112] + \left(8n^2\right)\ln[14], \\
\mathrm{GO}_2(\Upsilon) &= 16^{8n} \times 30^{12} \times 48^{(8n^2+8n-12)} \\
&= 3.552 \times 10^{-3}\left(768^{8n} \times 48^{8n^2}\right), \\
\ln[\mathrm{GO}_2(\Upsilon)] &= \ln\left[3.552 \times 10^{-3}\right] + (8n)\ln[768] \\
&\quad + \left(8n^2\right)\ln[48].
\end{aligned} \tag{19}$$

□

Theorem 3. *For $Saztec_n$, 1^{st} and 2^{nd} hyper-Gourava descriptors can be developed as*

$$\begin{aligned}
\ln[\mathrm{HGO}_1(\Upsilon)] &= \ln\left[3.0644 \times 10^{-3}\right] + (8n)\ln[12544] \\
&\quad + \left(8n^2\right)\ln[196], \\
\ln[\mathrm{HGO}_2(\Upsilon)] &= \ln\left[1.2621 \times 10^{-5}\right] + (8n)\ln[589824] \\
&\quad + \left(8n^2\right)\ln[2304].
\end{aligned} \tag{20}$$

Proof. With the help of Table 1, we infer

$$\begin{aligned}
\mathrm{HGO}_1(\Upsilon) &= \prod_{ef \in E(\Upsilon)} \left[\left(d_e + d_f\right) + \left(d_e d_f\right)\right]^2, \\
\mathrm{HGO}_2(\Upsilon) &= \prod_{ef \in E(\Upsilon)} \left[\left(d_e + d_f\right)\left(d_e d_f\right)\right]^2.
\end{aligned} \tag{21}$$

By making some computations, we get

$$\begin{aligned}
\mathrm{HGO}_1(\Upsilon) &= 64^{8n} \times 121^{12} \times 196^{(8n^2+8n-12)} \\
&= 3.0644 \times 10^{-3}\left(12544^{8n} \times 196^{8n^2}\right), \\
\ln[\mathrm{HGO}_1(\Upsilon)] &= \ln\left[3.0644 \times 10^{-3}\right] + (8n)\ln[12544] \\
&\quad + \left(8n^2\right)\ln[196], \\
\mathrm{HGO}_2(\Upsilon) &= 256^{8n} \times 900^{12} \times 2304^{(8n^2+8n-12)} \\
&= 1.2621 \times 10^{-5}\left(589824^{8n} \times 2304^{8n^2}\right), \\
\ln[\mathrm{HGO}_2(\Upsilon)] &= \ln\left[1.2621 \times 10^{-5}\right] + (8n)\ln[589824] \\
&\quad + \left(8n^2\right)\ln[2304].
\end{aligned} \tag{22}$$

□

Theorem 4. *For $Saztec_n$, GRI can be formulated as*

$$R_{\lambda_0'}(Saztec_n) = \begin{cases}
\ln[31.5692] - (8n)\ln[32] - \left(8n^2\right)\ln[8], & \text{for } \lambda_0' = -1, \\
\ln[0.03703] - (4n)\ln[48] - \left(4n^2\right)\ln[12], & \text{for } \lambda_0' = -\dfrac{1}{2}, \\
\ln[91.125] + (4n)\ln[128] + \left(4n^2\right)\ln[8], & \text{for } \lambda_0' = \dfrac{1}{2}, \\
\ln[0.0316] + (8n)\ln[32] + \left(8n^2\right)\ln[8], & \text{for } \lambda_0' = 1.
\end{cases} \tag{23}$$

Proof. We know that

$$R_{\lambda'}(\Upsilon) = \prod_{ef \in E(\Upsilon)} \left[d_e \times d_f\right]^{\lambda'}, \quad \text{for } \lambda' = \left\{\pm 1, \pm \frac{1}{2}\right\}. \tag{24}$$

□

Case 1. For $\lambda' = -1$, its RI can be formulated as

$$R_{-1}(\Upsilon) = \prod_{ef \in E(\Upsilon)} \frac{1}{d_e \times d_f}. \tag{25}$$

Using (24) and from Table 1, we get

$$R_{-1}(\Upsilon) = 4^{-8n} \times 6^{-12} \times 8^{-(8n^2+8n-12)}. \tag{26}$$

After some computations, we get

$$\ln[R_{-1}(\Upsilon)] = \ln[31.5692] - (8n)\ln[32] - \left(8n^2\right)\ln[8]. \tag{27}$$

Case 2. For $\lambda' = -(1/2)$, its Randic' index $R_{\lambda'}(\Upsilon)$ can be computed as

$$R_{-1/2}(\Upsilon) = \prod_{ef \in E(\Upsilon)} \frac{1}{\sqrt{\left(d_e \times d_f\right)}}. \tag{28}$$

Using (24) and from Table 1, we know

$$R_{-1/2}(\Upsilon) = \left[\frac{1}{\sqrt{4}}\right]^{8n} \times \left[\frac{1}{\sqrt{6}}\right]^{12} \times \left[\frac{1}{\sqrt{12}}\right]^{(8n^2+8n-12)}. \tag{29}$$

After some computations, we get

$$\ln[R_{-1/2}(\Upsilon)] = \ln[0.03703] - (4n)\ln[48] - \left(4n^2\right)\ln[12]. \tag{30}$$

Case 3. For $\lambda' = -(1/2)$, its Randic' index $R_{\lambda'}(\Upsilon)$ can be computed as

$$R_{1/2}(\Upsilon) = \prod_{ef\in E(\Upsilon)} \sqrt{\left(d_e \times d_f\right)}. \quad (31)$$

Using (24) and from Table 1, getting

$$R_{1/2}(\Upsilon) = (\sqrt{4})^{8n} \times (\sqrt{6})^{12} \times (\sqrt{8})^{\left(8n^2+8n-12\right)}. \quad (32)$$

By doing some calculations, we get

$$\ln\left[R_{1/2}(\Upsilon)\right] = \ln[91.125] + (4n)\ln[128] + \left(4n^2\right)\ln[8]. \quad (33)$$

Case 4. For $\lambda' = 1$, its Randic' index $R_{\lambda'}(\Upsilon)$ can be computed as

$$R_1(\Upsilon) = \prod_{ef\in E(\Upsilon)} \left(d_e \times d_f\right)^1. \quad (34)$$

Using (24) and from Table 1, we know

$$R_1(\Upsilon) = 4^{8n} \times 6^{12} \times 8^{\left(8n^2+8n-12\right)}. \quad (35)$$

By doing some calculations, we get

$$\ln\left[R_1(\Upsilon)\right] = \ln[0.0316] + (8n)\ln[32] + \left(8n^2\right)\ln[8]. \quad (36)$$

Theorem 5. *For Saztec$_n$, HI can be developed as*

$$\ln[\mathrm{HI}(\Upsilon)] = \ln[8.9161] - (8n)\ln[6] - \left(8n^2\right)\ln[3]. \quad (37)$$

Proof. With the help of Table 1, we infer

$$\mathrm{HI}(\Upsilon) = \prod_{ef\in E(\Upsilon)} \frac{2}{d_e + d_f}. \quad (38)$$

After simplifications, we obtain

$$\begin{aligned}\mathrm{HI}(\Upsilon) &= \left[\frac{1}{2}\right]^{8n} \times \left[\frac{2}{5}\right]^{12} \times \left[\frac{1}{3}\right]^{\left(8n^2+8n-12\right)} \\ &= 8.9161\left(6^{-8n} \times 3^{-8n^2}\right),\end{aligned} \quad (39)$$

$$\ln[\mathrm{HI}(\Upsilon)] = \ln[8.9161] - (8n)\ln[6] - \left(8n^2\right)\ln[3].$$

□

Theorem 6. *For Saztec$_n$, GHI can be developed as*

$$\ln\left[H_w I(\Upsilon)\right] = (12w)\ln[1.2] - (8nw)\ln[6] - \left(8wn^2\right)\ln[3]. \quad (40)$$

Proof. With the help of Table 1, we infer

$$H_w I(\Upsilon) = \prod_{ef\in E(\Upsilon)} \left[\frac{2}{d_e + d_f}\right]^w. \quad (41)$$

By making some computations, we get

$$\begin{aligned}H_w I(\Upsilon) &= \left[\frac{1}{2}\right]^{8nw} \times \left[\frac{2}{5}\right]^{12w} \times \left[\frac{1}{3}\right]^{\left(8n^2+8n-12\right)w} \\ &= (1.2)^{12w}\left(6^{-8nw} \times 3^{-8n^2 w}\right),\end{aligned}$$

$$\ln\left[H_w I(\Upsilon)\right] = (12w)\ln[1.2] - (8nw)\ln[6] - \left(8wn^2\right)\ln[3]. \quad (42)$$

□

Theorem 7. *For Saztec$_n$, the ABC index can be developed as*

$$\ln[\mathrm{ABC}(\Upsilon)] = (-4n)\ln[4] - \left(4n^2\right)\ln[2]. \quad (43)$$

Proof. With the help of Table 1, we infer

$$\mathrm{ABC}(\Upsilon) = \prod_{ef\in E(\Upsilon)} \sqrt{\frac{d_e + d_f - 2}{d_e \times d_f}}. \quad (44)$$

By making some computations, we get

$$\begin{aligned}\mathrm{ABC}(\Upsilon) &= \left[\frac{1}{\sqrt{2}}\right]^{8n} \times \left[\frac{1}{\sqrt{2}}\right]^{12} \times \left[\frac{1}{\sqrt{2}}\right]^{\left(8n^2+8n-12\right)} \\ &= 4^{-4n} \times 2^{-4n^2},\end{aligned} \quad (45)$$

$$\ln[\mathrm{ABC}(\Upsilon)] = (-4n)\ln[4] - \left(4n^2\right)\ln[2].$$

□

Theorem 8. *For Saztec$_n$, SK, SK1, and SK2 descriptors can be developed as*

$$\begin{aligned}\ln[\mathrm{SK}(\Upsilon)] &= \ln[0.8333] + (8n)\ln[6] + \left(8n^2\right)\ln[3], \\ \ln\left[\mathrm{SK}_1(\Upsilon)\right] &= \ln[0.75] + (8n)\ln[8] + \left(8n^2\right)\ln[4], \\ \ln\left[\mathrm{SK}_2(\Upsilon)\right] &= \ln[0.6944] + (8n)\ln[36] + \left(8n^2\right)\ln[9].\end{aligned} \quad (46)$$

Proof. With the help of Table 1, we infer

$$\begin{aligned}\mathrm{SK}(\Upsilon) &= \prod_{ef\in E(\Upsilon)} \frac{d_e + d_f}{2}, \\ \mathrm{SK}_1(\xi) &= \prod_{ef\in E(\xi)} \frac{d_e d_f}{2}, \\ \mathrm{SK}_2(\Upsilon) &= \prod_{ef\in E(\Upsilon)} \left[\frac{d_e + d_f}{2}\right]^2.\end{aligned} \quad (47)$$

By making some calculations, we get

$$\begin{aligned}
\mathrm{SK}(\Upsilon) &= 2^{8n} \times (2.5)^{12} \times 3^{\left(8n^2+8n-12\right)} \\
&= 0.8333\left(6^{8n} \times 3^{8n^2}\right), \\
\ln[\mathrm{SK}(\Upsilon)] &= \ln[0.8333] + (8n)\ln[6] + \left(8n^2\right)\ln[3], \\
\mathrm{SK}_1(\Upsilon) &= 2^{8n} \times (3)^{12} \times 4^{\left(8n^2+8n-12\right)} \\
&= 0.75\left(8^{8n} \times 4^{8n^2}\right), \\
\ln\left[\mathrm{SK}_1(\Upsilon)\right] &= \ln[0.75] + (8n)\ln[8] \\
&\quad + \left(8n^2\right)\ln[4], \\
\mathrm{SK}_2(\Upsilon) &= 4^{8n} \times (6.25)^{12} \times 9^{\left(8n^2+8n-12\right)} \\
&= 0.6944\left(36^{8n} \times 9^{8n^2}\right), \\
\ln\left[\mathrm{SK}_2(\Upsilon)\right] &= \ln[0.6944] + (8n)\ln[36] + \left(8n^2\right)\ln[9].
\end{aligned} \tag{48}$$

□

Theorem 9. *For $Saztec_n$, SI can be developed as*

$$\ln\left[\mathrm{SI}_{-1/2}(\Upsilon)\right] = \ln[2.9859] - (4n)\ln[24] - \left(4n^2\right)\ln[6]. \tag{49}$$

Proof. With the help of Table 1, we infer

$$\begin{aligned}
\mathrm{SI}_{-1/2}(\Upsilon) &= \prod_{ef\in E(\Upsilon)} \left[d_e + d_f\right]^{-1/2}, \\
\mathrm{SI}_{-1/2}(\Upsilon) &= \left[\frac{1}{\sqrt{4}}\right]^{8n} \times \left[\frac{1}{\sqrt{5}}\right]^{12} \times \left[\frac{1}{\sqrt{6}}\right]^{\left(8n^2+8n-12\right)}.
\end{aligned} \tag{50}$$

By making some computations, we get

$$\begin{aligned}
\mathrm{SI}_{-1/2}(\Upsilon) &= 2.9859\left(24^{-4n} \times 6^{-4n^2}\right), \\
\ln\left[\mathrm{SI}_{-1/2}(\Upsilon)\right] &= \ln[2.9859] - (4n)\ln[24] - \left(4n^2\right)\ln[6].
\end{aligned} \tag{51}$$

□

Theorem 10. *For $Saztec_n$, GSI can be developed as*

$$\ln\left[\mathrm{GSI}_w(\Upsilon)\right] = (12w)\ln[0.8333] + (8nw)\ln[24] + \left(8wn^2\right)\ln[6]. \tag{52}$$

Proof. With the help of Table 1, we infer

$$\begin{aligned}
\mathrm{GSI}_w(\Upsilon) &= \prod_{ef\in E(\Upsilon)} \left[d_e + d_f\right]^{w}, \\
\mathrm{GSI}_w(\Upsilon) &= 4^{8nw} \times 5^{12w} \times 6^{\left(8n^2+8n-12\right)w}.
\end{aligned} \tag{53}$$

By making some computations, we get

$$\begin{aligned}
\mathrm{GSI}_w(\Upsilon) &= (0.8333)^{12w}\left(24^{8nw} \times 6^{8n^2w}\right), \\
\ln\left[\mathrm{GSI}_w(\Upsilon)\right] &= (12w)\ln[0.8333] + (8nw)\ln[24] \\
&\quad + \left(8wn^2\right)\ln[6].
\end{aligned} \tag{54}$$

□

Theorem 11. *For $Saztec_n$, 1st GZI can be developed as*

$$\begin{aligned}
\ln\left[{}^{w}M_1(\Upsilon)\right] &= (8nw)\ln[2] + 12\ln\left[2^{w-1} + 3^{w-1}\right] \\
&\quad + \left(8n^2 + 8n - 12\right)\ln\left[2^{w-1} + 4^{w-1}\right].
\end{aligned} \tag{55}$$

Proof. With the help of Table 1, we infer

$$\begin{aligned}
{}^{w}M_1(\Upsilon) &= \prod_{ef\in E(\Upsilon)} \left[d_e^{w-1} + d_f^{w-1}\right], \quad w > 1, \\
{}^{w}M_1(\Upsilon) &= 2^{8nw} \times \left(2^{w-1} + 3^{w-1}\right)^{12} \times \left(2^{w-1} + 4^{w-1}\right)^{\left(8n^2+8n-12\right)}, \\
\ln\left[{}^{w}M_1(\Upsilon)\right] &= (8nw)\ln[2] + 12\ln\left[2^{w-1} + 3^{w-1}\right] + \left(8n^2 + 8n - 12\right)\ln\left[2^{w-1} + 4^{w-1}\right].
\end{aligned} \tag{56}$$

□

Theorem 12. *For $Saztec_n$, F-index can be developed as*

$$\ln[F(\Upsilon)] = \ln\left[5.688 \times 10^{-3}\right] + (8n)\ln[160] + \left(8n^2\right)\ln[20]. \tag{57}$$

Proof. With the help of Table 1, we infer

$$\begin{aligned}
F(\Upsilon) &= \prod_{ef\in E(\Upsilon)} \left[d_e^2 + d_f^2\right], \\
F(\Upsilon) &= 8^{8n} \times 13^{12} \times 20^{\left(8n^2+8n-12\right)}.
\end{aligned} \tag{58}$$

By making some computations, we get

$$\begin{aligned}
F(\Upsilon) &= 5.688 \times 10^{-3}\left(160^{8n} \times 20^{8n^2}\right), \\
\ln[F(\Upsilon)] &= \ln\left[5.688 \times 10^{-3}\right] + (8n)\ln[160] + \left(8n^2\right)\ln[20].
\end{aligned} \tag{59}$$

□

4. Main Findings

(i) Consideration of the molecular structure of the Aztec diamond network

(ii) Getting the subdivided version of the Aztec diamond network for $n = 1, 2, 3, 4$ by inserting a new node at each edge and placing the name of the new derived molecular structure as *$Saztec_4$*

(iii) Association of the mathematical graph with the chemical structure
(iv) Vertex labelling for each vertex with their degrees
(v) Edge partition of edge set according to their degrees
(vi) Computations of the degree of each vertex by constructing the generalised formula
(vii) Construction of new closed formulas using many various topological attributes such as general Randic' index $R_{\lambda'}(\Upsilon)$ for $\lambda' = \{\pm 1, \pm 1/2\}$, GHI, OGAI, SHI, GSHI, 1st and 2nd Gourava and hyper-Gourava descriptors, ABC invariant, SKs' indices, and F-index

5. Conclusions

This work involves inventing many new formulas based on multiplicative graph invariants. We have used many indices such as GRI, OGA, GHI, SI, GSI, 1st and 2nd Gourava and hyper-Gourava indices, ABC index, SKs' indices, 1st GZI and forgotten index (FI) for $Saztec_n$ The above-evaluated formulas can be interspersed with the molecular compounds and their graphical structures to understand their physical and biological properties better. More applications can be investigated for these above-mentioned topological indices.

Data Availability

No data were used in this manuscript.

Additional Points

Future Work. The latest topological indices can be found and applied to more molecular and general mathematical networks.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The researchers would like to thank the Deanship of Scientific Research, Qassim University for funding the publication of this project.

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