Research Article

A Nonsingular Fractional Derivative Approach for Heat and Mass Transfer Flow with Hybrid Nanoparticles

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This paper deals with the study of MHD Brinkman type fluid flow containing hybrid titanium (TiO2) and silver (Ag) nanoparticles with nonlocal noninteger type Atangana-Baleanu (ABC) fractional differential operator. The problem is designed for the convective flow restrained in a microchannel. With the Mittag–Leffler kernel, the conventional governing equations are converted into dimensionless form and then generalised with noninteger order fractional operators. The solutions for temperature and velocity fields obtained via Laplace transform method and expressed in the series form. The effect of related parameters is dignified graphically with the help of Mathcad and presented in the graphical section. Finally, the results show that the AB fractional operator exhibited improved memory effect as compared to CF fractional operator. Furthermore, due to increasing the values volume fractional temperature can be enhanced and velocity decreases. In comparison between nanoparticles for different types of based fluid, velocity and temperature of water based (TiO2) and silver (Ag) is higher than other base fluids.

1. Introduction

Fractional calculus is a field of mathematics that deals with the integration and derivation of real and complex numbers. Despite the fact that calculus is an old subject, in recent decades, it has regained popularity due to its numerous applications. Researchers have recently investigated mathematical models of heat transfer fluids with considerable applications, which play an essential role in manufacturing. These simulations are usually represented as integer-order partial differential equations. Its important to note that traditional PDEs cannot account for the dynamical behavior of physical flow parameters or retention effects. To cater and eliminate the above mentioned flaws researchers concentrated on fractional dynamic systems of heat transmission in simple and complex fluid models. Nanoparticles are tiny particles with a diameter of 1 to 100 nanometers. Nanoparticles, which are invisible to the naked eye, can have drastically varying physical and chemical characteristics than their bigger material counterparts. Only a few hundred atoms make up most nanoparticles. Nanofluids are often used as coolants in heat transfer equipment such as heat exchangers, electronic cooling systems (such as solid surfaces), and radiators due to their better cooling characteristics [1]. Nanofluids have unique features that make them potentially helpful in a variety of heat transfer applications, such as microelectronics, battery storage, pharmaceutical processes, hybrid-powered engines, engine cooling/vehicle thermal management, domestic refrigerator, chiller, superheater, and crusher [2]. Das et al. [3] conducted a research nanofluids and their application in research and technology. Heat transmission procedures play a significant role in
general energy intake, that is why improvement of its effectiveness is a significant research area [4]. Nonrenewable assets such as natural gas, coal and petroleum are the core energy resources for international economy motion [5]. First of all, Caputo established the fractional derivative with Laplace transform technique. The (RL) fractional derivative issue was solved using this operator. In the literature, (RL) and (C) fractional operators were usually rejected in numerous fields of science therefore, a new fractional derivative (ABC) was introduced, which provided more accurate and promising results [6]. Caputo and Fabrizio introduced (CF) fractional derivative with a nonsingular kernel [7]. It has been effectively applied in many fields of science.

Atangana and Baleanu [8] developed a new operator in 2016 which was better than Caputo and Fabrizio’s model in describing the physical behaviour of natural phenomena. Saqib et al. [9] described MHD movement of Casson fluid in a microchannel by (ABC) fractional operator. Atangana and Koca [10] discussed (ABC) fractional operator in nonlinear bags and freedman model. Siddique et al. [11] explained unsteady free convection movements of an incompressible fluid over an infinite vertical plate via (ABC) and (CF) approaches. Imran et al. [12] provided a full description of fractional (ABC) and (CF) MHD viscous fluid convective flow. Mohammad et al. [13] used fractional derivatives to examine the outcomes of the uncertain fractional backward difference equations. (CF) and (ABC) fractional operators were used to discover the influences of carbon nanotubes on magnetohydrodynamic movement of methanol based nanofluid in [14]. Hammadou and Makkaoui [15] described the dynamical behavior of a novel 3D fractional-ordered chaotic system. Ikram et al. [16] analyzed the heat transfer of viscous nanofluid over an exponentially moving vertical plate via (ABC) approach. The thermal conductivity of Newtonian fluid with C and CF fractional operator was discussed in [17, 18].

In [19] nonsingular kernel inside the Atangana-Baleanu fractional operator, the Painlev as well as Bagley-Torvik calculations are answered according to the initial conditions. In [20] Imran et al. discussed the thermophoresis properties on simultaneous mass as well as heat transfer in MHD fluid over a porous medium by even heat flux with the help of (AB) fractional operator. In [21] paper emphasis on conditions that a fresh high-order system for the arithmetic results of fractional instruction Volterra integro-DE’s with the help of AB operator. In [22] Khan discussed the novel idea of ABC fractional operator on human blood pour in Nanofluids. Sania and Yusuf [23] discussed the modeling and results of chickenpox infection with different fractional operators such as C, CF and ABC operators. Raza and Ullah [24] presented a comparison of C and CF fractional operators with heat transfer analysis of fractional Maxwell fluid. They used Tzous and Stehfests algorithms to find the solutions numerically. Imran et al. [25] studied the unsteady natural convection flow of Maxwell fluid with fractional derivative over an exponentially accelerated infinite vertical plate using Stehfests and Tzous algorithms. Chu et al. [26] explained a nonlocal fractional model of hybrid nanofluid with MHD free convection flow in microchannel. Imran et al. [27] used constant proportional Caputo (CPC) fractional operator to study the heat transfer flow of caly-water base nanofluids over an infinite vertical surface moving with constant velocity. Ikram et al. [28] calculated the heat transfer flow of hybrid nanofluid between parallel plates via (CPC) fractional approach and offered a comparison with CF fractional derivative. Imran et al. [29] discussed the heat and mass transfer flow for adhesive fluid between two upright plates through fractional approach. Dawar et al. [30] analyzed the heat transfer of 3D magnetohydrodynamic and sodium alginate based on iron oxide nanofluid moving through a spinning disk with homotopy analysis method. Vaidya et al. [31] calculated the influence of homogeneous and heterogeneous responses on the peristaltic flow of MHD Jeffrey fluid in a nonuniform vertical channel with slip conditions. Sabu et al. [32] presented the analysis of hydromagnetic alumina-water nanoliquid flow due to a rotating rigid disk numerically. They considered different nanoparticle shapes and the thermo-hydrodynamic slip conditions. The following references show a wide range of fractional models and their applications in applied sciences [33–36].

Freshly, In [37] the fractional derivative of (BTF) along with hybrid nanofluid is explained with the help of (CF) fractional operator. We applied the LT technique to acquire the logical results for velocity as well as temperature fields and is indicated as M-function. After that a few figures were plotted to show the impression of parameters on velocity and temperature. The physical properties of hybrid nanofluids are presented in Table 1 and relation of the physical values of nano and hybrid nanofluids can be seen in Tables 2 and 3.

The significance of the present work is that a nonsingular fractional operator is applied in the heat and mass transfer phenomena containing nanoparticles of the type Ag and TiO2. As the nanoparticles used in the base fluid to improve the physical quantities of base fluid, this paper contains some new comparisons between fractional operators as well as different kinds of nanoparticles which can be useful in the heat transfer process where we can make a choice with different base fluids according to desired targets. These objectives have already been justified physically in this paper.

2. Mathematical Formulation

Suppose that magnetohydrodynamic natural convection movement of Brinkman fluid happens in microchannel of a comprehensive, electrically conductive (Ag − TiO2 − H2O) hybrid nanofluid.

The hypotheses forgoes as shown in Figure 1:

(a) Width and length of microchannel is \( l \) and infinite, respectively
(b) Channel is beside horizontal line and normal to vertical line
(c) \( T_0 \) is the temperature of framework, when \( t \leq 0 \)
(d) Subsequently \( t = 0^+ \), temperature rises \( T_0 \) into \( T_W \)
(e) \( C_0 \) is the concentration of framework, when \( t \leq 0 \)
Subsequently $t = 0^+$, concentration rises $C_0$ into $C_W$.

Motion is along $x$-axis.

MF of strength $B_0$ works diagonally into the direction of flow.

Flow of electrically conductive Ag–TiO$_2$–H$_2$O hybrid nanofluid undergoes electromotive energy, which generate current. Reduced Reynolds number. The electromagnetic energy revolves on the power of electric instability [18].

Velocity component of hybrid nanofluid is of the form $v(y, t)$. The electromagnetic force $B_{em}$ is merged into the momentum equation of the natural convection flow of an incompressible Ag–TiO$_2$–H$_2$O hybrid nanofluid in the absence of pressure gradient and an inclined magnetic field is imposed.

The governing equations are [37]

$$\rho_{nf}v_t = \frac{\rho_{nf}}{\rho_f}v_t^2,$$

$$\mu_{nf}v_{yy} - \sigma_{nf}B_0^2v + g(\rho\beta T)_{nf}(T(y, t) - T_0) + g(\rho\beta T)_{nf}(C(y, t) - C_0),$$

$$\rho_{C_P}T_t = K_{nf}T_{yy} + Q_0(T(y, t) - T_0),$$

with the following initial and boundary conditions

$$\nu(y, 0) = 0,$$

$$T(y, 0) = T_0,$$

$$C(y, 0) = C_0,$$

$$y \geq 0, t < 0,$$

$$\nu(l, t) = 0,$$

$$T(l, t) = T_w,$$

$$C(l, t) = C_w,$$

$$t > 0.$$
Table 3: Expressions of hybrid nanofluid [37].

<table>
<thead>
<tr>
<th>Properties</th>
<th>Hybrid nanofluid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>( \rho_{hnf} = \phi_{Ag} \rho_{Ag} + \phi_{TiO_2} \rho_{TiO_2} + (1 - \phi_{hnf}) \rho_f )</td>
</tr>
<tr>
<td>Dynamic viscosity</td>
<td>( \mu_{hnf} = \phi_{Ag} \mu_{Ag} + \phi_{TiO_2} \mu_{TiO_2} + (1 - \phi_{hnf}) \mu_f )</td>
</tr>
<tr>
<td>Thermal expansion</td>
<td>( (\rho \beta_T)<em>{hnf} = \phi</em>{Ag} (\rho \beta_T)<em>{Ag} + \phi</em>{TiO_2} (\rho \beta_T)<em>{TiO_2} + (1 - \phi</em>{hnf}) (\rho \beta_T)_f )</td>
</tr>
<tr>
<td>Heat capacitance</td>
<td>( (\rho C_p)<em>{hnf} = \phi</em>{Ag} (\rho C_p)<em>{Ag} + \phi</em>{TiO_2} (\rho C_p)<em>{TiO_2} + (1 - \phi</em>{hnf}) (\rho C_p)_f )</td>
</tr>
<tr>
<td>Electric conductivity</td>
<td>( (\sigma_{hnf}/\sigma_f) = 1 + (3 ((\phi_{Ag} \sigma_{Ag} + \phi_{TiO_2} \sigma_{TiO_2})/\sigma_f) - \phi) )</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>( (K_{hnf}/K_f) = ((\phi_{Ag} k_{Ag} + \phi_{TiO_2} k_{TiO_2})/\phi_{hnf}) + 2k_f + 2(\phi_{Ag} k_{Ag} + \phi_{TiO_2} k_{TiO_2}) - 2\phi_{hnf} k_f/(\phi_{Ag} k_{Ag} + \phi_{TiO_2} k_{TiO_2})/\phi_{hnf} + 2k_f + (\phi_{Ag} k_{Ag} + \phi_{TiO_2} k_{TiO_2}) - \phi_{hnf} k_f )</td>
</tr>
</tbody>
</table>
$C_u \frac{D}{D_t^\alpha} U(\omega, \tau) + \beta_b U(\omega, \tau) = C_1 U_{\omega 0}(\omega, \tau) - C_2 M U(\omega, \tau) + C_3 G \rho \varphi(\omega, \tau) + C_4 G_m C(w, \tau)$,

$C_5 \frac{d}{d\tau} \varphi(\omega, \tau) = \lambda_{\text{inf}} \varphi_{\text{inf}}(\omega, \tau) + Q \varphi(\omega, \tau)$,

where $\text{Sc}$ is Schmidt number (dimensionless).

Initial and boundary conditions which are dimensionless,

$U(\omega, 0) = 0$,
$\varphi(\omega, 0) = 0$,
$C(\omega, 0) = 0$,
$\omega \geq 0$, $\tau_1 < 0$,
$U(l, \tau) = 0$,
$\varphi(l, \tau) = 1$,
$C(l, \tau) = 1$, $\tau_1 > 0$,

where

$\beta_b = \frac{I^2 \beta_{\text{inf}}}{\mu}$,
$M = \frac{I^2 \sigma_f B_0^2}{\mu}$,
$\lambda_{\text{inf}} = \frac{k_{\text{inf}}}{k_f}$,
$Q = Q_d I^2$,
$Pr = \frac{(\mu C_P)}{k_f}$,
$Gr = I^2 g (\beta_T f)(T_w - T_0)$,
$Sc = \frac{\nu}{D}$,
$C_0 = 1 - \phi_{\text{inf}} + \frac{\phi_{Ag} p_{Ag} + \phi_{TiO_2} p_{TiO_2}}{\rho_f}$,
$C_1 = \frac{1}{[1 - (\phi_{Ag} + \phi_{TiO_2})]^{\alpha \omega}}$,
$C_2 = \frac{\sigma_{\text{inf}}}{\sigma_f}$,
$C_3 = 1 - \phi_{\text{inf}} + \frac{\phi_{Ag} (\rho_{Ag})_f + \phi_{TiO_2} (\rho_{TiO_2})_f}{(\rho_{Ag})_f}$,
$C_4 = 1 - \phi_{\text{inf}} + \frac{\phi_{Ag} (\rho_{Ag})_f + \phi_{TiO_2} (\rho_{Ag})_f}{(\rho_{Ag})_f}$,
$C_5 = 1 - \phi_{\text{inf}} + \frac{\phi_{Ag} (\rho_{Ag})_f + \phi_{TiO_2} (\rho_{Ag})_f}{(\rho_{Ag})_f}$.

3. Result of the Problem

Fractional model obtained by equations (7)–(9) using the definition of ABC defined in [8].

$U_{\omega 0}(\omega, \tau_1) - D_1^{ABC} D_{t_1}^{\alpha} U(\omega, \tau_1) - D_2 U(\omega, \tau_1) + D_3 \varphi(\omega, \tau_1) + D_4 C(\omega, \tau_1) = 0$,

$\varphi_{\omega 0}(\omega, \tau_1) - D_3^{ABC} D_{t_1}^{\alpha} \varphi(\omega, \tau_1) + D_6 \varphi(\omega, \tau_1) = 0$,

$S_{\omega}^{ABC} D_{t_1}^{\alpha} C(\omega, \tau_1) = C_{\omega 0}(\omega, \tau_1)$. 

Figure 1: Formation of microchannel and coordinate system.
where as $D_1 = (C_0/C_1), D_2 = (C_2M + \beta/C_1), D_3 = (C_3Gr/C_1), D_4 = (C_4Gr/C_1), D_5 = (C_5Pr/\lambda_{sf}),$ and $D_6 = (Q/\lambda_{sf})$.

3.1. Formulation of Concentration. Implementing LT on equation (14) and using initial and boundary conditions, we have

$$\left[ D^2 - S_C \cdot \frac{q^a}{(1-\alpha)q^a + \alpha} \right] \bar{C}(\omega, q) = 0, \quad (15)$$

satisfying

$$\bar{C}(0, q) = 0,$$

$$\bar{C}(1, q) = \frac{1}{q} \quad (16)$$

The general solution of equation (15) subject to equation (16)

$$\bar{C}(\omega, q) = \frac{\sinh(\omega \sqrt{(S_Cq^a/(1-\alpha)q^a + \alpha)})}{q \sinh(\sqrt{(S_Cq^a/(1-\alpha)q^a + \alpha)})} \quad (17)$$

Equation (17) can be written in suitable form by using formula [28].

By converting into series form as in [28],

$$\bar{C}(\omega, q) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{(-2i + 1 - \omega)}{(i_j/2) + 1} \sqrt{S_C} \Gamma((i_j/2) + 1) q^{i+j+k} \Gamma((i_j/2) + 1) - \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{(-2i + 1 - \omega)}{(i_k/2) + 1} \sqrt{S_C} \Gamma((i_k/2) + 1) q^{i+j+k} \Gamma((i_k/2) + 1) \quad (19)$$

Taking inverse LT

$$\bar{C}(\omega, r_1) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{(-2i + 1 - \omega)}{(i_j/2) + 1} \sqrt{S_C} \Gamma((i_j/2) + 1) q^{i+j+k} \Gamma((i_j/2) + 1) - \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{(-2i + 1 - \omega)}{(i_k/2) + 1} \sqrt{S_C} \Gamma((i_k/2) + 1) q^{i+j+k} \Gamma((i_k/2) + 1) \quad (20)$$

3.2. Formulation of Temperature Field. By implementing LT on equation (13) and using initial condition, we obtain

$$\left[ D^2 - D_g \cdot \frac{q^a}{(1-\alpha)q^a + \alpha} + D_6 \right] \bar{\phi}(\omega, q) = 0, \quad (21)$$

which satisfies

$$\bar{\phi}(0, q) = 0,$$

$$\bar{\phi}(1, q) = \frac{1}{q} \quad (22)$$

The general solution of equation (21) subject to equation (22)
\[ \bar{\phi}(\omega, q) = \frac{1}{q} \left[ \sinh \left( \frac{\omega \sqrt{(D_2 q^3 / (1 - \alpha) q^a + \alpha) - D_6}}{\sinh \left( \frac{\sqrt{(D_2 q^3 / (1 - \alpha) q^a + \alpha) - D_6}}{2} \right)} \right) \right], \quad (23) \]

Suitably written in equivalent form

\[ \bar{\phi}(\omega, q) = \frac{1}{q} \sum_{\xi=0}^{\infty} \sum_{\xi_1=1}^{\infty} \sum_{\xi_2=1}^{\infty} \left[ \frac{(-2\xi + 1 + \omega)^{\xi_1} (-1)^{\xi_2} \xi_1 \xi_2 (D_3)^{\xi_3}}{\xi_1 ! \xi_2 ! \xi_3 ! (D_6)^{\xi_3} - (\xi_3/2)^{\xi_3} (1 - \alpha)^{\xi_3} q^{\xi_3 + 1} \Gamma((\xi_3/2) + 1 - \xi_3)^{\xi_3}} \right] \]

\[ \phi(\omega, \tau_1) = \frac{1}{\tau_1} \sum_{\xi=0}^{\infty} \sum_{\xi_1=1}^{\infty} \sum_{\xi_2=1}^{\infty} \left[ \frac{(\alpha - 2\xi - 1)^{\xi_1} (-1)^{\xi_2} \xi_1 \xi_2 (D_3)^{\xi_3}}{\xi_1 ! \xi_2 ! \xi_3 ! (D_6)^{\xi_3} - (\xi_3/2)^{\xi_3} (1 - \alpha)^{\xi_3} \Gamma((\xi_3/2) + 1 - \xi_3)^{\xi_3}} \right] \]

Taking inverse LT on equation (26) and expressing in \( M \)-function defined in [7], we have

\[ N_u = -\frac{\partial \phi(0, \tau_1)}{\partial \omega} \quad (27) \]

3.3 Nusselt Number. We have computed the heat transfer rate in terms of Nusselt number through the following relation and presented in Table 4.

\[ (D_2 - D_1) \left( \frac{q}{(1 - \alpha) q^a + \alpha} - D_2 \right) \quad (28) \]

which satisfies

3.4 Formulation of Velocity Field. By utilizing the LT on equation (12) and using initial condition, we obtain

\[ U(\omega, q) = -D_3 \bar{\phi}(\omega, q) - D_3 \bar{c}(w, q), \]
The current study of MHD Brinkman type fluid flow is the focus of this article, which contain silver (Ag) and hybrid titanium (TiO₂) nanoparticles with nonlocal noninteger type fractional differential operators the problem is created for the convective flow restrained in a microchannel. With the help of LT temperature and velocity solutions are obtained and then converted to the series form. In order insight the physical significance of flow parameters some graphs are plotted.

### 4. Results and Discussion

The current study of MHD Brinkman type fluid flow is the focus of this article, which contain silver (Ag) and hybrid titanium (TiO₂) nanoparticles with nonlocal noninteger type fractional differential operators the problem is created for the convective flow restrained in a microchannel. With the help of LT temperature and velocity solutions are obtained and then converted to the series form. In order insight the physical significance of flow parameters some graphs are plotted.

#### Table 4: Numerically analysis of Nusselt number for the effect of fractional parameter with comparison of (ABC) and (CF).

<table>
<thead>
<tr>
<th>α</th>
<th>Nu (ABC) t = 0.5</th>
<th>Nu (ABC) t = 2</th>
<th>Nu (CF) t = 0.5</th>
<th>Nu (CF) t = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>2.057</td>
<td>2.041</td>
<td>2.065</td>
<td>2.000</td>
</tr>
<tr>
<td>0.2</td>
<td>2.065</td>
<td>2.041</td>
<td>2.065</td>
<td>2.000</td>
</tr>
<tr>
<td>0.3</td>
<td>2.080</td>
<td>2.041</td>
<td>2.080</td>
<td>2.000</td>
</tr>
<tr>
<td>0.4</td>
<td>2.100</td>
<td>2.041</td>
<td>2.100</td>
<td>2.000</td>
</tr>
<tr>
<td>0.5</td>
<td>2.124</td>
<td>2.041</td>
<td>2.124</td>
<td>2.000</td>
</tr>
<tr>
<td>0.6</td>
<td>2.144</td>
<td>2.041</td>
<td>2.144</td>
<td>2.000</td>
</tr>
<tr>
<td>0.7</td>
<td>2.144</td>
<td>2.041</td>
<td>2.144</td>
<td>2.000</td>
</tr>
<tr>
<td>0.8</td>
<td>2.085</td>
<td>2.041</td>
<td>2.085</td>
<td>2.000</td>
</tr>
<tr>
<td>0.9</td>
<td>1.891</td>
<td>2.041</td>
<td>1.891</td>
<td>2.000</td>
</tr>
</tbody>
</table>

The fractional operator was used to measure the hybrid nanofluid (TiO₂ – H₂O). The (BTF) model with conflict along with affected by magnetic field was distinguished with the help of PDEs. Appraisal between the (ABC) fractional derivative was applied to determine the performance of hybrid nanofluid with the help of LT procedure. The results of the temperature and velocity fields are exposed as M-function. The parameters influences of α, β, φ, Gr, Q and M on temperature as well as velocity fields are also calculated and shown graphically with their physical impact.

Figures 2-5 will compare the (ABC) derivative with the (CF) derivative, which is argued in [37], to analyze the impact of α on temperature and velocity fields. The velocity as well as temperature is decreasing functions of α as a result of reduction in momentum and heat boundary layers. It is due to that the fractional parameter controls the phenomena.
of fluid flow physically. The rate of heat transfer is also calculated numerically in the form of Nusselt number by comparing (ABC) and (CF) fractional operators as shown in Table 4. It is observed that (ABC) derivatives show better retention than (CF) derivatives. Figures 6 and 7 signify the appraisal of velocity and temperature fields for nano fluid particles ($\text{Ag} - \text{H}_2\text{O}$) and ($\text{TiO}_2 - \text{H}_2\text{O}$). Meanwhile TiO$_2$ is semiconductor and Ag is good conductor, consequently ($\text{Ag} - \text{H}_2\text{O}$) had high temperature profile than ($\text{TiO}_2 - \text{H}_2\text{O}$). However the nanofluid concentration has more significance in the velocity field. Because of antiparticles adding in base fluid, the resultant nanofluid creates much denser which decrease the velocity. ($\text{Ag} - \text{H}_2\text{O}$) decreases velocity more than ($\text{TiO}_2 - \text{H}_2\text{O}$) relatively.

Figures 8 and 9 are designed to see the impact of $Q$ which is very predictable. By increasing values of $Q$, both velocity and temperature increased. This is owing to heat producing by system through rising $Q$. Figures 10 and 11 display the influence of $\phi_{\text{nf}}$ on the velocity and temperature fields. The velocity is a decreasing function of $\phi_{\text{nf}}$, whereas temperature is increasing function. This is because of $\phi_{\text{nf}}$ increases the viscosity and density of fluid from Table 2. The similar trend for $\phi_{\text{nf}}$ was concluded by [34].

Figures 12 and 13 are made to show the evaluation of changed base fluids (engine oil, water, kerosene) for the temperature and velocity arenas. Because of the changed thermal conductivity of water and base fluids by hybrid nanoparticles has unusual temperature due to greater heat
Figure 6: Correlation between nanofluid for velocity, when Pr = 6.2, t = 0.4, Gr = 40, Q = 0.5, M = 10, β = 2 and α = 0.2.

Figure 7: Correlation among nanofluids for temperature, when t = 3, α = 0.2, Q = 3, and Pr = 6.2.

Figure 8: Heat source Q Impact on velocity when Pr = 6.2, φ_{in} = 0.04, t = 0.2, β = 0.5, Gr = 30, M = 3, and α = 0.5.
Figure 9: Effect of heat source Q on temperature when $Pr = 6.2$, $t = 0.4$, $\phi_{inf} = 0.2$ and $\alpha = 0.5$.

Figure 10: Impact of $\phi_{inf}$ on velocity when $Pr = 6.2$, $t = 0.1$, $M = 3$, $\beta = 0.5$, $Q = 2$, $Gr = 30$, and $\alpha = 0.5$.

Figure 11: Effect of $\phi_{inf}$ on temperature after $Pr = 6.2$, $t = 0.4$, $Q = 1.2$ and $\alpha = 0.5$. 
conductivity than further base fluids, it is observed that water-based hybrid nanoparticles has higher velocity as it moves away from the plate, whereas the velocity of engine oil-based fluid decrees near the plate. Figure 14 represent the effect of $\beta$ on the velocity field. After increasing $\beta$, the fluid velocity shrinks. This is due to higher values of $\beta$ make the drag forces stronger physically, so velocity decreases. The impression of $M$ is verified in Figure 15. It guides that by rising $M$, velocity diminishes. Meanwhile values of $M$ transmit to resistive type forces, called Lorentz forces. Thus the velocity was declined. The effect of $Gr$ on the velocity field represented in Figure 16. It is comprehended that by
Figure 14: The velocity effect of the BT fluid $\beta$, $\Pr=6.2$, $Gr=30$, $\phi_{inf}=0.08$ when $t=3$, $Q=0.5$, and $\alpha=0.5$.

Figure 15: Effect of Magnetic field M on velocity after $\Pr=6.2$, $t=3$, $\beta=0.2$, $Q=0.5$, $\alpha=0.5$, $\phi_{inf}=0.08$ and $Gr=30$.

Figure 16: The influence of Gron velocity when $\beta=0$, $t=3$, $Pr=6.2$, $Q=0.5$, $M=4$, $\phi_{inf}=0.08$, $\beta=0.2$ and $\alpha=0.5$. 
rising Gr, the velocity way quicker. As a result, Gr is linked to buoyancy forces, which elevate conventional convection and hence increase velocity.

5. Conclusions

In present research work, the characteristics of \((Ag - TiO_2 - H_2O)\) hybrid nanofluid has been studied. In microchannel the flow of the hybrid nanofluid was calculated. The (MHD) effects and heat source \(Q\) were measure likewise. After converting into fractional model, (ABC) fractional operator used to solve the problem. The exact solutions were attained with the help of LT method and listed in the term of M-function, velocity fields.

The significance of the present work is the nonsingular fractional operator is applied in the heat and mass transfer phenomena containing nanoparticles of the type Ag and TiO_2. As the nanoparticles used in the base fluid to improve the physical quantities of base fluid. This paper contains some new comparisons between fractional operators as well as different kinds of nanoparticles which can be useful in the heat transfer process where we can make a choice with different base fluids according to desired targets. These objectives have already been justified physically in this study.

Major outcomes of present paper are as per the following:

(i) The results show that the Atangana–Baleanu fractional operator exhibited improved memory effect as compared to Caputo–Fabrizio fractional operator.

(ii) Further, due to increasing the values volume fractional temperature can be enhanced and velocity decreases.

(iii) In comparison between nanoparticles for different types of based fluid, velocity and temperature of water based \((TiO_2)\) and silver \((Ag)\) is higher than other base fluids.

(iv) Fractional parameter used to control the thermal and momentum boundary layer thickness.

(v) For larger values of concentration of nanoparticles, thermal as well as momentum boundary layer decrease.

Abbreviations

RL: Riemann–Liouville
C: Caputo
AB: Atangana–Baleanu
CF: Caputo–Fabrizio
\(B^*\): Magnetic fluidity force
\(\mu_0\): Magnetic absorptivity
\(E^*\): Electrical field strength
\(J\): Current density
\(\sigma_0\): Electrical conductivity
\(U\): Velocity field \((ms^{-1})\)
\(\rho_{hnf}\): Density of hybrid nanofluid \((kgm^{-3})\)
\(v\): Fluid velocity

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F_{em})</td>
<td>Electromagnetic force</td>
</tr>
<tr>
<td>(\beta_0)</td>
<td>Brinkman parameter</td>
</tr>
<tr>
<td>(\rho_{hnf})</td>
<td>Dynamic viscosity ((kgm^{-1}s^{-1}))</td>
</tr>
<tr>
<td>(\sigma_{hnf})</td>
<td>Electrical conductivity</td>
</tr>
<tr>
<td>(\beta_{hnf})</td>
<td>Thermal expansion</td>
</tr>
<tr>
<td>(B_0^*)</td>
<td>Magnetic field</td>
</tr>
<tr>
<td>((C_p)_{hnf})</td>
<td>Specific heat ((jkg^{-1}K^{-1}))</td>
</tr>
<tr>
<td>(T)</td>
<td>Temperature ((K))</td>
</tr>
<tr>
<td>(k_{hnf})</td>
<td>Thermal conductivity ((Wm^{-2}K^{-1}))</td>
</tr>
<tr>
<td>(T_w)</td>
<td>Fluid temperature at the plate ((K))</td>
</tr>
<tr>
<td>Gr</td>
<td>Thermal Grashof number (dimensionless)</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number (dimensionless)</td>
</tr>
<tr>
<td>M</td>
<td>Magnetic parameter (dimensionless)</td>
</tr>
<tr>
<td>t</td>
<td>Time ((s))</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Fractional parameter</td>
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<tr>
<td>Q</td>
<td>Heat generation parameter</td>
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<tr>
<td>q</td>
<td>Laplace transform parameter</td>
</tr>
<tr>
<td>BTF</td>
<td>Brinkman type fluid</td>
</tr>
<tr>
<td>CCE</td>
<td>Clasius–Clapeyron equation</td>
</tr>
<tr>
<td>LHE</td>
<td>Latent heat evaporation</td>
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<tr>
<td>MF</td>
<td>Magnetic field</td>
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<tr>
<td>LT</td>
<td>Laplace transform</td>
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</tbody>
</table>

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare no conflicts of interest in submission of this paper.

Authors’ Contributions

All of the authors substantially contributed to this work.

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References


[34] N. A. Shah, M. I. Asjad, and F. Miraj, “Exact solutions of time fractional free convection flows of viscous fluid over an isothermal vertical plate with Caputo and Caputo-fabrizio

