Hindawi
Journal of Mathematics
Volume 2022, Article ID 9252860, 9 pages
https://doi.org/10.1155/2022/9252860

Research Article

Weakly 2-Absorbing Ideals in Almost Distributive Lattices

Natnael Teshale Amare

Department of Mathematics, University of Gondar, Gondar, Ethiopia

Correspondence should be addressed to Natnael Teshale Amare; yenatnaelteshale@gmail.com

Received 4 July 2022; Accepted 6 September 2022; Published 7 October 2022

Academic Editor: Faranak Farshadifar

The concepts of weakly 2-absorbing ideal and weakly 1-absorbing prime ideal in an almost distributive lattice (ADL) are introduced, and the necessary conditions for a weakly 1-absorbing prime ideal to become a weakly 2-absorbing ideal in algebraic form are proved. Also, weakly 2-absorbing ideals are characterized in terms of weakly prime ideals and 2-absorbing ideals. Finally, the lattice epimorphic images and inverse images of the weakly 2-absorbing ideal and weakly 1-absorbing prime ideal are discussed.

1. Introduction

Badawi [1] was introduced the concept of 2-absorbing ideals on a commutative ring and assume that all rings are commutative with $1 \neq 0$, which is a generalization of prime ideals and some properties of these were studied. Subsequently, several researchers worked on 2-absorbing ideals in different cases (refer to [2–5]). Later in the paper [6], Yassine et al. obtained on 1-absorbing prime ideals of a commutative ring. In the papers [7–10] (Figures 1and 2), the concept of weakly prime ideals were introduced and some of its properties are investigated. The concept of an almost distributive lattice (ADL) was introduced by Swamy and Rao [11] (Figures 3 and 4) as a common abstraction to most of the existing ring theoretic generalizations of a Boolean algebra and which is an algebra $(A,\wedge,\vee,0)$ of type $(2,2,0)$ satisfying all the axioms of a distributive lattice with zero except $\wedge$ commutative, $\vee$ commutative, and right distributivity of $\vee$ over $\wedge$. The concept of prime ideal is a vital role in the study of structure theory of distributive lattices in general and of Boolean algebras in particular, see [11].

In this paper, we introduce the concept of 2-absorbing ideal in an almost distributive lattice which is a generalization of prime ideals in an ADL. A proper ideal P of an ADL A is called a 2-absorbing ideal (2-AI for short) of A if whenever $x, y, z \in A$ and $x \wedge y \wedge z \in P$, then $x \wedge y \in P$ or $y \wedge z \in P$ or $x \wedge z \in P$. It is shown that a proper ideal P of A is a 2-AI of A if and only if whenever $P_1 \cap P_2 \cap P_3 \subseteq P$ for some ideals P_i of A, for $1 \leq i \leq 3$, then $P_1 \cap P_2 \subseteq P$ or $P_1 \cap P_3 \subseteq P$ or $P_2 \cap P_3 \subseteq P$. In addition to this, it is observed that the lattice epimorphic image and inverse image of 2-AI and n-AI are also 2-AI and n-AI, respectively. Next, we introduce the concepts of weakly 2-AI of an ADL which is weaker than that of weakly prime ideal and 2-AI of an ADL. The Cartesian product of weakly 2-AIs is also discussed here, and some equivalent conditions for the set of all weakly 2-AIs to become 2-AIs under the Cartesian product are derived. Mainly, we have proved that all prime ideals are 2-AIs and weakly prime ideals, that weakly prime ideals are weakly 2-AIs, and also that all 2-AIs are weakly 2-AIs and vice versa is not true; examples were given to shown these. It is further demonstrated that all prime ideals are 1-absorbing prime ideals, that 1-absorbing prime ideals are 2-AIs, and that all weakly prime ideals and 1-absorbing prime ideals are weakly 1-absorbing prime ideals and that weakly 1-absorbing prime ideals are weakly 2-AIs, and there are examples that show that the converse of these is not true. Finally, it is also shown that the image and inverse image of weakly 2-AI, 1-absorbing prime ideal, and weakly 1-absorbing prime ideal under lattice epimorphism are again weakly 2-AI, 1-absorbing prime ideal, and weakly 1-absorbing prime ideal, respectively.

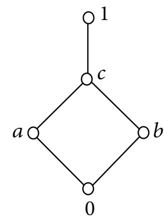

Figure 1: The complete lattice diagram.

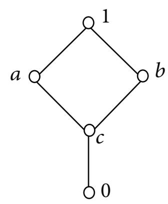

Figure 2: The Complete Lattice diagram in Example 5.

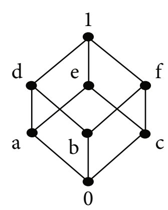

Figure 3: The Boolean lattice diagram.

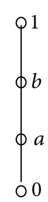

Figure 4: The chain 4 diagram.

Throughout this paper, A stands for an ADL $(A,\wedge,\vee,0)$ with a maximal element and L stands for a complete lattice $(L,\wedge,\vee,0,1)$ satisfying the infinite meet distributive law, that is,

$$a\wedge\left(\bigvee_{b\in S} b\right) = \bigvee_{b\in S}(a\wedge b), \tag{1}$$

for any $S\subseteq L$ and $a\in L$.

2. Preliminaries

In this section, we recall certain definitions, results, and notations which will be needed later on are presented, see [1, 11, 12].

Definition 1. An algebra $A=(A,\wedge,\vee,0)$ of type $(2,2,0)$ is called an ADL if it satisfies the following conditions for all a, b and $c\in A$.

(1) $0\wedge a=0$

(2) $a\vee 0=a$

(3) $a\wedge(b\vee c)=(a\wedge b)\vee(a\wedge c)$

(4) $a\vee(b\wedge c)=(a\vee b)\wedge(a\vee c)$

(5) $(a\vee b)\wedge c=(a\wedge c)\vee(b\wedge c)$

(6) $(a\vee b)\wedge b=b$

Each of the axioms (1) through (6) above is independent from the others. The element 0 is called the zero element.

Any bounded below distributive lattice is an ADL.

Example 1. Let X be a nonempty set. Fix an arbitrary element $x_0\in X$. For any $x,y\in X$, define $\wedge$ and $\vee$ on X by,

$$\begin{aligned} x\wedge y &= \begin{cases} y, & \text{if } x\neq x_0,\\ x_0, & \text{if } x=x_0,\end{cases}\\ x\vee y &= \begin{cases} y, & \text{if } x\neq x_0,\\ x_0, & \text{if } x=x_0.\end{cases}\end{aligned} \tag{2}$$

Then $(X,\wedge,\vee,x_0)$ is an ADL with x_0 as its zero element. This ADL is called the discrete ADL.

Several ring theoretic generalizations of Boolean algebras (other than Boolean rings which are precisely Boolean algebras) can be made as an ADL. The following example is one such.

Example 2. Let R be a commutative regular ring with identity (that is, R is a commutative ring with unity in which, for each $a\in R$, there exists an (unique) idempotent $a_0\in R$ such that $aR=a_0R$). For any $a,b\in R$, define

$$\begin{aligned} a\wedge b &= a_0 b,\\ a\vee b &= a+b-a_0 b.\end{aligned} \tag{3}$$

Then, $(R,\wedge,\vee,0)$ is an ADL with the additive identity 0 as the zero element.

Theorem 2. *Let $(A,\wedge,\vee,0)$ be an ADL. For any a and $b\in A$, we have*

(1) $a\wedge 0=0=0\wedge a$ and $a\vee 0=a=0\vee a$

(2) $a\wedge a=a=a\vee a$

(3) $(a\wedge b)\vee b=b$

(4) $a\vee(b\wedge a)=a$

(5) $a\wedge(a\vee b)=a$

(6) $a \wedge b = a \Leftrightarrow a \vee b = b$

(7) $a \wedge b = b \Leftrightarrow a \vee b = a$

(8) $a \vee (b \vee a) = a \vee b$

Definition 3. Let $(A, \wedge, \vee, 0)$ be an ADL. For any a and $b \in A$, define $a \leq b$ if $a = a \wedge b$ (equivalently $a \vee b = b$).

Then $\leq$ is a partial order on A.

Theorem 4. *The following hold good for any elements $a, b, c,$ and d of an ADL $(A, \wedge, \vee, 0)$.*

(1) $a \wedge b \leq b \leq b \vee a$

(2) $a \leq b \Rightarrow a \wedge b = a = b \wedge a$ and $a \vee b = b = b \vee a$

(3) $(a \vee b) \wedge c = (b \vee a) \wedge c$

(4) $(a \wedge b) \wedge c = a \wedge (b \wedge c)$ (i.e., $\wedge$ is associative on A)

(5) $a \wedge b \wedge c = b \wedge a \wedge c$

(6) The set $\{x \wedge a : x \in A\} = \{y \in A : y \leq a\}$ is a bounded distributive lattice under the induced operations $\wedge$ and $\vee$ with 0 as the smallest element and a as the largest element

(7) $a \vee b = b \vee a$ whenever $a \wedge b = 0$

(8) $a \wedge b = 0 \Leftrightarrow b \wedge a = 0$

(9) $a \leq b \Rightarrow a \wedge c \leq b \wedge c$ and $c \wedge a \leq c \wedge b$

(10) $a \leq b \Rightarrow c \vee a \leq c \vee b$

(11) $(a \vee (b \vee c)) \wedge d = ((a \vee b) \vee c) \wedge d$

Theorem 5. *For any elements a and b of an ADL $(A, \wedge, \vee, 0)$, the following are equivalent to each other.*

(1) $(a \wedge b) \vee a = a$

(2) $a \wedge (b \vee a) = a$

(3) $a \wedge b = b \wedge a$

(4) $a \vee b = b \vee a$

(5) $Sup\{a, b\}$ exists in $(A, \leq)$ and is equal to $a \vee b$

(6) There exists $x \in A$ such that $a \leq x$ and $b \leq x$

(7) $\inf \{a, b\}$ exists in $(A, \leq)$ is equal to $a \wedge b$

Theorem 6. *The following statements are equivalent for any ADL A.*

(1) $a \wedge b = b \wedge a$ for all $a, b \in A$

(2) $a \vee b = b \vee a$ for all $a, b \in A$

(3) $(A, \wedge, \vee)$ is a distributive lattice bounded below

(4) $(a \wedge b) \vee c = (a \vee c) \wedge (b \vee c)$ for all $a, b, c \in A$

(5) $b \wedge (a \vee b) = b$ (i.e., $b \leq a \vee b$) for all $a, b \in A$

(6) $(a \wedge b) \vee a = a$ (i.e., $a \wedge b \leq a$) for all $a, b \in A$

(7) For any $a, b, c \in A$, $a \leq b \Rightarrow a \vee c \leq b \vee c$

As a consequence, for any ideal I of A, $x \wedge a \in I$ for all $a \in I$ and $x \in A$. An element $m \in A$ is said to be maximal if, for any $x \in A$, $m \leq x$ implies $m = x$. It can be easily observed that m is maximal if and only if $m \wedge x = x$ for all $x \in A$.

Definition 7. Let I be a nonempty subset of an ADL A. Then, I is called an ideal of A if a, $b \in I \Rightarrow a \vee b \in I$ and $a \wedge x \in I$ for all $x \in A$.

Definition 8. Let $A = (A, \wedge, \vee, 0)$ be an *ADL* and for any subset S of A, let

$$\langle S] = \cap \{I \in \mathcal{I}(A) \colon S \subseteq I\}. \tag{4}$$

Then, $\langle S]$ is the smallest ideal of A containing S and is called the ideal generated by S in A. Also,

$$\langle S] = \left\{ \left(\bigvee_{i=1}^{n} s_i \right) \wedge a : n \geq 0, s_i \in S \, \text{and} \, a \in A \right\}. \tag{5}$$

When $S = \{x\}$, then we simply write $\langle x]$ for $\langle \{x\}]$ and call this the principal ideal generated by x in A. The principal ideal generated by x in A is given by

$$\langle x] = \{a \in A : x \wedge a = a\} = \{x \wedge a : a \in A\}. \tag{6}$$

Theorem 9. *Let $A = (A, \wedge, \vee, 0)$ be an ADL and a and $b \in A$. Then, the following holds good.*

(1) $\langle a] \cap \langle b] = \langle a \wedge b]$

(2) $\langle a] \vee \langle b] = \langle a \vee b]$

(3) $\langle a \wedge b] = \langle b \wedge a]$ and $\langle a \vee b] = \langle b \vee a]$

Theorem 10. *Let $A = (A, \wedge, \vee, 0)$ be an ADL and I and J be ideals of A. Then, in the lattice $(\mathcal{I}(A), \subseteq)$, $I \wedge J = I \cap J$, and $I \vee J = \{s \vee t : s \in I \, \text{and} \, t \in J\}$. Also, the lattice $(\mathcal{I}(A), \subseteq)$ is distributive.*

Definition 11. A nonzero proper ideal I of R is called a 2-absorbing ideal of R if for any $a, b, c \in R$ and $abc \in I$, then $ab \in I$ or $ac \in I$ or $bc \in I$.

3. 2-Absorbing Ideal

In this section, we introduce the notion of 2-absorbing ideal (2-AI) and n-absorbing (n-AI) of a given almost distributive lattice (ADL) $A = (A, \wedge, \vee, 0)$ and prove several structural properties of these.

Definition 12. Let $A = (A,\wedge,\vee,0)$ be an ADL. A proper ideal P of A is said to be a 2 -absorbing ideal of A and denoted by 2-AI if for any $x, y, z \in A$

$$x\wedge y\wedge z \in P \Rightarrow x\wedge y \in P \text{ or } y\wedge z \in P \text{ or } x\wedge z \in P. \tag{7}$$

Theorem 13. *Let P be a 2-AI of A. For $x, y, z \in A$ such that $x\wedge y\wedge z \in P$, we have*

$$y\wedge x \in P \text{ or } z\wedge y \in P \text{ or } z\wedge x \in P. \tag{8}$$

Proof. For $x, y, z \in A$ such that $x\wedge y\wedge z \in P$. Then $y\wedge x = y\wedge(x\wedge x.)$
$= (y\wedge x)\wedge x$ (by 2.7(4))
$= (x\wedge y)\wedge x$. (again by 2.7(4))

Since P is a 2-AI of A and $x\wedge y \in P$, hence $(x\wedge y)\wedge x \in P$. Thus $y\wedge x \in P$. Similarly, $z\wedge y \in P$ or $z\wedge x \in P$. □

Lemma 14. *Let P_1 and P_2 be ideals of A and P be a 2-AI of A. The following assertions hold for any $x, y \in A$.*

(1) $\langle x\wedge y] \cap P_1 \subseteq P \Rightarrow \langle x\wedge y] \subseteq P$ or $\langle x] \cap P_1 \subseteq P$ or $\langle y] \cap P_1 \subseteq P$

(2) $\langle x] \cap (P_1 \cap P_2) \subseteq P \Rightarrow \langle x] \cap P_1 \subseteq P$ or $\langle x] \cap P_2 \subseteq P$ or $P_1 \cap P_2 \subseteq P$

Proof. Let P be a 2-AI of A and P_1 and P_2 ideals of A.

(1) Suppose $\langle x\wedge y] \cap P_1 \subseteq P$. Let $t \in \langle x\wedge y]$. Then, $t = x\wedge y \wedge t$. Since P is a 2-AI of A, if whenever $x, y, z \in A$ and $x\wedge y\wedge z \in P$, then $x\wedge y \in P$ or $y\wedge z \in P$ or $x\wedge z \in P$. From this, we have $t = x\wedge y\wedge t \in P$. Hence, $\langle x\wedge y] \subseteq P$. Suppose $\langle x] \cap P_1 UP$ and $\langle y] \cap P_1$UP. Now, $(\langle x] \cap P_1) \cap (\langle y] \cap P_1) = (\langle x] \cap \langle y]) \cap P_1 = \langle x\wedge y] \cap P_1 \subseteq P$ (by given); this a gives a contradiction. Thus, $\langle x] \cap P_1 \subseteq P$ or $\langle y] \cap P_1 \subseteq P$

(2) Suppose $\langle x] \cap (P_1 \cap P_2) \subseteq P$. Let $x \in A$ such that $\langle x] \cap P_1 UP$ and $\langle x] \cap P_2$UP. It follows that $(\langle x] \cap P_1) \cap (\langle x] \cap P_2)$UP; that is, $\langle x] \cap (P_1 \cap P_2)UP$, a contradiction. Therefore, $\langle x] \cap P_1 \subseteq P$ or $\langle x] \cap P_2 \subseteq P$

□

Theorem 15. *Let P be a proper ideal of A. The following statements are equivalent:*

(1) P is a 2-AI of A

(2) For ideals P_1, P_2, P_3 of A, $P_1 \cap P_2 \cap P_3 \subseteq P \Rightarrow P_1 \cap P_2 \subseteq P$ or $P_1 \cap P_3 \subseteq P$ or $P_2 \cap P_3 \subseteq P$

(3) For ideals P_1, P_2, P_3 of A, $P = P_1 \cap P_2 \cap P_3 \Rightarrow P = P_1 \cap P_2$ or $P = P_1 \cap P_3$ or $P = P_2 \cap P_3$

Proof. $(1) \Leftrightarrow (2)$: suppose P is a 2-absorbing ideal of A. Let $P_1 \cap P_2 \cap P_3 \subseteq P$, for some proper ideals P_1, P_2, and P_3 of A. Let $x, y \in A$ such that $\langle x] \cap P_2$UP and $\langle y] \cap P_3$UP and put $P_1 = \langle x]$. It follows that, $\langle x] \cap P_3 \subseteq P$ and $\langle y] \cap P_2 \subseteq P$. By Lemma 14(2), $\langle x\wedge y] \cap (P_2 \cap P_3) \subseteq P$, we get either $\langle x\wedge y] \cap P_2 \subseteq P$ or $\langle x\wedge y] \cap P_3 \subseteq P$. If $\langle x\wedge y] \cap P_2 \subseteq P$, then $(x\wedge y)\wedge z = y\wedge(x\wedge z)$ (by Theorem 5(4)), for all $z \in P_2$, which implies that $x\wedge z \in P$, so $\langle x] \cap P_2 \subseteq P$, contradiction. Similarly, if $\langle x\wedge y] \cap P_3 \subseteq P$, then $(x\wedge y)\wedge t = y\wedge(x\wedge t)$ (by 2.7(4)), for all $t \in P_3$, implies that $x\wedge t \in P$, so $\langle x] \cap P_3 \subseteq P$, contradiction. Thus, either $P_1 \cap P_2 \subseteq P$ or $P_1 \cap P_3 \subseteq P$. Conversely suppose $P_1 \cap P_2 \cap P_3 \subseteq P$ implies that either $P_1 \cap P_2 \subseteq P$ or $P_1 \cap P_3 \subseteq P$ or $P_2 \cap P_3 \subseteq P$, for any ideals P_1, P_2, P_3 of A. Let $x, y, z \in A$, we have $x\wedge y\wedge z \in P$. Suppose also that $x\wedge y \notin P$ and $y\wedge z \notin P$. Let $P_1 = \langle x], P_2 = \langle y]$ and $P_3 = \langle z]$. Since $P_1 \cap P_2 \cap P_3 = \langle x] \cap \langle y] \cap \langle z] = \langle x\wedge y] \cap \langle z] \subseteq P$, it follows that $P_1 \cap P_2 UP$ and $P_2 \cap P_3 UP$. Then by the above lemma, $P_1 \cap P_3 = \langle x] \cap \langle z] = \langle x\wedge z] \subseteq P$; that is, $x\wedge z \in P$. Thus, P is a 2-absorbing ideal of A.

$(2)\Leftrightarrow(3)$ and $(1) \Leftrightarrow (3)$ are clear □

Definition 16. Let $(A_1,\wedge,\vee,0)$ and $(A_2,\wedge,\vee,0)$ be ADLs and form the set $A_1 \times A_2$ by $A_1 \times A_2 = \{(a, b)\colon a \in A_1 \text{ and } b \in A_2\}$. Define $\wedge$ and $\vee$ in $A_1 \times A_2$ by,

$$(a, b)\wedge(c, d) = (a\wedge c, b\wedge d), \tag{9}$$

and $(a, b)\vee(c, d) = (a\vee c, b\vee d)$, for any $(a, b), (c, d) \in A_1 \times A_2$.

Then, $(A_1 \times A_2,\wedge,\vee,0)$ is an ADL under the pointwise operations and $0 = (0, 0)$ is the zero element in $A_1 \times A_2$.

Let us recall from [11] that a proper ideal P of A is said to be a prime ideal if, for any x and $y \in A, x\wedge y \in P \Rightarrow$ either $x \in P$ or $y \in P$. Now, we have the following.

Theorem 17. *Every prime ideal of A is a 2-AI of A.*

Proof. Assume that P is a prime ideal of A. Let $x, y, z \in A$, $x\wedge y\wedge z \in P$. Then, either $x\wedge y \in P$ or $z \in P$, or $y\wedge z \in P$ or $x \in P$, and hence $x\wedge z \in P$ (since P is an ideal and by 2.10). If $x\wedge y \in P$, then it is obvious and if $z \in P$, $x\wedge z \in P$ and $y\wedge z \in P$. Thus, P is a 2-AI of A. □

The following example show that the converse of Theorem 44 is not true.

Example 3. Let $D = \{0, x, y\}$ be a discrete ADL with 0 as its zero element defined in Example1and$L = \{0, a, b, c, 1\}$ be the lattice represented by the Hasse diagram given below:

Consider $D \times L = \{(t, s)\colon t \in D \text{ and } s \in L\}$. Then, $(D \times L, \wedge,\vee,0)$ is an ADL (note that $D \times L$ is not a lattice) under the pointwise operations $\wedge$ and $\vee$ on $D \times L$ and $0 = (0, 0)$, the zero element in $D \times L$. Then, $P = \{(0, 0)\}$ is a 2-AI of $D \times L$ but P is an ideal which is not prime, since $(0, a)\wedge (x, b) = (0, 0)$, for all $(0, a), (x, b) \in D \times L$.

Theorem 18. *Let P and Q be prime ideals of A. Then, $P \cap Q$ is a 2-AI of A.*

Proof. Let $a, b, c \in A$ and $a \wedge b \wedge c \in P \cap Q$. Then, $a \wedge b \wedge c \in P$ and $a \wedge b \wedge c \in Q$. Since P and Q are prime ideals of A, we have either $a \wedge b \in P$ or $c \in P$, or $b \wedge c \in P$ or $a \in P$, or $a \wedge c \in P$ or $b \in P$ (and $a \wedge b \in Q$ or $c \in Q$, or $b \wedge c \in Q$ or $a \in Q$, or $a \wedge c \in Q$ or $b \in Q$). Suppose that $a \wedge b \in P$ or $c \in P$ and $a \wedge b \in Q$ or $c \in Q$. If $a \wedge b \in P$ and $a \wedge b \in Q$, then $a \wedge b \in P \cap Q$. If $c \in P$ and $c \in Q$, then either $a \wedge c \in P$ and $a \wedge c \in Q$ or $b \wedge c \in P$ and $b \wedge c \in Q$. Hence the theorem. □

Definition 19. Let A_1 and A_2 be ADLs. A mapping $f : A_1 \longrightarrow A_2$ is called a lattice homomorphism if the following are satisfied, for any $x, y, z \in A_1$.

(1) $f(x \wedge y \wedge z) = f(x) \wedge f(y) \wedge f(z)$

(2) $f(x \vee y \vee z) = f(x) \vee f(y) \vee f(z)$

(3) $f(0) = 0$

Theorem 20. *Let A_1 and A_2 be ADLs and $f : A_1 \longrightarrow A_2$ be a lattice homomorphism. Then, the following holds.*

(1) If f is an epimorphism and Q is a 2-AI of A_2, then $f^{-1}(Q)$ is a $2-AI$ of A_1

(2) If f is an isomorphism and P is a 2-AI of A_1, then $f(P)$ is a $2-AI$ of A_2

Proof. Let A_1 and A_2 be ADLs and $f : A_1 \longrightarrow A_2$ be a lattice homomorphism.

(1) Let Q be a 2-AI of A_2 and $f^{-1}(Q) = \{a \in A_1 : f(a) \in Q \subseteq A_2\}$. Let $a, b, c \in f^{-1}(Q)$. Then, $f(a), f(b), f(c) \in Q$. Since Q is a 2-AI of A_2, $f(a) \wedge f(b) \wedge f(c) = f(a \wedge b \wedge c) \in Q$. Thus, $a \wedge b \wedge c \in f^{-1}(Q)$. Now if $a \in f^{-1}(Q)$ and $x \in A_1$, then $f(a) \in Q$ and $f(x) \in A_2$ and hence $f(a) \wedge f(x) = f(a \wedge x) \in Q$. Thus, $a \wedge x \in f^{-1}(Q)$. Therefore, $f^{-1}(Q)$ is a 2-AI of A_1

(2) Let $a, b, c \in A_1$ such that $f(a) = x, f(b) = y$ and $f(c) = z$, for some $x, y, z \in A_2$. As $x \wedge y \wedge z \in f(P)$, we have $f(a \wedge b \wedge c) \in f(P)$. Since P is a 2-AI of A_1, $a \wedge b \wedge c \in P$ implies that either $a \wedge b \in P$ or $b \wedge c \in P$ or $a \wedge c \in P$. That is either $f(a \wedge b) = f(a) \wedge f(b) = x \wedge y \in f(P)$ or $y \wedge z \in f(P)$ or $x \wedge z \in f(P)$. Thus, $f(P)$ is a 2-AI of A_2

□

Theorem 21. *Let A_1 and A_2 be ADLs. If P is a 2-AI of A_1, then $P \times A_2$ is a 2-AI of $A_1 \times A_2$. Also, if Q is a 2-AI of A_2, then $A_1 \times Q$ is a 2-AI of $A_1 \times A_2$.*

Proof. Let P be a 2-AI of A_1 and $a, b, c \in A_1$ such that $(a, t) \wedge (b, t) \wedge (c, t) \in P \times A_2$, for every $t \in A_2$. Then, $(a, t) \wedge (b, t) \wedge (c, t) = (a \wedge b \wedge c, t) \in P \times A_2$. Since P is a 2-AI of A_1, we have either $a \wedge b \in P$ or $a \wedge c \in P$ or $b \wedge c \in P$. So that, either $(a \wedge b, t) \in P \times A_2$ or $(a \wedge c, t) \in P \times A_2$ or $(b \wedge c, t) \in P \times A_2$, for every $t \in A_2$. Thus, $P \times A_2$ is a 2-AI of $A_1 \times A_2$. Similarly, $A_1 \times Q$ is a 2-AI of $A_1 \times A_2$. □

Definition 22. A proper ideal P of A is a weakly prime ideal of A if for any $x, y \in A$,

$$0 \neq x \wedge y \in P \Rightarrow \text{either } x \in P \text{ or } y \in P. \tag{10}$$

Lemma 23. *Every prime ideal of A is a weakly prime ideal of A.*

Proof. It is clear. □

The converse of the above lemma is not true; consider the following example.

Example 4. Let $D \times L = \{(t, s) : t \in D \text{ and } s \in L\}$ be an ADL discussed in Example 3. Let $R = \{(0, 0)\}$. Clearly, R is a weakly prime ideal of A, while R is not a prime ideal of $D \times L$, since $(0, a) \wedge (x, b) \in R$ implies that $(0, a) \notin R$ and $(x, b) \notin R$, for all $(0, a), (x, b) \in D \times L$. Thus, every weakly prime ideal of $D \times L$ is not a prime ideal of $D \times L$.

Definition 24. A proper ideal P of A is a weakly 2-AI of A if for any $x, y, z \in A$,

$$0 \neq x \wedge y \wedge z \in P \Rightarrow \text{either } x \wedge y \in P \text{ or } y \wedge z \in P \text{ or } x \wedge z \in P. \tag{11}$$

Lemma 25. *Every weakly prime ideal of A is a weakly 2-AI of A.*

Proof. It is clear. □

The following example show that the converse of Lemma 25 is not true.

Example 5. Let $D = \{0, x, y\}$ be a discrete ADL with 0 as its zero element defined in Example1and$L = \{0, a, b, c, 1\}$ be the lattice represented by the Hasse diagram given below.

Consider $D \times L = \{(t, s) | t \in D \text{ and } s \in L\}$. Then, $(D \times L, \wedge, \vee, 0)$ is an ADL (which is not a lattice) under the pointwise operations $\wedge$ and $\vee$ on $D \times L$ and $0 = (0, 0)$, the zero element in $D \times L$. Let $P = \{(x, 0), (y, c)\}$. Then, $(0, 0) \neq (x, a) \wedge (y, b) \wedge (y, c) \in P$ implies $(x, a) \wedge (y, b) \in P$, $(y, b) \wedge (y, c) \in P$ and $(x, a) \wedge (y, c) \in P$, for all $(x, a), (y, b), (y, c) \in D \times L$. Thus, P is a weakly 2-AI of $D \times L$. But P is neither prime ideal nor weakly prime ideal of $D \times L$, since $(x, a) \wedge (y, b) \in P((0, 0) \neq (x, a) \wedge (y, b) \in P) \Rightarrow (x, a) \notin P$ and $(y, b) \notin P$.

Theorem 26. *Every 2-AI of A is a weakly 2-AI of A.*

Proof. It is clear. □

The following example show that the converse of Theorem 26 is not true.

Example 6. Let $D = \{0, x, y\}$ be a discrete ADL with 0 as its zero element defined in Example1 and $L = \{0, a, b, c, d, e, f, 1\}$ be a lattice whose Hasse diagram is given below.

Consider $D \times L = \{(t, s) | t \in D \text{ and } s \in L\}$. Then, $(D \times L, \wedge, \vee, 0)$ is an ADL (which is not a lattice) under the pointwise operations $\wedge$ and $\vee$ on $D \times L$ and $0 = (0, 0)$, the zero element in $D \times L$. Let $P = \{(0, 0)\}$. Clearly P is a weakly 2-AI of $D \times L$. On the other hand, consider $(0, d) \wedge (x, e) \wedge (y, f) = (0, 0) \in P$ which implies that $(0, d) \wedge (x, e) = (0, a) \notin P$, $(x, e) \wedge (y, f) = (y, c) \notin P$ and $(0, d) \wedge (y, f) = (0, b) \notin P$, for all $(0, d), (x, e), (y, f) \in D \times L$. Thus P is not a 2-AI of $D \times L$. Therefore, every weakly 2-AI of $D \times L$ is not a 2-AI of $D \times L$.

As a consequence of Theorem 18 and Lemmas 23 and 25, we have the following.

Theorem 27. *Let P and Q be weakly prime ideals of A. Then, the intersection of P and Q is also a weakly 2-AI of A.*

Theorem 28. *Let P be a proper ideal of A and* $P \neq \{0\}$ *in A. Then, P is a 2-AI of A if and only if P is a weakly 2-AI of A.*

Proof. It is clear. □

As a consequence of Theorems 20 and 26, we have the following.

Theorem 29. *Let* $f : A_1 \longrightarrow A_2$ *be a lattice homomorphism. Then, the following holds.*

(1) If f is an epimorphism and Q is a weakly 2 – AI of A_2*, then* $f^{-1}(Q)$ *is a weakly 2-AI of* A_1

(2) If f is an isomorphism and P is a weakly 2 – AI of A_1*, then* $f(P)$ *is a weakly 2-AI of* A_2

Theorem 30. *Let* $A = A_1 \times A_2$ *be an ADL, where* A_1 *and* A_2 *are ADLs. Let P and Q be proper ideal of* A_1 *and* A_2*, respectively. If* $P \times Q$ *is a weakly 2-AI of A, then P and Q are weakly 2-AI of* A_1 *and* A_2*, respectively.*

Proof. Suppose that $P \times Q$ is a weakly 2-AI of A. Let $a, b, c \in A_1$ and $x, y, z \in A_2$ such that $0 \neq x \wedge y \wedge z \in Q$. Then, $0 \neq (a, x \wedge y \wedge z) \in P \times Q$ implies that either $(a, x \wedge y) \in P \times Q$ or $(a, x \wedge z) \in P \times Q$ or $(a, y \wedge z) \in P \times Q$. From this, either $x \wedge y \in Q$ or $x \wedge z \in Q$ or $y \wedge z \in Q$. Thus, Q is a weakly 2-AI of A_2. Similarly, P is a weakly 2-AI of A_1. □

The converse of the above theorem is not true, consider the following example.

Example 7. Let $A_1 = \{0, a, b, c, 1\}$ be the lattice discussed in Example 5 and $A_2 = \{0, x, y, 1\}$ be a chain represented by the diagram given below.

Consider $A_1 \times A_2 = \{(u, v) | u \in A_1 \text{ and } v \in A_2\}$. Let $P = (c]$ and $Q = (0]$ be ideals of A_1 and A_2, respectively. Then, $P \times Q = ((c, 0)]$. We note that, for all $(1, 0), (a, x), (b, x) \in A_1 \times A_2$, $(1, 0) \wedge (a, x) \wedge (b, x) = (1 \wedge a \wedge b, 0 \wedge x) = (c, 0) \in P \times Q$. Now, $(1, 0) \wedge (a, x) = (a, 0) \notin P \times Q$, $(a, x) \wedge (b, x) = (a \wedge b, x) = (c, x) \notin P \times Q$ and $(1, 0) \wedge (b, x) = (b, 0) \notin P \times Q$. It follows that $P \times Q$ is not a weakly 2-AI of $A_1 \times A_2$.

Theorem 31. *Let* $A = A_1 \times A_2$ *be ADL, where* A_1 *and* A_2 *be ADLs and* $P(\neq \{0\})$ *be a proper ideal of* A_1*. Then, the following are equivalent.*

(1) $P \times A_2$ *is a weakly 2-AI of A*

(2) $P \times A_2$ *is a 2-AI of A*

(3) P is a 2-AI of A_1

Proof. $(1) \Rightarrow (2)$: assume (1). Let $a, b, c \in A_1$ such that $(a, x) \wedge (b, x) \wedge (c, x) \in P \times A_2$, for every $x \in A_2$. Then, $(a, x) \wedge (b, x) \wedge (c, x) = (a \wedge b \wedge c, x) \in P \times A_2$, implies either $(a \wedge b, x) \in P \times A_2$ or $(a \wedge c, x) \in P \times A_2$ or $(b \wedge c, x) \in P \times A_2$, for every $x \in A_2$ (since $P \times A_2$ is a weakly 2-AI of A). Thus, $P \times A_2$ is a 2-AI of A.

$(2) \Rightarrow (3)$: assume (2). Let $a, b, c \in A_1$ such that $a \wedge b \wedge c \in P$. Since $P \times A_2$ is a 2-AI of A, $(a \wedge b \wedge c, x) \in P \times A_2$, for every $x \in A_2$, which implies that either $(a \wedge b, x) \in P \times A_2$ or $(a \wedge c, x) \in P \times A_2$ or $(b \wedge c, x) \in P \times A_2$. From this, we have that either $a \wedge b \in P$ or $a \wedge c \in P$ or $b \wedge c \in P$. Therefore, P is a 2-AI of A_1.

$(3) \Rightarrow (1)$. Suppose P is a 2-AI of A_1 and $0 \neq (a, x) \wedge (b, x) \wedge (c, x) \in P \times A_2$, for every $x \in A_2$ and $a, b, c \in A_1$. Then, $(a, x) \wedge (b, x) \wedge (c, x) = (a \wedge b \wedge c, x) \in P \times A_2$, implies either $(a \wedge b, x) \in P \times A_2$ or $(a \wedge c, x) \in P \times A_2$ or $(b \wedge c, x) \in P \times A_2$, for every $x \in A_2$ (since either $a \wedge b \in P$ or $a \wedge c \in P$ or $b \wedge c \in P$). Thus, $P \times A_2$ is a weakly 2-AI of A. □

Theorem 32. *Let* $A = A_1 \times A_2$ *be ADL, where* A_1 *and* A_2 *be ADLs. Let* $P(\neq \{0\})$ *and* $Q(\neq \{0\})$ *be proper ideal of* A_1 *and* A_2*, respectively. Then, the following are equivalent.*

(1) $P \times Q$ *is a weakly 2-AI of A*

(2) $Q = A_2$ *and P is a 2-AI of* A_1 *or Q is a prime ideal of* A_2 *and P is a prime ideal of* A_1

(3) $P \times Q$ *is a 2-AI of A*

Proof. $(1) \Rightarrow (2)$: assume (1). If $Q = A_2$, then P is a 2-AI of A_1 (by the above theorem). Suppose that $Q \neq A_2$. Let $x, y \in A_2$ such that $x \wedge y \in Q$ and let $1 \neq t \in P$. Then, $(t, 1) \wedge (1, x) \wedge (1, y) = (t, x \wedge y) \in P \times Q - \{(0, 0)\}$. Since $(1, x) \wedge (1, y) = (1, x \wedge y) \notin P \times Q$, we conclude that either $(t, 1) \wedge (1, x) = (t, x) \in P \times Q$ or $(t, 1) \wedge (1, y) = (t, y) \in P \times Q$ and hence either $x \in Q$ or $y \in Q$. Thus, Q is a prime ideal of A_2. Similarly, P is a prime ideal of A_1.

$(2) \Rightarrow (3)$: assume (2). Then, by the above theorem, $P \times A_2$ is a 2-AI of A. Suppose that P is a prime ideal of A_1 and Q is a prime ideal of A_2. Then, clearly $P \times Q$ is a prime ideal of A. Let $(x, y), (z, t), (a, b) \in A$ such that $(x, y) \wedge (z, t) \wedge (a, b) \in P \times Q$. Then, either $(x, y) \wedge (z, t) \in P \times Q$ or $(a, b) \in P \times Q$, or $(x, y) \wedge (a, b) \in P \times Q$ or $(z, t) \in P \times$

Q, or $(x, y) \in P \times Q$ or $(z, t) \wedge (a, b) \in P \times Q$. Thus, $P \times Q$ is a 2-AI of A.

(3)⇒(1) is clear by Theorem 26 □

In the following, we introduce the concept of n-AI of an ADL A.

Definition 33. Let P be a proper ideal of A and $n \in Z^+$. Then, P is an n -AI of A if whenever $x_1 \wedge x_2 \wedge \cdots \wedge x_{n+1} \in P$, for $x_i \in A$, $1 \leq i \leq n+1$, then there are n of the $x_i's$ whose meet is in P.

Corollary 34. *Let P be a proper ideal of A and $n, m \in Z^+$. Then,*

(1) *P is n-AI if and only if whenever $x_1 \wedge x_2 \wedge \cdots \wedge x_m \in P$, for $x_1, \cdots, x_m \in A$ with $m > n$, then there are n of the $x_i's$ whose meet is in P*

(2) *If P is n-AI, then P is an m-AI, for all $m \geq n$*

Corollary 35. *Let $g : A \longrightarrow B$ be a lattice homomorphism. Then, the following holds.*

(1) *If g is an epimorphism and Q is an n-AI of B, then $g^{-1}(Q)$ is an n-AI of A*

(2) *If g is an isomorphism and P is an n-AI of A, then $g(P)$ is an n-AI of B*

Theorem 36. *If $\{P_\alpha\}_{\alpha \in \Delta}$ is a nonempty chain of n-AI of A, then $\bigwedge_{\alpha \in \Delta} P_\alpha$ is an n-AI of A.*

Proof. Let $P = \bigwedge_{\alpha \in \Delta} P_\alpha$ and $x_1, x_2, \cdots, x_{n+1} \in A$ such that $\bigwedge_{i=1}^{n+1} x_i \in P$. Let $x_i = \bigwedge_{j \neq i} x_j$ and $x_i \notin P$, for all $1 \leq i \leq n$. Then, for each $1 \leq i \leq n$, there exist an n-AI P_{α_i} such that $x_i \notin P_{\alpha_i}$. Assume that $P_{\alpha_1} \subseteq P_{\alpha_2} \subseteq \cdots \subseteq P_{\alpha_n}$. Let $\beta \in \Delta$. If $P_\beta \subseteq P_{\alpha_1} \subseteq \cdots \subseteq P_{\alpha_n}$, then $x_i \notin P_\beta$, for each $1 \leq i \leq n$. Since $x_1 \wedge x_2 \wedge \cdots \wedge x_{n+1} \in P$ and P_β is n-AI of A, we have $x_{n+1} \in P_\beta$. Again, $x_1 \wedge x_2 \wedge \cdots \wedge x_{n+1} \in P_{\alpha_1}$ and P_{α_1} is n-AI of A, then $x_{n+1} \in P_{\alpha_1}$. So, $x_{n+1} \in P_\beta$, for every $\beta \in \Delta$. Thus, $x_{n+1} \in P$. Hence the theorem. □

4. 1-Absorbing Prime Ideals

In this section, we introduce the 1-absorbing prime ideals of ADLs.

Definition 37. Let A be an ADL. A proper ideal P of A is a 1-absorbing prime ideal if for $x, y, z \in A$ such that $x \wedge y \wedge z \in P$, then either $x \wedge y \in P$ or $z \in P$.

Theorem 38. *Let P be a proper ideal of A. Then, every prime ideal of A is a 1-absorbing prime ideal and every 1-absorbing prime ideal of A is a 2-absorbing ideal of A.*

The following example show that every 2-AI of A is not 1-absorbing prime ideal of A.

Example 8. Let $D \times L = \{(t, s) | t \in D \text{ and } s \in L\}$ be an ADL discussed in Example 6. Let $Q = \{(0, 0), (x, b), (y, c)\}$. Then, for all $(0, d), (x, e), (y, f) \in D \times L$,

$$(0, d) \wedge (x, e) \wedge (y, f) \in Q \Rightarrow (x, e) \wedge (y, f) = (y, c) \in Q. \quad (12)$$

Thus, Q is a 2-AI of $D \times L$. On the other hand, $(0, d) \wedge (x, e) \wedge (y, f) = (0, 0) \in Q$ implies $(0, d) \wedge (x, e) = (0, a) \notin Q$ and $(y, f) \notin Q$. From this, Q is not a 1-absorbing prime ideal of $D \times L$. Therefore, every 2-AI of $D \times L$ is not 1-absorbing prime ideal of $D \times L$.

Next, we have the following result.

Theorem 39. *Let $A = A_1 \times A_2$ be an ADL where A_1 and A_2 be ADLs with 0, proper ideal P of A and Q a proper ideal of A_2. Then, P is a 1-absorbing prime ideal of A if and only if $P = R \times A_2$ or $P = A_1 \times Q$, where R and Q are prime ideals of A_1 and A_2, respectively.*

Lemma 40. *Let P be a 1-absorbing prime ideal of A. If $\langle x \wedge y] \cap Q \subseteq P$, for all proper ideal Q of A and for $x, y \in A$, then $\langle x \wedge y] \subseteq P$ or $Q \subseteq P$.*

Theorem 41. *Let P be a proper ideal of A. Then, the following are equivalent.*

(1) *P is a 1-absorbing prime ideal of A*

(2) *If $P_1 \cap P_2 \cap P_3 \subseteq P$ for some proper ideals P_1, P_2, and P_3 of A, then either $P_1 \cap P_2 \subseteq P$ or $P_3 \subseteq P$*

Proof. (1) ⇒ (2): suppose P is a 1-absorbing prime ideal of A. Let $P_1 \cap P_2 \cap P_3 \subseteq P$ for some proper ideals P_1, P_2, and P_3 of A. Let $P_1 \cap P_2 U P$. Then, there exists $x \in P_1$ and $y \in P_2$ such that $x \wedge y \notin P$ and hence $\langle x \wedge y] U P$. Since $\langle x] \cap \langle y] \cap P_3 = \langle x \wedge y] \cap P_3 \subseteq P$, it follows that $P_3 \subseteq P$ (by the above lemma).

(2) ⇒ (1): assume (2) hold. Suppose that $x \wedge y \wedge z \in P$, for $x, y, z \in A$ and let $x \wedge y \notin P$. Suppose also that $P_1 = \langle x]$, $P_2 = \langle y]$ and $P_3 = \langle z]$. Then $P_1 \cap P_2 \cap P_3 = \langle x] \cap \langle y] \cap \langle z] = \langle x \wedge y] \cap \langle z] \subseteq P$ and $P_1 \cap P_2 U P$ (by assumption, $x \wedge y \notin P$). Thus by the above lemma, $\langle z] \subseteq P$, that is, $P_3 \subseteq P$ and thus $z \in P$. Therefore, P is a 1-absorbing prime ideal of A. □

Theorem 42. *Let A_1 and A_2 be ADLs and $f : A_1 \longrightarrow A_2$ be a lattice homomorphism. Then, the following hold.*

(1) *If P is a 1-absorbing prime ideal of A_2, then $f^{-1}(P)$ is a 1-absorbing prime ideal of A_1*

(2) *If f is onto and P is a 1-absorbing prime ideal of A_1 with $\ker(f) \subseteq P$, then $f(P)$ is a 1-absorbing prime ideal of A_2*

Proof.

(1) Let P be a 1-absorbing prime ideal of A_2 and $x \wedge y \wedge z \in f^{-1}(P)$ for some $x, y, z \in A_1$. Then $f(x \wedge y \wedge z) = f(x) \wedge f(y) \wedge f(z) \in P$ which implies that $f(x) \wedge f(y) \in P$ or $f(z) \in P$. It follows that $x \wedge y \in f^{-1}(P)$ or $z \in f^{-1}(P)$. Hence $f^{-1}(P)$ is a 1-absorbing prime ideal of A_1

(2) Let P be a 1-absorbing prime ideal of A_1 with $\ker(f) \subseteq P$, f be onto and $x \wedge y \wedge z \in f(P)$ for some $x, y, z \in A_2$. Since f is onto, then there exists $a, b, c \in A_2$ such that $a = f(x)$, $b = f(y)$ and $c = f(z)$. Therefore, $f(x \wedge y \wedge z) = f(x) \wedge f(y) \wedge f(z) = a \wedge b \wedge c \in f(P)$. Since $\ker(f) \subseteq P$, we conclude that $x \wedge y \wedge z \in P$. Thus, $x \wedge y \in P$ or $z \in P$ and so, $a \wedge b \in f(P)$ or $c \in f(P)$. Therefore, $f(P)$ is a 1-absorbing prime ideal of A_2

□

Definition 43. Let P be a proper ideal of A. Then, P is said to be a weakly 1-absorbing prime ideal of A if $0 \neq x \wedge y \wedge z \in P$ for some $x, y, z \in A$, then either $x \wedge y \in P$ or $z \in P$.

Theorem 44. *Every weakly prime ideal of A is a weakly 1-absorbing prime ideal of A.*

Proof. It is clear. □

The following example show that the converse of Theorem 44 is not true.

Example 9. Let $D \times L = \{(t, s) | t \in D \text{ and } s \in L\}$ be an ADL discussed in Example 5. Let $P = \{(0, c), (x, 0), (y, c)\}$. Let $(0, a), (x, b), (y, c) \in D \times L$. Then,

$$(0,0) \neq (0,a) \wedge (x,b) \wedge (y,c) \in P \Rightarrow (0,a) \wedge (x,b) \in P \text{ and } (y,c) \in P. \quad (13)$$

Thus, P is a weakly 1-absorbing prime ideal of $D \times L$, but P is not weakly prime ideal of $D \times L$, since $(0,0) \neq (0,a) \wedge (x,b) \in P$ implies that $(0,a) \notin P$ and $(x,b) \notin P$. From this, we conclude that, every weakly 1-absorbing prime ideal of $D \times L$ is not weakly prime ideal of $D \times L$.

Theorem 45. *Every weakly 1-absorbing prime ideal of A is a weakly 2-absorbing ideal of A.*

Proof. It is clear. □

The following example show that every weakly 1-absorbing prime ideal of A is not 1-absorbing prime ideal of A.

Example 10. Let $D \times L = \{(t, s) | t \in D \text{ and } s \in L\}$ be an ADL discussed in Example 6. Let $Q = \{(0,0)\}$. Clearly Q is a weakly 1-absorbing prime ideal of $D \times L$. But Q is not 1-absorbing prime ideal of $D \times L$, since $(0,d) \wedge (x,e) \wedge (y,f) \in P$ implies $(0,d) \wedge (x,e) = (0,a) \notin P$ and $(y,f) \notin P$, for all $(0,d), (x,e), (y,f) \in D \times L$. Thus, every weakly 1-absorbing prime ideal of $D \times L$ is not 1-absorbing prime ideal of $D \times L$.

Theorem 46. *Let A and B be ADLs and $f : A \longrightarrow B$ be a lattice homomorphism. Then, the following holds.*

(1) If f is a monomorphism and P is a weakly 1-absorbing prime ideal of B, then $f^{-1}(P)$ is a weakly 1-absorbing prime ideal of A

(2) If f is an epimorphism and Q is a weakly 1-absorbing prime ideal of A such that $\ker(f) \subseteq Q$, *then $f(Q)$ is a weakly 1-absorbing prime ideal of B*

Proof.

(1) Let $0 \neq x \wedge y \wedge z \in f^{-1}(P)$ for some $x, y, z \in A$. Then by assumption, $f(x \wedge y \wedge z) = f(x) \wedge f(y) \wedge f(z) \in P$, for some $f(x), f(y), f(z) \in B$. Since f is monomorphism, we have $f(x \wedge y \wedge z) \neq 0$. Also, since P is a weakly 1-absorbing prime ideal of B, we conclude either $f(x) \wedge f(y) = f(x \wedge y) \in P$ or $f(z) \in P$ which implies that $x \wedge y \in f^{-1}(P)$ or $z \in f^{-1}(P)$. Thus, $f^{-1}(P)$ is a weakly 1-absorbing prime ideal of A.

(2) Assume that $0 \neq a \wedge b \wedge c \in f(Q)$ for some $a, b, c \in B$. Since f is an epimorphism, then there exists $x, y, z \in A$ such that $a = f(x)$, $b = f(y)$ and $c = f(z)$. Then, $0 \neq f(x) \wedge f(y) \wedge f(z) = f(x \wedge y \wedge z) \in f(Q)$. Since $\ker(f) \subseteq Q$, then we get that $0 \neq x \wedge y \wedge z \in Q$. As Q is a weakly 1-absorbing prime ideal of A, we have either $x \wedge y \in Q$ or $z \in Q$ and which implies that $f(x \wedge y) = a \wedge b \in f(Q)$ or $f(z) = c \in f(Q)$. Therefore, $f(Q)$ is a weakly 1-absorbing prime ideal of B

□

5. Conclusion

In this paper, the concepts of 2-absorbing ideal, 1-absorbing prime ideal, weakly 1-absorbing prime ideal, and weakly 2-absorbing ideal of an almost distributive lattice are introduced and obtain certain results of these.

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares that he/she has no conflicts of interest.

References

[1] A. Badawi, "On 2-absorbing ideals of commutative rings," *Bulletin of the Australian Mathematical Society*, vol. 75, no. 3, pp. 417–429, 2007.

[2] B. J. Abadi and H. F. Moghimi, "On (m, n)-absorbing ideals of commutative rings," *Proceedings-Mathematical Sciences*, vol. 127, no. 2, pp. 251–261, 2017.

[3] D. F. Anderson and A. Badami, "On *n*-absorbing ideals of commutative rings," *Communications in Algebra*, vol. 39, no. 5, pp. 1646–1672, 2011.

[4] J. N. Chuadhari, "2-Absorbing ideals in semirings," *International Journal of Algebra*, vol. 6, no. 6, pp. 265–270, 2012.

[5] S. Payrovi and S. Babali, "On the 2-absorbing ideals," *International Mathematical Forum*, vol. 7, no. 6, pp. 265–271, 2012.

[6] A. Yassine, M. J. Nikmehr, and R. Nikandish, "On 1-absorbing prime ideals of commutative rings," *Journal of Algebra and its Applications*, vol. 20, no. 10, article 2150175, 2021.

[7] A. Badawi and A. Y. Darani, "On weakly 2-absorbing ideals of commutative rings," *Houston Journal of Mathematics*, vol. 39, no. 2, pp. 441–452, 2013.

[8] C. Beddani and W. Messirdi, "2-Prime ideals and their applications," *Journal of Algebra and Its Applications*, vol. 15, no. 3, article 1650051, 2016.

[9] M. K. Dubey, "Prime and weakly prime ideals," *Quasi-groups and Related Systems*, vol. 20, pp. 197–202, 2012.

[10] M. P. Wasadikar and K. T. Gaikevad, "On 2-absorbing ideals and weakly 2-absorbing ideals of lattice," *Mathematical Sciences International Research Journal*, vol. 4, no. 2, pp. 82–85, 2015.

[11] U. M. Swamy and G. C. Rao, "Almost distributive lattices," *Journal of the Australian Mathematical Society*, vol. 31, no. 1, pp. 77–91, 1981.

[12] G. C. Rao, *Almost distributive lattices [PhD Thesis]*, Andhra University, Waltair, 1980.