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Research Article

A New Notion of Classical Mean Graphs Based on Duplicating Operations

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Classical mean labeling of a graph G with p vertices and q edges is an injective function from the vertex set of G to the set $\{1, 2, 3, \ldots, q + 1\}$ such that the edge labels obtained from the flooring function of the average of mean of arithmetic, geometric, harmonic, and root square of the vertex labels of each edge's end vertices is distinct, and the set of edge labels is $\{1, 2, 3, \ldots, q\}$. One of the graph operations is to duplicate the graph. The classical meanness of graphs formed by duplicating an edge and a vertex of numerous standard graphs is discussed in this study.

1. Introduction and Preliminaries

The term "graph" refers to a finite, undirected, and basic graph used throughout this paper. Let us consider a graph with p vertices and q edges, such as $G(V, E)$. [1–6] are the notations and terms we utilize. For a comprehensive look into graph labeling, we recommend [7]. The corona $G_1 \circ G_2$ is a graph created by linking the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2 using one G_1 copy of order p_1 and p_1 copies of G_2. A pair of nearby vertices on a graph is referred to as neighbors. The neighborhood set is defined as the set of all neighbors of a vertex v and is denoted by $N(v)$ [8].

2. Literature Survey

Somosundaram and Ponraj [9] proposed the idea of mean labeling. The F-harmonic mean graph was defined by Durai Baskar and Arockiaraj [10]. Arockiaraj et al. introduced the concept of F-root square mean graphs [11] and evaluated the meanness of some graphs by duplicating graph components [12]. Vaidya and Barasara thoroughly examined the harmonic mean [8] and geometric mean [13] labeling for a variety of graphs emerging from graph element duplication. In [14], Maya and Nicholas researched the duplication of divisor cordial graphs. Durai Baskar et al. researched the geometric meanness of graphs [15] and Durai Baskar and Arockiaraj talked about the F-geometric mean graphs [16]. Prajapati and Gajjar developed the cordiality in the context of duplication in web and armed helm [17]. Exponential mean labeling of graphs of some standard graphs by duplication of graph elements is developed by Rajesh Kannan et al. [18]. Muhiuddin et al. defined classical mean labeling of graphs [19] and Alanazi et al. extended its meanness for specific ladder graphs [20]. We explore a conventional mean labeling of graphs based on some duplicating techniques [21–25], which has been produced by a significant number of creators in the domain of graph labeling.

3. Methodology

A labeling Φ on a graph G with $\Delta = q + 1$ is called classical mean labeling if the injective function

$\Phi: V \longrightarrow N - \{\Delta+1, \Delta+2, \ldots, \infty\}$ and an induced bijective function $\Phi^*: V \longrightarrow N - \{\Delta, \Delta+1, \ldots, \infty\}$ is defined by

$$\Phi^*(uv) = \left\lfloor \frac{1}{4}\left(\frac{\Phi(u)+\Phi(v)}{2} + \sqrt{\Phi(u)\Phi(v)} + \frac{2\Phi(u)\Phi(v)}{\Phi(u)+\Phi(v)} + \sqrt{\frac{\Phi(u)^2+\Phi(v)^2}{2}}\right)\right\rfloor, \quad (1)$$

for all uv $\in E(G)$. A classical mean graph is one that facilitates classical mean labeling. In this article, we essentially address our topic's flooring function and try to rationalize the classical meanness of some of these graphs created by duplicating procedures.

Figure 1 depicts classical mean labeling of $2ST_4$ [20].

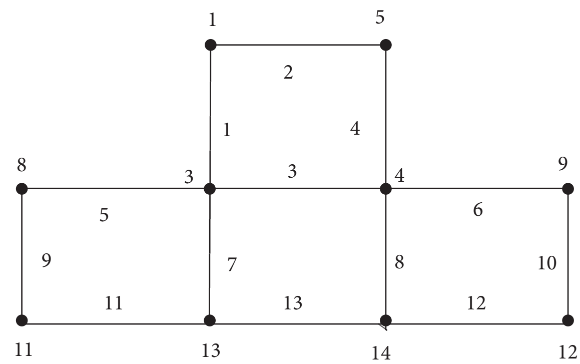

FIGURE 1: A classical mean labeling of $2ST_4$.

4. Main Results

4.1. Classical Meanness of Graphs Obtained from Edge Duplicating Operation

Theorem 1. *If a graph G is formed by duplicating one edge of another graph G', then G is a classical mean graph, where G' represents the path P_n for $n \geq 3$.*

Proof. Let $e' = v'_\gamma v_{\gamma+1}{}'$ be the duplicating edge of $e = v_\gamma v_{\gamma+1}$, $1 \leq \gamma \leq n-1$.

Case 1. $\gamma = -1+n$ or $\gamma = 1$, and $\Delta = n+3$.

Let us construct $\Phi: V \longrightarrow N - \{\Delta, \Delta+1, \ldots, \infty\}$ (see Table 1).

Hence,

$$\Phi^*(v'_2 v_\delta) = 2\delta - 1, \quad \text{for } \delta = 1, 3. \quad (2)$$

and (see Table 2).

Case 2. $n \geq 4$, $\gamma = 2$, and $\Delta = n+4$.

Let us construct $\Phi: V \longrightarrow N - \{\Delta+1, \Delta+2, \ldots, \infty\}$ (see Table 3).

Hence,

$$\begin{aligned} &\Phi^*(v'_2 v_\delta) = 2\delta - 2, \quad \text{for } \delta = 1 \text{ and } 3, \\ &\Phi^*(v'_3 v_4) = 6. \end{aligned} \quad (3)$$

and (see Table 4).

Case 3. $n \geq 5$, $3 \leq \gamma \leq n-2$, and $\Delta = n+4$.

Let us construct $\Phi: V \longrightarrow N - \{\Delta+1, \Delta+2, \ldots, \infty\}$ (see Table 5).

Hence,

$$\begin{aligned} \Phi^*\left(v'_\gamma v_{\gamma-1}\right) &= \gamma - 1, \\ \Phi^*\left(v'_\gamma v_{\gamma-1}\right) &= 1 + \gamma, \\ \Phi^*\left(v_{\gamma+1}{}' v_{\gamma+2}\right) &= 3 + \gamma, \end{aligned} \quad (4)$$

and (see Table 6).

Figure 2 depicts classical mean labeling of G in the preceding circumstances. □

TABLE 1: Vertex labeling of $\Phi(v)$.

$\Phi(v)$	$1 \leq \delta \leq 3$	$4 \leq \delta \leq n$
$\Phi(v_\delta)$	$2\delta - 1$	$\delta + 2$
$\Phi(v'_\delta)$	2δ	–

TABLE 2: Edge labeling of $\Phi^*(e)$.

$\Phi^*(e)$	$1 \leq \delta \leq 3$	$4 \leq \delta \leq n-1$
$\Phi^*(v_\delta v_{\delta+1}$	$2\delta - 1$	$\delta + 2$

TABLE 3: Vertex labeling of $\Phi(v)$.

$\Phi(v)$	$\delta = 1$	$2 \leq \delta \leq 3$	$\delta = 4$	$5 \leq \delta \leq n$
$\Phi(v_\delta)$	-12δ	$-1 + 2\delta$	$-1 + 2\delta$	$3 + \delta$
$\Phi(v'_\delta)$	–	2δ	–	–

TABLE 4: Edge labeling of $\Phi^*(e)$.

$\Phi^*(e)$	$1 \leq \delta \leq 4$	$5 \leq \delta \leq n-1$
$\Phi^*(v_\delta v_{\delta+1})$	$2\delta - 1$	$\delta + 3$

TABLE 5: Vertex labeling of $\Phi(v)$.

$\Phi(v)$	$1 \leq \delta \leq \gamma - 1$	$\delta = \gamma$	$\delta = \gamma + 1$	$\gamma + 2 \leq \delta \leq n$
$\Phi(v_\delta)$	δ	$2 + \gamma$	$3 + \delta$	$3 + \delta$
$\Phi(v'_\gamma)$	–	$1 + \delta$	–	–
$\Phi(v_{\gamma+1}{}')$	–	–	$3 + \gamma$	–

TABLE 6: Edge labeling of $\Phi^*(e)$.

$\Phi^*(e)$	$1 \leq \delta \leq \gamma - 2$	$\delta = \gamma - 1$	$\delta = \gamma$	$\gamma + 1 \leq \delta \leq n-1$
$\Phi^*(v_\delta v_{\delta+1}{}')$	δ	γ	$\gamma + 2$	$\delta + 3$

Theorem 2. *If a graph G is formed by duplicating one edge of another graph G', then G is a classical mean graph, where G' represents the path $P_n \circ K_1$.*

Proof. Suppose $n \geq 3$.

Case 1. $1 \leq \gamma \leq n$, $e = u_\gamma v_\gamma$, and $n \geq 3$.

Subcase (1). $\gamma = n$ or $\gamma = 1$, and $\Delta = 2n + 3$.

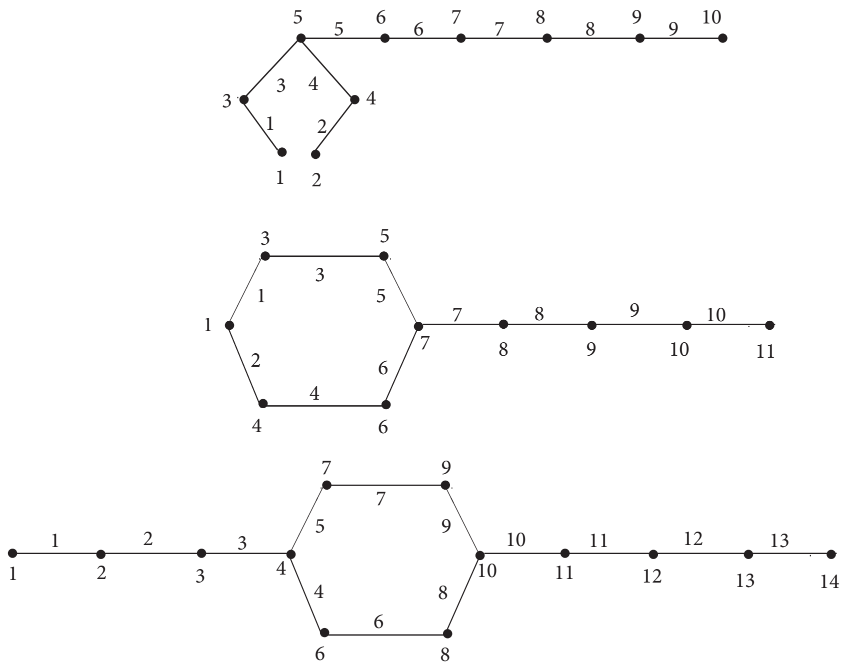

Figure 2: A classical mean labeling of G obtained by duplicating an edge of a path.

Let us construct $\Phi: V \longrightarrow N - \{\Delta+1, \Delta+2, \ldots, \infty\}$.

$$\begin{aligned}\Phi\left(u_1'\right) &= 3,\\ \Phi\left(v_1'\right) &= 1.\end{aligned} \tag{5}$$

and (see Table 7).

Hence,

$$\begin{aligned}\Phi^*\left(u_1'v_1'\right) &= 1,\\ \Phi^*\left(u_1'u_1'\right) &= 3.\end{aligned} \tag{6}$$

and (see Table 8).

Subcase (2). $\gamma = 2$ and $\Delta = 2n+4$.

Let us construct $\Phi: V \longrightarrow N - \{\Delta+1, \Delta+2, \ldots, \infty\}$.

$$\begin{aligned}\Phi\left(u_2'\right) &= 7,\\ \Phi\left(v_2'\right) &= 6.\end{aligned} \tag{7}$$

and (see Table 9).

Hence,

$$\begin{aligned}\Phi^*\left(u_1u_2'\right) &= 4,\\ \Phi^*\left(u_2'u_3\right) &= 7,\\ \Phi^*\left(u_2'v_2'\right) &= 6.\end{aligned} \tag{8}$$

and (see Table 10).

Subcase (3). $n \geq 4$, $3 \leq \gamma \leq n-1$, and $\Delta = 2n+4$.

Let us construct $\Phi: V \longrightarrow N - \{\Delta+1, \Delta+2, \ldots, \infty\}$.

$$\begin{aligned}\Phi\left(u_\gamma'\right) &= 2\gamma+3,\\ \Phi\left(v_\gamma'\right) &= 2\gamma+2.\end{aligned} \tag{9}$$

and (see Table 11).

Table 7: Vertex labeling of $\Phi(v)$.

$\Phi(v)$	$1 \leq \delta \leq 2$	$3 \leq \delta \leq n$
$\Phi(u_\delta)$	$\delta+3$	$2\delta+2$
$\Phi(v_\delta')$	$4\delta-2$	$2\delta+1$

Table 8: Edge labeling of $\Phi^*(e)$.

$\Phi^*(e)$	$\delta = 1$	$2 \leq \delta \leq n-1$	$\delta = n$
$\Phi^*(u_\delta u_{1+\delta})$	4	$2\delta+2$	–
$\Phi^*(u_\delta v_{\delta+1})$	2	$2\delta+1$	$2n+1$

Table 9: Vertex labeling of $\Phi(v)$.

$\Phi(v)$	$\delta = 1$	$2 \leq \delta \leq 3$	$4 \leq \delta \leq n$
$\Phi(u_\delta)$	3	$4\delta-4$	$2\delta+3$
$\Phi(v_\delta)$	1	$7\delta-12$	$2\delta+2$

Table 10: Edge labeling of $\Phi^*(e)$.

$\Phi^*(e)$	$1 \leq \delta \leq 2$	$3 \leq \delta \leq n-1$	$\delta = n$
$\Phi^*(u_\delta u_{1+\delta})$	$2\delta+1$	$2\delta+3$	–
$\Phi^*(u_\delta v_\delta)$	δ	$2\delta+2$	$2n+2$

Table 11: Vertex labeling of $\Phi(v)$.

$\Phi(v)$	$1 \leq \delta \leq i$	$\delta = \gamma+1$	$\gamma+2 \leq \delta \leq n$
$\Phi(u_\delta)$	2δ	$2\delta+4$	$2\delta+3$
$\Phi(v_\delta)$	$2\delta-1$	$2\delta+5$	$2\delta+2$

Hence,

$$\Phi^*\left(u_{\gamma-1}u'_{\gamma}\right) = 2\gamma,$$
$$\Phi^*\left(u'_{\gamma}u_{\gamma+1}\right) = 2\gamma + 3, \quad (10)$$
$$\Phi^*\left(u'_{\gamma}v'_{\gamma}\right) = 2\gamma + 2,$$

and (see Table 12).

Hence, Φ is a classical meanness of G.

Case 2. $1 \le \gamma \le n-1$, $e = u_\gamma u_{\gamma+1}$, and $n \ge 3$.

Subcase (1). $\gamma = n-1$ or $\gamma = 1$, and $\Delta = 2n+5$.

Let us construct $\Phi\colon V \longrightarrow N - \{\Delta+1, \Delta+2, \ldots, \infty\}$.

$$\Phi\left(u'_1\right) = 4,$$
$$\Phi\left(u'_2\right) = 5. \quad (11)$$

and (see Table 13).

Hence,

$$\Phi^*\left(u'_1u'_2\right) = 4,$$
$$\Phi^*\left(u'_2v_2\right) = 5,$$
$$\Phi^*\left(u'_1v_1\right) = 2, \quad (12)$$
$$\Phi^*\left(u'_2u_3\right) = 7.$$

and (see Table 14).

Subcase (2). $n \ge 4$, $\gamma = 2$, and $\Delta = 2n+6$.

Let us construct $\Phi\colon V \longrightarrow N - \{\Delta+1, \Delta+2, \ldots, \infty\}$.

$$\Phi\left(u'_2\right) = 3,$$
$$\Phi\left(u'_3\right) = 11. \quad (13)$$

and (see Table 15).

Hence,

$$\Phi^*\left(u_1u'_2\right) = 2,$$
$$\Phi^*\left(u'_2u'_3\right) = 6,$$
$$\Phi^*\left(u'_3u_4\right) = 11, \quad (14)$$
$$\Phi^*\left(u'_2v_2\right) = 3,$$
$$\Phi^*\left(u'_3v_3\right) = 10.$$

and (see Table 16).

Subcase (3). $n \ge 5$, $3 \le \gamma \le n-2$, and $\Delta = 2n+5$.

Let us construct $\Phi\colon V \longrightarrow N - \{\Delta+1, \Delta+2, \ldots, \infty\}$.

$$\Phi\left(u'_{\gamma}\right) = 2\gamma - 1,$$
$$\Phi\left(u_{\gamma+1}{}'\right) = 2\gamma + 7. \quad (15)$$

and (see Table 17)

Hence (see Table 18),

Table 12: Edge labeling of $\Phi^*(e)$.

$\Phi^*(e)$	$1 \le \delta \le \gamma-1$	$\delta = \gamma$	$\gamma+1 \le \delta \le n-1$
$\Phi^*(u_\delta u_{1+\delta})$	2δ	$2\gamma+1$	$2\delta+3$
$\Phi^*(u_\delta v_\delta)$	$2\delta-1$	$2\delta-1$	$2\delta+2$

Table 13: Vertex labeling of $\Phi(v)$.

$\Phi(v)$	$\delta = 1$	$2 \le \delta \le 3$	$4 \le \delta \le n$
$\Phi(u_\delta)$	1	$2\delta+4$	$2\delta+4$
$\Phi(v_\delta)$	2	3δ	$2\delta+3$

Table 14: Edge labeling of $\Phi^*(e)$.

$\Phi^*(e)$	$\delta = 1$	$\delta = 2$	$3 \le \delta \le n-1$	$\delta = n$
$\Phi^*(u_\delta u_{1+\delta})$	3	$2\delta+4$	$2\delta+4$	–
$\Phi^*(u_\delta v_\delta)$	$5\delta-4$	$5\delta-4$	$2\delta+3$	$2\delta+3$

Table 15: Vertex labeling of $\Phi(v)$.

$\Phi(v)$	$1 \le \delta \le 2$	$3 \le \delta \le 4$	$5 \le \delta \le n$
$\Phi(u_\delta)$	$5\delta-3$	$4\delta-4$	$2\delta+5$
$\Phi(v_\delta)$	$3\delta-2$	$3\delta+1$	$2\delta+4$

Table 16: Edge labeling of $\Phi^*(e)$.

$\Phi^*(e)$	$\delta = 1$	$2 \le \delta \le 3$	$4 \le \delta \le n-1$	$\delta = n$
$\Phi^*(u_\delta u_{1+\delta})$	4	$2\delta+3$	$2\delta+5$	–
$\Phi^*(u_\delta v_\delta)$	1	$3\delta-1$	$2\delta-4$	$2n-4$

Table 17: Vertex labeling of $\Phi(v)$.

$\Phi(v)$	$1 \le \delta \le \gamma-1$	$\delta = \gamma$	$\delta = \gamma+1$	$\delta = \gamma+2$	$\gamma+3 \le \delta \le n$
$\Phi(u_\delta)$	2δ	$2\delta+3$	$2\delta+4$	$2\delta+8$	$2\delta+5$
$\Phi(v_\delta)$	$2\delta-1$	2δ	$2\delta+6$	$2\delta+9$	$2\delta+4$

$$\Phi^*\left(u'_{\gamma}u'_{\gamma+1}\right) = 2\gamma + 2,$$
$$\Phi^*\left(u_{\gamma-1}u'_{\gamma}\right) = 2\gamma - 2,$$
$$\Phi^*\left(u'_{\gamma+1}v_{\gamma+1}\right) = 2\gamma + 6, \quad (16)$$
$$\Phi^*\left(u'_{\gamma}u_{\gamma+2}\right) = 2\gamma + 7,$$
$$\Phi^*\left(u'_{\gamma}v_{\gamma}\right) = 2\gamma - 1.$$

Figure 3 depicts the classical mean labeling of G in the preceding situations.

Case 3. $e = u_1v_1$, $e = u_1u_2$, and $n = 2$.

Figure 4 depicts classical mean labeling of G where $e = u_1v_1$ and $e = u_1u_2$ for $n = 2$. □

Theorem 3. *If a graph G is formed by duplicating one edge of another graph G', then G is a classical mean graph, where G' represents the path C_n for $n \ge 3$.*

Proof. Case 1. $n \ge 6$ *and* $\Delta = n+5$.

Table 18: Edge labeling of $\Phi^*(e)$.

$\Phi^*(e)$	$1 \leq \delta \leq \gamma - 2$	$\delta = \gamma - 1$	$\delta = \gamma$	$\delta = \gamma + 1$	$\delta + 2 \leq \delta \leq n - 1$
$\Phi^*(u_\delta u_{1+\delta})$	2δ	2δ	$2\delta + 3$	$2\delta + 5$	$2\delta + 5$
$\Phi^*(u_\delta v_\delta)$	$2\delta - 1$	$2\delta - 1$	$2\delta + 1$	$2\delta + 4$	$2\delta + 4$

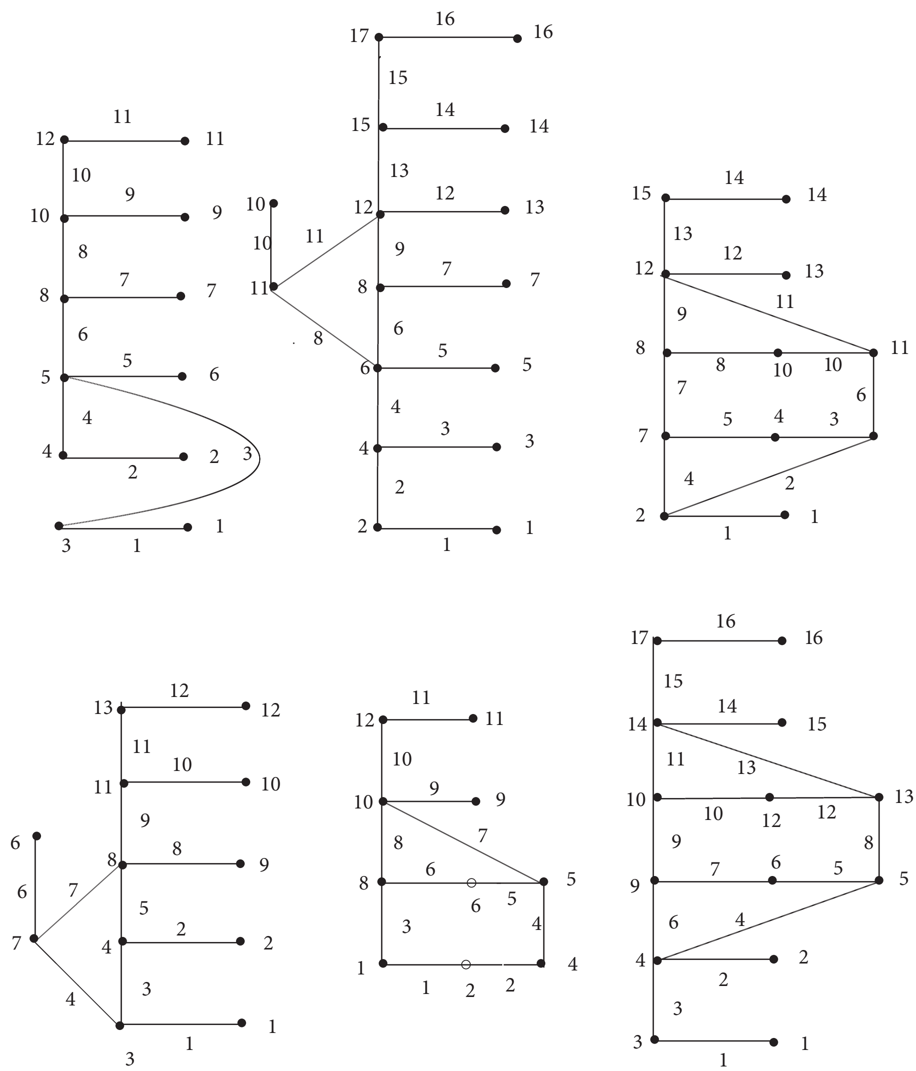

Figure 3: A classical mean labeling of G obtained by duplicating an edge of a graph $P_n \circ K_1$.

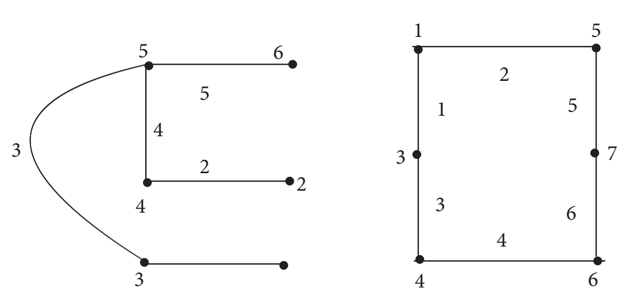

Figure 4: A classical mean labeling of G.

Let us construct $\Phi: V \longrightarrow N - \{\Delta + 1, \Delta + 2, \ldots, \infty\}$ (see Table 19).

$$\Phi(v_i') = n + 2\gamma - 1 \quad \text{for } 1 \le \gamma \le 2, \tag{17}$$

$$\Phi(v_\gamma) = \begin{cases} \gamma - 3, & 4 \le \gamma \le \left\lfloor \dfrac{1}{4}\left(\dfrac{n^2 + 14n + 41}{2n + 10} + \sqrt{n + 4} + \sqrt{\dfrac{n^2 + 8n + 17}{2}}\right)\right\rfloor + 2, \\ \gamma - 2, & \left\lfloor \dfrac{1}{4}\left(\dfrac{n^2 + 14n + 41}{2n + 10} + \sqrt{n + 4} + \sqrt{\dfrac{n^2 + 8n + 17}{2}}\right)\right\rfloor + 3 \le \gamma \le n. \end{cases} \tag{18}$$

Hence, (see Table 20).

Let us take $X = ((n^2 + 14n + 41)/(2n + 10))$, $Y = \sqrt{4 + n}$, and $Z = \sqrt{((n^2 + 8n + 17)/2)}$.

$$\begin{aligned} \Phi^*(v_1'v_n) &= -1 + n, \\ \Phi^*(v_1v_n) &= -2 + n, \\ \Phi^*(v_3v_4) &= \left\lfloor \frac{1}{4}(X + Y + Z) \right\rfloor, \\ \Phi^*(v_\gamma v_{\gamma+1}) &= \begin{cases} 3 + \gamma, & 4 \le \gamma \le 2 + \left\lfloor \dfrac{1}{4}(X + Y + Z) \right\rfloor, \\ -2 + \gamma, & 3 + \left\lfloor \dfrac{1}{4}(X + Y + Z) \right\rfloor \le \gamma \le n - 1. \end{cases} \end{aligned} \tag{19}$$

Figure 5 depicts classical mean labeling of C_{15} in the preceding circumstances.

Case 2. $n = 3, 4, 5$.

Figure 6 depicts classical mean labeling of a graph for Case 2.

4.2. Classical Meanness of Graphs Obtained from Vertex Duplicating Operation

Theorem 4. *If a graph G is created by duplicating a vertex of another graph G′, then G is a classical mean graph, where G′ is the path P_n, for $n \ge 3$.*

Proof. Let $V(P_n)$ be $v_1, v_2, \ldots, v_n$. Let the duplicating a vertex v_δ by a vertex k_δ, for some δ of the graph G'.

Case 1. $\delta = n$ or $\delta = 1$.

Let us take $\Delta = n + 2$.

Let us construct $\Phi: V \longrightarrow N - \{\Delta + 1, \Delta + 2, \ldots, \infty\}$ (see Table 21).

$$\Phi(k_1) = 1. \tag{20}$$

Hence (see Table 22),

$$\Phi^*(v_2k_1) = 1. \tag{21}$$

Case 2. $2 \le \delta \le n - 1$ and $\Delta = n + 2$.

Let us construct $\Phi: V \longrightarrow N - \{\Delta + 1, \Delta + 2, \ldots, \infty\}$.

$$\Phi(k_\delta) = 1 + \delta. \tag{22}$$

and (see Table 23).

Hence,

$$\begin{aligned} \Phi^*(v_{\delta-1}k_\delta) &= \delta - 1, \\ \Phi^*(v_{\delta+1}k_\delta) &= \delta + 1. \end{aligned} \tag{23}$$

and (see Table 24).

Figure 7 depicts classical mean labeling of G in the preceding circumstances. □

Theorem 5. *If a graph G is created by duplicating a vertex of another graph G′, then G is a classical mean graph, where G′ is the path $P_n \circ K_1$, for $n \ge 3$.*

Proof. Case 1. $v = u_\gamma$, for $1 \le \gamma \le n$.

Let its duplication be $v\prime = u_\gamma'$.

Subcase 1. $\gamma = n$ or $\gamma = 1$ and $\Delta = 3 + 2n$.

Table 19: Vertex labeling of $\Phi(v)$.

$\Phi(v)$	$\delta = 1$	$2 \leq \delta \leq 3$
$\Phi(v_\delta)$	$n - 1$	$n + 2\delta - 2$

Table 20: Edge labeling of $\Phi^*(e)$.

$\Phi^*(e)$	$1 \leq \gamma \leq 2$
$\Phi^*(v_\gamma v_{\gamma+1})$	$n + 2\gamma - 2$
$\Phi^*(v'_\gamma v'_{\gamma+1})$	$n + 2\gamma - 1$

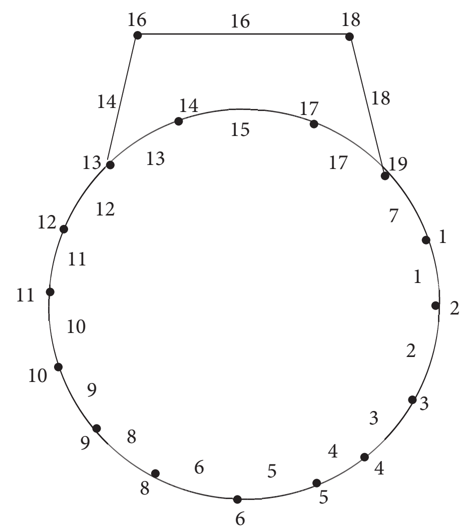

Figure 5: A classical mean labeling of the graph obtained by duplicating an edge of a graph C_{15}.

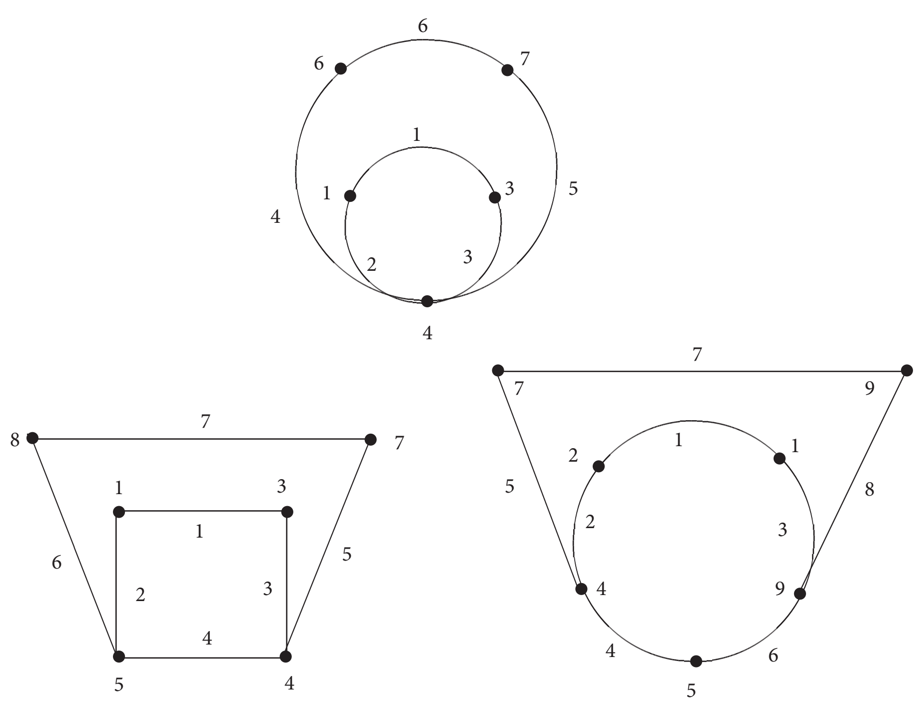

Figure 6: A classical mean labeling of the graph obtained by duplicating an edge of a graph C_3, C_4, and C_5.

Table 21: Vertex labeling of $\Phi(v)$.

$\Phi(v)$	$1 \le \gamma \le n$
$\Phi(v_\gamma)$	$\gamma + 1$

Table 22: Edge labeling of $\Phi^*(e)$.

$\Phi^*(e)$	$1 \le \gamma \le n-1$
$\Phi^*(v_\gamma v_{\gamma+1})$	$\gamma + 1$

Table 23: Vertex labeling of $\Phi(v)$.

$\Phi(v)$	$1 \le \gamma \le \delta - 1$	$\delta \le \gamma \le n$
$\Phi(v_\gamma)$	γ	$\gamma + 2$

Table 24: Edge labeling of $\Phi^*(e)$.

$\Phi^*(e)$	$1 \le \gamma \le \gamma - 2$	$\gamma = \delta - 1$	$\delta \le \gamma \le n-1$
$\Phi^*(v_\gamma v_{\gamma+1})$	γ	$\gamma + 1$	$\gamma + 2$

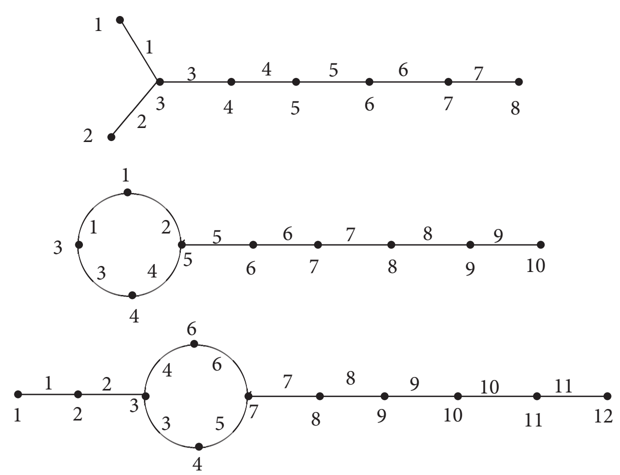

Figure 7: A classical mean labeling of G obtained by duplicating a vertex of a path.

Let us construct Φ: $V \longrightarrow N - \{\Delta + 1, \Delta + 2, \ldots, \infty\}$ (see Table 25).

$$\Phi(u_1') = 4. \tag{24}$$

Hence (see Table 26),

Subcase 2. $n \ge 3$, $2 \le \gamma \le n - 1$, and take $\Delta = 4 + 2n$.

Let us construct Φ: $V \longrightarrow N - \{\Delta + 1, \Delta + 2, \ldots, \infty\}$ (see Table 27).

$$\Phi(u_\gamma') = 2\delta + 2 \tag{25}$$

Hence,

Table 25: Vertex labeling of $\Phi(v)$.

$\Phi(v)$	$\delta = 1$	$2 \le \delta \le n$
$\Phi(u_\delta)$	2	$2\delta + 2$
$\Phi(v_\delta)$	1	$2\delta + 1$

Table 26: Edge labeling of $\Phi^*(e)$.

$\Phi^*(e)$	$\delta = 1$	$2 \le \delta \le n-1$
$\Phi^*(u_\delta u_{1+\delta})$	3	$2\delta + 2$
$\Phi^*(u_\delta v_\delta)$	1	$2\delta + 1$
$\Phi^*(e)$	$1 \le \gamma \le 2$	
$\Phi^*(u_\gamma' v_\gamma)$	2γ	

Table 27: Vertex labeling of $\Phi(v)$.

$\Phi(v)$	$1 \le \delta \le \gamma - 1$	$\delta = \gamma$	$\gamma + 1 \le \delta \le n$
$\Phi(u_\delta)$	2δ	2δ	$2\delta + 3$
$\Phi(v_\delta)$	$2\delta - 1$	$2\delta + 1$	$2\delta + 2$

$$\begin{aligned}\Phi^*\left(u_{\gamma-1}u_\gamma'\right) &= 2\gamma - 1,\\ \Phi^*\left(u_\gamma' u_{\gamma+1}\right) &= 2\gamma + 3,\\ \Phi^*\left(u_\gamma' v_\gamma\right) &= 2\gamma + 1.\end{aligned} \tag{26}$$

and (see Table 28).

Case 2. $1 \le \gamma \le n$, $v = v_\gamma$, and $\Delta = 2 + 2n$.

Let us construct Φ: $V \longrightarrow N - \{\Delta + 1, \Delta + 2, \ldots, \infty\}$.

$$\Phi(v_\delta') = 2\gamma + 1. \tag{27}$$

and (see Table 29).

Hence,

$$\Phi^*\left(u_\gamma v_\gamma'\right) = 2\gamma. \tag{28}$$

and (see Table 30).

Figure 8 depicts classical mean labeling of G in the preceding circumstances. □

Theorem 6. *If a graph G is created by duplicating a vertex of another graph G', then G is a classical mean graph, where G' is the path C_n, for $n \ge 3$.*

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of the cycle C_n and let $v = v_1$ and its duplicated vertex is v_1'.

Case 1. $n \ge 5$ and $\Delta = 4 + n$.

Let us construct Φ: $V \longrightarrow N - \{\Delta + 1, \Delta + 2, \ldots, \infty\}$.

Table 28: Edge labeling of $\Phi^*(e)$.

$\Phi^*(e)$	$1 \leq \delta \leq \gamma - 1$	$\delta = \gamma$	$\gamma + 1 \leq \delta \leq n - 1$	$\gamma = n$
$\Phi^*(u_\delta u_{1+\delta})$	2δ	$2\delta + 2$	$2\delta + 3$	–
$\Phi^*(u_\delta v_\delta)$	$2\delta - 1$	2δ	$2\delta + 2$	$2n + 2$

Table 29: Vertex labeling of $\Phi(v)$.

$\Phi(v)$	$1 \leq \delta \leq \gamma$	$\delta + 1 \leq n$
$\Phi(u_\delta)$	2δ	$2\delta + 1$
$\Phi(v_\delta)$	$2\delta - 1$	2δ

Table 30: Edge labeling of $\Phi^*(e)$.

$\Phi^*(e)$	$1 \leq \delta \leq \gamma - 1$	$\delta = \gamma$	$\delta = \gamma + 1$	$\gamma + 1 \leq \delta \leq n - 1$	$\delta = n$
$\Phi^*(u_\delta u_{1+\delta})$	2δ	2δ	$2\delta + 1$	$2\delta + 1$	–
$\Phi^*(u_\delta v_\delta)$	$2\delta - 1$	$2\delta - 1$	2δ	2δ	2δ

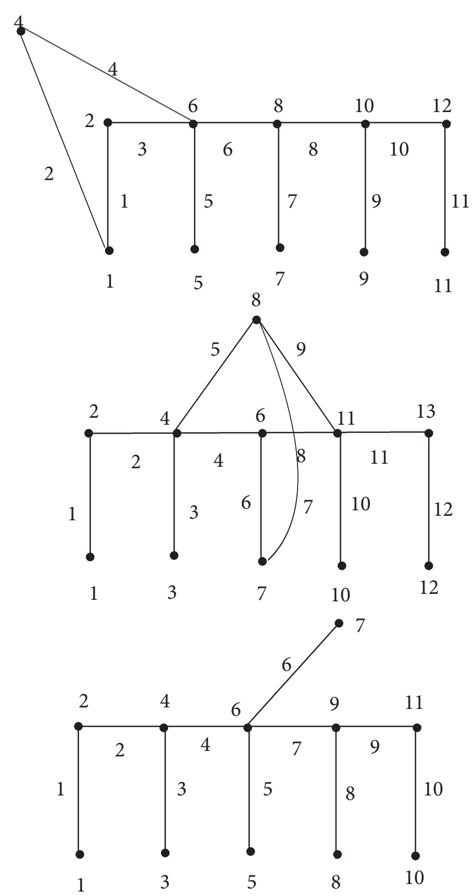

Figure 8: A classical mean labeling of G obtained by duplicating a vertex of the graph $P_n \circ K_1$.

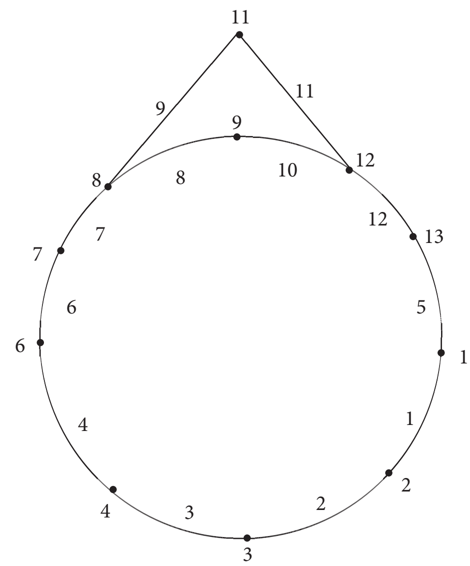

Figure 9: A classical mean labeling of G obtained by duplicating a vertex of cycle C_{10}.

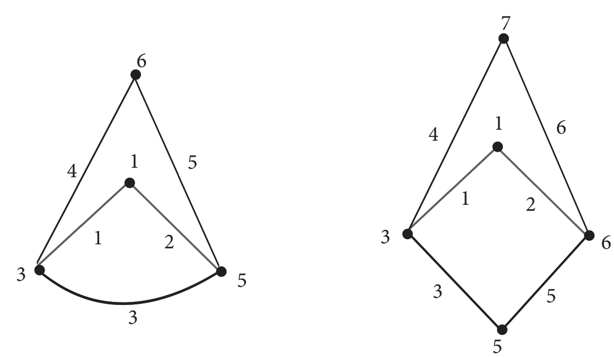

Figure 10: A classical mean labeling of the graph obtained by duplicating a vertex of a graph C_3 and C_4.

$$
\begin{aligned}
&\Phi(v_1') = n - 1, \quad \text{for } 2 \le \delta \le 3,\\
&\Phi(v_1) = n - 1,\\
&\Phi(v_\gamma) = n + 2\gamma - 2,\\
&\Phi(v_\gamma) = \begin{cases} \gamma - 3, & 4 \le \gamma \le \left\lfloor \dfrac{1}{4}\left(\dfrac{n^2 + 12n + 28}{2n + 8} + \sqrt{n + 3} + \sqrt{\dfrac{n^2 + 6n + 10}{2}}\right)\right\rfloor + 2,\\ \gamma - 2, & \left\lfloor \dfrac{1}{4}\left(\dfrac{n^2 + 12n + 28}{2n + 8} + \sqrt{n + 3} + \sqrt{\dfrac{n^2 + 6n + 10}{2}}\right)\right\rfloor + 3 \le \gamma \le n. \end{cases}
\end{aligned} \tag{29}
$$

Hence,

$$\begin{aligned}\Phi^*(v_\gamma v_{\gamma+1}) &= n+2\gamma-2, \quad \text{for } 1\le\delta\le 2,\\ \Phi^*(v_1v_n) &= n-2,\\ \Phi^*(v_1'v_2) &= n+1,\\ \Phi^*(v_1'v_n) &= n-1.\end{aligned} \tag{30}$$

Let us take $X = ((28+12n+n^2)/(8+2n))$, $Y=\sqrt{3+n}$, and $Z=\sqrt{((10+6n+n^2)/2)}$.

$$\Phi^*(v_3v_4) = \left\lfloor \frac{1}{4}(X+Y+Z)\right\rfloor,$$

$$\Phi^*(v_\gamma v_{\gamma+1}) = \begin{cases} -3+\gamma, & 4\le\gamma\le\left\lfloor \frac{1}{4}(X+Y+Z)\right\rfloor+2,\\ -2+\gamma, & \left\lfloor \frac{1}{4}(X+Y+Z)\right\rfloor+3\le\gamma\le n-1.\end{cases} \tag{31}$$

Figure 9 depicts classical mean labeling of G in the preceding circumstances.

Case 2. $n=3,4$.

Figure 10 depicts classical mean labeling of a graph for Case 2. □

Theorem 7. *The graph $S'(P_n)$ is a classical mean graph, for $n\ge 2$.*

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of the path P_n and $v_1, v_2, \ldots, v_n, v_1', v_2', \ldots, v_n'$ be the vertices of $S\prime(P_n)$.

Case 1. n is odd. Let us take $\Delta = -1+3n$.

Let us construct $\Phi\colon V \longrightarrow N-\{\Delta+1, \Delta+2, \ldots, \infty\}$ (see Table 31).

(see Table 32).

Hence (see Table 33).

(see Table 34).

(see Table 35).

Case 2. n is even and $n\ge 4$. Let us take $\Delta=-1+3n$.

Let us construct $\Phi\colon V \longrightarrow N-\{\Delta+1, \Delta+2, \ldots, \infty\}$ (see Table 36).

(see Table 37).

Hence (see Table 38).

(see Table 39).

(see Table 40).

Figure 11 depicts classical mean labeling of $S\prime(P_9)$ in the preceding circumstances.

The graph $S'(P_2)$ is P_4 when $n=2$ and it is also a classical mean labeling graph. □

Table 31: Vertex labeling of $\Phi(v)$.

$\Phi(v)$	$\gamma=1$	$\gamma=2$	$\gamma=3$
$\Phi(v_\gamma)$	$5\gamma-4$	$5\gamma-4$	5
$\Phi(v_\gamma')$	4	$4\gamma-5$	$4\gamma-5$

Table 32: Vertex labeling of $\Phi(v)$.

$\Phi(v)$	$2\le\gamma\le\lfloor n/2\rfloor$
$\Phi(v_{2\gamma})$	$6\gamma+1$
$\Phi(v_{2\gamma+1})$	6γ
$\Phi(v_{2\gamma}')$	$6\gamma-2$
$\Phi(v_{2\gamma+1}')$	$6\gamma-1$

Table 33: Edge labeling of $\Phi^*(e)$.

$\Phi^*(e)$	$\gamma=1$	$\gamma=2$	$\gamma=3$
$\Phi^*(v_\gamma v_{\gamma+1})$	$3\gamma-1$	$3\gamma-1$	$3\gamma-1$
$\Phi^*(v_\gamma v_{\gamma+1}')$	$5\gamma-4$	$5\gamma-4$	7
$\Phi^*(v_{\gamma+1}v_\gamma')$	4	$6\gamma-9$	$6\gamma-9$

Table 34: Edge labeling of $\Phi^*(e)$.

$\Phi^*(e)$	$2\le\gamma\le\lfloor n/2\rfloor$
$\Phi^*(v_{2\gamma}v_{2\gamma+1})$	6γ
$\Phi^*(v_{2\gamma}'v_{\gamma+1})$	$6\gamma-2$
$\Phi^*(v_{2\gamma}v_{2\gamma+1}')$	$6\gamma+2$

Table 35: Edge labeling of $\Phi^*(e)$.

$\Phi^*(e)$	$2\le\gamma\le\lfloor n/2\rfloor-1$
$\Phi^*(v_{2\gamma+1}v_{2\gamma+2})$	$6\gamma+3$
$\Phi^*(v_{2\gamma+1}'v_{\gamma+2})$	$6\gamma+2$
$\Phi^*(v_{2\gamma+2}v_{2\gamma+2}')$	$6\gamma+1$

Table 36: Vertex labeling of $\Phi(v)$.

$\Phi(v)$	$1\le\gamma\le 2$
$\Phi(v_\gamma)$	$2\gamma-1$
$\Phi(v_\gamma')$	$6-\gamma$

Table 37: Vertex labeling of $\Phi(v)$.

$\Phi(v)$	$2\le\gamma\le\lfloor n/2\rfloor$
$\Phi(v_{2\gamma-1})$	$6\gamma-2$
$\Phi(v_{2\gamma})$	$6\gamma-3$
$\Phi(v_{2\gamma-1}')$	$6\gamma-5$
$\Phi(v_{2\gamma}')$	$6\gamma-4$

Table 38: Edge labeling of $\Phi^*(e)$.

$\Phi^*(e)$	$1\le\gamma\le 2$
$\Phi^*(v_\gamma v_{\gamma+1})$	$4\gamma-3$
$\Phi^*(v_\gamma v_{\gamma+1}')$	2γ
$\Phi^*(v_\gamma' v_{\gamma+1})$	3γ

Table 39: Edge labeling of $\Phi^*(e)$.

$\Phi^*(e)$	$2 \le \gamma \le \lfloor n/2 \rfloor$
$\Phi^*(v_{2\gamma-1}v_{2\gamma})$	$6\gamma - 3$
$\Phi^*(v_{2\gamma-1}'v_{2\gamma})$	$6\gamma - 5$
$\Phi^*(v_{2\gamma-1}v_{2\gamma'})$	$6\gamma - 4$

Table 40: Edge labeling of $\Phi^*(e)$.

$\Phi^*(e)$	$2 \le \gamma \le \lfloor n/2 \rfloor - 1$
$\Phi^*(v_{2\gamma}v_{2\gamma+1})$	6γ
$\Phi^*(v_{2\gamma}'v_{2\gamma+1})$	$6\gamma - 1$
$\Phi^*(v_{2\gamma}v_{2\gamma+1'})$	$6\gamma - 2$

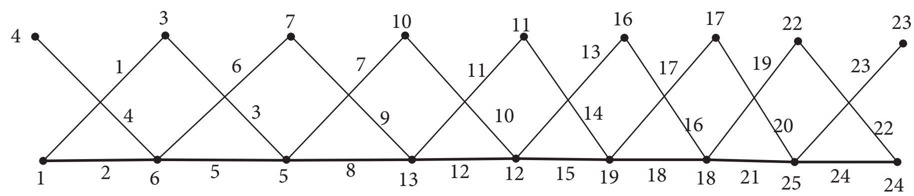

Figure 11: A classical mean labeling of $S'(P_9)$.

5. Conclusion

The classical meanness of some graphs generated from duplicating operations is addressed here, along with sufficient examples to aid comprehension. It is feasible to look into comparable outcomes for a variety of other graphs.

Data Availability

No data ware used in this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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