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Research Article

Multiattribute Group Decision-Making Method in terms of Linguistic Neutrosophic Z-Number Weighted Aggregation Operators

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To make a fuzzy value more reliable, Zadeh presented the notion of Z-number, which reflects a fuzzy value related to its reliability measure. Since linguistic expression conforms to human thinking habits, linguistic neutrosophic decision-making is one of the key research topics in linguistic indeterminate and inconsistent setting. In order to ensure the reliability of multiattribute group decision-making (MAGDM) problems in the linguistic environment of truth, falsehood, and indeterminacy, we require a new linguistic neutrosophic framework that combines the decision-maker's linguistic neutrosophic judgment with its reliability measure. Inspired by the linguistic Z-numbers of the truth, falsehood, and indeterminacy, this article first proposes a linguistic neutrosophic Z-number (LNZN) to make the truth, falsehood, and indeterminacy linguistic values more reliable. Then, we define the operational relations, score and accuracy functions, and sorting laws of LNZNs. Next, we establish the LNZN weighted arithmetic mean (LNZNWAM) and LNZN weighted geometric mean (LNZNWGM) operators and indicate their properties. Furthermore, an MAGDM approach is developed based on the two aggregation operators and the score and accuracy functions of LNZNs in the LNZN setting. Lastly, an MAGDM example of industrial robot selection and comparison with existing related methods are provided to verify the applicability and efficiency of the developed MAGDM method in the setting of LNZNs. In general, the developed MAGDM approach not only makes the MAGDM information more reliable but also solves MAGDM problems under the environment of LNZNs.

1. Introduction

Decision-making is a hotspot of current research problems. It is very significant to establish reasonable information representation and decision-making models. Linguistic representation may be more suitable for human thinking habits, especially reflecting its advantages in qualitative assessment of complex objective things. In this case, linguistic decision-making indicates its importance. Thus, linguistic multiattribute (group) decision-making (MADM/MAGDM) research has attracted the attention of many researchers in the past few decades. Since Zadeh [1] first introduced the concept of linguistic variables, various linguistic MADM/MAGDM methods have been utilized to solve various decision-making problems [2–5]. In terms of a membership/truth linguistic variable and a nonmembership/falsity linguistic variable, Chen et al. [6] proposed linguistic intuitionistic fuzzy numbers (LIFNs) and used them for MAGDM problems. Then, Yager [7] presented the ordinal LIFN aggregation operators, and Zhang et al. [8] used LIFNs to indicate the preferred and nonpreferred qualitative judgments of decision-makers in linguistic MADM problems. Next, some LIFN aggregation operators and their decision-making approaches [9–11] have been proposed and applied in MADM issues with LIFN information. Regarding the truth, falsehood, and indeterminacy linguistic variables, Fang and Ye [12] defined linguistic neutrosophic numbers (LNNs) and their operations; then, they presented the LNN weighted arithmetic and geometric mean operators and their MAGDM approach to solve MAGDM issues with LNN information. After that, various aggregation operators of LNNs and their MAGDM methods [12–16] have been applied in MAGDM issues with LNNs.

However, LIFN is a special case of LNN. LNN can describe indeterminacy and inconsistent linguistic information as its highlighting advantage, while LIFN cannot do it. More recently, according to the conceptual generalization of Z-numbers [18], Ding et al. [19] presented the linguistic Z-number QUALIFLEX (Qualitative Flexible Multiple Criteria) method for MAGDM. Next, Du et al. and Ye proposed neutrosophic Z-numbers (NZNs), their weighted arithmetic and geometric mean operators [20], and their similarity measures [21] and then applied them to MADM problems in the environment of NZNs. Yong et al. [22] presented trapezoidal neutrosophic Z-numbers and their weighted arithmetic and geometric mean operators for MADM issues with trapezoidal NZNs. Although NZN and trapezoidal NZN contain the information of the truth, falsehood, and indeterminacy Z-numbers, they cannot represent LNN information. Furthermore, existing LNN lacks the reliability measure of the truth, falsehood, and indeterminacy linguistic values, which shows its flaw. To make up for this flaw, we should introduce the reliability measure to the truth, falsehood, and indeterminacy linguistic values in LNN, propose a linguistic neutrosophic Z-number (LNZN) so as to strengthen the reliability of LNN, and then present some operations and sorting laws of LNZNs to solve MAGDM issues with the information of LNZNs. Therefore, this study aims (a) to propose LNZNs and their operational relations, (b) to define the score and accuracy functions and sorting laws of LNZNs, (c) to establish LNZN weighted arithmetic mean (LNZNWAM) and LNZN weighted geometric mean (LNZNWGM) operators, (d) to develop an MAGDM approach by using the LNZNWAM and LNZNWGM operators and score and accuracy functions of LNZNs, and (e) to apply the developed MAGDM approach to an MAGDM problem of industrial robot selection in the environment of LNZNs.

Generally, the critical contributions of this original study are summarized as follows:

(a) The new notion of LNZN proposed in terms of linguistic Z-numbers of the truth, falsehood, and indeterminacy can make the linguistic values more reliable

(b) The defined operational relations, score and accuracy functions, and sorting laws of LNZNs and the proposed LNZNWAM and LNZNWGM operators provide the necessary mathematical tools for modeling of MAGDM issues in the setting of LNZNs

(c) The proposed MAGDM approach can solve MAGDM issues with LNZNs

(d) The proposed MAGDM method can efficiently handle the MAGDM problem of industrial robot selection in the LNZN setting and show its usability

The rest of this study is composed of the following structures: in Section 2, some basic concepts of LNNs are reviewed as preliminaries of this study. Section 3 proposes LNZNs and their operational relations, score and accuracy functions, and sorting laws. Section 4 develops the LNZNWAM and LNZNWGM operators and indicates their properties. In Section 5, an MAGDM approach is developed using the LNZNWAM and LNZNWGM operators and score and accuracy functions to carry out MAGDM issues in the LNZN environment. Section 6 applies the developed MAGDM approach to an MAGDM example of industrial robot selection for a manufacturing company under the environment of LNZNs and then presents a comparison with existing related MAGDM methods to reflect the efficiency of the developed approach. Lastly, conclusions and future research are indicated in Section 7.

2. Preliminaries of LNNs

Set $U = \{\delta_0, \delta_1, \ldots, \delta_v\}$ as a linguistic term set (LTS) with odd cardinality $v + 1$. Fang and Ye [12] first defined the LNN $le = <\delta_T, \delta_I, \delta_F>$ on U such that $\delta_T, \delta_I, \delta_F \in U$ and T, I, F $\in [0, v]$, where δ_T, δ_I, and δ_F express the truth, indeterminacy, and falsehood linguistic variables, respectively.

Regarding two LNNs $le_1 = \left\langle \delta_{T(1)}, \delta_{I(1)}, \delta_{F(1)} \right\rangle$ and $le_2 = \left\langle \delta_{T(2)}, \delta_{I(2)}, \delta_{F(2)} \right\rangle$ on U and any real number $p > 0$, the following operational relations [12] are defined as follows:

(i) $le_1 \oplus le_2 = \left\langle \delta_{T(1)}, \delta_{I(1)}, \delta_{F(1)} \right\rangle \oplus \left\langle \delta_{T(2)}, \delta_{I(2)}, \delta_{F(2)} \right\rangle = \left\langle \delta_{T(1)+T(2)-T(1)T(2)/v}, \delta_{I(1)I(2)/v}, \delta_{F(1)F(2)/v} \right\rangle$

(ii) $le_1 \otimes le_2 = \left\langle \delta_{T(1)}, \delta_{I(1)}, \delta_{F(1)} \right\rangle \otimes \left\langle \delta_{T(2)}, \delta_{I(2)}, \delta_{F(2)} \right\rangle = \left\langle \delta_{T(1)T(2)/v}, \delta_{I(1)+I(2)-I(1)I(2)/v}, \delta_{F(1)+F(2)-F(1)F(2)/v} \right\rangle$

(iii) $p \cdot le_1 = p \cdot \left\langle \delta_{T(1)}, \delta_{I(1)}, \delta_{F(1)} \right\rangle = \left\langle \delta_{v-v(1-T(1)/v)^p}, \delta_{v(I(1)/v)^p}, \delta_{v(F(1)/v)^p} \right\rangle$

(iv) $le_1^p = \left\langle \delta_{T(1)}, \delta_{I(1)}, \delta_{F(1)} \right\rangle^p = \left\langle \delta_{v(T(1)/v)^p}, \delta_{v-v(1-I(1)/v)^p}, \delta_{v-v(1-F(1)/v)^p} \right\rangle$

Regarding a series of LNNs $le_k = <\delta_{T(k)}, \delta_{I(k)}, \delta_{F(k)}>$ with their weights p_k ($k = 1, 2, \ldots, n$) for $p_k \in [0, 1]$ and $\sum_{k=1}^{n} p_k = 1$, the LNN weighted arithmetic mean (LNNWAM) and LNN weighted geometric mean (LNNWGM) operators [12] are proposed as follows:

$$\mathrm{LNNWAM}(le_1, le_2, \ldots, le_n) = \sum_{k=1}^{n} p_k \cdot le_k = \left\langle \delta_{v-v\prod_{k=1}^{n}(1-T(k)/v)^{p_k}}, \delta_{v\prod_{k=1}^{n}(I(k)/v)^{p_k}}, \delta_{v\prod_{k=1}^{n}(F(k)/v)^{p_k}} \right\rangle, \quad (1)$$

$$\mathrm{LNNWGM}(le_1, le_2, \ldots, le_n) = \prod_{k=1}^{n} le_k^{p_k} = \left\langle \delta_{v\prod_{k=1}^{n}(T(k)/v)^{p_k}}, \delta_{v-v\prod_{k=1}^{n}(1-I(k)/v)^{p_k}}, \delta_{v-v\prod_{k=1}^{n}(1-F(k)/v)^{p_k}} \right\rangle. \quad (2)$$

Then, Fang and Ye [12] defined the score and accuracy functions of $le_k = <\delta_{T(k)}, \delta_{I(k)}, \delta_{F(k)}>$:

$$D(le_k) = \frac{(2v + T(k) - I(k) - F(k))}{(3le_k)}, \quad \text{for } D(le_k) \in [0,1], \tag{3}$$

$$E(le_k) = \frac{(T(k) - F(k))}{le_k}, \quad \text{for } E(le_k) \in [-1,1]. \tag{4}$$

Regarding two LNNs, $le_1 = \langle \delta_{T(1)}, \delta_{I(1)}, \delta_{F(1)} \rangle$ and $le_2 = \langle \delta_{T(2)}, \delta_{I(2)}, \delta_{F(2)} \rangle$, their sorting laws [12] are defined as follows:

(i) $le_1 \succ le_2$ if $D(le_1) > D(le_2)$

(ii) $le_1 \succ le_2$ if $D(le_1) = D(le_2)$ and $E(le_1) > E(le_2)$

(iii) $le_1 = le_2$ if $D(le_1) = D(le_2)$ and $E(le_1) = E(le_2)$

3. LNZNs

To make LNN more reliable, this section proposes LNZNs in terms of the truth, falsehood, and indeterminacy Z-numbers and then defines the operational relations, score and accuracy functions, and sorting laws of LNZNs.

Definition 1. Set $U = \{\delta_0, \delta_1, \ldots, \delta v\}$ as LTS with odd cardinality $v+1$. Then, LNZN on U is defined as $lz = <(\delta_{RT}, \delta_{MT}), (\delta_{RI}, \delta_{MI}), (\delta_{RF}, \delta_{MF})>$ for $\delta_{RT}, \delta_{RI}, \delta_{RF}, \delta_{MT}, \delta_{MI}, \delta_{MF} \in U$ and $RT, RI, RF, MT, MI, MF \in [0, v]$, where $(\delta_{RT}, \delta_{MT})$ is the truth linguistic Z-number that combines the truth linguistic term value δ_{RT} with the linguistic reliability measure δ_{MT} for δ_{RT} specified from the LTS U; $(\delta_{RI}, \delta_{MI})$ is the indeterminacy linguistic Z-number that combines the indeterminacy linguistic term value δ_{RI} with the linguistic reliability measure δ_{MI} for δ_{RI} specified from the LTS U; $(\delta_{RF}, \delta_{MF})$ is the falsehood linguistic Z-number that combines the falsity linguistic term value δ_{RF} with the linguistic reliability measure δ_{MF} for δ_{RF} specified from the LTS U.

Definition 2. Let two LNZNs be $lz_k = <(\delta_{RT(k)}, \delta_{MT(k)}), (\delta_{RI(k)}, \delta_{MI(k)}), (\delta_{RF(k)}, \delta_{MF(k)})>$ $(k = 1, 2)$ on U and any real number $p > 0$. Then, their operational relations are defined as follows:

(i) $lz_1 \oplus lz_2 = \langle(\delta_{RT(1)}, \delta_{MT(1)}), (\delta_{RI(1)}, \delta_{MI(1)}), (\delta_{RF(1)}, \delta_{MF(1)})\rangle \oplus \langle(\delta_{RT(2)}, \delta_{MT(2)}), (\delta_{RI(2)}, \delta_{MI(2)}), (\delta_{RF(2)}, \delta_{MF(2)})\rangle$
$= \langle(\delta_{RT(1)+RT(2)-RT(1)RT(2)/v}, \delta_{MT(1)+MT(2)-MT(1)MT(2)/v}), (\delta_{RI(1)RI(2)/v}, \delta_{MI(1)MI(2)/v}), (\delta_{RF(1)RF(2)/v}, \delta_{MF(1)MF(2)/v})\rangle$

(ii) $lz_1 \otimes lz_2 = \langle(\delta_{RT(1)}, \delta_{MT(1)}), (\delta_{RI(1)}, \delta_{MI(1)}), (\delta_{RF(1)}, \delta_{MF(1)})\rangle \otimes \langle(\delta_{RT(2)}, \delta_{MT(2)}), (\delta_{RI(2)}, \delta_{MI(2)}), (\delta_{RF(2)}, \delta_{MF(2)})\rangle$
$= \langle(\delta_{RT(1)RT(2)/v}, \delta_{MT(1)MT(2)/v}), (\delta_{RI(1)+RI(2)-RI(1)RI(2)/v}, \delta_{MI(1)+MI(2)-MI(1)MI(2)/v}), (\delta_{RF(1)+RF(2)-RF(1)RF(2)/v}, \delta_{MF(1)+MF(2)-MF(1)MF(2)/v})\rangle$

(iiii) $p \cdot lz_1 = p\langle(\delta_{RT(1)}, \delta_{MT(1)}), (\delta_{RI(1)}, \delta_{MI(1)}), (\delta_{RF(1)}, \delta_{MF(1)})\rangle$
$= \langle(\delta_{v-v(1-RT(1)/v)^p}, \delta_{v-v(1-MT(1)/v)^p}), (\delta_{v(RI(1)/v)^p}, \delta_{v(MI(1)/v)^p}), (\delta_{v(RF(1)/v)^p}, \delta_{v(MF(1)/v)^p})\rangle$

(iv) $lz_1^p = \langle(\delta_{RT(1)}, \delta_{MT(1)}), (\delta_{RI(1)}, \delta_{MI(1)}), (\delta_{RF(1)}, \delta_{MF(1)})\rangle^p$
$= \langle(\delta_{v(RT(1)/v)^p}, \delta_{v(MT(1)/v)^p}), (\delta_{v-v(1-RI(1)/v)^p}, \delta_{v-v(1-MI(1)/v)^p}), (\delta_{v-v(1-RF(1)/v)^p}, \delta_{v-v(1-MF(1)/v)^p})\rangle$

Clearly, the above operational results are still LNZNs.

Example 1. Set two LNZNs as $lz_1 = <(\delta_6, \delta_7), (\delta_2, \delta_6), (\delta_3, \delta_7)>$ and $lz_2 = <(\delta_7, \delta_7), (\delta_2, \delta_5), (\delta_3, \delta_6)>$ on $U = \{\delta_0, \delta_1, \ldots, \delta_8\}$ with $v = 8$ and $p = 0.6$. Then, based on the above operational relations, one obtains their operational results:

(i) $lz_1 \oplus lz_2 = \langle(\delta_{RT(1)+RT(2)-RT(1)RT(2)/v}, \delta_{MT(1)+MT(2)-MT(1)MT(2)/v}), (\delta_{RI(1)RI(2)/v}, \delta_{MI(1)MI(2)/v}), (\delta_{RF(1)RF(2)/v}, \delta_{MF(1)MF(2)/v})\rangle$
$= \langle(\delta_{6+7-6\times7/8}, \delta_{7+7-7\times7/8}), (\delta_{2\times2/8}, \delta_{6\times5/8}), (\delta_{3\times3/8}, \delta_{7\times6/8})\rangle = \langle(\delta_{7.75}, \delta_{7.875}), (\delta_{0.5}, \delta_{3.75}), (\delta_{1.125}, \delta_{5.25})\rangle$

(ii) $lz_1 \otimes lz_2 = \left\langle \begin{matrix}(\delta_{RT(1)RT(2)/v}, \delta_{MT(1)MT(2)/v}), (\delta_{RI(1)+RI(2)-RI(1)RI(2)/v}, \delta_{MI(1)+MI(2)-MI(1)MI(2)/v}),\\ (\delta_{RF(1)+RF(2)-RF(1)RF(2)/v}, \delta_{MF(1)+MF(2)-MF(1)MF(2)/v})\end{matrix} \right\rangle$
$= \langle(\delta_{6\times7/8}, \delta_{7\times7/8}), (\delta_{2+2-2\times2/8}, \delta_{6+5-6\times5/8}), (\delta_{3+3-3\times3/8}, \delta_{7+6-7\times6/8})\rangle = \langle(\delta_{5.25}, \delta_{6.125}), (\delta_{3.5}, \delta_{7.25}), (\delta_{4.875}, \delta_{7.75})\rangle$

(iiii) $p \cdot lz_1 = \langle(\delta_{v-v(1-RT(1)/v)^p}, \delta_{v-v(1-MT(1)/v)^p}), (\delta_{v(RI(1)/v)^p}, \delta_{v(MI(1)/v)^p}), (\delta_{v(RF(1)/v)^p}, \delta_{v(MF(1)/v)^p})\rangle$
$= \langle(\delta_{8-8(1-6/8)^{0.6}}, \delta_{8-8(1-7/8)^{0.6}}), (\delta_{8(2/8)^{0.6}}, \delta_{8(6/8)^{0.6}}), (\delta_{8(3/8)^{0.6}}, \delta_{8(7/8)^{0.6}})\rangle$
$= \langle(\delta_{4.5178}, \delta_{5.7026}), (\delta_{3.4822}, \delta_{6.7317}), (\delta_{4.4413}, \delta_{7.3841})\rangle$

(iv) $lz_1^p = \langle(\delta_{v(RT(1)/v)^p}, \delta_{v(MT(1)/v)^p}), (\delta_{v-v(1-RI(1)/v)^p}, \delta_{v-v(1-MI(1)/v)^p}), (\delta_{v-v(1-RF(1)/v)^p}, \delta_{v-v(1-MF(1)/v)^p})\rangle$
$= \langle(\delta_{8(6/8)^{0.6}}, \delta_{8(7/8)^{0.6}}), (\delta_{8-8(1-2/8)^{0.6}}, \delta_{8-8(1-6/8)^p}), (\delta_{8-8(1-3/8)^{0.6}}, \delta_{8-8(1-7/8)^{0.6}})\rangle$
$= \langle(\delta_{6.7317}, \delta_{7.3841}), (\delta_{1.2683}, \delta_{4.5178}), (\delta_{1.9658}, \delta_{5.7026})\rangle$

To compare LNZNs, we can present the score and accuracy functions and sorting laws of LNZNs.

Definition 3. Set LNZN as $lz = <(\delta_{RT}, \delta_{MT}), (\delta_{RI}, \delta_{MI}), (\delta_{RF}, \delta_{MF})>$ on U. Then, the score and accuracy functions of lz are presented as follows:

$$Y(lz) = \frac{(2v^2 + RT \times MT - RI \times MI - RF \times MF)}{(3v^2)}, \quad \text{for } Y(lz) \in [0,1], \tag{5}$$

$$Z(lz) = \frac{(RT \times MT - RF \times MF)}{v^2}, \quad \text{for } Z(lz) \in [-1,1]. \tag{6}$$

Definition 4. Set two LNZNs as $lz_k = <(\delta_{RT(k)}, \delta_{MT(k)}), (\delta_{RI(k)}, \delta_{MI(k)}), (\delta_{RF(k)}, \delta_{MF(k)})>$ for $k = 1, 2$ on U. Then, their sorting laws are defined as follows:

(i) $lz_1 \succ lz_2$ if $Y(lz_1) > Y(lz_2)$

(ii) $lz_1 \succ lz_2$ if $Y(lz_1) = Y(lz_2)$ and $Z(lz_1) > Z(lz_2)$

(iii) $lz_1 = lz_2$ If $Y(lz_1) = Y(lz_2)$ and $Z(lz_1) = Z(lz_2)$

Example 2. Set three LNZNs as $lz_1 = <(\delta_6, \delta_6), (\delta_3, \delta_7), (\delta_3, \delta_6)>$, $lz_2 = <(\delta_6, \delta_6), (\delta_3, \delta_6), (\delta_3, \delta_7)>$, and $lz_3 = <(\delta_7, \delta_6), (\delta_3, \delta_7), (\delta_3, \delta_5)>$ on $U = \{\delta_0, \delta_1, \ldots, \delta_8\}$ with $v = 8$. Then, by equations (5) and (6), the values of their score and accuracy functions are yielded as follows: $Y(lz_1) = (2 \times 8^2 + 6 \times 6 - 3 \times 7 - 3 \times 6)/192 = 0.651$, $Y(lz_2) = (2 \times 8^2 + 6 \times 6 - 3 \times 6 - 3 \times 7)/192 = 0.651$, and $Y(lz_3) = (2 \times 8^2 + 7 \times 6 - 3 \times 7 - 3 \times 5)/192 = 0.6979$; $Z(lz_1) = (6 \times 6 - 3 \times 6)/64 = 0.2813$ and $Z(lz_2) = (6 \times 6 - 3 \times 7)/64 = 0.2344$.

According to the sorting laws in Definition 4, their sorting order is $lz_3 \succ lz_1 \succ lz_2$.

4. LNZNWAM and LNZNWGM Operators

4.1. LNZNWAM Operator

Definition 5. Set $lz_k = <(\delta_{RT(k)}, \delta_{MT(k)}), (\delta_{RI(k)}, \delta_{MI(k)}), (\delta_{RF(k)}, \delta_{MF(k)})>$ $(k = 1, 2, \ldots, n)$ as a series of LNZNs on U. Then, the LNZNWAM operator can be defined as

$$\text{LNZNWAM}(lz_1, lz_2, \ldots, lz_n) = \sum_{k=1}^{n} p_k \cdot lz_k, \tag{7}$$

where p_k is the weight of lz_k $(k = 1, 2, \ldots, n)$ for $p_k \in [0, 1]$ and $\sum_{k=1}^{n} p_k = 1$.

Thus, the following theorem can be presented corresponding to the operational relations in Definition 2 and equation (7).

Theorem 1. *Set* $lz_k = <(\delta_{RT(k)}, \delta_{MT(k)}), (\delta_{RI(k)}, \delta_{MI(k)}), (\delta_{RF(k)}, \delta_{MF(k)})>$ $(k = 1, 2, \ldots, n)$ *as a series of LNZNs on U. Then, the aggregation result yielded by equation (7) is also LNZN, which is calculated by the following equation:*

$$
\begin{aligned}
\mathrm{LNZNWAM}\left(lz_1, lz_2, \ldots, lz_n\right) &= \sum_{k=1}^{n} p_k \cdot lz_k \\
&= \left\langle \left(\delta_{v - v \prod_{k=1}^{n} \left(1 - RT(k)/v\right)^{p_k}}, \delta_{v - v \prod_{k=1}^{n} \left(1 - MT(k)/v\right)^{p_k}} \right), \right. \\
&\quad \left. \cdot \left(\delta_{v \prod_{k=1}^{n} \left(RI(k)/v\right)^{p_k}}, \delta_{v \prod_{k=1}^{n} \left(MI(k)/v\right)^{p_k}} \right), \left(\delta_{v \prod_{k=1}^{n} \left(RF(k)/v\right)^{p_k}}, \delta_{v \prod_{k=1}^{n} \left(MF(k)/v\right)^{p_k}} \right) \right\rangle,
\end{aligned}
\tag{8}
$$

where p_k *is the weight of* lz_k $(k = 1, 2, \ldots, n)$ *for* $p_k \in [0, 1]$ *and* $\sum_{k=1}^{n} p_k = 1$.

In the following, Theorem 1 can be verified by means of mathematical induction.

Proof

(1) Set $n = 2$. By the operational relations in Definition 2, there is the following result:

$$
\begin{aligned}
\mathrm{LNZNWAM}\left(lz_1, lz_2\right) &= p_1 \cdot lz_1 \oplus p_2 \cdot lz_2 \\
&= \left\langle \begin{pmatrix} \delta_{v - v(1 - RT(1)/v)^{p_1} + v - v(1 - RT(2)/v)^{p_2} - \left(v - v(1 - RT(1)/q)^{p_1}\right)\left(v - v(1 - RT(2)/v)^{p_2}\right)/v}, \\ \delta_{v - v(1 - MT(1)/v)^{p_1} + v - v(1 - MT(2)/v)^{p_2} - \left(v - v(1 - MT(1)/v)^{p_1}\right)\left(v - v(1 - MT(2)/v)^{p_2}\right)/v} \end{pmatrix}, \right. \\
&\quad \left. \left(\delta_{v(RI(1)/v)^{p_1}(RI(2)/v)^{p_2}}, \delta_{v(MI(1)/v)^{p_1}(MI(2)/v)^{p_2}} \right), \left(\delta_{v(RF(1)/v)^{p_1}(RF(2)/v)^{p_2}}, \delta_{v(MF(1)/v)^{p_1}(MF(2)/v)^{p_2}} \right) \right\rangle \\
&= \left\langle \begin{pmatrix} \delta_{v - v(1 - RT(1)/v)^{p_1} + v - v(1 - RT(2)/v)^{p_2} - \left(v - v(1 - RT(1)/v)^{p_1} - v(1 - RT(2)/v)^{p_2} + v(1 - RT(1)/v)^{w_1}(1 - RT(2)/v)^{p_2}\right)}, \\ \delta_{v - v(1 - MT(1)/v)^{p_1} + v - v(1 - MT(2)/v)^{p_2} - \left(v - v(1 - MT(1)/v)^{p_1} - v(1 - MT(2)/v)^{p_2} + v(1 - MT(1)/v)^{w_1}(1 - MT(2)/v)^{p_2}\right)} \end{pmatrix}, \right. \\
&\quad \left. \left(\delta_{v(RI(1)/v)^{p_1}(RI(2)/v)^{p_2}}, \delta_{v(MI(1)/v)^{p_1}(MI(2)/v)^{p_2}} \right), \left(\delta_{v(RF(1)/v)^{p_1}(RF(2)/v)^{p_2}}, \delta_{v(MF(1)/v)^{p_1}(MF(2)/v)^{p_2}} \right) \right\rangle \\
&= \left\langle \left(\delta_{v - v(1 - RT(1)/v)^{p_1}(1 - RT(2)/v)^{p_2}}, \delta_{v - v(1 - MT(1)/v)^{p_1}(1 - MT(2)/v)^{p_2}} \right), \left(\delta_{v(RI(1)/v)^{p_1}(RI(2)/v)^{p_2}}, \delta_{v(MI(1)/v)^{p_1}(MI(2)/v)^{p_2}} \right), \right. \\
&\quad \left. \left(\delta_{v(RF(1)/v)^{p_1}(RF(2)/v)^{p_2}}, \delta_{v(MF(1)/v)^{p_1}(MF(2)/v)^{p_2}} \right) \right\rangle \\
&= \left\langle \left(\delta_{v - v \prod_{k=1}^{2} \left(1 - RT(k)/v\right)^{p_k}}, \delta_{v - v \prod_{k=1}^{2} \left(1 - MT(k)/v\right)^{p_k}} \right), \left(\delta_{v \prod_{k=1}^{2} \left(RI(k)/v\right)^{p_k}}, \delta_{v \prod_{k=1}^{2} \left(MI(k)/v\right)^{p_k}} \right), \right. \\
&\quad \left. \left(\delta_{v \prod_{k=1}^{2} \left(RF(k)/v\right)^{p_k}}, \delta_{v \prod_{k=1}^{2} \left(MF(k)/v\right)^{p_k}} \right) \right\rangle.
\end{aligned}
\tag{9}
$$

(2) Set $n = m$. Equation (8) can keep the following result:

$$
\begin{aligned}
\mathrm{LNZNWAM}\left(lz_1, lz_2, \ldots, lz_m\right) &= \sum_{k=1}^{m} p_k \cdot lz_k \\
&= \left\langle \left(\delta_{v-v\prod_{k=1}^{m}(1-RT(k)/v)^{p_k}}, \delta_{v-v\prod_{k=1}^{m}(1-MT(k)/v)^{p_k}} \right), \right. \\
&\quad \left. \cdot \left(\delta_{v\prod_{k=1}^{m}(RI(k)/v)^{p_k}}, \delta_{v\prod_{k=1}^{m}(MI(k)/v)^{p_k}} \right), \left(\delta_{v\prod_{k=1}^{m}(RF(k)/v)^{p_k}}, \delta_{v\prod_{k=1}^{m}(MF(k)/v)^{p_k}} \right) \right\rangle.
\end{aligned} \tag{10}
$$

(3) Set $n = m + 1$. By equations (9) and (10), the operational result is given as follows:

$$
\begin{aligned}
&\mathrm{LNZNWAM}\left(lz_1, lz_2, \ldots, lz_m, lz_{m+1}\right) = \sum_{k=1}^{m} p_k \cdot lz_k \oplus p_{m+1} \cdot lz_{m+1} \\
&= \left\langle \begin{matrix} \left(\begin{matrix} \delta_{v-v\prod_{k=1}^{m}(1-RT(k)/v)^{p_k}+v-v(1-RT(m+1)/v)^{p_{m+1}}-\left(v-v\prod_{k=1}^{m}(1-RT(k)/v)^{p_k}\right)(v-v(1-RT(m+1)/v)^{p_{m+1}})/v}, \\ \delta_{v-v\prod_{k=1}^{m}(1-MT(k)/v)^{p_k}+v-v(1-MT(m+1)/v)^{p_{m+1}}-\left(v-v\prod_{k=1}^{m}(1-MT(k)/v)^{p_k}\right)(v-v(1-MT(m+1)/v)^{p_{m+1}})/v} \end{matrix} \right), \\ \left(\delta_{v\prod_{k=1}^{m}(RI(k)/v)^{p_k}(RI(m+1)/v)^{p_{m+1}}}, \delta_{v\prod_{k=1}^{m}(MI(k)/v)^{p_k}(MI(m+1)/v)^{p_{m+1}}} \right), \left(\delta_{v\prod_{k=1}^{m}(RF(k)/v)^{p_k}(RF(m+1)/v)^{p_{m+1}}}, \delta_{v\prod_{k=1}^{m}(MF(k)/v)^{p_k}(MF(m+1)/v)^{p_{m+1}}} \right) \end{matrix} \right\rangle \\
&= \left\langle \begin{matrix} \left(\begin{matrix} \delta_{v-v\prod_{k=1}^{m}(1-RT(k)/v)^{p_k}+v-v(1-RT(m+1)/v)^{p_{m+1}}-\left(v-v\prod_{k=1}^{m}(1-RT(k)/v)^{p_k}-v(1-RT(m+1)/v)^{p_{m+1}}+v\prod_{k=1}^{m}(1-RT(k)/v)^{p_k}(1-RT(m+1)/v)^{p_{m+1}}\right)}, \\ \delta_{v-v\prod_{k=1}^{m}(1-MT(k)/v)^{p_k}+v-v(1-MT(m+1)/v)^{p_{m+1}}-\left(v-v\prod_{k=1}^{m}(1-MT(k)/v)^{p_k}-v(1-MT(m+1)/v)^{p_{m+1}}+v\prod_{k=1}^{m}(1-MT(k)/v)^{p_k}(1-MT(m+1)/v)^{p_{m+1}}\right)} \end{matrix} \right), \\ \left(\delta_{v\prod_{k=1}^{m}(RI(k)/v)^{p_k}(RI(m+1)/v)^{p_{m+1}}}, \delta_{v\prod_{k=1}^{m}(MI(k)/v)^{p_k}(RI(m+1)/v)^{p_{m+1}}} \right), \left(\delta_{v\prod_{k=1}^{m}(RF(k)/v)^{p_k}(RF(m+1)/v)^{p_{m+1}}}, \delta_{v\prod_{k=1}^{m}(MF(k)/v)^{p_k}(MF(m+1)/v)^{p_{m+1}}} \right) \end{matrix} \right\rangle \\
&= \left\langle \begin{matrix} \left(\delta_{q-q\prod_{k=1}^{m}(1-RT(k)/q)^{p_k}(1-RT(m+1)/q)^{p_{m+1}}}, \delta_{q-q\prod_{k=1}^{m}(1-MT(k)/q)^{p_k}(1-MT(m+1)/q)^{p_{m+1}}} \right), \\ \left(\delta_{v\prod_{k=1}^{m}(RI(k)/v)^{p_k}(RI(m+1)/v)^{p_{m+1}}}, \delta_{v\prod_{k=1}^{m}(MI(k)/v)^{p_k}(MI(m+1)/v)^{p_{m+1}}} \right), \left(\delta_{v\prod_{k=1}^{m}(RF(k)/v)^{p_k}(RF(m+1)/v)^{p_{m+1}}}, \delta_{v\prod_{k=1}^{m}(MF(k)/v)^{p_k}(MF(m+1)/v)^{p_{m+1}}} \right) \end{matrix} \right\rangle \\
&= \left\langle \left(\delta_{v-v\prod_{k=1}^{m+1}(1-RT(k)/v)^{p_k}}, \delta_{v-v\prod_{k=1}^{m+1}(1-MT(k)/v)^{p_k}} \right), \left(\delta_{v\prod_{k=1}^{m+1}(RI(k)/v)^{p_k}}, \delta_{v\prod_{k=1}^{m+1}(MI(k)/v)^{p_k}} \right), \left(\delta_{v\prod_{k=1}^{m+1}(RF(k)/v)^{p_k}}, \delta_{v\prod_{k=1}^{m+1}(MF(k)/v)^{p_k}} \right) \right\rangle.
\end{aligned} \tag{11}
$$

Based on the above results, equation (8) holds for any n. Thus, the proof is completed.

Then, the LNZNWAM operator implies the following properties:

(i) Idempotency: set lz_k $(k = 1, 2, \ldots, n)$ as a series of LNZNs on U. If $lz_k = lz$ for $k = 1, 2, \ldots, n$, then $\mathrm{LNZNWAM}\left(lz_1, lz_2, \ldots, lz_n\right) = lz$.

(ii) Boundedness: set lz_k $(k = 1, 2, \ldots, n)$ as a series of LNZNs on U and then set the minimum and maximum LNZNs as

$$
\begin{aligned}
lz^{-} &= \left\langle \left(\min_k\left(\delta_{RT(k)}\right), \min_k\left(\delta_{MT(k)}\right) \right), \left(\max_k\left(\delta_{RI(k)}\right), \max_k\left(\delta_{MI(k)}\right) \right), \left(\max_k\left(\delta_{RF(k)}\right), \max_k\left(\delta_{MF(k)}\right) \right) \right\rangle, \\
lz^{+} &= \left\langle \left(\max_k\left(\delta_{RT(k)}\right), \max_k\left(\delta_{MT(k)}\right) \right), \left(\min_k\left(\delta_{RI(k)}\right), \min_k\left(\delta_{MI(k)}\right) \right), \left(\min_k\left(\delta_{RF(k)}\right), \min_k\left(\delta_{MF(k)}\right) \right) \right\rangle,
\end{aligned} \tag{12}
$$

and then there is $lz^- \leq \mathrm{LNZNWAM}(lz_1, lz_2, \ldots, lz_n) \leq lz^+$.

(iii) Monotonicity: set lz_k ($k = 1, 2, \ldots, n$) as a series of LNZNs on U. If $lz_k \leq lz_k^*$ for $k = 1, 2, \ldots, n$, there is $\mathrm{LNZNWAM}(lz_1, lz_2, \ldots, lz_n) \leq \mathrm{LNZNWAM}(lz_1^*, lz_2^*, \ldots, lz_n^*)$.

(iv) Commutativity: set the LNZN sequence $(lz_1', lz_2', \ldots, lz_n')$ as an arbitrary permutation of $(lz_1, lz_2, \ldots, lz_n)$. Then, there is $\mathrm{LNZNWAM}(lz_1', lz_2', \ldots, lz_n') = \mathrm{LNZNWAM}(lz_1, lz_2, \ldots, lz_n)$. □

Proof

(i) Because $lz_k = lz$ for $k = 1, 2, \ldots, n$, there is the following operational result:

$$
\begin{aligned}
\mathrm{LNZNWAM}(lz_1, lz_2, \ldots, lz_n) &= \sum_{k=1}^{n} p_k \cdot lz_k \\
&= \left\langle \left(\delta_{v - v\prod_{k=1}^{n}(1 - RT(k)/v)^{p_k}}, \delta_{v - v\prod_{k=1}^{n}(1 - MT(k)/v)^{p_k}} \right), \right. \\
&\quad \left. \cdot \left(\delta_{v\prod_{k=1}^{n}(RI(k)/v)^{p_k}}, \delta_{v\prod_{k=1}^{n}(MI(k)/v)^{p_k}} \right), \left(\delta_{v\prod_{k=1}^{n}(RF(k)/v)^{p_k}}, \delta_{v\prod_{k=1}^{n}(MF(k)/v)^{p_k}} \right) \right\rangle \\
&= \left\langle \left(\delta_{v - v(1 - RT/v)^{\sum_{k=1}^{n} p_k}}, \delta_{v - v(1 - RT/v)^{\sum_{k=1}^{n} p_k}} \right), \left(\delta_{v(RI/v)^{\sum_{k=1}^{n} p_k}}, \delta_{v(RI/v)^{\sum_{k=1}^{n} p_k}} \right), \right. \\
&\quad \left. \cdot \left(\delta_{v(RF/v)^{\sum_{k=1}^{n} p_k}}, \delta_{v(RF/v)^{\sum_{k=1}^{n} p_k}} \right) \right\rangle \\
&= \left\langle \left(\delta_{v - v(1 - RT/v)}, \delta_{v - v(1 - MT/v)} \right), \left(\delta_{v(RI/v)}, \delta_{v(MI/v)} \right), \left(\delta_{v(RF/v)}, \delta_{v(MF/v)} \right) \right\rangle \\
&= \left\langle \left(\delta_{RT}, \delta_{MT} \right), \left(\delta_{RI}, \delta_{MI} \right), \left(\delta_{RF}, \delta_{MF} \right) \right\rangle = lz.
\end{aligned}
\tag{13}
$$

(ii) Since the minimum and maximum LNZNs are lz^- and lz^+, there is $lz^- \leq lz_j \leq lz^+$. Thus, there exists the inequality $\sum_{k=1}^{n} p_k lz^- \leq \sum_{k=1}^{n} p_k lz_k \leq \sum_{k=1}^{n} p_k lz^+$. Regarding property (i), there is the inequality $lz^- \leq \sum_{k=1}^{n} p_k lz_k \leq lz^+$, namely, $lz^- \leq \mathrm{LNZNWAM}(lz_1, lz_2, \ldots, lz_n) \leq lz^+$.

(iii) Because $lz_k \leq lz_k^*$ for $k = 1, 2, \ldots, n$, there exists the inequality $\sum_{k=1}^{n} p_k lz_k \leq \sum_{k=1}^{n} p_k lz_k^*$, namely, $\mathrm{LNZNWAM}(lz_1, lz_2, \ldots, lz_n) \leq \mathrm{LNZNWAM}(lz_1^*, lz_2^*, \ldots, lz_n^*)$.

(iv) The commutativity of the LNZNWAM operator is straightforward.

Hence, the proof of these properties is finished.

Especially when $p_k = 1/n$ for $k = 1, 2, \ldots, n$, the LNZNWAM operator reduces into the LNZN arithmetic mean operator. □

4.2. LNNWGM Operator

Definition 6. Set $lz_k = <(\delta_{RT(k)}, \delta_{MT(k)}), (\delta_{RI(k)}, \delta_{MI(k)}), (\delta_{RF(k)}, \delta_{MF(k)})>$ ($k = 1, 2, \ldots, n$) as a series of LNZNs on U. Then, we can define the LNZNWGM operator:

$$
\mathrm{LNZNWGM}(lz_1, lz_2, \ldots, lz_n) = \prod_{k=1}^{n} lz_k^{p_k}, \tag{14}
$$

where p_k is the weight of lz_k ($k = 1, 2, \ldots, n$) for $p_k \in [0, 1]$ and $\sum_{k=1}^{n} p_k = 1$.

Based on the operational relations in Definition 2 and equation (14), we can give the following theorem.

Theorem 2. *Set $lz_k = <(\delta_{RT(k)}, \delta_{MT(k)}), (\delta_{RI(k)}, \delta_{MI(k)}), (\delta_{RF(k)}, \delta_{MF(k)})>$ ($k = 1, 2, \ldots, n$) as a series of LNZNs on U. Then, the* *aggregation result yielded by equation (14) is also LNZN, which is calculated by the aggregation equation:*

$$\begin{aligned}\mathrm{LNZNWGM}(lz_1, lz_2, \ldots, lz_n) &= \prod_{k=1}^{n} lz_k^{p_k}\\ &= \left\langle \left(\delta_{v\prod_{k=1}^{n}(RT(k)/v)^{p_k}}, \delta_{v\prod_{k=1}^{n}(MT(k)/v)^{p_k}} \right), \right.\\ &\quad \cdot \left(\delta_{v-v\prod_{k=1}^{n}(1-RI(k)/v)^{p_k}}, \delta_{v-v\prod_{k=1}^{n}(1-MI(k)/v)^{p_k}} \right),\\ &\quad \left. \cdot \left(\delta_{v-v\prod_{k=1}^{n}(1-RF(k)/v)^{p_k}}, \delta_{v-v\prod_{k=1}^{n}(1-MF(k)/v)^{p_k}} \right) \right\rangle,\end{aligned} \tag{15}$$

where p_k is the weight of lz_k ($k = 1, 2, \ldots, n$) for $p_k \in [0, 1]$ and $\sum_{k=1}^{n} p_k = 1$. Especially when $p_k = 1/n$ for $k = 1, 2, \ldots, n$, the LNZNWGM operator reduces into the geometric mean operator of LNZNs.

By the similar proof way of Theorem 1, one can verify Theorem 2, which is not repeated here.

Similarly, the LNZNWGM operator also implies the following properties:

(i) Idempotency: set lz_k ($k = 1, 2, \ldots, n$) as a series of LNZNs on U. If $lz_k = lz$ ($k = 1, 2, \ldots, n$), there exists $\mathrm{LNZNWGM}(lz_1, lz_2, \ldots, lz_n) = lz$.

(ii) Boundedness: set lz_k ($k = 1, 2, \ldots, n$) as a series of LNZNs on U and set the minimum and maximum LNZNs as

$$\begin{aligned} lz^- &= \left\langle \left(\min_k\left(\delta_{RT(k)}\right), \min_k\left(\delta_{MT(k)}\right) \right), \left(\max_k\left(\delta_{RI(k)}\right), \max_k\left(\delta_{MI(k)}\right) \right), \left(\max_k\left(\delta_{RF(k)}\right), \max_k\left(\delta_{MF(k)}\right) \right) \right\rangle,\\ lz^+ &= \left\langle \left(\max_k\left(\delta_{RT(k)}\right), \max_k\left(\delta_{MT(k)}\right) \right), \left(\min_k\left(\delta_{RI(k)}\right), \min_k\left(\delta_{MI(k)}\right) \right), \left(\min_k\left(\delta_{RF(k)}\right), \min_k\left(\delta_{MF(k)}\right) \right) \right\rangle, \end{aligned} \tag{16}$$

and then there exists the inequality $lz^- \le \mathrm{LNZNWGM}(lz_1, lz_2, \ldots, lz_n) \le lz^+$.

(iii) Monotonicity: set lz_k ($k = 1, 2, \ldots, n$) as a series of LNZNs on U. If $lz_k \le lz_k^*$ ($k = 1, 2, \ldots, n$), there exists the inequality $\mathrm{LNZNWGM}(lz_1, lz_2, \ldots, lz_n) \le \mathrm{LNZNWGM}(lz_1^*, lz_2^*, \ldots, lz_n^*)$.

(iv) Commutativity: set the LNZN sequence $(lz_1', lz_2', \ldots, lz_n')$ as an arbitrary permutation of $(lz_1, lz_2, \ldots, lz_n)$. Then, there is $\mathrm{LNZNWGM}(lz_1', lz_2', \ldots, lz_n') = \mathrm{LNZNWGM}(lz_1, lz_2, \ldots, lz_n)$.

By the similar proof of the properties of the LNZNWAM operator, one can verify these properties, which are not repeated here.

5. MAGDM Approach in terms of the LNZNWAM and LNZNWGM Operators

This section develops an MAGDM method by utilizing the LNZNWAM and LNZNWGM operators and score and accuracy functions to perform MAGDM issues in the setting of LNZNs.

Regarding an MAGDM problem, experts/decision-makers preliminarily propose a set of alternatives $N = \{N_1, N_2, \ldots, N_m\}$, which needs to satisfy the requirements of n attributes in a set of attributes $H = \{h_1, h_2, \ldots, h_n\}$ with the weight vector of the attributes $P = (p_1, p_2, \ldots, p_n)$. Then, a group of decision-makers/experts $G = \{G_1, G_2, \ldots, G_s\}$ is invited along with their corresponding weight vector $D = (d_1, d_2, \ldots, d_s)$ to assess the alternatives over the attributes by LNZNs from the predefined LTS $U = \{\delta_0, \delta_1, \ldots, \delta_v\}$ with odd cardinality v+1. In the assessment process of each alternative N_j ($j = 1, 2, \ldots, m$) over each attribute h_k ($k = 1, 2, \ldots, n$), each decision-maker can provide the truth, falsehood, and indeterminacy linguistic term values and their corresponding linguistic reliability measure values from U, which are constructed as LNZN. Thus, the LNZNs provided by each decision-maker G_i ($i = 1, 2, \ldots, s$) can be constructed as each LNZN decision matrix $M^i = (lz_{jk}^i)_{m*n}$, where $lz_{jk}^i = \left\langle (\delta_{RT^i(jk)}, \delta_{MT^i(jk)}), (\delta_{RI^i(jk)}, \delta_{MI^i(jk)}), (\delta_{RF^i(jk)}, \delta_{MF^i(jk)}) \right\rangle$ ($i = 1, 2, \ldots, s$; $k = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$) are LNZNs.

Thus, we can develop an MAGDM method by using the LNZNWAM and LNZNWGM operators and score and accuracy functions to perform the MAGDM issue with LNZN information and introduce the decision steps below.

Step 1: obtain the aggregated matrix $M = (lz_{jk})_{m\times n}$, where $lz_{jk} = \left\langle (\delta_{RT(jk)}, \delta_{MT(jk)}), (\delta_{RI(jk)}, \delta_{MI(jk)}), (\delta_{RF(jk)}, \delta_{MF(jk)})\right\rangle$ $(k=1, 2, \ldots, n;\ j=1, 2, \ldots, m)$ is an aggregated LNZN, by using the LNZNWAM operator:

$$lz_{jk} = \mathrm{LNZNWAM}\left(lz_{jk}^1, lz_{jk}^2, \ldots, lz_{jk}^s\right) = \sum_{i=1}^{s} d_i \cdot lz_{jk}^i = \left\langle \begin{array}{c} \left(\delta_{v - v\prod_{i=1}^{s}\left(1 - RT^i(jk)/v\right)^{d_i}}, \delta_{v - v\prod_{i=1}^{s}\left(1 - MT^i(jk)/v\right)^{d_i}}\right) \\ \left(\delta_{v\prod_{i=1}^{s}\left(RI^i(jk)/v\right)^{d_i}}, \delta_{v\prod_{i=1}^{s}\left(MI^i(jk)/v\right)^{d_i}}\right) \\ \left(\delta_{v\prod_{i=1}^{s}\left(RF^i(jk)/v\right)^{d_i}}, \delta_{v\prod_{i=1}^{s}\left(MF^i(jk)/v\right)^{d_i}}\right) \end{array} \right\rangle \tag{17}$$

Step 2: obtain the aggregated LNZN lz_j for N_j $(j=1, 2, \ldots, m)$ by using the LNZNWAM or LNZNWGM operator:

$$lz_j = \mathrm{LNZNWAM}\left(lz_{j1}, lz_{j2}, \ldots, lz_{jn}\right) = \sum_{k=1}^{n} p_k \cdot lz_{jk} = \left\langle \begin{array}{c} \left(\delta_{v - v\prod_{k=1}^{n}(1 - RT(jk)/v)^{p_k}}, \delta_{v - v\prod_{k=1}^{n}(1 - MT(jk)/v)^{p_k}}\right) \\ \left(\delta_{v\prod_{k=1}^{n}(RI(jk)/v)^{p_k}}, \delta_{v\prod_{k=1}^{n}(MI(jk)/v)^{p_k}}\right) \\ \left(\delta_{v\prod_{k=1}^{n}(RF(jk)/v)^{p_k}}, \delta_{v\prod_{k=1}^{n}(MF(jk)/v)^{p_k}}\right) \end{array} \right\rangle, \tag{18}$$

$$lz_j = \mathrm{LNZNWGM}\left(lz_{j1}, lz_{j2}, \ldots, lz_{jn}\right) = \prod_{k=1}^{n} lz_{jk}^{p_k} = \left\langle \begin{array}{c} \left(\delta_{v\prod_{k=1}^{n}(RT(jk)/v)^{p_k}}, \delta_{v\prod_{k=1}^{n}(MT(jk)/v)^{p_k}}\right) \\ \left(\delta_{v - v\prod_{k=1}^{n}(1 - RI(jk)/v)^{p_k}}, \delta_{v - v\prod_{k=1}^{n}(1 - MI(jk)/v)^{p_k}}\right) \\ \left(\delta_{v - v\prod_{k=1}^{n}(1 - RF(jk)/v)^{p_k}}, \delta_{v - v\prod_{k=1}^{n}(1 - MF(jk)/v)^{p_k}}\right) \end{array} \right\rangle, \tag{19}$$

Step 3: calculate the score values of $Y(lz_j)$ (the accuracy values of $Z(lz_j)$ if necessary) ($j = 1, 2, \ldots, m$) by equation (5) (equation (6)).

Step 4: sort the alternatives based on the score (accuracy) values and sorting laws of LNZNs and then choose the best one.

Step 5: end.

6. MAGDM Example and Comparison

6.1. MAGDM Example of Industrial Robot Selection. Because of the complexity, advanced features, and facilities of industrial robots, selecting an industrial robot for a specific application is a multifaceted task. This requires decision-makers to select the most suitable robot for a specific application in terms of the various features, costs, and benefits, which is an MAGDM issue. To illustrate the applicability and efficiency of the proposed MAGDM approach, this section provides an MAGDM application of industrial robot selection in order to choose the most suitable industrial robot for the flexible manufacturing system of a manufacturing company.

A manufacturing company needs to select the most suitable type of industrial robots from robot suppliers. Some experts preliminarily choose four types of industrial robots, which are denoted as a set of alternatives $N = \{N_1, N_2, N_3, N_4\}$ from robot suppliers. Meanwhile, they must satisfy four indices (attributes): the operation dexterity (h_1), the payload capacity (h_2), the programming versatility (readability, coordination, and intelligent control capacity) (h_3), and the man-machine interface (h_4). The weight vector of the four attributes h_k for $k = 1, 2, 3, 4$ is given by $P = (0.27, 0.23, 0.26, 0.24)$ to indicate the importance of the attributes. A group of three experts/decision-makers $G = \{G_1, G_2, G_3\}$ with the weight vector $D = (0.37, 0.35, 0.28)$ is requested to assess the four alternatives over the four attributes by LNZNs from the predefined LTS $U = \{\delta_0$(extremely low), δ_1(very low), δ_2(low), δ_3(slightly high), δ_4(medium), δ_5(slightly high), δ_6(high), δ_7(very high), δ_8(extremely high)$\}$ with $\nu = 8$. Therefore, the LNZNs specified by each decision-maker G_i ($i = 1, 2, 3$) can form the following LNZN decision matrix M^i ($i = 1, 2, 3$):

$$
\begin{gathered}
M^1 = \begin{bmatrix}
\langle(\delta_6,\delta_7),(\delta_1,\delta_6),(\delta_2,\delta_6)\rangle & \langle(\delta_7,\delta_6),(\delta_1,\delta_5),(\delta_2,\delta_5)\rangle & \langle(\delta_5,\delta_6),(\delta_2,\delta_6),(\delta_2,\delta_7)\rangle & \langle(\delta_6,\delta_5),(\delta_2,\delta_5),(\delta_2,\delta_6)\rangle \\
\langle(\delta_7,\delta_6),(\delta_1,\delta_5),(\delta_1,\delta_6)\rangle & \langle(\delta_7,\delta_7),(\delta_3,\delta_6),(\delta_2,\delta_4)\rangle & \langle(\delta_7,\delta_6),(\delta_3,\delta_6),(\delta_2,\delta_5)\rangle & \langle(\delta_6,\delta_6),(\delta_2,\delta_6),(\delta_2,\delta_5)\rangle \\
\langle(\delta_6,\delta_6),(\delta_2,\delta_6),(\delta_2,\delta_6)\rangle & \langle(\delta_6,\delta_5),(\delta_1,\delta_5),(\delta_1,\delta_6)\rangle & \langle(\delta_6,\delta_7),(\delta_2,\delta_4),(\delta_1,\delta_5)\rangle & \langle(\delta_6,\delta_5),(\delta_2,\delta_6),(\delta_3,\delta_6)\rangle \\
\langle(\delta_7,\delta_7),(\delta_1,\delta_6),(\delta_2,\delta_5)\rangle & \langle(\delta_7,\delta_6),(\delta_1,\delta_6),(\delta_3,\delta_7)\rangle & \langle(\delta_7,\delta_5),(\delta_2,\delta_5),(\delta_2,\delta_6)\rangle & \langle(\delta_5,\delta_6),(\delta_1,\delta_7),(\delta_1,\delta_6)\rangle
\end{bmatrix}, \\
M^2 = \begin{bmatrix}
\langle(\delta_5,\delta_6),(\delta_1,\delta_6),(\delta_2,\delta_6)\rangle & \langle(\delta_6,\delta_7),(\delta_3,\delta_5),(\delta_2,\delta_5)\rangle & \langle(\delta_6,\delta_6),(\delta_2,\delta_6),(\delta_2,\delta_7)\rangle & \langle(\delta_6,\delta_5),(\delta_2,\delta_5),(\delta_2,\delta_6)\rangle \\
\langle(\delta_6,\delta_5),(\delta_3,\delta_5),(\delta_1,\delta_7)\rangle & \langle(\delta_6,\delta_7),(\delta_2,\delta_6),(\delta_3,\delta_4)\rangle & \langle(\delta_6,\delta_6),(\delta_1,\delta_6),(\delta_3,\delta_5)\rangle & \langle(\delta_7,\delta_6),(\delta_1,\delta_6),(\delta_3,\delta_5)\rangle \\
\langle(\delta_5,\delta_5),(\delta_2,\delta_6),(\delta_2,\delta_6)\rangle & \langle(\delta_5,\delta_5),(\delta_2,\delta_5),(\delta_1,\delta_7)\rangle & \langle(\delta_5,\delta_7),(\delta_2,\delta_5),(\delta_4,\delta_5)\rangle & \langle(\delta_7,\delta_6),(\delta_3,\delta_6),(\delta_1,\delta_7)\rangle \\
\langle(\delta_6,\delta_6),(\delta_1,\delta_7),(\delta_2,\delta_5)\rangle & \langle(\delta_7,\delta_6),(\delta_1,\delta_7),(\delta_2,\delta_6)\rangle & \langle(\delta_6,\delta_5),(\delta_1,\delta_7),(\delta_3,\delta_6)\rangle & \langle(\delta_5,\delta_5),(\delta_3,\delta_5),(\delta_1,\delta_6)\rangle
\end{bmatrix}, \\
M^3 = \begin{bmatrix}
\langle(\delta_5,\delta_5),(\delta_1,\delta_6),(\delta_1,\delta_6)\rangle & \langle(\delta_6,\delta_6),(\delta_1,\delta_6),(\delta_3,\delta_5)\rangle & \langle(\delta_5,\delta_6),(\delta_2,\delta_5),(\delta_1,\delta_7)\rangle & \langle(\delta_5,\delta_4),(\delta_3,\delta_5),(\delta_1,\delta_6)\rangle \\
\langle(\delta_7,\delta_6),(\delta_2,\delta_5),(\delta_1,\delta_7)\rangle & \langle(\delta_5,\delta_6),(\delta_2,\delta_7),(\delta_3,\delta_6)\rangle & \langle(\delta_5,\delta_5),(\delta_2,\delta_6),(\delta_1,\delta_6)\rangle & \langle(\delta_6,\delta_6),(\delta_2,\delta_6),(\delta_2,\delta_5)\rangle \\
\langle(\delta_4,\delta_5),(\delta_1,\delta_6),(\delta_2,\delta_6)\rangle & \langle(\delta_5,\delta_6),(\delta_1,\delta_6),(\delta_2,\delta_5)\rangle & \langle(\delta_7,\delta_6),(\delta_1,\delta_7),(\delta_2,\delta_5)\rangle & \langle(\delta_7,\delta_5),(\delta_2,\delta_5),(\delta_3,\delta_7)\rangle \\
\langle(\delta_3,\delta_6),(\delta_1,\delta_6),(\delta_3,\delta_5)\rangle & \langle(\delta_5,\delta_5),(\delta_2,\delta_5),(\delta_1,\delta_5)\rangle & \langle(\delta_6,\delta_6),(\delta_3,\delta_6),(\delta_2,\delta_6)\rangle & \langle(\delta_5,\delta_5),(\delta_3,\delta_4),(\delta_1,\delta_7)\rangle
\end{bmatrix}.
\end{gathered} \tag{20}
$$

Then, the developed MAGDM approach can be used in this MAGDM problem and depicted by the following steps:

Step 1: by equation (17), we obtain the aggregated matrix $M = (lz_{jk})_{4\times 4}$:

$$M = \begin{bmatrix} \langle(\delta_{5.4179},\delta_{6.2664}),(\delta_{1.0000},\delta_{6.0000}),(\delta_{1.6472},\delta_{6.0000})\rangle & \langle(\delta_{6.4524},\delta_{6.4308}),(\delta_{1.4689},\delta_{5.2619}),(\delta_{2.2405},\delta_{5.0000})\rangle & \langle(\delta_{5.3969},\delta_{6.0000}),(\delta_{2.0000},\delta_{5.7014}),(\delta_{1.6472},\delta_{7.0000})\rangle & \langle(\delta_{5.7595},\delta_{4.7483}),(\delta_{2.2405},\delta_{5.0000}),(\delta_{1.6472},\delta_{6.0000})\rangle \\ \langle(\delta_{6.7254},\delta_{5.6950}),(\delta_{1.7835},\delta_{5.0000}),(\delta_{1.0000},\delta_{6.6119})\rangle & \langle(\delta_{6.2664},\delta_{6.7858}),(\delta_{2.3237},\delta_{6.2646}),(\delta_{2.5821},\delta_{4.4809})\rangle & \langle(\delta_{6.2664},\delta_{5.7595}),(\delta_{1.8232},\delta_{6.0000}),(\delta_{1.8983},\delta_{5.2619})\rangle & \langle(\delta_{6.4308},\delta_{6.0000}),(\delta_{1.5692},\delta_{6.0000}),(\delta_{1.5692},\delta_{6.0000})\rangle \\ \langle(\delta_{5.2013},\delta_{5.4179}),(\delta_{1.6472},\delta_{6.0000}),(\delta_{2.0000},\delta_{6.0000})\rangle & \langle(\delta_{5.4179},\delta_{5.3220}),(\delta_{1.2746},\delta_{5.2619}),(\delta_{1.2142},\delta_{6.0174})\rangle & \langle(\delta_{6.1017},\delta_{6.7858}),(\delta_{1.6472},\delta_{5.0586}),(\delta_{1.9725},\delta_{5.0000})\rangle & \langle(\delta_{6.7076},\delta_{5.3969}),(\delta_{2.3050},\delta_{5.7014}),(\delta_{2.0423},\delta_{6.6119})\rangle \\ \langle(\delta_{5.9998},\delta_{6.4524}),(\delta_{1.0000},\delta_{6.3326}),(\delta_{2.2405},\delta_{5.0000})\rangle & \langle(\delta_{6.6398},\delta_{5.7595}),(\delta_{1.2142},\delta_{6.0174}),(\delta_{1.9138},\delta_{6.0360})\rangle & \langle(\delta_{6.4524},\delta_{5.3220}),(\delta_{1.7578},\delta_{5.9195}),(\delta_{2.3050},\delta_{6.0000})\rangle & \langle(\delta_{5.0000},\delta_{5.4179}),(\delta_{1.9980},\delta_{5.3199}),(\delta_{1.0000},\delta_{6.2646})\rangle \end{bmatrix}. \tag{21}$$

Step 2: by equation (18), we obtain the aggregated LNZNs of lz_j for N_j ($j=1, 2, \ldots, m$): lz_1 = <($\delta_{5.7769}$, $s_{5.9550}$), ($\delta_{1.5877}$, $s_{5.4988}$), ($\delta_{1.7679}$, $s_{5.9889}$)>, lz_2 = <(δ6.4423, s6.0917), (δ1.8486, s5.7688), (δ1.6372, s5.5662)>, lz_3 = <(δ5.9368, s5.8558), (δ1.6832, s5.5011), (δ1.7857, s5.8611)>, and lz_4 = <(δ6.1127, s5.8027), (δ1.4296, s5.8980), (δ1.7936, s5.7792)>, or by equation (19), we obtain the aggregated LNZNs of lz_j for N_j ($j=1$, 2, . . ., m): lz1 = <(δ5.7177, s5.8316), (δ1.6841, s5.5431), (δ1.7888, s6.1666)>, lz_2 = <(δ6.4270, s6.0216), (δ1.8726, s5.8403), (δ1.7604, s5.7607)>, lz_3 = <(δ5.8174, s5.7156), (δ1.7300, s5.5426), (δ1.8309, s5.9682)>, and lz_4 = <(δ5.9907, s5.7334), (δ1.4983, s5.9404), (δ1.9052, s5.8523)>.

Step 3: by equation (5), we obtain the score values: $Y(lz_1)=0.7452$, $Y(lz_2)=0.7681$, $Y(lz_3)=0.7450$, and $Y(lz_4)=0.7535$, or $Y(lz_1)=0.7343$, $Y(lz_2)=0.7585$, $Y(lz_3)=0.7330$, and $Y(lz_4)=0.7411$.

Step 4: the sorting order of the four alternatives is $N_2 \succ f$ $N_4 \succ N_1 \succ N_3$, and then the best one is N_2.

Sorting orders of the four alternatives regarding the developed MAGDM approach using the LNZNWAM and LNNWGM operators are shown in Figure 1. It is obvious that the sorting orders of the alternatives and the best one in terms of the LNZNWAM and LNNWGM operators are identical.

6.2. Comparison with Related Methods. This part compares the proposed MAGDM approach with the related LNN and NZN decision-making approaches [12,20] to indicate the suitability and efficiency of the proposed MAGDM approach.

To conveniently compare the proposed MAGDM approach with the existing related MAGDM approach [12] for the robot selection problem, we only use LNNs in M^1, M^2, and M^3 without considering the linguistic reliability measures in LNZNs since the linguistic neutrosophic MAGDM approach [12] cannot handle such an MAGDM problem with the information of LNZNs. As a special case, the above LNZN decision matrices of the three decision-makers are reduced to the following LNN decision matrices:

$$M'^1 = \begin{bmatrix} \langle\delta_6,\delta_1,\delta_2\rangle & \langle\delta_7,\delta_1,\delta_2\rangle & \langle\delta_5,\delta_2,\delta_2\rangle & \langle\delta_6,\delta_2,\delta_2\rangle \\ \langle\delta_7,\delta_1,\delta_1\rangle & \langle\delta_7,\delta_3,\delta_2\rangle & \langle\delta_7,\delta_3,\delta_2\rangle & \langle\delta_6,\delta_2,\delta_2\rangle \\ \langle\delta_6,\delta_2,\delta_2\rangle & \langle\delta_6,\delta_1,\delta_1\rangle & \langle\delta_6,\delta_2,\delta_1\rangle & \langle\delta_6,\delta_2,\delta_3\rangle \\ \langle\delta_7,\delta_1,\delta_2\rangle & \langle\delta_7,\delta_1,\delta_3\rangle & \langle\delta_7,\delta_2,\delta_2\rangle & \langle\delta_5,\delta_1,\delta_1\rangle \end{bmatrix},$$

$$M'^2 = \begin{bmatrix} \langle\delta_5,\delta_1,\delta_2\rangle & \langle\delta_6,\delta_3,\delta_2\rangle & \langle\delta_6,\delta_2,\delta_2\rangle & \langle\delta_6,\delta_2,\delta_2\rangle \\ \langle\delta_6,\delta_3,\delta_1\rangle & \langle\delta_6,\delta_2,\delta_3\rangle & \langle\delta_6,\delta_1,\delta_3\rangle & \langle\delta_7,\delta_1,\delta_3\rangle \\ \langle\delta_5,\delta_2,\delta_2\rangle & \langle\delta_5,\delta_2,\delta_1\rangle & \langle\delta_5,\delta_2,\delta_4\rangle & \langle\delta_7,\delta_3,\delta_1\rangle \\ \langle\delta_6,\delta_1,\delta_2\rangle & \langle\delta_7,\delta_1,\delta_2\rangle & \langle\delta_6,\delta_1,\delta_3\rangle & \langle\delta_5,\delta_3,\delta_1\rangle \end{bmatrix},$$

$$M'^3 = \begin{bmatrix} \langle\delta_5,\delta_1,\delta_1\rangle & \langle\delta_6,\delta_1,\delta_3\rangle & \langle\delta_5,\delta_2,\delta_1\rangle & \langle\delta_5,\delta_3,\delta_1\rangle \\ \langle\delta_7,\delta_2,\delta_1\rangle & \langle\delta_5,\delta_2,\delta_3\rangle & \langle\delta_5,\delta_2,\delta_1\rangle & \langle\delta_6,\delta_2,\delta_2\rangle \\ \langle\delta_4,\delta_1,\delta_2\rangle & \langle\delta_5,\delta_1,\delta_2\rangle & \langle\delta_7,\delta_1,\delta_2\rangle & \langle\delta_7,\delta_2,\delta_3\rangle \\ \langle\delta_3,\delta_1,\delta_3\rangle & \langle\delta_5,\delta_2,\delta_1\rangle & \langle\delta_6,\delta_3,\delta_2\rangle & \langle\delta_5,\delta_3,\delta_1\rangle \end{bmatrix}. \tag{22}$$

Thus, we utilize the existing MAGDM approach using the LNNWAM and LNNWGM operators of equations (1) and (2) and the score function of equation (3) [12] for this MAGDM example in the setting of LNNs.

First, by equation (1), we obtain the following aggregated matrix:

$$M\prime = \begin{bmatrix} \langle\delta_{5.4179},\delta_{1.0000},\delta_{1.6472}\rangle & \langle\delta_{6.4524},\delta_{1.4689},\delta_{2.2405}\rangle & \langle\delta_{5.3969},\delta_{2.0000},\delta_{1.6472}\rangle & \langle\delta_{5.7595},\delta_{2.2405},\delta_{1.6472}\rangle \\ \langle\delta_{6.7254},\delta_{1.7835},\delta_{1.0000}\rangle & \langle\delta_{6.2664},\delta_{2.3237},\delta_{2.5821}\rangle & \langle\delta_{6.2664},\delta_{1.8232},\delta_{1.8983}\rangle & \langle\delta_{6.4308},\delta_{1.5692},\delta_{1.5692}\rangle \\ \langle\delta_{5.2013},\delta_{1.6472},\delta_{2.0000}\rangle & \langle\delta_{5.4179},\delta_{1.2746},\delta_{1.2142}\rangle & \langle\delta_{6.1017},\delta_{1.6472},\delta_{1.9725}\rangle & \langle\delta_{6.7076},\delta_{2.3050},\delta_{2.0423}\rangle \\ \langle\delta_{5.9998},\delta_{1.0000},\delta_{2.2405}\rangle & \langle\delta_{6.6398},\delta_{1.2142},\delta_{1.9138}\rangle & \langle\delta_{6.4524},\delta_{1.7578},\delta_{2.3050}\rangle & \langle\delta_{5.0000},\delta_{1.9980},\delta_{1.0000}\rangle \end{bmatrix}. \tag{23}$$

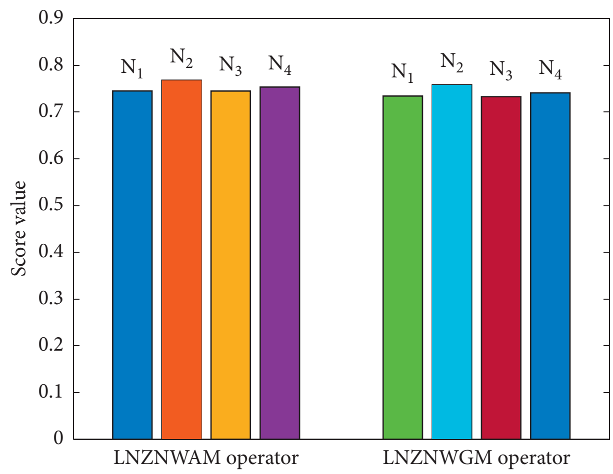

FIGURE 1: Sorting orders of the four alternatives regarding the developed MAGDM approach using the LNZNWAM and LNNWGM operators.

TABLE 1: Decision results of different MAGDM methods.

MAGDM method	Score value	Sorting order	The best one
Existing MAGDM method using equation (1) [12]	0.7676, 0.7899, 0.7695, 0.7871	$N_2 \succ N_4 \succ N_3 \succ N_1$	N_2
Existing MAGDM method using equation (2) [12]	0.7602, 0.7831, 0.7607, 0.7745	$N_2 \succ N_4 \succ N_3 \succ N_1$	N_2
Proposed MAGDM method using equation (18)	0.7452, 0.7681, 0.7450, 0.7535	$N_2 \succ N_4 \succ N_1 \succ N_3$	N_2
Proposed MAGDM method using equation (19)	0.7343, 0.7585, 0.7330, 0.7411	$N_2 \succ N_4 \succ N_1 \succ N_3$	N_2

Then, by equations (1)–(4), the decision results are shown in Table 1. For the convenient comparison, the decision results corresponding to the proposed MAGDM approach are also contained in Table 1.

In Table 1, there is the sorting difference between the existing MAGDM method [12] and the proposed MAGDM method; then, the best one is the same. However, this sorting difference reflects that the new method utilizes the LNZN information constructed by the truth, falsehood, and indeterminacy linguistic Z-numbers, while the existing method [12] only contains the LNN information without containing their reliability measures. Moreover, LNZNs imply much more useful information than LNNs and strengthen the reliability of LNNs. Thus, different MAGDM information and methods may affect the sorting order of alternatives, which illustrates the efficiency and applicability of the new MAGDM method in the LNZN environment. Therefore, the new MAGDM method is superior to the existing one [12].

Furthermore, the existing decision-making method in the setting of NZNs [20] cannot carry out such a linguistic decision-making issue with the LNZN information. On the contrary, the new MAGDM approach with the LNZN information especially suits such an MAGDM issue with the LNZN information. Then, in the MAGDM problem with qualitative attributes, the new MAGDM approach shows its main merit since it contains the LNN assessments related to their reliability measures in the LNZN environment.

However, the original study reveals the following main advantages:

(a) The proposed LNZN information can express more useful information to strengthen the reliability measure of LNNs and avoid the insufficiency of missing reliability measures in the existing methods

(b) The developed MAGDM approach not only makes the MAGDM process more reliable and reasonable but also provides a new way for linguistic MAGDM problems in the LNZN setting

(c) The proposed MAGDM approach can effectively solve the MAGDM issue of selecting industrial robots with the evaluation information of LNZNs and make the decision result more reliable

7. Conclusion

To make the LNN information more reliable, this study proposed an LNZN notion in terms of the truth, falsehood, and indeterminacy linguistic Z-numbers as a new linguistic neutrosophic framework. Then, the proposed operational relations and score and accuracy functions of LNZNs are to realize reasonable operations and sorting rules of LNZNs in the setting of LNZNs. The proposed LNZNWAM and LNZNWGM operators provided useful information aggregation tools in MAGDM problems with the LNZN information. Then, the established MAGDM approach in terms of the proposed LNZNWAM and LNZNWGM operators and score and accuracy functions can solve MAGDM problems under the environment of LNZNs. Through the application of the proposed MAGDM method in the industrial robot selection problem and the comparison of existing decision-making methods, the decision results demonstrated not only the suitability and efficiency of the proposed MAGDM approach in the LNZN setting but also the superiority of the new MAGDM method over the existing ones.

However, the limitations of this study lie in the lack of flexible decision-making methods and quantitative algorithms for reliability measures of LNNs. To overcome the limitations, we shall further study new aggregation operators with a changeable parameter, flexible MAGDM methods, and some quantitative algorithms of the reliability measures. Then, we shall use them in engineering areas such as environmental risk assessment and management, slope stability/risk assessment, and construction engineering management in the LNZN environment.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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