Research Article

Wave Breaking for the Rotational Camassa–Holm Equation on the Circle

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In this paper, we consider the rotational Camassa–Holm equation on the circle. Sufficient conditions on the initial data to guarantee wave breaking are established.

1. Introduction

Recently, a rotational Camassa–Holm equation was proposed in [1–4], which reads as

$$m_t + um_x + 2u_x m + cu_x - \frac{\beta_0}{\beta} u_{xxx} + \frac{\omega_1}{\alpha^2} u^2 u_x + \frac{\omega_2}{\alpha^3} u^3 u_x = 0,$$

$$m = u - u_{xx},$$
(1)

with

$$\begin{aligned} c &= \sqrt{1 + \Omega^2} - \Omega, \\ \alpha &= \frac{c^2}{1 + c^2}, \\ \beta_0 &= \frac{c(c^4 + 6c^2 - 1)}{6(c^2 + 1)^2}, \\ \beta &= \frac{3c^4 + 8c^2 - 1}{6(c^2 + 1)^2}, \\ \omega_1 &= \frac{3c(c^2 - 1)(c^2 - 2)}{2(c^2 + 1)^3}, \\ \omega_2 &= \frac{(c^2 - 1)^2(c^2 - 2)(8c^2 - 1)}{2(c^2 + 1)^5}, \end{aligned}$$
(2)

where Ω is a parameter related to the Coriolis effect. In the whole space, Gui et al. [3] established the local well-posedness in $H^s(\mathbb{R})$, $s > 3/2$. Sufficient conditions to guarantee wave breaking phenomena were also studied in [3]. Tu et al. [4] established the global existence and uniqueness of the energy conservative weak solutions. In [5], they found some explicit solutions by elliptic integrals. They also classified the members of the equation describing pseudo-spherical surfaces. Generic regularity of conservative solutions was investigated in [6]. Moon [7] studied the question of nonexistence of periodic peaked traveling wave solution for rotational Camassa–Holm equation.

If $\Omega = 0$, it follows $\omega_1 = \omega_2 = 0$, and then system (1) reduces to the Dullin–Gottwald–Holm equation [8]. Some mathematical studies can be found in [9–12].

Another highly related model is the well-known Camassa–Holm equation [13] $m_t + um_x + 2u_x m = 0$, $m = u - u_{xx}$. This equation has a physical background with shallow water propagation. The Camassa–Holm equation [14,15] has infinitely many conservation laws. In [16,17], they established the local well-posedness. Wave breaking phenomena were widely studied in [16–19]. McKean [20] (see also [21] for a simple proof) established a necessary and sufficient condition on the initial datum u_0 , which depends on the shape of $y = u - u_{xx}$. In [22], the orbital stability of the peakons was proved. In [23, 24], they studied persistence properties and unique continuation of solutions. The long-time behavior for the support of

momentum density of the Camassa–Holm equation was discussed in [25]. Mathematical studies for the related models can be found in [26–28].

For the convenience of research, in this paper, we consider the rotational Camassa–Holm equation as the following form on the circle:

$$m_t + um_x + 2u_x m = \alpha u_x + \beta u^2 u_x + \gamma u^3 u_x + \Gamma u_{xxx}, \quad (3)$$

where $m = u - u_{xx}$ and $\alpha, \beta, \gamma, \Gamma$ are real constants. $x \in \mathbb{S}$, where $\mathbb{S}$ denotes the unit circle, i.e., $\mathbb{S} = \mathbb{R}/\mathbb{Z}$.

The paper is organized as follows. In Section 2, we introduce some useful lemmas. The main result and its proof will be shown in Section 3.

2. Preliminaries

Let $\Lambda = (1 - \partial_x^2)^{1/2}$; then, the operator Λ^{-2} can be expressed by its associated Green's function as

$$u = \Lambda^{-2} m(x, t) = G * m(x, t), \quad (4)$$

$$G(x) = \frac{\cosh(x - [x] - (1/2))}{2 \sinh(1/2)}.$$

Then, (3) can be rewritten as

$$u_t + (u + \Gamma)u_x + \partial_x G * \left(u^2 + \frac{u_x^2}{2} \right) = \partial_x G * \left((\alpha + \Gamma)u + \frac{\beta}{3}u^3 + \frac{\gamma}{3}u^4 \right). \quad (5)$$

By applying Kato's semigroup theory [29] and similar arguments in [3], we can also have the local well-posedness on the circle.

Theorem 1 (see [3]). *Given $u(x, t = 0) = u_0 \in H^s(\mathbb{S})$, $s > 3/2$, then there exist a maximal $T = T(u_0) > 0$ and a unique solution u to the rotational Camassa–Holm equation (3) such that*

$$u = u(\cdot, u_0) \in C([0, T]; H^s(\mathbb{S})) \cap C^1([0, T]; H^{s-1}(\mathbb{S})). \quad (6)$$

Then, the precise blow-up scenario will be shown as follows.

Theorem 2 (see [3]). *Assume that $u_0 \in H^2(\mathbb{S})$ and let T be the maximal existence time of the solution $u(x, t)$ to equation (3) with the initial data $u_0(x)$. Then, the corresponding solution of the rotational Camassa–Holm equation (3) blows up in finite time if and only if*

$$\lim_{t \rightarrow T} \inf_{x \in \mathbb{S}} \{u_x(x, t)\} = -\infty. \quad (7)$$

Then, we introduce some useful inequality in the circle.

Lemma 1 (see [30]). *For all $f \in H^1(\mathbb{S})$, the following inequality holds:*

$$\max_{x \in [0, 1]} f^2(x) \leq C_0 \|f\|_{H^1(\mathbb{S})}^2, \quad (8)$$

where $C_0 = e^{1/2} + e^{-1/2}/2(e^{1/2} - e^{-1/2})$.

Lemma 2 (see [30]). *Let $f \in H^1(\mathbb{S})$. If $\int_{\mathbb{S}} f dx = 0$, then the following inequalities hold:*

$$\|f\|_{L^\infty(\mathbb{S})}^2 \leq \frac{1}{12} \int_{\mathbb{S}} f_x^2 dx \leq \frac{1}{12} \|f\|_{H^1(\mathbb{S})}^2,$$

$$\int_{\mathbb{S}} f^2(x) dx \leq \frac{1}{12} \int_{\mathbb{S}} f_x^2(x) dx, \quad (9)$$

$$\int_{\mathbb{S}} f^2(x) f_x^2(x) dx \leq \frac{1}{12} \left(\int_{\mathbb{S}} f_x^2(x) dx \right)^2.$$

3. Main Results

In this section, we firstly establish a sufficient condition to guarantee the blow up of the solution to the rotational Camassa–Holm equation (3). We give the particle trajectory as

$$\begin{cases} q_t(x, t) = u(q(x, t), t) + \Gamma, & 0 < t < T, x \in \mathbb{S}, \\ q(x, 0) = x, & x \in \mathbb{S}, \end{cases} \quad (10)$$

where T is the lifespan of the solution. Taking derivative (10) with respect to x , we obtain

$$\frac{dq_t(x, t)}{dx} = q_{tx} = u_x q_x(q(x, t), t), \quad t \in (0, T). \quad (11)$$

Therefore,

$$\begin{cases} q_x = \exp \left\{ \int_0^t u_x(q, s) ds \right\}, & 0 < t < T, x \in \mathbb{S}, \\ q_x(x, 0) = 1, & x \in \mathbb{S}, \end{cases} \quad (12)$$

which is always positive before the blow-up time.

Theorem 3. *Assume that $u_0 \in H^2(\mathbb{S})$ and there exists $x_0 \in \mathbb{S}$ such that*

$$u_{0x}(x_0) < -\sqrt{2C^*}, \quad (13)$$

where $C^* = (1/2)C_0 \|u_0\|_{H^1}^2 + 2(|\alpha| + |\Gamma|)\sqrt{C_0} \|u_0\|_{H^1} + (2|\beta|/3)C_0^{3/2} \|u_0\|_{H^1}^3 + (|\gamma|/4) \|u_0\|_{H^1}^4$; then, the corresponding solution $u(x, t)$ to equation blows up at a finite time T bounded by

$$T \leq \frac{1}{-(1/2)u_{0x}(x_0) + (C^*/u_{0x}(x_0))}. \quad (14)$$

Proof. Let $\mathcal{H}(u) = (\alpha + \Gamma)u + (\beta/3)u^3 + (\gamma/4)u^4$. Differentiating (5) with respect to x yields

$$u_{xt} + (u + \Gamma)u_{xx} + u_x^2 + \partial_x G * \left(u^2 + \frac{u_x^2}{2} \right) = \partial_x G * \mathcal{H}. \quad (15)$$

Then, we have u_{xt} at the point $(q(x, t), t)$ as

$$u_{xt}(q(x, t), t) = \left(u^2 - \frac{u_x^2}{2} - G * \left(u^2 + \frac{u_x^2}{2} \right) - \mathcal{H}(u) + G * \mathcal{H} \right) \cdot (q(x, t), t). \quad (16)$$

Without loss of generality, we choose $0 \leq q(x, t) \leq 1$, and we have

$$\begin{aligned} G * \left(u^2 + \frac{u_x^2}{2} \right) (q(x, t), t) &= \frac{1}{2\sinh(1/2)} \int_0^{q(x,t)} \frac{e^{q(x,t)-\eta-1/2} + e^{1/2+\eta-q(x,t)}}{2} \left(u^2 + \frac{u_x^2}{2} \right) (\eta, t) d\eta \\ &+ \frac{1}{2\sinh(1/2)} \int_{q(x,t)}^1 \frac{e^{q(x,t)-\eta+1/2} + e^{-1/2+\eta-q(x,t)}}{2} \left(u^2 + \frac{u_x^2}{2} \right) (\eta, t) d\eta. \end{aligned} \quad (17)$$

Note that

$$\int_0^{q(x,t)} e^{-\eta} (u^2 + u_x^2) (\eta, t) d\eta \geq -2 \int_0^{q(x,t)} e^{-\eta} u u_x (\eta, t) d\eta = -e^{-\eta} u^2 (\eta, t) \Big|_0^{q(x,t)} - \int_0^{q(x,t)} e^{-\eta} u^2 (\eta, t) d\eta. \quad (18)$$

We have

$$\int_0^{q(x,t)} e^{-\eta} \left(u^2 + \frac{1}{2} u_x^2 \right) q(x, t) d\eta \geq -\frac{1}{2} e^{-\eta} u^2 (\eta, t) \Big|_0^{q(x,t)}. \quad (19)$$

Similar argument yields that

$$\begin{aligned} \int_0^{q(x,t)} e^{\eta} \left(u^2 + \frac{1}{2} u_x^2 \right) (\eta, t) d\eta &\geq \frac{1}{2} e^{\eta} u^2 (\eta, t) \Big|_0^{q(x,t)}, \\ \int_{q(x,t)}^1 e^{-\eta} \left(u^2 + \frac{1}{2} u_x^2 \right) (\eta, t) d\eta &\geq -\frac{1}{2} e^{-\eta} u^2 (\eta, t) \Big|_{q(x,t)}^1, \\ \int_{q(x,t)}^1 e^{\eta} \left(u^2 + \frac{1}{2} u_x^2 \right) (\eta, t) d\eta &\geq \frac{1}{2} e^{\eta} u^2 (\eta, t) \Big|_{q(x,t)}^1. \end{aligned} \quad (20)$$

Then, we have

$$G * \left(u^2 + \frac{u_x^2}{2} \right) (q(x, t), t) \geq \frac{1}{2} u^2 (q(x, t), t). \quad (21)$$

Combining (21) into (16), we obtain

$$u_{xt}(q(x, t), t) \leq \left(\frac{u^2}{2} - \frac{u_x^2}{2} - \mathcal{H}(u) + G * \mathcal{H} \right) (q(x, t), t). \quad (22)$$

A direct calculation gives

$$\begin{aligned} |G * \mathcal{H}(q(x, t), t)| &= \left| \frac{1}{2\sinh(1/2)} \int_0^{q(x,t)} \frac{e^{q(x,t)-\eta-1/2} + e^{1/2+\eta-q(x,t)}}{2} \mathcal{H}(\eta, t) d\eta + \frac{1}{2\sinh(1/2)} \int_{q(x,t)}^1 \frac{e^{q(x,t)-\eta+1/2} + e^{-1/2+\eta-q(x,t)}}{2} \mathcal{H}(\eta, t) d\eta \right| \\ &\leq \max_{x \in \mathbb{S}} |\mathcal{H}| \left\{ \frac{1}{2\sinh(1/2)} \int_0^{q(x,t)} \frac{e^{q(x,t)-\eta-1/2} + e^{1/2+\eta-q(x,t)}}{2} d\eta + \frac{1}{2\sinh(1/2)} \int_{q(x,t)}^1 \frac{e^{q(x,t)-\eta+1/2} + e^{-1/2+\eta-q(x,t)}}{2} d\eta \right\} \\ &= \max_{x \in \mathbb{S}} |\mathcal{H}|. \end{aligned} \quad (23)$$

By Lemma 1, we have

$$\frac{u^2}{2} \leq \frac{1}{2} C_0 \|u_0\|_{H^1}^2. \quad (24)$$

For $\gamma \geq 0$, we have

$$\begin{aligned} -\mathcal{H}(u) &\leq -(\alpha + \Gamma)u - \frac{\beta}{3}u^3 \leq (|\alpha| + |\Gamma|)\sqrt{C_0}\|u_0\|_{H^1} + \frac{|\beta|}{3}C_0^{3/2}\|u_0\|_{H^1}^3, \\ G * \mathcal{H} &\leq \max_{x \in \mathbb{S}} |\mathcal{H}| \leq (|\alpha| + |\Gamma|)\sqrt{C_0}\|u_0\|_{H^1} + \frac{|\beta|}{3}C_0^{3/2}\|u_0\|_{H^1}^3 + \frac{|\gamma|}{4}C_0^2\|u_0\|_{H^1}^4. \end{aligned} \quad (25)$$

For $\gamma < 0$, we have

$$\begin{aligned} -\mathcal{H}(u) &\leq (|\alpha| + |\Gamma|)\sqrt{C_0}\|u_0\|_{H^1} + \frac{|\beta|}{3}C_0^{3/2}\|u_0\|_{H^1}^3 + \frac{|\gamma|}{4}C_0^2\|u_0\|_{H^1}^4, \\ G * \mathcal{H} &\leq G * \left((\alpha + \Gamma)u + \frac{\beta}{3}u^3 \right) \leq (|\alpha| + |\Gamma|)\sqrt{C_0}\|u_0\|_{H^1} + \frac{|\beta|}{3}C_0^{3/2}\|u_0\|_{H^1}^3. \end{aligned} \quad (26)$$

By (22) and the definition of C^* , we have

$$u_{xt}(q(x, t), t) \leq -\frac{u_x^2}{2}(q(x, t), t) + C^*. \quad (27)$$

This is a Riccati type inequality. By the fundamental ODE methods, the proof is completed by the initial condition. $\square$

Theorem 4. Assume that $u_0 \in H^2(\mathbb{S})$, $\int_{\mathbb{S}} u_0 dx = 0$, and there exists $x_0 \in \mathbb{S}$ such that

$$u_{0x}(x_0) < -\sqrt{2C^*}, \quad (28)$$

where $C^* = (1/24)\|u_0\|_{H^1}^2 + (2(|\alpha| + |\Gamma|)/\sqrt{12})\|u_0\|_{H^1} + (|\beta|/18\sqrt{12})\|u_0\|_{H^1}^3 + (|\gamma|/576)\|u_0\|_{H^1}^4$; then, the corresponding solution $u(x, t)$ to equation blows up at a finite time T bounded by

$$T \leq \frac{1}{-(1/2)u_{0x}(x_0) + (C^*/u_{0x}(x_0))}. \quad (29)$$

Proof. Recall that

$$u_{xt}(q(x, t), t) \leq \left(\frac{u^2}{2} - \frac{u_x^2}{2} - \mathcal{H}(u) + G * \mathcal{H} \right)(q(x, t), t). \quad (30)$$

With the initial condition $\int_{\mathbb{S}} u_0 dx = 0$, by Lemma 2, we have

$$\frac{u^2}{2} \leq \frac{1}{24}\|u_0\|_{H^1}^2. \quad (31)$$

For $\gamma \geq 0$, we have

$$\begin{aligned} -\mathcal{H}(u) &\leq -(\alpha + \Gamma)u - \frac{\beta}{3}u^3 \leq \frac{(|\alpha| + |\Gamma|)}{\sqrt{12}}\|u_0\|_{H^1} \\ &\quad + \frac{|\beta|}{36\sqrt{12}}\|u_0\|_{H^1}^3, \\ G * \mathcal{H} &\leq \max_{x \in \mathbb{S}} |\mathcal{H}| \leq \frac{(|\alpha| + |\Gamma|)}{\sqrt{12}}\|u_0\|_{H^1} \\ &\quad + \frac{|\beta|}{36\sqrt{12}}\|u_0\|_{H^1}^3 + \frac{|\gamma|}{576}\|u_0\|_{H^1}^4. \end{aligned} \quad (32)$$

For $\gamma < 0$, we have

$$\begin{aligned} -\mathcal{H}(u) &\leq \frac{(|\alpha| + |\Gamma|)}{\sqrt{12}}\|u_0\|_{H^1} + \frac{|\beta|}{36\sqrt{12}}\|u_0\|_{H^1}^3 + \frac{|\gamma|}{576}\|u_0\|_{H^1}^4, \\ G * \mathcal{H} &\leq G * \left((\alpha + \Gamma)u + \frac{\beta}{3}u^3 \right) \leq \frac{(|\alpha| + |\Gamma|)}{\sqrt{12}}\|u_0\|_{H^1} \\ &\quad + \frac{|\beta|}{36\sqrt{12}}\|u_0\|_{H^1}^3. \end{aligned} \quad (33)$$

Combining the above arguments into (30), we have

$$u_{xt}(q(x, t), t) \leq -\frac{u_x^2}{2}(q(x, t), t) + C^*, \quad (34)$$

where C^* is defined in Theorem 4. The proof of Theorem 4 is completed by the fundamental ODE methods. $\square$

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares that there are no conflicts of interest.

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