Hindawi
Journal of Mathematics
Volume 2022, Article ID 9959564, 10 pages
https://doi.org/10.1155/2022/9959564

Research Article

General Solution for Unsteady MHD Natural Convection Flow with Arbitrary Motion of the Infinite Vertical Plate Embedded in Porous Medium

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Received 31 March 2021; Accepted 11 March 2022; Published 23 April 2022

Academic Editor: Efthymios G. Tsionas

This article has concentrated on heat transfer analysis in the unsteady MHD natural convection flow of viscous fluid under radiation and uniform heat flux over an infinite vertical plate embedded in a porous medium. Overall solutions are found for temperature as well as velocity by the Laplace transform techniques. In the literature, the solutions that have been achieved are rare, meet with all the initial and boundary conditions imposed, and can make general solutions for any problem with motion with this form's methodological relevance. Also, few different cases of engineering applications are discussed. Solutions are plotted graphically through the use of the Mathcad software to analyze how the variation is taking place in the physical behavior of the viscous fluid flow with respect to the change in a distinct physical parameter.

1. Introduction

Magnetohydrodynamics is the study of the properties of magnetic characteristics and electrically conducting fluid performance. It occurs naturally and has numerous applications in the polymer and metallurgy industries. According to astrophysics, sunspots are caused by the sun's magnetic field. MHD has a wide range of applications in geophysics, particularly in earthquake investigation. The prominent authors who used magnetic field and its outcomes in their works are Soundalgekar et al. [1], Raptis and Singh [2], and Mansour [3], and they have also proposed a similar solution. Samiulhaq et al. [4] obtained the exact solution in the study of porous medium by ramped wall temperature and thermal diffusion of unsteady magnetohydrodynamic free convection flow. Khan et al. [5] reviewed the unsteady magnetohydrodynamic flow in sodium alginate-based Casson kind nanofluid by Newtonian heating with a porous medium. Gaffar et al. [6] studied the MHD free convection flow which was shown from a vertical surface by Hall/ion slip currents and Eyring–Powell fluid ohmic intemperance in a porous medium. Khan et al. [7] studied the MHD free convection flow past an oscillating plate embedded in a porous medium. Fetecau et al. [8] investigated the unstable solution for the MHD natural convection flow by the radiative special outcome. Seth et al. [9] studied the radiative heat transfer past an unwisely flow of the plate through ramped wall temperature by MHD free convection flow. Zeeshan et al. [10], under the presence of MHD, studied the normal convection flow past a porous medium, and their findings were recognized mathematically and graphically. Ghara and Das [11] investigated the MHD free convection flow using an unwisely moving vertical plate with ramped wall temperature. The unsteady magnetohydrodynamic free convection flow past a plate of porous below oscillatory force velocity was investigated by Reddy [12]. Ahmed and Kalita studied the magnetohydrodynamic transient flux in the existence of radiation restricted by a hot vertical plate over a porous medium [13]. Ambethkar [14] studied the numerical solutions and properties of heat and mass transfer of unsteady magnetohydrodynamic free convective flow of infinite vertical plate. The unsteady flow of hydromagnetic free convection through a flux of heat and rapid motion of boundary was calculated by Chandran et al. [15]. Chandran et al. also studied the ramped wall temperature of free convection flow near a vertical plate [16]. Seth et al. studied natural convection flow with radiative heat transfer past an accelerated moving vertical plate with ramped temperature through a porous medium [17]. Similarly, Das and Jana studied the natural convective magneto-nanofluid flow and radiative heat transfer past a moving vertical plate [18]. Moreover, Erdogan worked on unsteady motions over a plane wall of a second-order fluid [19]. Natural convection flow past an impulsively moving vertical plate with Newtonian heating in a rotating system was investigated by Seth et al. [20]. Fetecau et al. worked on slip effects over a shifting plate on unstable radiative MHD natural convection flow [21], while a mathematical model with properties for MHD free convection flow in a revolving flat channel was studied by Ghosh et al. [22]. Kelleher studied the free convection with irregular wall temperature from a vertical plate [23]. Das et al. [24] worked on the impact of mass transfer on the flow past a vertical infinite plate with heat flux, and chemical reaction began impulsively. Kim [25] worked on unsteady convective heat transfer of MHD past a semi-infinite vertical porous variable suction moving plate. An analysis of the effects on sublimation of a semi-infinite, frozen porous medium of the Darcy and Fick laws was studied by Fey et al. [26]. Similarly, Ram and Takhar [27] worked on MHD free convection on a revolving fluid with Hall and ion slip currents over an infinite vertical plate. An analysis of the heat transfer in a nanofluid occupied elliptic inside cylinder enclosure was conducted by Sheikholeslami et al. [28]. Cheng and Minkowycz [29] studied the free convection flow of a vertical flat plate with heat transfer from a dike. Heat transfer of free convection to steady radiating non-Newtonian MHD flow past a vertical porous plate was investigated by Bestman [30]. Heat transfer in flow by a porous medium restricted by a vertical plate which is infinite under the action of a magnetic field was studied by Raptis and Kafousias [31]. Makinde and Aziz [32] studied the MHD mix convection with convective boundary condition from a vertical plate embedded in a porous medium. Similarly, Ibrahim et al. [33] studied the existence of thermal and mass diffusion with a constant source of heat, unsteady magnetohydrodynamic micropolar fluid flow, and heat transfer over a vertical porous plate through a porous medium. Natural laminar convection by a vertical plate with a progressive change in wall temperature was investigated by Lee and Yovanovich [34]. In a porous medium, MHD varied, and convective flow, heat, and mass transfer past a vertical plate were studied by Makinde and Sibanda [35]. Hayday et al. [36] studied the free convection flow from a vertical flat plate with step discontinuities in surface temperature. Gamal and Azzam [37] investigated the radiation effects on the MHD mixed free-forced convective flow past a semi-infinite moving vertical plate for high-temperature differences. The impact of viscous dissipation plus radiation on MHD natural convection flow past an infinite heated vertical plate in a porous medium was studied by Israel-Cookey et al. [38]. Jha and Prasad [39] studied the MHD free convection flow on the exponentially accelerated vertical plate. The best, fresh, and exciting results were obtained by researchers [40–57].

With our prior discussion in mind, the purpose of this article is to find the solution for unsteady MHD free convection flow. The dimensionless governing equations are solved using the Laplace transform approach. General velocity solutions are presented as exponential and complementary error functions. Knowing the velocity offers a dimensionless depiction of skin friction. The temperature is lowered to the form reported in a comparable reference by Ghosh et al. [22]. When it comes to velocity, there are a few options to consider. Finally, graphical diagrams are widely used to show how device characteristics affect fluid velocity.

2. Statement of the Problem

Consider an incompressible flow of a viscous fluid under the influence of a magnetic field with strength B_0 that is applied transversely on the plate. The produced magnetic field because of the motion of the fluid is considered to be so small in contrast to the generated magnetic field. Because of the Reynolds magnetic number, this limit is appropriate for metallic liquids and partly ionized fluids. Likewise, the effect of fluid polarization is small if no extraneous electric field is applied. At the original instant $t = 0$, the fluid and the plate are at rest at constant temperature T_∞. On time $t = 0+$, the plate in its own plane begins to oscillate ($y = 0$), so

$$v = U\cos(\omega t)i, \quad t > 0, \tag{1}$$

where U is the constant amplitude of the movement while i denotes the unit vector in direction of flow and ω denotes the frequency because of vibrations. Since fluid is progressively moving, this velocity is

$$v = v(y,t) = u(y,t)i. \tag{2}$$

Under common Boussinesq's approximation, the flow is governed through given system of equations

$$\rho\frac{\partial u}{\partial t} = \mu\frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 u - \frac{\mu}{K}u + \rho g\beta(T - T_\infty), \tag{3}$$

$$\rho\frac{\partial T}{\partial t} = \frac{k}{c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{c_p}\frac{\partial q_r}{\partial y}, \tag{4}$$

where ρ is the fluid density, μ is the dynamic viscosity of fluid, T is the fluid temperature, K represents the permeability, σ denotes the electrical conductivity, g represents acceleration because of gravity, β is the thermal expansion volumetric coefficient, k is the thermal conductivity, c_p is the specific heat at any constant pressure, and q_r is the radiative heat flux, in direction of y.

Consider that no slipping happens among the plate as well as the fluid, with initial and boundary conditions being

$$u(y,0)=0, T(y,0)=T_\infty,$$
$$u(0,t)=UH(t), \frac{\partial T(0,t)}{\partial y}=-\frac{q}{k}, \quad (5)$$
$$u\longrightarrow 0, T\longrightarrow T_\infty, y\longrightarrow\infty, t>0,$$

where $u(y,t)$ is for velocity while $T(y,t)$ is for the temperature; then q is for constant heat flux. Adapting the Rosseland approximation for radiative flux q_r,

$$q_r=-\frac{4\sigma^*\partial T^4}{3k^*\partial y}, \quad (6)$$

where σ^* is the Stefan–Boltzmann constant while k^* is the coefficient of mean spectral absorption. It is imagined that difference of temperature inside the flow is appropriately minor, and thus it can be linearized through expanding T^4 into the Taylor series around T_∞; also, ignoring advanced order terms, we conclude that

$$T^4=4T_\infty^3T-3T_\infty^4, \quad (7)$$

and using (7) in (4), we get

$$\rho c_p\frac{\partial T}{\partial t}=k\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma^* T_\infty^3\partial^2 T}{3k^*\partial y^2}. \quad (8)$$

Presenting the dimensionless variables,

$$\begin{aligned}
u^*&=\frac{u}{U},\\
y^*&=\frac{U}{\nu}y,\\
t^*&=\frac{U^2}{\nu}t,\\
\omega^*&=\frac{\nu}{U^2}\omega,\\
\theta&=\frac{Uk}{\nu q}\left(T-T_\infty\right),\\
\mathrm{Gr}&=\left(\frac{\nu}{U^2}\right)^2\frac{g\beta q}{k},\\
\mathrm{Pr}&=\frac{\mu c_p}{k},\\
N_r&=\frac{16\sigma^* T_\infty^3}{3kk^*},\\
M^2&=\frac{\sigma\nu B_0^2}{\rho U^2},\\
K^*&=\frac{U^2}{\nu^2}K.
\end{aligned} \quad (9)$$

Reducing the star representation from t, y, ω, u, and K, then equations (3) and (8) take this system

$$\frac{\partial u(y,t)}{\partial t}=\frac{\partial^2 u(y,t)}{\partial y^2}-hu(y,t)+\mathrm{Gr}\theta(y,t);\quad y,t>0, \quad (10)$$

$$\mathrm{Pr}\frac{\partial\theta(y,t)}{\partial t}=\left(1+N_r\right)\frac{\partial^2\theta(y,t)}{\partial y^2};\quad y,t>0, \quad (11)$$

where $h = M^2 + (1/k)$.

In (9), Gr is the Grashof number, Pr is the Prandtl number, and Nr is the parameter of radiation. The non-dimensional initial and boundary conditions (4) reduces to $u(y,0) = 0$, $\theta(y,0) = 0$, for $y \geq 0$,

$$u(0,t) = f(t),$$
$$\frac{\partial \theta(y,t)}{\partial y} = -1, \tag{12}$$
$$t > 0.$$

3. Solution of the Problem

3.1. Temperature Field. We take Laplace transform technique into equation (11), and utilizing the transform initial and boundary conditions, we obtained

$$\overline{\theta}(y,s) = \frac{1}{\sqrt{\mathrm{Pr}_{\mathrm{eff}}} s^{3/2}} e^{-y\sqrt{s\mathrm{Pr}_{\mathrm{eff}}}}. \tag{13}$$

Using the inverse Laplace transform of that above equation, we obtained

$$\theta(y,t) = \frac{2\sqrt{t}}{\sqrt{\mathrm{Pr}_{\mathrm{eff}}}} \times \left[\frac{1}{\sqrt{\pi}} \exp\left(-\frac{y^2 \mathrm{Pr}_{\mathrm{eff}}}{4t}\right) - \frac{y\sqrt{\mathrm{Pr}_{\mathrm{eff}}}}{2\sqrt{t}} erfc\left(\frac{y\sqrt{\mathrm{Pr}_{\mathrm{eff}}}}{2\sqrt{t}}\right)\right], \tag{14}$$

where $\mathrm{Pr}_{\mathrm{eff}} = \mathrm{Pr}/(1+N_r)$ is the effective Prandtl number.

3.2. Nusselt Number. Nu is the Nusselt number; it is the rate of heat transfer at the plate and can be found by differentiating the expression of temperature given in equation (17). With respect to "y" and using $y = 0$, we get

$$\mathrm{Nu}(t) = -1. \tag{15}$$

3.3. Velocity Field. We take the Laplace transform technique into equation (10), and using the transform initial and boundary conditions, we obtained

$$\overline{u}(y,s) = F(s)e^{-y\sqrt{h+s}} + \left(\frac{Gr}{\sqrt{\mathrm{Pr}_{\mathrm{eff}}} s^{3/2}(s\mathrm{Pr}_{\mathrm{eff}} - s - h)}\right)e^{-y\sqrt{h+s}} - \left(\frac{Gr}{\sqrt{\mathrm{Pr}_{\mathrm{eff}}} s^{3/2}(s\mathrm{Pr}_{\mathrm{eff}} - s - h)}\right)e^{-y\sqrt{s\mathrm{Pr}_{\mathrm{eff}}}}. \tag{16}$$

Inverse Laplace transform is applied on the above equation, and we develop

$$u(y,t) = \Phi(y,t) + k_j(y,t). \tag{17}$$

Also,

$$\Phi(y,t) = \int_0^t \left(\frac{U(t)y}{2t\sqrt{\pi t}} e^{-ht-(y^2/4t)}\right) f(t-\xi)\mathrm{d}\xi. \tag{18}$$

Here $U(t)$ is the Heaviside unit function.

$$k_j(y,t) = \frac{Gr}{h}\left[-\frac{1}{\pi}\left[\int_0^\infty \frac{\cos(y\sqrt{x})}{(x+h)^{3/2}}\left[1 - e^{-(x+h)t}\right]\mathrm{d}x\right] + \int_0^h \frac{e^{-y\sqrt{x+h}}}{x^{3/2}}\left[1 - e^{-xt}\right]\mathrm{d}x\right] + 2\sqrt{\frac{t}{\pi}}\exp\left(-\frac{y^2}{4t}\right) - y\,erfc\left(\frac{y}{2\sqrt{t}}\right). \tag{19}$$

4. Applications

To determine the application of this problem solution in (14) and (17), we consider some limiting and special cases.

4.1. Limiting Case

(1) During the non-appearance of radiation.

In the non-appearance of thermal radiation, it corresponds to $N_r = 0$; then, the dimensionless temperature $\theta(y,t)$ is

$$\theta(y,t) = \frac{2\sqrt{t}}{\sqrt{\mathrm{Pr}}}\left[\frac{1}{\sqrt{\pi}}\exp\left(-\frac{y^2\mathrm{Pr}}{4t}\right) - \frac{y\sqrt{\mathrm{Pr}}}{2\sqrt{t}} erfc\left(\frac{y\sqrt{\mathrm{Pr}}}{2\sqrt{t}}\right)\right]. \tag{20}$$

4.2. Special Cases

Case 1. Plate motion with constant velocity $f(t) = U(t)$.

In this case, single impulse of plate is taken, so we take $f(t) = U(t)$.

Now by substitution, we get

$$u(y,t) = \frac{U(t)}{2}\left\{e^{-z\sqrt{h}}\, erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{ht}\right) + e^{z\sqrt{h}}\, erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{ht}\right)\right\} + K_j(y,t), \tag{21}$$

and using (21), we get skin friction

$$\begin{aligned}\tau &= \frac{\partial u}{\partial y}\Big|_{y=0} \\ &= -U(t)\sqrt{h}\, erf\sqrt{ht} - \frac{U(t)}{\sqrt{\pi t}}e^{-ht} + \tau_{0j},\end{aligned} \tag{22}$$

where $\tau_{0j}, j = 1, 2$.

$$\tau_{0j} = \frac{\partial A_j(y,t)}{\partial y}\Big|_{y=0}. \tag{23}$$

For $\Pr_{\text{eff}} \neq 1, \tau_{01}$ and for $\Pr_{\text{eff}} = 1, \tau_{02}$,

$$\begin{aligned}\tau_{01} &= \frac{Gr}{h\sqrt{\Pr_{\text{eff}}}}\left[\left(1 - e^{bt}\right)\sqrt{\Pr_{\text{eff}}} - \frac{1}{\pi}\int_0^h \frac{\sqrt{h+x}}{x^{3/2}}\left[1 - e^{-tx}\right]dx + \frac{e^{bt}}{\pi}\int_0^h \frac{\sqrt{h+x}}{(b+x)\sqrt{x}}\left[1 - e^{-t(b+x)}dx\right]\right], \\ \tau_{02} &= \frac{Gr}{h}\left[1 - \frac{1}{\pi}\int_0^h \frac{\sqrt{h+x}}{x^{3/2}}\left[1 - e^{-tx}\right]dx\right].\end{aligned} \tag{24}$$

Case 2. For accelerated plate motion

$$f(t) = \frac{t}{t_0}U(t), \tag{25}$$

t_0 is a constant, so in this situation, this expression provides complex velocity field

$$u(y,t) = \frac{U(t)}{t_0}\left\{\left(t - \frac{y}{2\sqrt{h}}\right)e^{-y\sqrt{h}}\, erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{ht}\right) + \left(t + \frac{y}{2\sqrt{h}}\right)e^{y\sqrt{h}}\, erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{ht}\right)\right\} + K_j(y,t). \tag{26}$$

The expression for the skin friction is given by

$$\tau = -\frac{U(t)}{t_0}\left\{\left(\frac{1}{2\sqrt{h}} + t\sqrt{h}\right)erf(\sqrt{ht}) + \sqrt{\frac{t}{\pi}}e^{-ht}\right\} + \tau_{0j}. \tag{27}$$

Case 3. For decaying oscillatory motion

$$f(t) = \text{Re}\left[U(t)e^{-(\lambda^2 - i\omega)t}\right], \tag{28}$$

λ and ω are dimensionless constants. Put (28) into (17), and we have

$$\begin{aligned}u(y,t) = \frac{U(t)}{2}\text{Re}\Bigg[e^{-(\lambda^2 - i\omega)t}\Bigg\{&e^{-y\sqrt{(h-\lambda^2+i\omega)t}} \times erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{(h - \lambda^2 + i\omega)t}\right) + \\ &\times e^{y\sqrt{(h-\lambda^2+i\omega)t}} \times erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{(h - \lambda^2 + i\omega)t}\right)\Bigg\}\Bigg] + K_j(y,t).\end{aligned} \tag{29}$$

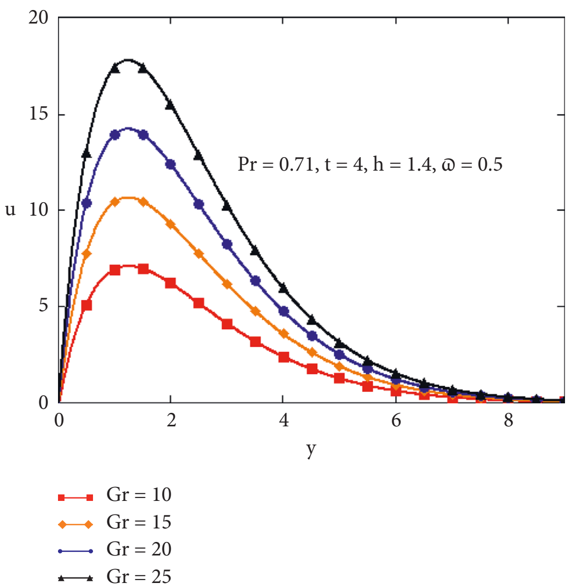

Figure 1: Velocity profile for different values of Gr.

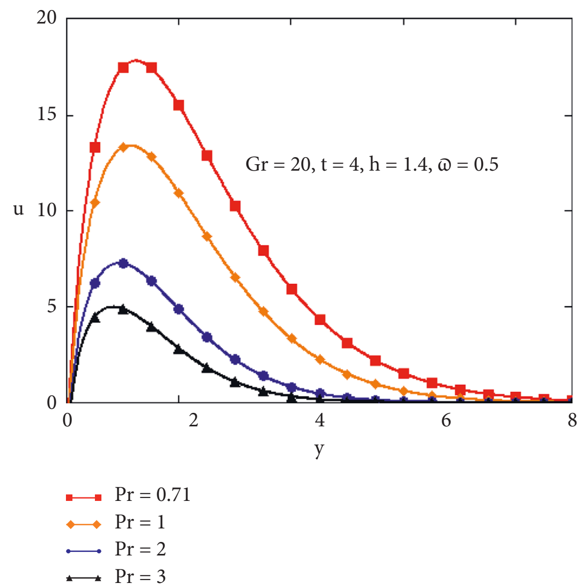

Figure 3: Velocity profile for different values of Pr.

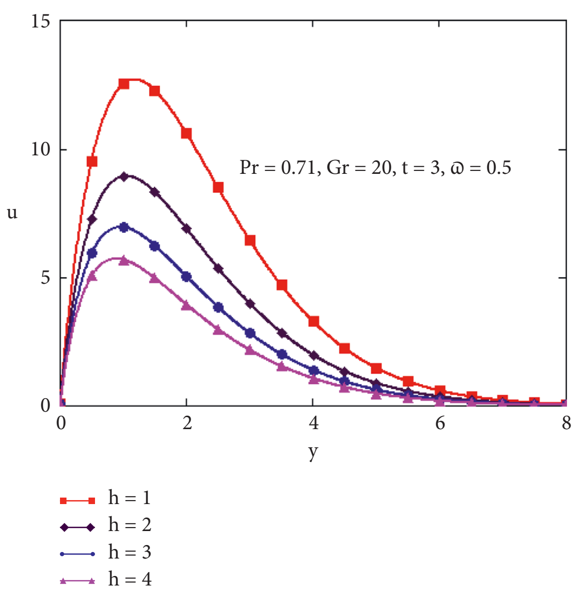

Figure 2: Velocity profile for different values of h.

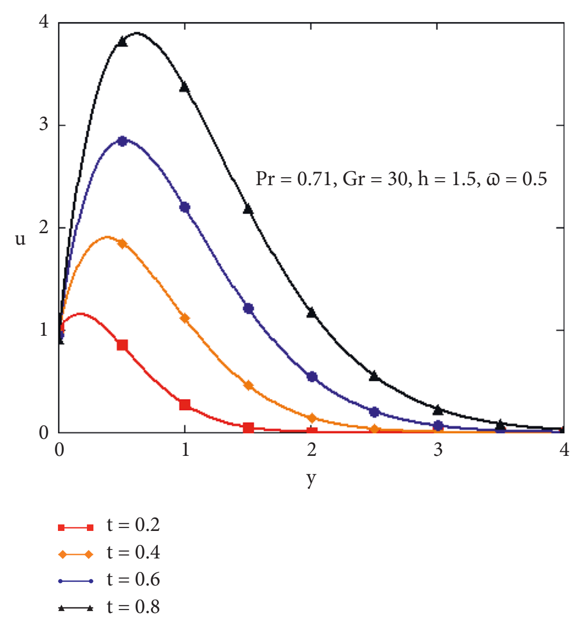

Figure 4: Velocity profile for different values of t.

The skin friction expression is

$$\tau = -\frac{U(t)}{2}\mathrm{Re}\left(e^{-(\lambda^2 - i\omega)t}\sqrt{\left(h-\lambda^2+i\omega\right)}\, erf\sqrt{\left(h-\lambda^2+i\omega\right)t}\right) + \tau_{0j}. \tag{30}$$

5. Results and Discussion

We can study the general solution for unsteady MHD natural convection flow with arbitrary motion of the infinite vertical plate. The governing equations which are dimensionless can be explained with the techniques of Laplace transform, and we can obtain a solution of temperature and velocity in the form of special function (error and complementary error functions). It satisfies all the given conditions.

For understanding different parameters for the given problem, different calculations can be done for temperature,

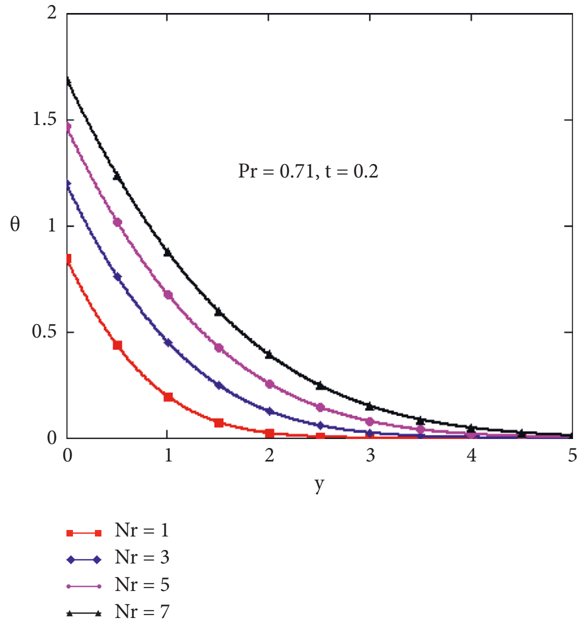

Figure 5: Temperature profile for different values of Nr.

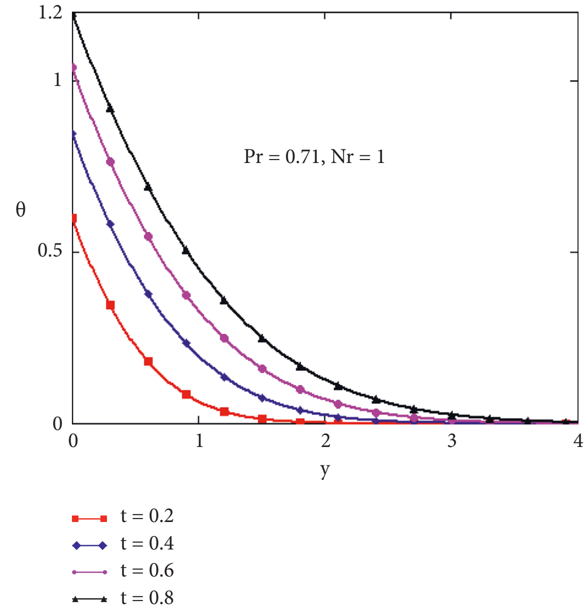

Figure 7: Temperature profile for different values of t.

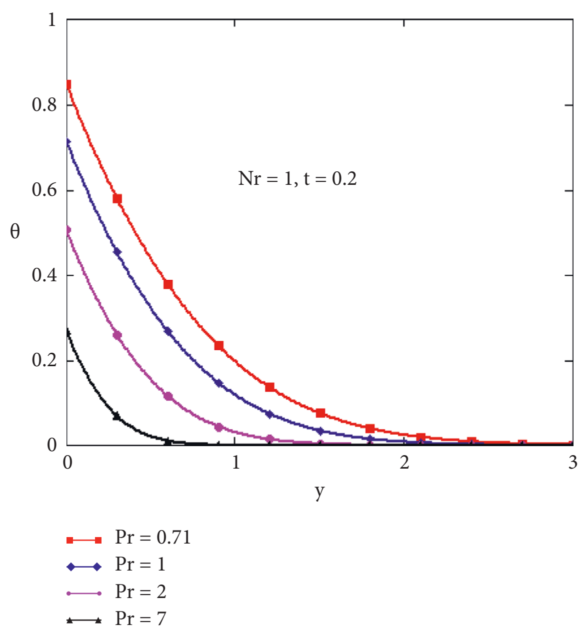

Figure 6: Temperature profile for different values of Pr.

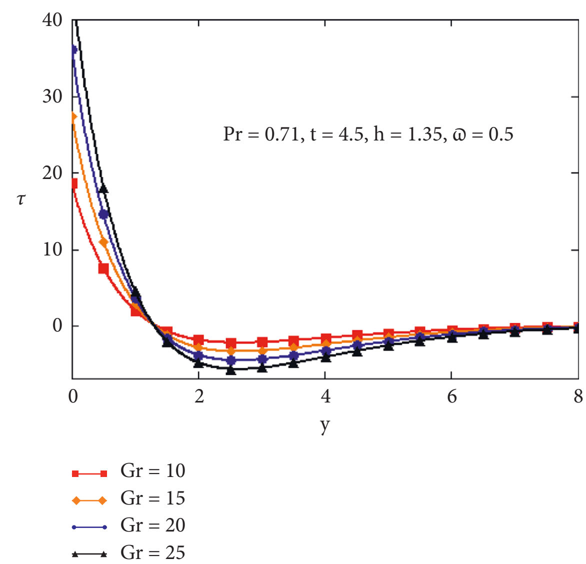

Figure 8: Skin friction profile for different values of Gr.

velocity, and skin friction. For changed values of pertinent parameters, i.e., Grashof number, radiation parameter, phase angle, magnetic parameter, Prandtl number, and time, Gr equal to zero links to the non-appearance of free convection currents, whereas Gr greater than zero links to externally cooled plate. Here we cannot discuss Gr = 0, and we can start our graph journey with Grashof number. In Figure 1, we get that velocity increases with the increase of Grashof number Gr. Here we cannot discuss Gr for negative values, but similarly, velocity decreases with the decrease of Grashof number Gr. In Figure 2, the fluid velocity falls by increasing parameter h. Figure 3 shows the motion of fluid for different values of Pr. For a large value of Pr, velocity is an increasing function of time and decays fluid motion. Physically, this is correct since an increase in the Pr increases the viscosity of the fluid, making it thicker and causing a drop in the fluid's velocity. In Figure 4, velocity profiles because of variations in time t are shown for heating and cooling of the plate; it can be understood that velocity increases with time for the occasion of plate heating; similarly, velocity decreases with time for the situation of cooling of the plate.

Figures 5–7 show the temperature profiles for different values of N_r, Pr, and t. Clearly, Figure 5 indicates that when

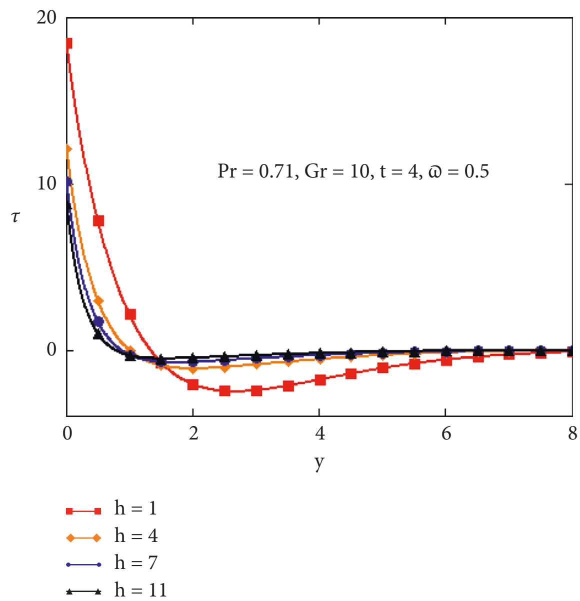

Figure 9: Skin friction profile for different values of h.

the value of the radiation parameter N_r increases, then the temperature will be decreased. Due to the existence of radiation, the existing thermal boundary layer will thicken. This is actually because radiations provide an extra means for diffuse energy. From Figure 6, we can see that the temperature decreases with the increase of Pr, thickness of air thermal layer of boundary (Pr = 0.71) is larger, and the temperature shared through the thermal boundary layer is more uniform relative to water (Pr = 7) and electrolytic solution (Pr = 1.0). The explanation is that lower Prandtl number values contribute to an increase in thermal conductivity, which makes it easier to disperse heat farther than higher from the heated surface Prandtl number values. Therefore, the water temperature decreases faster than that of air as well as an electrolytic solution. From Figure 7, it is found that with increased time in the existence of the radiation, the temperature would rise. It is also noted that by increasing time values, the thermal boundary layer thickens.

The skin friction graph is shown in Figures 8 and 9. From Figure 8, we can see that when the Gr value is negative, the skin friction at some time tends to go up, but after some time, it comes down and asymptotically reaches near the surface. Then, when the value of Gr is positive, the skin friction moves downward; after some time, it moves upward. In Figure 9, a similar behaviour is shown by varying values of h.

6. Conclusion

In this study, we can obtain a general solution for unsteady MHD natural convection flow with arbitrary motion of the infinite vertical plate. The governing equations are analytically resolved by using the method of Laplace transform. Finally, we obtained the solution for temperature, velocity, and skin friction. In velocity profiles, we observe that velocity increases with increasing Grashof numbers Gr and permeability of porous medium K. While for increasing M, velocity decreases. It has been observed that temperature increases by increasing time t and radiation parameter Nr. The behaviour in all the figures corresponding to the Skin friction is such that at some stage it reaches upward; then after some time, it comes downward.

Nomenclature

T: Fluid temperature
g: Acceleration due to gravity
M: Magnetic parameter
q: Constant heat flux
K: Permeability of porous medium
s: Laplace transform parameter
U: Amplitude of the motion
Gr: Thermal Grashof number
k: Thermal conductivity of the fluid
Pr: Prandtl number
Nu: Nusselt number
C_p: Specific heat at a constant pressure
q_r: Radiative heat flux in y−direction
T_∞: Fluid temperature far away from the plate
C_∞: Fluid concentration far away from the plate
N_r: Radiation parameter
T_w: Temperature at the plate
Pr_{eff}: Effective Prandtl number
B_0: Uniform magnetic field
t_0: Characteristic time
ν: Kinematic viscosity
μ: Dynamic viscosity
β: Volumetric coefficient of thermal expansion
ρ: Fluid density
σ: Electrical conductivity
ω: Frequency of vibrations.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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