Hindawi
Journal of Mathematics
Volume 2023, Article ID 2017167, 21 pages
https://doi.org/10.1155/2023/2017167

Research Article

A New Conformable Fractional-Order Time-Delay Grey Bernoulli Model with the Arithmetic Optimization Algorithm and Its Application in Rural Regional Economy

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Received 24 June 2022; Revised 3 November 2022; Accepted 24 November 2022; Published 11 January 2023

Academic Editor: Wendong Yang

To further promote the development of the grey system theory, this paper develops a novel conformable fractional-order grey Bernoulli model with a time-delay effect, namely, the CFTDNGBM (1, 1) model. In addition, the arithmetic optimization algorithm (AOA) is incorporated into the system of the model to solve the hyperparameters existing in the model. Compared with the previous grey prediction models, the CFTDNGBM (1, 1) model with a conformable fractional-order accumulation operation (CFAO), time-delay factor, and Bernoulli parameter has stronger compatibility in structure. The proposed model and its nine competitive models with excellent performance are used to predict and analyze the consumption level and per capita consumption expenditure of rural residents in China to verify the feasibility of the proposed method. The case results show that in both cases, the seven descriptive indicators of the CFTDNGBM (1, 1) model are higher than those of its competing models. Therefore, the CFTDNGBM (1, 1) model has a certain application value.

1. Introduction

In recent years, due to the development of deep learning, people pay more attention to the deep learning models based on large sample data sets and ignore the small sample learning method, which is more serious in time series analysis. Prediction is the most important part of time series analysis. An accurate prediction can enable decision-makers to make correct decisions based on the prediction results. For time series with small data sets, the deep learning model based on large samples is obviously no longer applicable. Based on this, we turn our attention to the grey prediction model which can realize the small sample modeling model.

The grey system model, as a small sample learning method, has been praised for its simple modeling mechanism and high predictive performance [1–5]. With the development of society, many derivative models and optimization measures based on the most basic GM (1, 1) model have been proposed. These methods can be roughly divided into three parts, namely, structural optimization, hybrid optimization, and optimization based on data enhancement methods. Table 1 shows some important work.

From Table 1, we can see that the structural optimization of the grey model mainly focuses on the optimization of the grey action of the model. Scholars hope to enhance the adaptive performance of the model by using functions with strong fitting performance to replace the traditional constant grey action. This class of models can be further divided into two parts, namely, polynomial function-based expansion models and time-delay term-based expansion models. Up to now, the polynomial function-based expansion model has gradually converged to perfection because the PTGM (1, 1, α) model proposed by Liu et al. is a unified expression of most existing grey prediction models. In contrast, the time-delay term-based extended model has received less attention because of its more complex structure, which has led to its

Table 1: Some important work on grey models.

Author	Model	Details
Structural optimization		
Cui et al. [6]	NGM (1, 1, k, c)	A linear function is used instead of the constant as the grey action in the traditional grey model
Qian et al. [7]	GM (1, 1, t^{α})	A grey prediction model with a power term with hyperparameter α as grey action
Wei et al. [8]	GMP (1, 1, N)	A model is developed by using polynomial as the grey action of the grey model
Liu et al. [9]	PTGM (1, 1, α)	Combination of GMP (1, 1, N) and GM (1, 1, t^{α})
Saxena [10]	OFOPGM	A new data-driven grey prediction model using the time item with two hyperparameters as the action
Wu et al. [11]	NGBM (1, 1, k, c)	A NGM (1, 1, k, c) model with nonlinear Bernoulli operators
Liu et al. [12]	NGBM (1, 1, N)	A GMP (1, 1, N) model with nonlinear Bernoulli operators
Ma and Liu [13]	TDPGM (1, 1)	A grey model with the function called time-delayed polynomial as the grey action
Ma et al. [14]	FTDGM	A model is developed by using fractional time delayed term as the grey action of the grey model
Xiang et al. [15]	HTGM (1, 1)	A prediction algorithm using hyperbolic time delayed polynomial as grey action
Salehi and Dehnavi [16]	NGBM (1, 1)	Grey prediction model with the Bernoulli parameter
Khan and Osinska [17]	ONGBM (1, 1)	A model that optimizes the background value of the model by using the integral mean value theorem
Hybrid optimization		
Ofosu-Adarkwa et al. [3]	Verhulst-GM (1, N)	A new prediction algorithm by combining Verhulst and GM (1, N) models
Nguyen et al. [18]	Fourier-NGBM (1, 1)	The Fourier transformation is used to correct the residuals of the NGBM (1, 1) model
Wang et al. [19]	Hybrid model	A new hybrid forecasting model based on the Super-SBM DEA and GM (1, 1) model
Guefano et al. [20]	Hybrid model	A combination of GM (1, 1) and VAR (1)
Zhou et al. [21]	Hybrid model	A combination of GM (1, 1) and ARIMA
Saxena [2]	IOGM	Grey forecasting models based on internal optimization
Optimization based on data enhancement methods		
Wu et al. [22]	FGM (1, 1)	The author extends the traditional first-order accumulation operation to fractional-order accumulation operation and establishes GM (1, 1) model with fractional accumulation operation on this basis
Ma et al. [23]	CFGM (1, 1)	Similar to the FGM model, the difference is that the fractional accumulation operation is replaced by the conformable fractional accumulation operation
Chen et al. [24]	FHGM (1, 1)	Hausdorff fractional GM (1, 1) model
Javed and Cudjoe [25]	DGM (1, 1, α)	Discrete GM (1, 1) with conformable fractional accumulation operation
Şahin [26]	ROFANGBM (1, 1)	A novel optimized fractional nonlinear grey Bernoulli model with rolling mechanism
Liu et al. [27]	WFDPGM (1, 1, t^{α})	DiscretePTGM (1, 1, α) with weighted fractional accumulation

slower progress. It is important to note that extended models based on time-delay terms are not time-delay models in the traditional sense; they are simply prediction algorithms that use a so-called time-delay polynomial instead of the grey action of the GM (1,1) model. Although the extended models based on time delay have received little attention, some existing studies show that the extended models based on time delay usually outperform the extended models based on polynomial functions. Therefore, the time-delay term-based extended models seem to be more desirable, both in terms of refining the grey prediction domain and building efficient prediction models for predictive analysis. In addition, most of the grey models based on structural optimization have corresponding discrete forms, which appear to address the shortcomings of traditional grey forecasting models that do not satisfy unbiasedness. Therefore, their discrete forms are more reasonable and reliable than the traditional grey prediction models based on ordinary differential equations.

Hybrid optimization focuses on combining grey forecasting algorithms with other forecasting models or correction measures, with the aim of developing an efficient new forecasting algorithm by combining the advantages of two or more methods. However, such optimization methods are heavily influenced by the performance of the model used for the combination. Therefore, the establishment of an efficient grey forecasting model is still the top priority. Optimization based on data enhancement methods mainly uses various fractional-order accumulation operations to enhance the initial data and then build the model, which usually has better prediction performance than traditional models due to the strong adaptive performance of fractional-order accumulation operations. However, the method of determining the hyperparameters in such models is still open to discussion.

According to the previous description, we can know that the attention of the time delay grey prediction model is less than that of other models, which may lead to the unbalanced development of this field. In addition, data enhancement and discretization measures can effectively improve the prediction performance of the model. To promote the development of grey system theory, this paper develops a new grey prediction model with a time-delay polynomial based on discrete operations, the conformable fractional accumulation operator (CFAO), and the Bernoulli operator. It can be seen that the constructed model combines all the advantages of the existing optimization methods, which makes it have stronger fitting performance than the earlier grey prediction models.

In particular, to reduce the complexity of the model, we set the conformable fractional accumulation operator and the time-delay operator to the same hyperparameter. In addition, the arithmetic optimization algorithm (AOA) with a relatively simple mechanism is used to solve the hyperparameters existing in the model. The specific contributions of this paper are as follows:

(1) We establish a discrete time delay polynomial grey prediction model with the Bernoulli operator
(2) The AOA is used to solve the planning problem based on the proposed model
(3) The proposed model is used to study the development of China's rural regional economies

The other parts of this paper are arranged as follows: Section 2 introduces the proposed model, including its modeling steps and solving methods. Section 3 introduces the application of the proposed model in rural regional economies. Section 4 is the summary of this article.

2. Methods

In this section, we will develop a new time-delay polynomial grey prediction model based on the conformable fractional accumulation operator (CFAO) [28], the NGM (1,1,*k*,*c*) [6], and the FTDGM model [14] and introduce its modeling mechanism and solution method in detail.

2.1. Conformable Fractional Accumulation Operator-CFAO. Let $L^{(0)} = [l^{(0)}(1), \ldots, l^{(0)}(k)], k = 1, \ldots, n$ be a time series composed of original data, then its conformable fractional accumulation generation sequence is as follows:

$$L^{(\gamma)} = \left[l^{(\gamma)}(1), \ldots, l^{(\gamma)}(k)\right], l^{(\gamma)}(k) = \sum_{j=1}^{k} \begin{pmatrix} k-j-1+[\gamma] \\ k-j \end{pmatrix} \frac{l^{(0)}(j)}{j^{[\gamma]-\gamma}}, \tag{1}$$

where $\begin{pmatrix} k-j-1+[\gamma] \\ k-j \end{pmatrix} = (k-j+[\gamma]-1)!/(k-j)!([\gamma]-1)!$, $\gamma > 0$, $[\gamma]$ means forward rounding, such as $[1.7] = 2$[28].

According to [28], the inverse accumulative generating operator of $L^{(0)}$ can be represented by the following way:

$$l^{(0)}(k) = k^{[\gamma]-\gamma} \cdot \sum_{j=1}^{k} (-1)^{k-j} \begin{pmatrix} [\gamma] \\ k-j \end{pmatrix} \cdot l^{(\gamma)}(j), \tag{2}$$

where $\begin{pmatrix} [\gamma] \\ k-j \end{pmatrix} = [\gamma]!/([\gamma]-k+j)!(k-j)$. In particular, when $r = 1$, the CFAO degenerates into the 1-order accumulation operator (1-AGO).

2.2. Conformable Fractional Grey Bernoulli Model with Time-Delay Polynomial. If $x^{(\gamma)}(k), k = 1, \ldots, n$ is the conformable fractional accumulation sequence of $x^{(0)}(k)$, the following nonlinear ordinary differential equation

$$\frac{dx^{(\gamma)}(t)}{dt} + ax^{(\gamma)}(t) = \left(b\sum_{\tau=1}^{t} \tau^{(\gamma)} + ct + d\right) \cdot \left[x^{(\gamma)}(t)\right]^{\eta}, \gamma \geq 0, \eta \neq 1, \tag{3}$$

is called the whitening differential equation of the conformable fractional grey Bernoulli model with a time-delay polynomial (CFTNGBM (1,1) model), where $\tau^{(\gamma)} = \sum_{j=1}^{\tau} \begin{pmatrix} \tau-j+\gamma-1 \\ \tau-j \end{pmatrix} \cdot j = \Gamma(\tau+\gamma+1)/\Gamma(\tau) \cdot \Gamma(2+\gamma)$, $\sum_{\tau=1}^{t} \tau^{(\gamma)}$ is the time-delay polynomial and η is a nonlinear Bernoulli parameter since the general solution of the equation does not exist when $\eta = 1$, $\eta \neq 1$.

By multiplying both sides of Equation (3) by $[x^{(\gamma)}(t)]^{-\eta}$, we can get

$$\frac{dx^{(\gamma)}(t)}{dt} \cdot \left[x^{(\gamma)}(t)\right]^{-\eta} + a\left[x^{(\gamma)}(t)\right]^{1-\eta} = b\sum_{\tau=1}^{t} \tau^{(\gamma)} + ct + d. \tag{4}$$

We set $Q^{(\gamma)}(t) = [x^{(\gamma)}(t)]^{1-\eta}$, then equation (4) converts to

$$\frac{dQ^{(\gamma)}(t)}{dt} \cdot \frac{1}{1-\eta} + a \cdot Q^{(\gamma)}(t) = b\sum_{\tau=1}^{t} \tau^{(\gamma)} + ct + d, \tag{5}$$

namely,

$$\frac{dQ^{(\gamma)}(t)}{dt} + a_1 \cdot Q^{(\gamma)}(t) = b_1 \cdot \sum_{\tau=1}^{t} \tau^{(\gamma)} + c_1 \cdot t + d_1, \tag{6}$$

where $a_1 = a \cdot (1-\eta); b_1 = b \cdot (1-\eta); c_1 = c \cdot (1-\eta); d_1 = d \cdot (1-\eta)$.

The integral form of Equation (6) on an interval $[t-1, t]$ is used to estimate the least square parameters of the model, namely,

$$\left[\hat{a}_1, \hat{b}_1, \hat{c}_1, \hat{d}_1\right]^T = \left(\Xi^T \Xi\right)^{-1} \Xi^T \Psi, \tag{7}$$

where

$$\Xi = \begin{bmatrix} -0.5 \cdot \left[Q^{(\gamma)}(2) + Q^{(\gamma)}(1)\right] & 0.5 \cdot \left[\sum_{\tau=1}^{2} \tau^{(\gamma)} + \sum_{\tau=1}^{1} \tau^{(\gamma)}\right] & \frac{2^2 - 1^2}{2} & 1 \\ -0.5 \cdot \left[Q^{(\gamma)}(3) + Q^{(\gamma)}(2)\right] & 0.5 \cdot \left[\sum_{\tau=1}^{3} \tau^{(\gamma)} + \sum_{\tau=1}^{2} \tau^{(\gamma)}\right] & \frac{3^2 - 2^2}{2} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ -0.5 \cdot \left[Q^{(\gamma)}(n) + Q^{(\gamma)}(n-1)\right] & 0.5 \cdot \left[\sum_{\tau=1}^{n} \tau^{(\gamma)} + \sum_{\tau=1}^{n-1} \tau^{(\gamma)}\right] & \frac{n^2 - (n-1)^2}{2} & 1 \end{bmatrix}, \Psi = \begin{bmatrix} Q^{(\gamma-1)}(2) \\ Q^{(\gamma-1)}(3) \\ \vdots \\ Q^{(\gamma-1)}(n) \end{bmatrix}. \tag{8}$$

Taking $x^{(\gamma)}(1) = x^{(0)}(1)$ as the initial condition to solve Equation (6), we can get

$$\hat{Q}^{(\gamma)}(t) = Q^{(\gamma)}(1)e^{-a_1(t-1)} + \int_1^t \left(b_1 \cdot \sum_{\tau=1}^{v} \tau^{(\gamma)} + c_1 \cdot v + d_1\right) e^{-a_1(t-v)} dv. \tag{9}$$

Furthermore, we can obtain the time response function of the model, namely,

$$\hat{x}^{(\gamma)}(t) = \left[Q^{(\gamma)}(1)e^{-a_1(t-1)} + \sum_{v=2}^{t} e^{-a_1(t-v+0.5)} \cdot 0.5 \cdot \left[f(v) + f(v-1)\right]\right]^{1/1-\eta}, \tag{10}$$

where $f(v) = b_1 \cdot \sum_{\tau=1}^{v} \tau^{(\gamma)} + c_1 \cdot v + d_1$.

The output values of this model can be obtained by using Equation (2), namely,

$$\hat{x}^{(0)}(k) = k^{[\gamma]-\gamma} \cdot \sum_{j=1}^{k} (-1)^{k-j} \binom{[\gamma]}{k-j} \cdot \hat{x}^{(\gamma)}(j). \tag{11}$$

2.3. Discrete CFTNGBM (1, 1) Model Satisfying Unbiasedness. In this section, we will discretize the CFTNGBM (1, 1) model to make it unbiased. First, we conduct an unbiased analysis of the CFTNGBM (1, 1) model, as shown in the following paragraph.

If the proposed model satisfies unbiasedness, then we have the following relationship:

$$\begin{aligned} L(t) &= \left[\widehat{Q}^{(\gamma)}(t) - \widehat{Q}^{(\gamma)}(t-1)\right] + a_1 \cdot 0.5 \cdot \left[\widehat{Q}^{(\gamma)}(t) + \widehat{Q}^{(\gamma)}(t-1)\right] \\ &= b_1 \cdot \sum_{\tau=1}^{t} \tau^{(\gamma)} + c_1 \cdot t + d_1 \\ &= R(t). \end{aligned} \quad (12)$$

Obviously, we can see from Equation (9) that the previous relationship is not tenable, so the proposed model does not satisfy the unbiasedness. Based on this, we will discretize the model to make it unbiased.

Based on the first-order backward difference, Equation (6) can be expressed as follows:

$$Q^{(\gamma)}(t) - Q^{(\gamma)}(t-1) + a_1 \cdot Q^{(\gamma)}(t) = b_1 \cdot \sum_{\tau=1}^{t} \tau^{(\gamma)} + c_1 \cdot t + d_1, \quad (13)$$

that is,

$$Q^{(\gamma)}(t) = \frac{1}{(1+a_1)} Q^{(\gamma)}(t-1) + \frac{b_1}{(1+a_1)} \sum_{\tau=1}^{t} \tau^{(\gamma)} + \frac{c_1}{(1+a_1)} t + \frac{d_1}{(1+a_1)}. \quad (14)$$

We set $\alpha = 1/(1+a_1); \beta = b_1/(1+a_1); C = c_1/(1+a_1); D = d_1/(1+a_1)$, then Equation (14) converts to the following:

$$Q^{(\gamma)}(t) = \alpha \cdot Q^{(\gamma)}(t-1) + \beta \cdot \sum_{\tau=1}^{t} \tau^{(\gamma)} + Ct + D. \quad (15)$$

which is called the expression of the discrete CFTNGBM (1,1) model-CFTDNGBM (1,1).

To minimize the estimation error, we present the following unconstrained programming problem:

$$P = \sum_{j=2}^{n} \left[Q^{(\gamma)}(j) - \widehat{Q}^{(\gamma)}(j)\right]^2 = (U - Z\widehat{\iota})^T (U - Z\widehat{\iota}), \quad (16)$$

where

$$Z = \begin{bmatrix} Q^{(\gamma)}(1) & \sum_{\tau=1}^{2} \tau^{(\gamma)} & 2 & 1 \\ Q^{(\gamma)}(2) & \sum_{\tau=1}^{3} \tau^{(\gamma)} & 3 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ Q^{(\gamma)}(n-1) & \sum_{\tau=1}^{n} \tau^{(\gamma)} & n & 1 \end{bmatrix}, \quad U = \begin{bmatrix} Q^{(\gamma)}(2) \\ Q^{(\gamma)}(3) \\ \vdots \\ Q^{(\gamma)}(n) \end{bmatrix}, \quad (17)$$

$Q^{(\gamma)}(j)$ and $\widehat{Q}^{(\gamma)}(j)$ are the actual values and estimated values, respectively. According to the extreme value existence condition, we have $\partial P/\partial \iota = 0$, namely,

$$\begin{aligned}
\frac{\partial P}{\partial \iota} &= \frac{\partial\left[(U - Z\hat{\iota})^T (U - Z\hat{\iota})\right]}{\partial \iota} \\
&= \frac{2(U - Z\hat{\iota})\partial(U - Z\hat{\iota})^T}{\partial \iota} \\
&= 2\left[\frac{\partial U^T}{\partial \hat{\iota}} - \frac{\partial\left(\hat{\iota}^T Z^T\right)}{\partial \hat{\iota}}\right](U - Z\hat{\iota}) \\
&= -2Z^T (U - Z\hat{\iota}) \\
&= 2Z^T Z\hat{\iota} - 2Z^T U \\
&= 0.
\end{aligned} \tag{18}$$

Therefore, we know that the least square estimate of the model is $\hat{\iota} = \left[\hat{\alpha}\ \hat{\beta}\ \hat{C}\ \hat{D}\right]^T = (Z^T Z)^{-1} Z^T U$.

When $t = 2$, Equation (15) converts to the following:

$$Q^{(\gamma)}(2) = \alpha Q^{(\gamma)}(1) + \beta \sum_{\tau=1}^{2} \tau^{(\gamma)} + 2C + D. \tag{19}$$

When $t = 3$, we have

$$\begin{aligned}
Q^{(\gamma)}(3) &= \alpha Q^{(\gamma)}(2) + \beta \sum_{\tau=1}^{3} \tau^{(\gamma)} + 3C + D \\
&= \alpha^2 Q^{(\gamma)}(1) + \alpha\beta \sum_{\tau=1}^{2} \tau^{(\gamma)} + 2\alpha C + \alpha D + \beta \sum_{\tau=1}^{3} \tau^{(\gamma)} + 3C + D \\
&= \alpha^2 Q^{(\gamma)}(1) + \beta\left[\alpha \sum_{\tau=1}^{2} \tau^{(\gamma)} + \sum_{\tau=1}^{3} \tau^{(\gamma)}\right] + C[2\alpha + 3] + D[\alpha + 1] \\
&= \alpha^{3-1} Q^{(\gamma)}(1) + \beta \sum_{j=0}^{3-2} \alpha^j \sum_{\tau=1}^{3-j} \tau^{(\gamma)} + C \sum_{j=0}^{3-2} \alpha^j (3 - j) + D \sum_{j=0}^{3-2} \alpha^j.
\end{aligned} \tag{20}$$

When $t = 4$, we can get

$$\begin{aligned}
Q^{(\gamma)}(4) &= \alpha Q^{(\gamma)}(3) + \beta \sum_{\tau=1}^{4} \tau^{(\gamma)} + 4C + D \\
&= \alpha^3 Q^{(\gamma)}(1) + \beta\left[\alpha^2 \sum_{\tau=1}^{2} \tau^{(\gamma)} + \alpha \sum_{\tau=1}^{3} \tau^{(\gamma)}\right] + C\left[2\alpha^2 + 3\alpha\right] \\
&\quad + D\left[\alpha^2 + \alpha\right] + \beta \sum_{\tau=1}^{4} \tau^{(\gamma)} + 4C + D = \alpha^3 Q^{(\gamma)}(1) \\
&\quad + \beta\left[\alpha^2 \sum_{\tau=1}^{2} \tau^{(\gamma)} + \alpha \sum_{\tau=1}^{3} \tau^{(\gamma)} + \sum_{\tau=1}^{4} \tau^{(\gamma)}\right] + C\left[2\alpha^2 + 3\alpha + 4\right] + D\left[\alpha^2 + \alpha + 1\right] \\
&= \alpha^{4-1} Q^{(\gamma)}(1) + \beta \sum_{j=0}^{4-2} \alpha^j \sum_{\tau=1}^{4-j} \tau^{(\gamma)} + C \sum_{j=0}^{4-2} \alpha^j (4 - j) + D \sum_{j=0}^{4-2} \alpha^j.
\end{aligned} \tag{21}$$

According to this law, we can obtain the recursive formula of Equation (15), namely,

$$Q^{(\gamma)}(k)=\alpha^{k-1}Q^{(\gamma)}(1)+\beta\sum_{j=0}^{k-2}\alpha^{j}\sum_{\tau=1}^{k-j}\tau^{(\gamma)}+C\sum_{j=0}^{k-2}\alpha^{j}(k-j)+D\sum_{j=0}^{k-2}\alpha^{j}, k=2,\ldots,n,\ldots,m, \tag{22}$$

then we can use $Q^{(\gamma)}(t)=[x^{(\gamma)}(t)]^{1-\eta}$ to restore Equation (22) to obtain the time response function of the CFTDNGBM (1,1) model, namely,

$$\widehat{x}^{(\gamma)}(k)=\left[\alpha^{k-1}Q^{(\gamma)}(1)+\beta\sum_{j=0}^{k-2}\alpha^{j}\sum_{\tau=1}^{k-j}\tau^{(\gamma)}+C\sum_{j=0}^{k-2}\alpha^{j}(k-j)+D\sum_{j=0}^{k-2}\alpha^{j}\right]^{1/1-\eta}, k=2,\ldots,n,\ldots,m. \tag{23}$$

According to Equation (2), we can know that the prediction formula of the CFTDNGBM (1,1) model is as follows:

$$\widehat{x}^{(0)}(k)=k^{[\gamma]-\gamma}\cdot\sum_{j=1}^{k}(-1)^{k-j}\binom{[\gamma]}{k-j}\cdot\widehat{x}^{(r)}(j). \tag{24}$$

Theorem 1. *CFTDNGBM (1,1) model satisfies unbiasedness.*

Proof. Suppose there is a time series $x^{(\gamma)}(t), t=1,\ldots,n$ satisfying Equation (22), namely,

$$Q^{(\gamma)}(k)=\alpha^{k-1}Q^{(\gamma)}(1)+\beta\sum_{j=0}^{k-2}\alpha^{j}\sum_{\tau=1}^{k-j}\tau^{(\gamma)}+C\sum_{j=0}^{k-2}\alpha^{j}(k-j)+D\sum_{j=0}^{k-2}\alpha^{j}, \tag{25}$$

where α,β,C,D are the given parameters, then we have

$$\begin{aligned}&Q^{(\gamma)}(t)-\alpha\cdot Q^{(\gamma)}(t-1)\\&=\beta\sum_{j=0}^{t-2}\alpha^{j}\sum_{\tau=1}^{t-j}\tau^{(\gamma)}-\beta\sum_{j=1}^{t-2}\alpha^{j}\sum_{\tau=1}^{t-j}\tau^{(\gamma)}+C\sum_{j=0}^{t-2}\alpha^{j}(t-j)-\alpha C\sum_{j=0}^{t-3}\alpha^{j}(t-1-j)+D\sum_{j=0}^{t-2}\alpha^{j}-D\sum_{j=1}^{t-2}\alpha^{j}\\&=\beta\cdot\sum_{\tau=1}^{t}\tau^{(\gamma)}+Ct+D,\end{aligned} \tag{26}$$

which means that $x^{(\gamma)}(t), t=1,\ldots,n$ satisfies the proposed model. We can easily give the matrix form of equation (26), namely, $Z'\iota=U'$, where the definitions of U', Z' are similar to equation (16). Furthermore, we can get $(Z')^{T}Z'\iota=(Z')^{T}U'$. Obviously, we have $\iota=\widehat{\iota}$, which means that our given parameters are consistent with the least squares estimate of the parameters of the model.

Based on $\iota=\widehat{\iota}$, we have

$$\begin{aligned}\widehat{Q}^{(\gamma)}(k)&=\widehat{\alpha}^{k-1}\widehat{Q}^{(\gamma)}(1)+\widehat{\beta}\sum_{j=0}^{k-2}\widehat{\alpha}^{j}\sum_{\tau=1}^{k-j}\tau^{(\gamma)}+\widehat{C}\sum_{j=0}^{k-2}\widehat{\alpha}^{j}(k-j)+\widehat{D}\sum_{j=0}^{k-2}\widehat{\alpha}^{j}\\&=\alpha^{k-1}Q^{(\gamma)}(1)+\beta\sum_{j=0}^{k-2}\alpha^{j}\sum_{\tau=1}^{k-j}\tau^{(\gamma)}+C\sum_{j=0}^{k-2}\alpha^{j}(k-j)+D\sum_{j=0}^{k-2}\alpha^{j}=Q^{(\gamma)}(k).\end{aligned} \tag{27}$$

which means that the model is unbiased. □

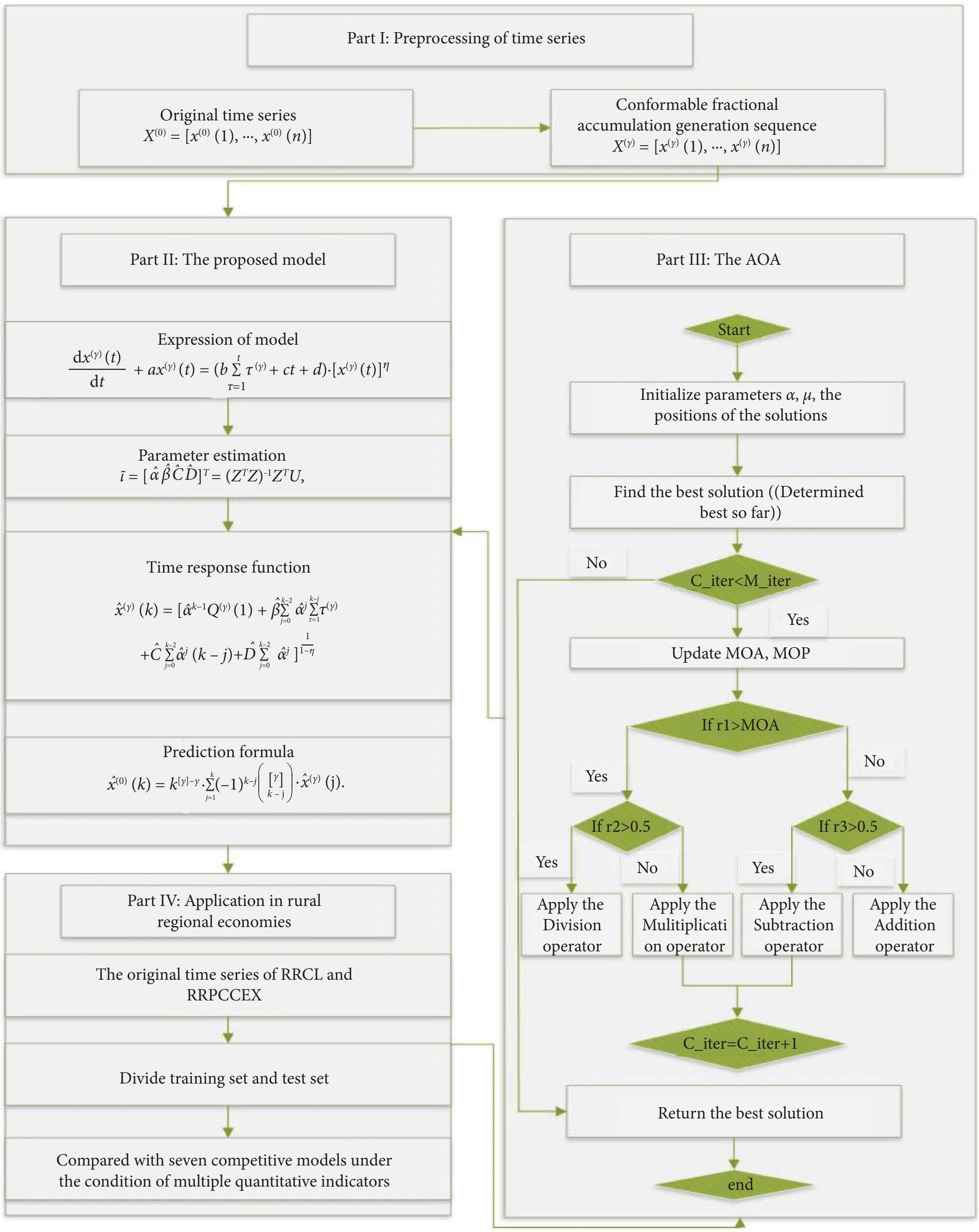

Figure 1: Flow chart of the proposed model.

Table 2: Raw data for RRPCCEX and RRPCCEX (Yuan).

Year	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
RRCL	2157	2292	2521	2784	3066	3538	3981	4295	4782	5880
Year	2012	2013	2014	2015	2016	2017	2018	2019	2020	—
RRCL	6573	7397	8365	9409	10609	12145	13985	15382	16046	—
Year	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
RRPCCEX	2050	2326	2749	3072	3536	4054	4464	4945	5892	6667
Year	2013	2014	2015	2016	2017	2018	2019	2020	2021	—
RRPCCEX	7485	8383	9223	10130	10955	12124	13328	13713	15916	—

2.4. The Solution Method of the Model. Obviously, due to the existence of hyperparameters γ and η, it is difficult for us to solve the CFTDNGBM (1, 1) model with traditional methods. Considering that the purpose of the model is to minimize the estimation error, here we take MAPE as the objective function and the modeling mechanism of the model as constraints to establish the following planning model.

$$\min_{\gamma,\eta} \text{fitness} = \frac{1}{n-1}\sum_{j=2}^{n}\left|\frac{x^{(\gamma)}(j)-\widehat{x}^{(\gamma)}(j)}{x^{(\gamma)}(j)}\right| \cdot 100\%,$$

$$\text{s.t.}\begin{cases} \gamma>0, \eta\neq 1, \widehat{\iota}=\left[\widehat{\alpha}\ \widehat{\beta}\ \widehat{C}\ \widehat{D}\right]^{T}=\left(Z^{T}Z\right)^{-1}Z^{T}U, \\ Z=\begin{bmatrix} Q^{(\gamma)}(1) & \sum_{\tau=1}^{2}\tau^{(\gamma)} & 2 & 1 \\ Q^{(\gamma)}(2) & \sum_{\tau=1}^{3}\tau^{(\gamma)} & 3 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ Q^{(\gamma)}(n-1) & \sum_{\tau=1}^{n}\tau^{(\gamma)} & n & 1 \end{bmatrix}, U=\begin{bmatrix} Q^{(\gamma)}(2) \\ Q^{(\gamma)}(3) \\ \vdots \\ Q^{(\gamma)}(n) \end{bmatrix}, \\ \widehat{x}^{(\gamma)}(k)=\left[\widehat{\alpha}^{k-1}Q^{(\gamma)}(1)+\widehat{\beta}\sum_{j=0}^{k-2}\widehat{\alpha}^{j}\sum_{\tau=1}^{k-j}\tau^{(\gamma)}+\widehat{C}\sum_{j=0}^{k-2}\widehat{\alpha}^{j}(k-j)+\widehat{D}\sum_{j=0}^{k-2}\widehat{\alpha}^{j}\right]^{1/1-\eta}, k=2,\ldots,n,\ldots,m, \\ \widehat{x}^{(0)}(k)=k^{[\gamma]-\gamma}\cdot\sum_{j=1}^{k}(-1)^{k-j}\binom{[\gamma]}{k-j}\cdot\widehat{x}^{(r)}(j). \end{cases} \tag{28}$$

In the field of grey systems, swarm intelligence optimization algorithms are usually used to solve such complex planning problems. Here, we choose an arithmetic optimization algorithm (AOA) with faster search speed to solve this planning problem to obtain the hyperparameters of the model and the corresponding prediction results. The AOA is an efficient swarm intelligence optimization algorithm developed by Abailigah et al. based on arithmetic operation symbols [29]. It is mainly composed of three parts: the initialization phase, the exploration phase, and the exploitation phase.

2.4.1. Initialization Phase. At this stage, the AOA algorithm iterates based on a matrix ϕ composed of a group of candidates, and the optimal candidate solution in each iteration is taken as the approximate optimal solution of the current stage, where

$$\phi=\begin{bmatrix} \varphi_{1,1} & \cdots & \varphi_{1,n-1} & \varphi_{1,n} \\ \varphi_{2,1} & \cdots & \varphi_{2,n-1} & \varphi_{2,n} \\ \vdots & \ddots & \vdots & \vdots \\ \varphi_{m-1,1} & \cdots & \varphi_{m-1,n-1} & \varphi_{m-1,n} \\ \varphi_{m,1} & \cdots & \varphi_{m,n-1} & \varphi_{m,n} \end{bmatrix}. \tag{29}$$

Table 3: The evaluation metrics of the seven algorithms in the two cases.

	WOA	MPA	GWO	GOA	EOA	ALO	AOA
				RRCL			
Training							
MAPE	1.4557	1.2950	1.2973	1.2950	1.2950	1.3612	1.3620
RMSE	96.7318	91.4569	91.5072	91.4569	91.4569	94.8716	94.8843
R^2	0.9976	0.9978	0.9978	0.9978	0.9978	0.9977	0.9977
rRMSE	0.0218	0.0206	0.0206	0.0206	0.0206	0.0214	0.0214
MAE	66.4270	60.2699	60.4633	60.2700	60.2699	62.1145	62.1306
IA	0.9994	0.9995	0.9995	0.9995	0.9995	0.9994	0.9994
MSE	9357.0423	8364.3704	8373.5728	8364.3716	8364.3704	9000.6279	9003.0324
Test							
MAPE	2.3508	22.4480	22.2082	22.4479	22.4480	2.3120	2.3113
RMSE	574.2689	4221.1269	4184.0688	4221.1219	4221.1270	453.6824	453.1245
R^2	0.9718	0.1028	0.0840	0.1028	0.1028	0.9788	0.9788
rRMSE	0.0444	0.3265	0.3236	0.3265	0.3265	0.0351	0.0350
MAE	339.9470	3290.0644	3256.6666	3290.0600	3290.0645	327.1120	327.0361
IA	0.9874	0.4507	0.4541	0.4507	0.4507	0.9908	0.9908
MSE	329784.8049	17817911.9373	17506431.5260	17817869.9763	17817912.8397	205827.7001	205321.7845
				RRPCCEX			
Train							
MAPE	1.1291	0.9944	1.0005	1.0004	1.0004	1.0004	1.0008
RMSE	76.7420	67.0344	70.0764	70.0840	70.0848	70.0847	70.0404
R^2	0.9989	0.9991	0.9990	0.9990	0.9990	0.9990	0.9990
rRMSE	0.0154	0.0134	0.0140	0.0141	0.0141	0.0141	0.0140
MAE	50.8588	43.4909	51.6357	51.6414	51.6420	51.6420	51.6080
IA	0.9997	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
MSE	5889.3287	4493.6150	4910.7057	4911.7717	4911.8755	4911.8709	4905.6618
Test							
MAPE	5.4232	5.3032	3.5414	3.5465	3.5470	3.5470	3.5174
RMSE	1052.7385	1132.5090	560.1814	560.8081	560.8691	560.8664	557.2025
R^2	0.9570	0.9385	0.9736	0.9736	0.9736	0.9736	0.9736
rRMSE	0.0829	0.0892	0.0441	0.0442	0.0442	0.0442	0.0439
MAE	767.2886	767.5576	452.6361	453.3208	453.3874	453.3845	449.3749
IA	0.8957	0.8711	0.9795	0.9794	0.9794	0.9794	0.9797
MSE	1108258.4180	1282576.7233	313803.1752	314505.6961	314574.1137	314571.0751	310474.6518

In addition to constructing matrix ϕ, determining the search stage is also an important step in the AOA algorithm. As described by Abualigah et al., the math optimizer accelerated function (MOAF) is used for the following search phase, which can be computed by, i.e.,

$$MOAF(C\_\text{iter}) = \frac{C\_\text{iter} \cdot (\text{MAX} - \text{MIN})}{M\_\text{iter}} + \text{MIN}, \tag{30}$$

where $MOAF(C\_\text{iter})$ represents the function value at the ith iteration, C_iter is the value between 1 and the maximum number of iterations M_iter, which represents the current iteration, and MAX, MIN represent the maximum and minimum values of the acceleration function.

2.4.2. Exploration Phase. The exploration stage is the most important part of AOA. In AOA, exploration operators randomly explore the search area over several areas and search for better solutions based on the two main search strategies (division (D) and multiplication (M) search strategy). This stage is constrained by the MOAF for the condition of $MOAF < r_1$ (r_1 is a random number). The first operator in this phase, D, is constrained by $0.5 > r_2$, and the second operator (M) is ignored until D completes its task. If conditions change, then the second operator participates in the task. This process can be expressed as follows:

$$\varphi_{i,j}(C\_\text{iter}+1) = \begin{cases} \dfrac{\text{BSET}(\varphi_j)}{(MOP + \varpi)} \cdot \ell_j, & r_2 < 0.5, \\ \text{BEST}(\varphi_j) \cdot MOP \cdot \ell_j, & \text{otherwise}, \end{cases} \tag{31}$$

where $\ell_j = [(ub_j - lb_j) \cdot \tau + lb_j]$,$\varphi_i(C\_\text{iter}+1)$ represents the ith solution in the next iteration, $\varphi_{i,j}(C\_\text{iter})$ describes the jth position of the ith solution at the current stage, and $\text{BEST}(\varphi_j)$ represents the jth position of the best solution obtained so far. ϖ is a small integer, ub_j and lb_j represent the upper and lower bounds of the jth position, respectively, and $\tau = 0.5$ is a parameter used to adjust the search process. $MOP(C\_\text{iter})$ represents the function value of the tth iteration, and its mathematical expression is as follows:

$$MOP(C\_\text{iter}) = \frac{M\_\text{iter}^{\beta} - C\_\text{iter}^{\beta}}{M\_\text{iter}^{\beta}}, \tag{32}$$

Table 4: The evaluation indicators of the ten models in two cases.

	CFTDNGBM	FDGM	FNDGM	FDGMP	FDGM (1, 1, t^{α})	FGDGMP	WDGM	FTDGM	LSSVR	CFDGM (1, 1)
RRCL										
Train										
MAPE	1.3620	1.7806	1.4525	1.4630	1.3943	1.4096	1.7957	1.5495	2.4392	2.6191
RMSE	94.8849	103.8036	99.0455	101.7698	97.9779	98.6626	119.3347	105.4968	160.6342	130.8161
R^2	0.9977	0.9972	0.9975	0.9974	0.9975	0.9975	0.9968	0.9971	0.0332	0.0295
rRMSE	0.0214	0.0234	0.0223	0.0230	0.0221	0.0223	0.0269	0.0238	116.2264	103.3162
MAE	62.1315	76.0923	69.7769	73.0246	68.1469	68.6023	75.6808	76.4423	0.9981	0.9989
IA	0.9994	0.9993	0.9994	0.9993	0.9994	0.9994	0.9991	0.9993	25803.3463	17112.8593
MSE	9003.1358	10775.1772	9810.0136	10357.0824	9599.6754	9734.3096	14240.7803	11129.5774	2.4392	2.6191
Test										
MAPE	2.3113	3.1480	6.6800	9.6739	5.4340	6.7895	3.8925	17.1083	5.5442	2.5684
RMSE	453.1107	821.3741	1359.0174	1824.4150	1103.7555	1353.0515	910.2267	3434.1013	964.5873	510.6199
R^2	0.9788	0.9666	0.9636	0.9613	0.9681	0.9643	0.9661	0.9331	0.0746	0.0395
rRMSE	0.0351	0.0635	0.1051	0.1411	0.0854	0.1047	0.0704	0.2656	794.8417	369.8183
MAE	327.0333	470.0250	964.6383	1383.1333	777.8067	975.2067	566.1450	2503.6217	0.9523	0.9898
IA	0.9908	0.9762	0.9431	0.9082	0.9596	0.9433	0.9713	0.7861	930428.6440	260732.6682
MSE	205309.3459	674655.4052	1846928.2315	3328490.0677	1218276.2584	1830748.4720	828512.7178	11793051.9895	5.5442	2.5684
RRPCCEX										
Train										
MAPE	1.0008	1.2976	1.2325	1.0481	1.2160	1.1185	1.2500	1.3130	1.5513	1.2496
RMSE	70.0404	90.7675	98.4981	76.3337	82.4327	88.7683	86.6319	93.3950	108.1011	89.4319
R^2	0.9990	0.9984	0.9982	0.9989	0.9987	0.9985	0.9985	0.9983	0.0197	0.0179
rRMSE	0.0140	0.0182	0.0198	0.0153	0.0165	0.0178	0.0174	0.0187	78.6891	68.7715
MAE	51.6080	70.8369	69.8069	56.2285	62.2608	63.2800	66.0585	70.5546	0.9993	0.9996
IA	0.9998	0.9996	0.9995	0.9997	0.9997	0.9996	0.9996	0.9996	11685.8522	7998.0700
MSE	4905.6618	8238.7322	9701.8670	5826.8320	6795.1469	7879.8179	7505.0835	8722.6162	1.5513	1.2496
Test										
MAPE	3.5174	12.8543	15.7291	6.4070	6.7780	12.7090	10.2211	13.2455	12.5090	12.3202
RMSE	557.2025	2024.5019	2444.3165	953.9553	1044.7826	1966.8477	1606.1281	2180.2284	2275.6704	1936.7940
R^2	0.9736	0.9775	0.9776	0.9748	0.9757	0.9774	0.9772	0.9780	0.1793	0.1526
rRMSE	0.0439	0.1595	0.1926	0.0752	0.0823	0.1549	0.1265	0.1718	1756.0850	1663.0717
MAE	449.3749	1736.5667	2119.8250	840.3600	901.1633	1708.6450	1377.1817	1810.1883	0.6031	0.8518
IA	0.9797	0.8427	0.7969	0.9483	0.9412	0.8471	0.8865	0.8306	5178675.7793	3751171.0137
MSE	310474.6518	4098607.7344	5974683.2700	910030.7244	1091570.6801	3868489.9384	2579647.5880	4753395.7882	12.5090	12.3202

where C_iter represents the current iteration and M_iter represents the maximum number of iterations. $\beta = 0.2$ is a sensitive parameter that defines the exploitation precision of the iterative process.

2.4.3. Exploitation Phase. This subsection will describe the exploitation phase of the AOA. This stage is constrained by the value of the MOAF. In AOA, the exploitation operator (subtraction (S) and addition (A)) of AOA deeply explores the search area in several dense regions and searches for better solutions based on two main search strategies. The mathematical expressions for this stage are shown as follows:

$$\varphi_{i,j}(C\_\text{iter}+1) = \begin{cases} \text{BEST}(\varphi_j) - MOP \cdot \ell_j, & r_3 < 0.5, \\ \text{BEST}(\varphi_j) + MOP \cdot \ell_j, & \text{otherwise}. \end{cases} \tag{33}$$

At this stage, when $r_3 < 0.5$, the first operator (S) starts working, otherwise another operator (A) will start working. Different from the previous stage, developers try to avoid falling into the local search area, which helps exploration search strategies to find the optimal solution and maintain the diversity of candidate solutions. In addition, the parameters of τ are carefully designed to produce a random value in each iteration, which allows operators to keep exploring not only during the first iteration but also during the last.

2.5. The Calculation Method of the Model. In order to facilitate readers to understand the calculation steps of the model, here we use a case (China's oil consumption (Exajoules)) to explain. The original data are the oil consumption from 2002 to 2020 obtained from the World Energy Statistics Review-2021 (https://www.bp.com/statisticalreview), which can be expressed as follows:

$$X^{(0)} = \begin{pmatrix} 10.58, 11.78, 13.75, 13.99, 15.03, 15.77, 16.09, 16.69, 18.99, 19.68, \\ 20.63, 21.54, 22.39, 24.24, 25.06, 26.20, 27.06, 27.94, 28.50 \end{pmatrix}, \tag{34}$$

where the data from 2002 to 2016 are used as the training set and the data from 2017 to 2020 are used to test the performance of the CFTDNGBM (1,1) model.

Firstly, we input the original sequence into the programming model in Subsection 2.4 and obtain the solution ($\gamma = 0.0660, \eta = 0.0747$) of this programming model through the AOA algorithm.

According to equation (1), we can obtain a conformable fractional accumulation generation sequence of the original time series, that is,

$$X^{(0.0660)} = \begin{pmatrix} 10.5800, 16.7457, 21.6737, 25.5063, 28.8492, 31.8075, 34.4211, 36.8142, 39.2535, 41.5445, 43.7415, \\ 45.8564, 47.8964, 49.9573, 51.9549 \end{pmatrix}, \tag{35}$$

then we can construct the matrix

$$Z=\begin{bmatrix} 8.8707 & 3.0660 & 2 & 1\\ 13.5668 & 6.2332 & 3 & 1\\ 17.2242 & 10.5258 & 4 & 1\\ 20.0249 & 15.9623 & 5 & 1\\ 22.4420 & 22.5579 & 6 & 1\\ 24.5635 & 30.3254 & 7 & 1\\ 26.4255 & 39.2757 & 8 & 1\\ 28.1212 & 49.4187 & 9 & 1\\ 29.8411 & 60.7630 & 10 & 1\\ 31.4493 & 73.3166 & 11 & 1\\ 32.9852 & 87.0868 & 12 & 1\\ 34.4583 & 102.0803 & 13 & 1\\ 35.8744 & 118.3032 & 14 & 1\\ 37.3004 & 135.7614 & 15 & 1 \end{bmatrix}, U=\begin{bmatrix} 13.5668\\ 17.2242\\ 20.0249\\ 22.4420\\ 24.5635\\ 26.4255\\ 28.1212\\ 29.8411\\ 31.4493\\ 32.9852\\ 34.4583\\ 35.8744\\ 37.3004\\ 38.6785 \end{bmatrix}, \tag{36}$$

used to estimate the least squares parameters.

Furthermore, we have $\hat{\iota} = (Z^T Z)^{-1} Z^T U = [\,0.63095 \;\; -0.00860 \;\; 0.63788 \;\; 6.74385\,]^T$.

By inputting the obtained least squares parameters into equation (23), we can obtain the time response sequence

$$\hat{x}^{(0.0660)} = \begin{pmatrix} 10.5800, 16.7769, 21.6116, 25.5320, 28.8497, 31.7704, 34.4269, 36.9040, 39.2556, \\ 41.5155, 43.7050, 45.8373, 47.9203, 49.9589, 51.9560, 53.9130, 55.8308, 57.7095, 59.5491 \end{pmatrix}. \tag{37}$$

Finally, according to $\hat{x}^{(0)}(k) = k^{[\gamma]-\gamma} \cdot \sum_{j=1}^{k} (-1)^{k-j} \binom{[\gamma]}{k-j} \cdot \hat{x}^{(r)}(j)$, we can get the final prediction results of the model, namely,

$$\hat{x}^{(0.0660)} = \begin{pmatrix} 10.5800, 11.8395, 13.4899, 14.3103, 14.9168, 15.5697, 16.3543, 17.2757, 18.3072, 19.4131, 20.5592, \\ 21.7165, 22.8622, 23.9787, 25.0532, 26.0763, 27.0414, 27.9438, 28.7800 \end{pmatrix}. \tag{38}$$

3. Application

This section describes the application of the proposed model and its competitors in the rural regional economy. To facilitate understanding, the application process of the proposed model in the cases is plotted in Figure 1.

3.1. Data Collection, Preprocessing, Description of the Models, and Evaluation Indicators. The accurate prediction of the rural residents' consumption level and rural per capita consumption index is helpful for the government to formulate corresponding policies for short-term guidance and regulation of rural residents' consumption according to the forecast results so as to improve rural residents' consumption levels and regulate the overall operation of the macroeconomy. Therefore, it is of great importance to study the future development trends of RRCL and RRPCCEX. The two sets of data used in this article are the consumption level of rural residents (RRCL) from 2002 to 2020 and the per capita consumption expenditure of rural residents (RRPCCEX) from 2003 to 2021. These two sets of data are from the National Bureau of Statistics of China (https://www.stats.gov.cn/,2020), and the details are shown in Table 2. In particular, the consumption level data of rural residents from 2002 to 2014 and the per capita consumption index of rural residents from 2003 to 2015 are used as

Table 5: Model parameters based on different algorithms.

	WOA	MPA	GWO	GOA	EOA	ALO	AOA
				RRCL			
η	0.407655	2.308542	2.308677	2.308541	2.308542	0.000000	0.000000
γ	0.844325	0.000000	0.000000	0.000000	0.000000	0.683049	0.686749
				RRPCCEX			
η	0.777573	0.824907	0.000000	0.000000	0.000000	0.000000	0.000000
γ	0.404488	1.000000	0.087175	0.087114	0.087173	0.087173	0.083635

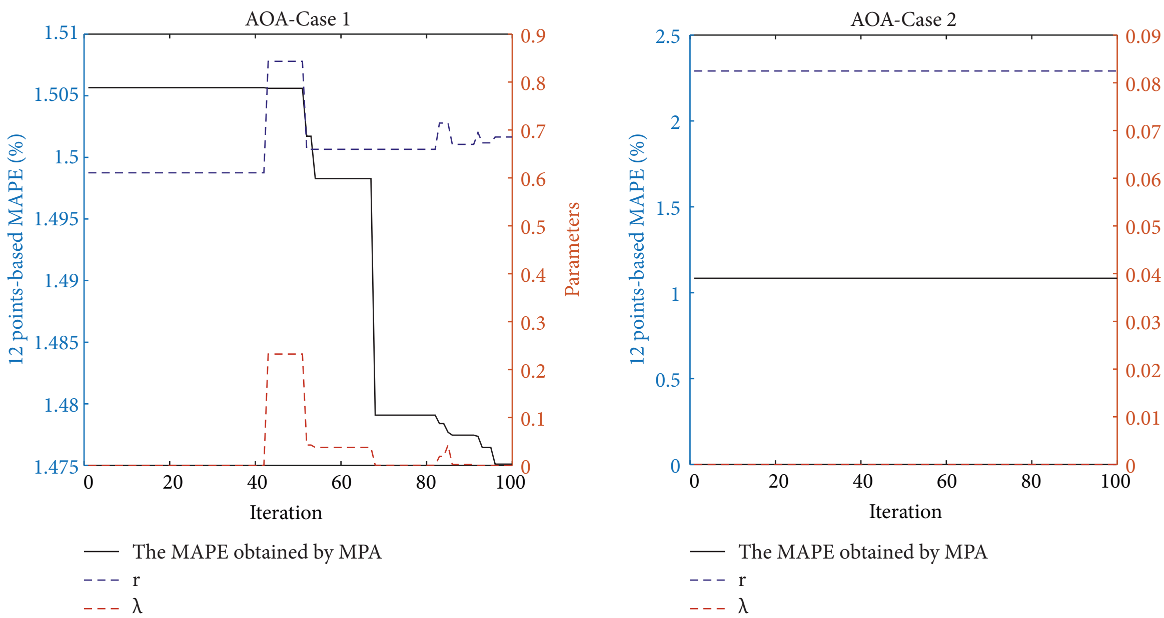

Figure 2: Iteration diagram of the proposed model based on AOA in two cases.

training sets, and the data from 2015 to 2020 and 2016 to 2021 are used to test the performance of the model.

According to [30], one of the reasons why the grey prediction model is ill-conditioned is that the original data dimension is too large and multiplicative transformations can alleviate the ill-condition of the system to a certain extent. Therefore, this paper selects multiplicative transformation as the preprocessing operation of the model, which can be expressed as follows:

$$x'^{(0)}(k) = x^{(0)}(k) \times 10^p, k = 1, \ldots, n, \ldots, m, \quad (39)$$

where p is an integer, and its function is mainly to reduce the dimensions of the original data to single digits.

It should be mentioned that if there are missing values in the sequence, then we need to carry out nonequidistant conformable fractional accumulation processing on the data and establish a nonequidistant CFTDNGBM (1, 1) model. The construction process of the nonequidistant conformable fractional order accumulation operation is similar to the nonequidistant fractional order accumulation operation proposed in [31]. For space reasons, it is not constructed here.

In addition to the model proposed in this paper, the models used for the comparative analysis also include other nine different types of discrete grey models with hyperparameters, which are LSSVR [32], CFDGM (1, 1) [33], FDGM (1, 1) [34], FNDGM (1, 1, k, c) [35], FDGMP (1, 1, N) [36], FDGM (1, 1, t^{α}) [37], FGDGMP (1, 1, N, α) [38], WDGM (1, 1) [39], and the FTDGM (1, 1) model [40]. The reasons why these models are selected for comparative analysis are that they all belong to excellent prediction models and all contain hyperparameters, which make them have strong competitive ability. It is worth mentioning that all models are implemented in the MATLAB 2019a environment.

To evaluate the performance of the models, we collected seven metrics used to quantify the performance of predictive models, which are

$$\begin{aligned}
\mathrm{MAPE}(\%) &= \frac{1}{l}\sum_{j=1}^{l}\left|\frac{x_j-\widehat{x}_j}{x_j}\right| \times 100\%,\\
\mathrm{RMSE} &= \sqrt{\frac{1}{l}\sum_{j=1}^{l}\left(x_j-\widehat{x}_j\right)^2},\\
R^2 &= 1-\frac{\sum_{j=1}^{l}\left(x_j-\widehat{x}_j\right)^2}{\sum_{j=1}^{l}\left(x_j-\overline{x}_j\right)^2},\\
r\mathrm{RMSE} &= \frac{\mathrm{RMSE}}{\overline{x}},\\
\mathrm{MAE} &= \frac{1}{l}\sum_{j=1}^{l}\left|x_j-\widehat{x}_j\right|,\\
\mathrm{MSE} &= \frac{1}{l}\sum_{j=1}^{l}\left(x_j-\widehat{x}_j\right)^2,\\
IA &= \frac{\sum_{j=1}^{l}\left(|\widehat{x}-\overline{x}|+|x-\overline{x}|\right)^2-\sum_{j=1}^{l}\left(\widehat{x}-x\right)^2}{\sum_{j=1}^{l}\left(|\widehat{x}-\overline{x}|+|x-\overline{x}|\right)^2},
\end{aligned} \tag{40}$$

where l is the number of data used to build the proposed model.

3.2. Comparison with Other Optimization Algorithms. In this subsection, we will establish CFTDNGBM (1, 1) models based on different optimization algorithms and compare their prediction results with those of the CFTDNGBM (1, 1) model based on AOA. The selected optimization algorithms for comparison include WOA (Whale Optimization Algorithm) [41], MPA (Marine Predators Algorithm) [42], GWO (Grey Wolf Optimization) [43], GOA (Grasshopper Optimization Algorithm) [44], EOA (Equilibrium Optimizer Algorithm) [45], and ALO (Ant lion optimizer) [46]. The reason why these algorithms are chosen as comparison models is that they have been used to solve grey prediction models and have reference significance.

Table 3 shows the evaluation metrics of the CFTDNGBM (1, 1) model based on the seven intelligent optimization algorithms in the two cases. From Table 3, we can see that in the training phase of the RRCL case, the performance of the seven algorithms is not very different, among which the performance of EOA is slightly better than the other six algorithms, and the performance of the AOA, ALO, and WOA algorithms in the testing phase is much higher than the other algorithms. Taking MAPE as an example, the MAPE of AOA, ALO, and WOA in the test phase are 2.3113, 2.3120, and 2.3508, respectively, which are much higher than the MAPE of the other four algorithms. It seems a strange phenomenon that the performance gap of the algorithm in the training set is small, but the performance gap in the test set is very large. This phenomenon shows that in some cases, when the loss function of the model based on some intelligent algorithms in the training set reaches a certain critical point, the performance of the model in the test set will become worse as the performance in the training set becomes better. Therefore, the best algorithm should be determined by multiple comparisons when selecting the intelligent algorithm for solving the model. In the case of RRPCCEX, although the fitting performance of MPA is better than that of other algorithms, the performance of AOA in the test set is better than that of the other six algorithms and its evaluation index in the training set is very close to that of MPA. Therefore, AOA outperforms the other algorithms in these two cases. By the way, the hyperparameter outputs by the seven algorithms in the two cases are shown in the appendix.

3.3. Comparison with Other Forecasting Models. The evaluation metrics of the CFTDNGBM (1, 1) model and the other nine forecasting models mentioned in Subsection 3.1 in the two cases are shown in Table 4. The modeling details of the ten models are shown in Tables 5–10. Figure 2 reflects the convergence of the AOA-based CFTDNGBM (1, 1) model.

Table 6: The solution formulas for eight models in two cases.

Model	Case	Formula
FTDGM	RRCL	$\widehat{x}^{(0.1098759391215)}(k+1) = (0.08094k - 0.66565)\widehat{x}^{(0.1098759391215)}(k) + 358.32754k + 3432.06707$
	RRPCCEX	$\widehat{x}^{(0.109021196584951)}(k+1) = (0.04053k - 0.01660)\widehat{x}^{(0.109021196584951)}(k) + 390.18705k + 2114.22632$
WDGM	RRCL	$\widehat{x}^{(0.093416248693326)}(k+1) = 1.13996\widehat{x}^{(0.093416248693326)}(k) - 44.00747$
	RRPCCEX	$\widehat{x}^{0.947491737254247}(k+1) = 1.109512\widehat{x}^{(0.947491737254247)}(k) + 1991.21134$
FDGMP	RRCL	$\widehat{x}^{(2.02890046949104)}(k) = 13.95091 + 2206.66582k - 65.1803k^2 + 1.16278\widehat{x}^{(2.02890046949104)}(k-1)$
	RRPCCEX	$\widehat{x}^{(0.996243380586089)}(k) = -101.90263 + 1940.10325k + 44.93859k^2 + 10.40266k^3 + 0.16110\widehat{x}^{(0.996243380586089)}(k-1)$
FNDGM	RRCL	$\widehat{x}^{(1.05796960827444)}(k+1) = 2169.17103 - 79.24915k + 1.15167\widehat{x}^{(1.05796960827444)}(k)$
	RRPCCEX	$\widehat{x}^{(1.96618970016966)}(k+1) = 1985.51629 + 2054.90451k - 1.12949\widehat{x}^{(1.96618970016966)}(k)$
FDGM	RRCL	$\widehat{x}^{(1.08841035831398)}(k+1) = 1.13722\widehat{x}^{(1.05796960827444)}(k) + 2136.3516$
	RRPCCEX	$\widehat{x}^{(0.909695302199426)}(k+1) = 1.12366\widehat{x}^{(0.909695302199426)}(k) + 1896.88462$
FDGM (1, 1, t^{α})	RRCL	$\widehat{x}^{(1.00363634664512)}(k+1) = 2090.02081 + 2.31073k^{2.64279951473824} + 1.0962\widehat{x}^{(1.00363634664512)}(k)$
	RRPCCEX	$\widehat{x}^{(0.970455316951867)}(k+1) = 1979.46960 - 0.02866k^{3.999889121019478} + 1.13951\widehat{x}^{(0.970455316951867)}(k)$
GDGMP	RRCL	$\widehat{x}^{(1.46509454027063)}(k) = -19219.04789 + 21321.6039k^{0.065276119698169} - 51.99760673k^{1.065276119698169} + 1.147742\widehat{x}^{(1.46509454027063)}(k)$
	RRPCCEX	$\widehat{x}^{(2.09000481662779)}(k) = -2071.48964 + 3341.40994k^{0.54012580504343} + 530.11256k^{1.54012580504343} + 1.11668\widehat{x}^{(2.09000481662779)}(k)$
CFTDNGBM	RRCL	$x^{(0.686839677391444)}(k) = [0.774929^{k-1}Q^{(0.686839677391444)}(1) + 16.101045\sum_{j=0}^{k-2}0.774929^{j}\sum_{\tau=1}^{k-j}\tau^{(0.686839677391444)} + 285.891112\sum_{j=0}^{k-2}0.774929^{j}(k-j) + 1701.997504\sum_{j=0}^{k-2}0.774929^{j}]^{1/1-0}$
	RRPCCEX	$x^{(0.094154087687521)}(k) = [0.633348^{k-1}Q^{(0.094154087687521)}(1) + 9.809083\sum_{j=0}^{k-2}0.633348^{j}\sum_{\tau=1}^{k-j}\tau^{(0.094154087687521)} + 198.366539\sum_{j=0}^{k-2}0.633348^{j}(k-j) + 1565.394824\sum_{j=0}^{k-2}0.633348^{j}]^{1/1-0}$
CFDGM (1, 1)	RRCL	$\widehat{x}^{(0.9708)}(k) = 1.1269^{k-1}\cdot\widehat{x}^{(0.9708)}(1) - 14090.7076\times(1 - 1.1269^{k-1})$.
	RRPCCEX	$\widehat{x}^{(0.9045)}(k) = 1.1166^{k-1}\cdot\widehat{x}^{(0.9045)}(1) - 16616.9475\times(1 - 1.1166^{k-1})$.

Table 7: The prediction results of ten models in the case of the rural residents' consumption level.

Year	Data	CFTDNGBM	FDGM	FNDGM	FDGMP	FDGM (1, 1, t^{α})	FGDGMP	WDGM	FTDGM	LSSVM	CFDGM (1, 1)
2002	2157.00	2157.00	2157.00	2157.00	2157.00	2157.00	2157.00	2157.00	2157.00	—	2157.00
2003	2292.00	2295.59	2241.63	2291.02	2298.33	2292.00	2295.99	2213.40	2292.16	—	2103.64
2004	2521.00	2535.55	2464.13	2505.42	2517.64	2520.95	2523.64	2483.29	2491.21	2574.23	2398.78
2005	2784.00	2783.73	2747.69	2774.79	2782.99	2788.65	2779.83	2790.97	2785.57	2807.03	2725.93
2006	3066.00	3079.79	3084.94	3096.27	3097.80	3102.93	3091.29	3141.71	3111.81	3116.19	3091.88
2007	3538.00	3443.08	3477.10	3473.39	3467.74	3472.13	3463.15	3541.54	3493.57	3457.46	3502.77
2008	3981.00	3884.14	3928.70	3912.38	3900.61	3905.14	3900.98	3997.33	3928.05	3939.04	3965.00
2009	4295.00	4409.05	4446.23	4421.40	4405.94	4411.52	4412.02	4516.91	4426.36	4505.73	4485.53
2010	4782.00	5021.43	5037.71	5010.30	4995.08	5001.55	5005.32	5109.22	4999.86	4965.60	5072.07
2011	5880.00	5723.44	5712.59	5690.69	5681.37	5686.34	5691.71	5784.43	5664.32	5468.39	5733.22
2012	6573.00	6516.40	6481.86	6476.08	6480.42	6477.98	6483.98	6554.14	6440.34	6467.86	6478.64
2013	7397.00	7401.10	7358.14	7382.17	7410.41	7389.55	7397.05	7431.59	7355.03	7501.49	7319.22
2014	8365.00	8377.99	8355.84	8427.07	8492.53	8435.35	8448.17	8431.84	8444.51	8378.97	8267.17
2015	9409.00	9447.35	9491.45	9631.71	9751.46	9630.94	9657.29	9572.10	9757.46	9374.64	9336.28
2016	10609.00	10609.31	10783.73	11020.24	11215.90	10993.29	11047.38	10871.95	11360.51	10425.96	10542.07
2017	12145.00	11863.96	12254.08	12620.49	12919.24	12540.96	12644.90	12353.74	13346.38	11516.26	11902.05
2018	13985.00	13211.33	13926.81	14464.54	14900.30	14294.22	14480.22	14042.92	15846.38	12716.01	13435.93
2019	15382.00	14651.44	15829.64	16589.34	17204.26	16275.22	16588.27	15968.52	19050.09	13953.57	15165.95
2020	16046.00	16184.27	17994.06	19037.51	19883.64	18508.21	19009.18	18163.64	23236.91	14820.51	17117.19

Table 8: The prediction results of ten models in the case of the per capita consumption expenditure of rural residents.

Year	Data	CFTDNGBM	FDGM	FNDGM	FDGMP	FDGM (1, 1, t^{α})	FGDGMP	WDGM	FTDGM	LSSVM	CFDGM (1, 1)
2003	2050.00	2050.00	2050.00	2050.00	2050.00	2050.00	2050.00	2050.00	2050.00		2050.00
2004	2326.00	2324.90	2335.51	2330.06	2329.21	2322.20	2333.60	2323.35	2329.98		2326.03
2005	2749.00	2748.56	2711.42	2719.81	2749.10	2678.51	2749.00	2682.34	2681.27	2753.51	2699.85
2006	3072.00	3107.67	3109.49	3100.46	3097.67	3072.08	3079.59	3080.64	3085.72	3133.66	3098.70
2007	3536.00	3498.85	3543.09	3522.79	3496.95	3511.40	3497.50	3522.57	3534.55	3553.94	3534.61
2008	4054.00	3960.58	4021.61	3995.32	3966.99	4002.61	3980.17	4012.88	4028.18	4007.08	4016.16
2009	4464.00	4504.34	4553.31	4525.98	4511.20	4550.99	4524.91	4556.90	4570.45	4585.83	4551.05
2010	4945.00	5129.76	5146.36	5123.08	5130.22	5161.36	5134.80	5160.49	5167.21	5141.80	5147.04
2011	5892.00	5832.06	5809.38	5795.71	5824.23	5838.30	5815.56	5830.18	5825.98	5676.22	5812.33
2012	6667.00	6605.41	6551.73	6553.97	6593.31	6586.19	6574.59	6573.20	6556.06	6522.72	6555.83
2013	7485.00	7444.44	7383.72	7409.14	7437.50	7409.25	7420.59	7397.60	7368.81	7515.05	7387.34
2014	8383.00	8344.73	8316.81	8373.91	8356.85	8311.54	8363.54	8312.28	8278.07	8372.56	8317.72
2015	9223.00	9302.84	9363.79	9462.56	9351.40	9296.96	9414.69	9327.13	9300.77	9207.63	9359.05
2016	10130.00	10316.25	10538.94	10691.18	10421.17	10369.23	10586.67	10453.11	10457.75	9937.08	10524.78
2017	10955.00	11383.13	11858.30	12077.92	11566.19	11531.87	11893.62	11702.41	11774.81	10535.18	11829.93
2018	12124.00	12502.21	13339.84	13643.26	12786.49	12788.20	13351.38	13088.51	13284.20	10981.27	13291.27
2019	13328.00	13672.63	15003.74	15410.30	14082.09	14141.28	14977.59	14626.41	15026.48	11318.45	14927.54
2020	13713.00	14893.80	16872.63	17405.12	15452.99	15593.89	16791.99	16332.73	17053.24	11464.78	16759.71
2021	15916.00	16165.36	18971.95	19657.17	16899.23	17148.51	18816.62	18225.92	19430.65	11392.73	18811.20

From Table 4, we can see that in the training stage of the RRCL case, the CFTDNGBM model has the best performance, while the LSSVR model has the worst performance. In the test stage, the seven indicators of the CFTDNGBM model are the smallest among the ten models. Although the performance of the CFDGM (1, 1) model in the test set is close to that of the CFTDNGBM model, its fitting performance is lower than that of the CFTDNGBM model. Therefore, the CFTDNGBM model performs best in the RRCL case. Different from the RRCL case, in the RRPCCEX case, the CFTDNGBM model significantly outperforms the other algorithms in both the training and test sets. For example, the MAPE of the CFTDNGBM model in the test set is 3.5174, while the smallest MAPE among the other nine models is 6.4070. In conclusion, the CFTDNGBM model outperforms the other nine models in two cases.

Table 9: The APE (%) of ten models in the case of the rural residents' consumption level.

Year	Data	CFTDNGBM	FDGM	FNDGM	FDGMP	FDGM (1, 1, t^{α})	FGDGMP	WDGM	FTDGM	LSSVM	CFDGM (1, 1)
2002	2157.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	—	0.0000
2003	2292.00	0.1565	2.1977	0.0429	0.2762	0.0000	0.1742	3.4295	0.0072	—	8.2182
2004	2521.00	0.5770	2.2560	0.6182	0.1334	0.0021	0.1048	1.4957	1.1816	2.1115	4.8482
2005	2784.00	0.0097	1.3041	0.3309	0.0362	0.1672	0.1499	0.2503	0.0565	0.8273	2.0857
2006	3066.00	0.4498	0.6178	0.9874	1.0370	1.2046	0.8249	2.4693	1.4940	1.6371	0.8440
2007	3538.00	2.6829	1.7214	1.8263	1.9858	1.8618	2.1156	0.1000	1.2557	2.2765	0.9958
2008	3981.00	2.4331	1.3137	1.7237	2.0194	1.9055	2.0101	0.4101	1.3299	1.0541	0.4018
2009	4295.00	2.6555	3.5212	2.9430	2.5829	2.7130	2.7246	5.1667	3.0585	4.9064	4.4362
2010	4782.00	5.0069	5.3473	4.7742	4.4558	4.5911	4.6699	6.8427	4.5558	3.8395	6.0658
2011	5880.00	2.6626	2.8471	3.2196	3.3780	3.2935	3.2022	1.6254	3.6680	7.0002	2.4963
2012	6573.00	0.8611	1.3865	1.4745	1.4085	1.4457	1.3543	0.2869	2.0183	1.5995	1.4355
2013	7397.00	0.0554	0.5254	0.2005	0.1812	0.1007	0.0006	0.4676	0.5674	1.4126	1.0515
2014	8365.00	0.1553	0.1095	0.7420	1.5246	0.8410	0.9942	0.7991	0.9505	0.1670	1.1695
2015	9409.00	0.4076	0.8763	2.3670	3.6397	2.3588	2.6388	1.7335	3.7035	0.3651	0.7729
2016	10609.00	0.0030	1.6470	3.8764	5.7206	3.6223	4.1322	2.4786	7.0837	1.7253	0.6308
2017	12145.00	2.3140	0.8981	3.9151	6.3749	3.2603	4.1161	1.7187	9.8920	5.1769	2.0004
2018	13985.00	5.5321	0.4161	3.4289	6.5449	2.2111	3.5411	0.4141	13.3098	9.0740	3.9261
2019	15382.00	4.7495	2.9102	7.8491	11.8467	5.8069	7.8421	3.8130	23.8467	9.2864	1.4045
2020	16046.00	0.8617	12.1405	18.6433	23.9165	15.3447	18.4668	13.1973	44.8144	7.6374	6.6758

Table 10: The (APE (%)) of ten models in the case of the per capita consumption expenditure of rural residents.

Year	Data	CFTDNGBM	FDGM	FNDGM	FDGMP	FDGM (1, 1, t^{α})	FGDGMP	WDGM	FTDGM	LSSVM	CFDGM (1, 1)
2003	2050.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	—	0.0000
2004	2326.00	0.0474	0.4090	0.1743	0.1378	0.1632	0.3268	0.1138	0.1713	—	0.0013
2005	2749.00	0.0158	1.3672	1.0618	0.0036	2.5642	0.0000	2.4248	2.4637	0.1639	1.7879
2006	3072.00	1.1610	1.2204	0.9263	0.8357	0.0025	0.2471	0.2814	0.4467	2.0071	0.8690
2007	3536.00	1.0507	0.2004	0.3735	1.1043	0.6958	1.0889	0.3799	0.0411	0.5074	0.0393
2008	4054.00	2.3043	0.7990	1.4474	2.1462	1.2676	1.8211	1.0142	0.6369	1.1574	0.9335
2009	4464.00	0.9036	2.0006	1.3883	1.0574	1.9487	1.3644	2.0810	2.3847	2.7293	1.9501
2010	4945.00	3.7364	4.0719	3.6011	3.7457	4.3754	3.8381	4.3577	4.4936	3.9798	4.0857
2011	5892.00	1.0173	1.4023	1.6343	1.1502	0.9114	1.2973	1.0493	1.1206	3.6623	1.3522
2012	6667.00	0.9238	1.7290	1.6954	1.1053	1.2121	1.3861	1.4069	1.6640	2.1640	1.6675
2013	7485.00	0.5418	1.3530	1.0135	0.6346	1.0120	0.8605	1.1677	1.5523	0.4015	1.3047
2014	8383.00	0.4566	0.7895	0.1084	0.3119	0.8524	0.2321	0.8436	1.2517	0.1246	0.7787
2015	9223.00	0.8657	1.5265	2.5974	1.3922	0.8019	2.0784	1.1290	0.8432	0.1667	1.4751
2016	10130.00	1.8386	4.0369	5.5398	2.8743	2.3616	4.5081	3.1897	3.2354	1.9045	3.8971
2017	10955.00	3.9081	8.2456	10.2503	5.5791	5.2658	8.5680	6.8225	7.4835	3.8322	7.9866
2018	12124.00	3.1195	10.0284	12.5310	5.4643	5.4784	10.1235	7.9554	9.5694	9.4253	9.6277
2019	13328.00	2.5858	12.5731	15.6235	5.6579	6.1020	12.3769	9.7420	12.7437	15.0777	12.0014
2020	13713.00	8.6108	23.0411	26.9243	12.6886	13.7161	22.4531	19.1040	24.3582	16.3948	22.2177
2021	15916.00	1.5667	19.2005	23.5057	6.1776	7.7439	18.2246	14.5132	22.0825	28.4196	18.1905

4. Conclusion

In this study, we propose a CFTDNGBM (1,1) model with all the characteristics of a time-delay polynomial, a conformable fractional accumulation operator, and a Bernoulli operator. This model has a simpler modeling mechanism than traditional time-delay grey prediction models. In addition, a novel AOA algorithm is introduced to solve the hyperparameters of the proposed model. In order to verify the validity of the model and expand the application scope of the grey system model, the CFTDNGBM (1,1) model and nine competitive models are used to study the regional economies in rural China. Numerical results show that in both cases, the seven quantitative indexes of the CFTDNGBM (1,1) model are superior to those of its competing models, which shows the superiority of the proposed model. It should be noted that the model proposed in this paper is not applicable to a large sample setting. According to the description in the literature [30], the pathological nature caused by large magnitude gaps in the coefficient matrix can make the prediction results of the model inaccurate, while the gaps in the elements of the coefficient matrix of the proposed model increase rapidly with the increase in the sample size. Therefore, the model proposed in this paper is suitable for a small-sample nonlinear time series.

The limitation of this study is that the randomness of the algorithm is not taken into account. Since the proposed model has two hyperparameters, the prediction results may be unstable. In the future, we will continue to improve this aspect of work in order to enrich the application of the swarm intelligence optimization algorithm in grey system theory.

Abbreviation

AOA: Arithmetic optimization algorithm
CFTDNGBM (1, 1): Conformable fractional-order grey Bernoulli model with time-delay effect
CFAO: Conformable fractional-order accumulation operation
GM (1, 1): The most basic grey prediction model
NGM (1, 1, *k*, *c*): Nonhomogeneous grey prediction model
GM (1, 1, t^{α}): Grey prediction model with time power term
GMP (1, 1, *N*): Grey prediction model with polynomial
PTGM (1, 1, α): Generalized grey prediction model with time power term
FTDGM: Fractional grey prediction model with a time delay term
GM (1, *N*): Multivariate grey prediction model
DGM (1, 1): Discrete grey model
VAR: Vector autoregressive
DEA: Data envelopment analysis
NGBM (1, 1, *k*, *c*): Nonlinear nonhomogeneous grey prediction model
NGBM (1, 1, *N*): Nonlinear grey prediction model with polynomial
1-AGO: 1-order accumulation operator
NGBM (1, 1): Nonlinear grey Bernoulli model
LSSVM: Least squares support vector machine
CFNGM (1, 1, *k*, *c*): Conformable fractional nonhomogeneous grey prediction model
CFGM (1, 1): Conformable fractional grey model
MOAF: Math optimizer accelerated function
RRCL: Consumption level of rural residents
RRPCCEX: Per capita consumption expenditure of rural residents
FDGM (1, 1): Fractional discrete GM (1, 1)
FNDGM (1, 1, *k*, *c*): Fractional discrete nonhomogeneous grey prediction model
FDGMP (1, 1, *N*): Fractional discrete grey prediction model with polynomial
FDGM (1, 1, t^{α}): Fractional discrete grey prediction model with time power term
FGDGMP (1, 1, *N*, α): Fractional discrete generalized grey prediction model with time power term
WDGM (1, 1): Discrete grey model with the weighted accumulation
FTDGM (1, 1): Fractional-order accumulative linear time-varying parameters discrete grey forecasting model
CFDGM (1, 1): Conformable fractional discrete grey model
WOA: Whale optimization algorithm
MPA: Marine predators algorithm
GWO: Grey wolf optimization
GOA: Grasshopper optimization algorithm
EOA: Equilibrium optimizer algorithm
ALO: Ant lion optimizer
AOA: Arithmetic optimization algorithm

Data Availability

The data used in this article come from the National Bureau of Statistics of China.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Ran Wang conducted writing of the original draft, methodology, and experimental data validation. Hongmei Du conducted conceptualization, methodology, supervision, writing, review, and editing. Qinwen Yang conducted investigation, software, and data collection. Hui Liu performed investigation, visualization, and collected resources.

Acknowledgments

This work was supported by the Natural Social Science Found of China (No. 20BJY046).

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