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*Research Article*

# Common Fixed Point Results for Intuitionistic Fuzzy Hybrid Contractions with Related Applications

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Over time, hybrid fixed point results have been examined merely in the framework of classical mathematics. This one way research has clearly dropped-off a great amount of important results, considering the fact that a fuzzy set is a natural enhancement of a crisp set. In order to entrench hybrid fixed notions in fuzzy mathematics, this paper focuses on introducing a new idea under the name intuitionistic fuzzy $p$-hybrid contractions in the realm of -metric spaces. Sufficient conditions for the existence of common intuitionistic fuzzy fixed points for such maps are established. In the instance where our presented results are slimmed down to their equivalent nonfuzzy counterparts, the concept investigated herein unifies and generalizes a significant number of well-known fixed point theorems in the setting of both single-valued and multivalued mappings in the corresponding literature. A handful of these special cases are highlighted and analysed as corollaries. A nontrivial example is put together to indicate that the hypotheses of our results are valid.

## 1. Introduction

In practical, if a model asserts that conclusions drawn from it have some bearings on reality, then two major complications are immediate, namely, real situations are often not crisp and deterministic; a complete description of real systems often requires more detailed data than human beings could recognize simultaneously, process, and understand. Whence, by using classical mathematical tools, as the difficulty of a practical system increases, our ability to come up with precise and significant statements reduces until a threshold is reached after which accuracy become an almost mutually exclusive characteristics, see [1]. These restrictions in everyday systems paved the way to the launching of the fuzzy set by Zadeh [2], which is a flexible mathematical device to design mathematical approaches in line with practical issues. At present, the primitive ideas of the fuzzy set have been upgraded in a multifarious framework. Following this development, Heilpern [3] employed the idea of the fuzzy set to initiate a class of fuzzy set-valued mappings and presented a fixed point(Fp) theorem which is a fuzzy version of the Fp result of Nadler [4]. Thereafter, a substantial number of authors have studied the existence of Fp of fuzzy set-valued maps, for example, see [5–10]. Following Zadeh [2], an intuitionistic fuzzy set (IFS) was brought up by Atanassov [11] as an additional refinement of the notions of fuzzy set. IFS gives relevant frames to take care of inaccuracy and hesitancy due to inadequate information. IFS is more useful than a fuzzy set as it evaluates the degrees of both membership and nonmembership. Whence, it has attracted enormous applications in several fields. At present, work on IFS have been rising at a faster speed and varying views have been discovered in different arms. Along this direction, Azam et al. [12] came up with a modern way for examining the existence of Fp of intuitionistic fuzzy set-valued maps defined on a complete metric space. Later after, [13] presented criteria for investigating coincidence points for intuitionistic fuzzy set-valued maps and employed their results to examine conditions for the existence of solutions to a system of integral equations. Of recent, [14, 15] coined the idea of Fp results for two intuitionistic fuzzy set-valued maps using $(\mathscr{T}, \mathscr{N}, \breve{\alpha})$-cut set.

The well-celebrated Banach contraction has laid a solid foundation for the development of the metric fixed point theory. The prototypical concept of the contraction mapping principle has been refined in several domains (e.g., see [16–20]). Along the lane, hybrid Fp theory emerged and has so far been studied only in the context of classical mathematics. This one way investigation has obviously neglected a great amount of useful results, considering the fact that a fuzzy set is a natural generalization of a crisp set. Whence, in order to entrench hybrid fixed point notions in fuzzy mathematics, the aim of this paper is to introduce new concepts under the name intuitionistic fuzzy $p$-hybrid contractions in the framework of $b$-metric space. Sufficient criteria for the existence of common intuitionistic fuzzy Fp for such mappings are established. It is observed that at the instance where our results are reduced to their corresponding crisp ideas, the concepts examined herein harmonize and generalize a significant number of Fp results in the setting of both point-valued and set-valued mappings in the related literature. A few of these particular cases are pinned down and discussed. A comparative example is designed to validate the hypotheses of our obtained results.

## 2. Preliminaries

We collect herewith specific fundamentals that will be needed later on. These basis are some extracts from [2, 4, 21, 22].

*Definition 1* [21]. Let $\mho$ be a nonempty set and $\widehat{\eta} \geq 1$ be a constant. Suppose that the mapping $\widehat{\mu}$: $\mho \times \mho \longrightarrow \mathbb{R}_+$ satisfies the following criteria for all $\varsigma, \omega, \xi \in \mho$:

(i) $\widehat{\mu}(\varsigma, \omega) = 0 \Leftrightarrow \varsigma = \omega$;

(ii) $\widehat{\mu}(\varsigma, \omega) = \widehat{\mu}(\omega, \varsigma)$;

(iii) $\widehat{\mu}(\varsigma, \omega) \leq \widehat{\eta}[\widehat{\mu}(\varsigma, \xi) + \widehat{\mu}(\xi, \omega)]$.

Then, $(\mho, \widehat{\mu}, \widehat{\eta})$ is called as a $b$-metric space.

*Definition 2* [23]. Consider a $b$-metric space $(\mho, \widehat{\mu}, \widehat{\eta})$. A sequence $\{\varsigma_\wp\}_{\wp \in \mathbb{N}}$ is called:

(i) convergent $\Leftrightarrow \varsigma \in \mho$ is such that $\widehat{\mu}(\varsigma_\wp, \varsigma) \longrightarrow 0$ as $\wp \longrightarrow \infty$.

(ii) Cauchy if $\widehat{\mu}(\varsigma_\wp, \varsigma_\varpi) \longrightarrow 0$ as $\wp, \varpi \longrightarrow \infty$.

(iii) complete if every Cauchy sequence in $\mho$ is convergent.

In a $b$-metric space, the limit of a sequence is not always unique. However, if a $b$-metric is continuous, then every convergent sequence has a unique limit.

*Definition 3* [23]. Consider a $b$-metric space $(\mho, \widehat{\mu}, \widehat{\eta})$. A subset $\widetilde{\nabla}$ of $\mho$ is called:

(i) compact $\Leftrightarrow$ for every sequence of elements of $\widetilde{\nabla}$, we can find a subsequence that converges to an element of $\widetilde{\nabla}$.

(ii) closed $\Leftrightarrow$ for every sequence $\{\varsigma_\wp\}_{\wp \in \mathbb{N}}$ of elements of $\widetilde{\nabla}$ that converges to an element $\varsigma$, we have $\varsigma \in \widetilde{\nabla}$.

*Definition 4* [24]. A nonempty subset $\widetilde{\nabla}$ of $\mho$ is called proximal if, for each $\varsigma \in \mho$, we can find $a \in \widetilde{\nabla}$ such that $\widehat{\mu}(\varsigma, a) = \widehat{\mu}(\varsigma, \widetilde{\nabla})$.

We denote by $\mathscr{N}(\mho)$, $CB(\mho)$, $P^r(\mho)$, $P_b^r(\mho)$ and $\mathscr{K}(\mho)$, the family of all nonempty subsets of $\mho$, the class of all nonempty closed and bounded subsets of $\mho$, the collection of all nonempty proximal subsets of $\mho$, the totality of all bounded proximal subsets of $\mho$ and the class of nonempty compact subsets of $\mho$, respectively.

Consider a $b$-metric space $(\mho, \widehat{\mu}, \widehat{\eta})$. For $\widetilde{\nabla}, \widetilde{\nabla} \in \mathscr{P}^r(\mho)$, the function $\aleph$: $\mathscr{P}^r(\mho) \times \mathscr{P}^r(\mho) \longrightarrow \mathbb{R}_+$, defined by

$$\aleph(\widetilde{\nabla}, \widetilde{\Delta}) = \max\left\{\sup_{\varsigma \in \widetilde{\nabla}} \widehat{\mu}(\varsigma, \widetilde{\Delta}), \sup_{\varsigma \in \widetilde{\Delta}} \widehat{\mu}(\varsigma, \widetilde{\nabla})\right\}, \quad (1)$$

is called a Hausdorff-Pompeiu $b$-metric on $\mathscr{P}^r(\mho)$ generated by $\widehat{\mu}$, where

$$\widehat{\mu}(\varsigma, \widetilde{\nabla}) = \inf_{\omega \in \widetilde{\nabla}} \widehat{\mu}(\varsigma, \omega). \quad (2)$$

*Remark 1.* Since every compact set is proximal and every proximal set is closed (see [24]), whence:

$$\mathscr{K}(\mho) \subseteq \mathscr{P}^r(\mho) \subseteq CB(\mho) \subseteq \mathscr{N}(\mho). \quad (3)$$

Let $\mho$ be a universal set. A fuzzy set in $\mho$ is a function with domain $\mho$ and values in $[0, 1] = I$. If $\widetilde{\nabla}_f$ is a fuzzy set in $\mho$, then the function value $\widetilde{\nabla}_f(\varsigma)$ is called the grade of membership of $\varsigma$ in $\widetilde{\nabla}_f$. The $\breve{\alpha}$-level set of a fuzzy set $\widetilde{\nabla}_f$ is denoted by $[\widetilde{\nabla}_f]_{\breve{\alpha}}$ and is given as follows:

$$[\widetilde{\nabla}_f]_{\breve{\alpha}} = \begin{cases} \overline{\{\varsigma \in \mho: \widetilde{\nabla}(\varsigma) > 0\}}, & \text{if } \breve{\alpha} = 0 \\ \{\varsigma \in \mho: \widetilde{\nabla}(\varsigma) \geq \breve{\alpha}\}, & \text{if } \breve{\alpha} \in (0,1], \end{cases} \quad (4)$$

where by $\overline{M}$, we mean the closure of the crisp set $M$. We denote the family of all fuzzy sets in $\mho$ by $I^{\mho}$.

A fuzzy set $\widetilde{\nabla}_f$ in a metric space $V$ is called an approximate quantity if and only if $[\widetilde{\nabla}_f]_{\breve{\alpha}}$ is compact and convex in $V$ and $\sup_{\varsigma \in V} \widetilde{\nabla}(\varsigma) = 1$. Denote the collection of all approximate quantities in $V$ by $W(V)$. If we can find an $\breve{\alpha} \in [0,1]$ such that $[\widetilde{\nabla}_f]_{\breve{\alpha}}, [\widetilde{\triangle}_f]_{\breve{\alpha}} \in \mathscr{P}_b^r(\mho)$, then define

$$\begin{aligned} D_{\breve{\alpha}}(\widetilde{\nabla}_f, \widetilde{\triangle}_f) &= \aleph([\widetilde{\nabla}_f]_{\breve{\alpha}}, [\widetilde{\triangle}_f]_{\breve{\alpha}}), \\ \widehat{\mu}_{\infty}(\widetilde{\nabla}_f, \widetilde{\triangle}_f) &= \sup_{\breve{\alpha}} D_{\breve{\alpha}}(\widetilde{\nabla}, \widetilde{\triangle}). \end{aligned} \quad (5)$$

*Definition 5* [3]. Let $\mho$ be a nonempty set. The mapping $\Xi: \mho \longrightarrow I^{\mho}$ is called a fuzzy set-valued map. A point $u \in \mho$ is called a fuzzy Fp of $\Xi$ if we can find an $\breve{\alpha} \in (0,1]$ such that $u \in [\Xi u]_{\breve{\alpha}}$.

*Definition 6* [11]. Let $\mho$ be a nonempty set. An IFS $\widetilde{\nabla}$ in $\mho$ is a set:

$$\widetilde{\nabla} = \{\langle J, \widehat{\mu}_{\widetilde{\nabla}}(J), \nu_{\widetilde{\nabla}}(J)\rangle: J \in \mho\}, \quad (6)$$

where $\widehat{\mu}_{\widetilde{\nabla}}: \mho \longrightarrow [0,1]$ and $\nu_{\widetilde{\nabla}}: \mho \longrightarrow [0,1]$ define the degrees of membership and non-membership, accordingly of $J$ in $\mho$ and fulfil $0 \leq \widehat{\mu}_{\widetilde{\nabla}} + \nu_{\widetilde{\nabla}} \leq 1$, for each $J \in \mho$.

We depict the set of all IFS in $\mho$ as $(IFS)^{\mho}$.

*Definition 7* [11]. Let $\widetilde{\nabla}$ be an IFS in $\mho$. Then the $\breve{\alpha}$-level set of $\widetilde{\nabla}$ is a crisp subset of $\mho$ denoted by $[\widetilde{\nabla}]_{\breve{\alpha}}$ and is given as follows:

$$[\widetilde{\nabla}]_{\breve{\alpha}} = \{J \in \mho: \widehat{\mu}_{\widetilde{\nabla}}(J) \geq \breve{\alpha} \text{ and } \nu_{\widetilde{\nabla}}(J) \leq 1 - \breve{\alpha}\}, \text{if } \breve{\alpha} \in [0,1]. \quad (7)$$

*Definition 8* [12]. Let $L = \{(\breve{\alpha}, \breve{\beta}): \breve{\alpha} + \breve{\beta} \leq 1, (\breve{\alpha}, \breve{\beta}) \in (0,1] \times t[0,1)\}$ and $\widetilde{\nabla}$ is an IFS in $\mho$. Then the $(\breve{\alpha}, \breve{\beta})$-level set of $\widetilde{\nabla}$ is given as follows:

$$[\widetilde{\nabla}]_{(\breve{\alpha}, \breve{\beta})} = \{J \in \mho: \widehat{\mu}_{\widetilde{\nabla}}(J) \geq \breve{\alpha} \text{ and } \nu_{\widetilde{\nabla}}(J) \leq \breve{\beta}\}. \quad (8)$$

A modification of Definition 2.9 in [13] is the following.

*Definition 9* [13]. The $(\widetilde{M}, \widetilde{\omega})$-level set of an intuitionistic fuzzy set $\widetilde{\nabla}$ in $\mho$ is given as follows:

$$[\widetilde{\nabla}]_{(\widetilde{M}, \widetilde{\omega})} = \{\varsigma \in \mho: \widehat{\mu}_{\widetilde{\nabla}}(\varsigma) = \widetilde{M} \text{ and } \nu_{\widetilde{\nabla}}(\varsigma) = \widetilde{\omega}\}, \quad (9)$$

with

$$\widetilde{M} = \max_{\varsigma \in \mho} \widehat{\mu}_{\widetilde{\nabla}}(\varsigma) \text{ and } \widetilde{\omega} = \min_{\varsigma \in \mho} \nu_{\widetilde{\nabla}}(\varsigma). \quad (10)$$

*Example 1.* Let $\mho = \{J_1, J_2, J_3, J_4, J_5\}$ and $\widetilde{\nabla}$ be an IFS in $\mho$ given by

$$\widetilde{\nabla} = \{(J_1, 0.6, 0.2), (J_2, 0.5, 0.4), (J_3, 0.1, 0.7), (J_4, 0.3, 0.5), (J_5, 0.4, 0.3)\}. \quad (11)$$

Then the $(\breve{\alpha}, \breve{\beta})$-level sets of $\widetilde{\nabla}$ are given by.
$[\widetilde{\nabla}]_{(0.4,0.3)} = \{J_1, J_5\}$.
$[\widetilde{\nabla}]_{(0.1,0.7)} = \{J_1, J_2, J_3, J_4, J_5\}$.
$[\widetilde{\nabla}]_{(0.3,0.5)} = \{J_1, J_2, J_4, J_5\}$.

*Definition 10* [12]. Let $\mho$ be a nonempty set. The map $\Upsilon = \langle \widehat{\mu}_{\Upsilon}, \nu_{\Upsilon}\rangle: \mho \longrightarrow (IFS)^{\mho}$ is called an intuitionistic fuzzy set-valued map. An element $u \in \mho$ is named an intuitionistic fuzzy Fp of $\Upsilon$ if we can find $(\breve{\alpha}, \breve{\beta}) \in (0,1] \times t[0,1)$ such that $u \in [\Upsilon u]_{(\breve{\alpha}, \breve{\beta})}$.

*Definition 11* [22, 25]. An increasing function $\widehat{\varphi}: \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ is called:

(i) a $c$-comparison function if $\widehat{\varphi}^{\wp}(t) \longrightarrow 0$ as $\wp \longrightarrow \infty$ for every $t \in \mathbb{R}_+$;

(ii) a $b$-comparison function if we can find $k_0 \in \mathbb{N}$, $\lambda \in (0,1)$ and a convergent non-negative series $\sum_{\wp=1}^{\infty} \varsigma_{\wp}$: $\widehat{\eta}^{k+1}\widehat{\varphi}^{k+1}(t) \leq \lambda \widehat{\eta}^k \widehat{\varphi}^k(t) + \varsigma_k$, for $\widehat{\eta} \geq 1, k \geq k_0$ and any $t \geq 0$, where $\widehat{\varphi}^{\wp}$ denotes the $\wp^{th}$ iterate of $\widehat{\varphi}$

Denote by $\Omega$, the class of functions $\widehat{\varphi}: \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ obeying:

(i) $\widehat{\varphi}$ is a $b$-comparison function;

(ii) $\widehat{\varphi}(t) = 0 \Leftrightarrow t = 0$;

(iii) $\widehat{\varphi}$ is continuous.

**Lemma 1** [25]. *For every comparison function $\widehat{\varphi}: \mathbb{R}_+ \longrightarrow \mathbb{R}_+$, we have the following points:*

*(i) each iterate $\widehat{\varphi}^{\wp}, \wp \in \mathbb{N}$ is also a comparison function;*

*(iii) $\widehat{\varphi}(t) < t$ for all $t > 0$.*

**Lemma 2** [25]. *Let $\widehat{\varphi}: \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ be a $b$-comparison function. Then, the series $\sum_{k=0}^{\infty} \widehat{\eta}^k \widehat{\varphi}^k(t)$ converges for every $t \in \mathbb{R}_+$.*

*Remark 2* [22]. In Lemma 2, every $b$-comparison function is a comparison function and thus, in Lemma 1, every $b$-comparison function satisfies $\widehat{\varphi}(t) < t$.

**Lemma 3** ([26]). *Consider a b-metric space $(\mho,\widehat{\mu},\widehat{\eta})$. For $\widetilde{\nabla},\widetilde{\triangle}\in\mathscr{K}(\mho)$ and $\varsigma,\omega\in\mho$, the following criteria hold:*

(i) $\widehat{\mu}(\varsigma,\widetilde{\triangle})\leq\aleph(\widetilde{\nabla},\widetilde{\triangle})$, *for any* $\varsigma\in\widetilde{\nabla}$.

(ii) $\widehat{\mu}(\varsigma,\widetilde{\nabla})\leq\widehat{\eta}[\widehat{\mu}(\varsigma,\omega)+\widehat{\mu}(\omega,\widetilde{\nabla})]$.

(iii) $\widehat{\mu}(\varsigma,\widetilde{\nabla})=0\Leftrightarrow\varsigma\in\widetilde{\nabla}$.

(iv) $\aleph(\widetilde{\nabla},\widetilde{\triangle})=0\Leftrightarrow\widetilde{\nabla}=\widetilde{\triangle}$.

(v) $\aleph(\widetilde{\nabla},\widetilde{\triangle})=\aleph(\widetilde{\triangle},\widetilde{\nabla})$.

(vi) $\aleph(\widetilde{\nabla},\widetilde{\triangle})\leq\widehat{\eta}[\aleph_b(\widetilde{\nabla},C)+\aleph_b(C,\widetilde{\triangle})]$.

## 3. Main Results

We commence this section with the notion of intuitionistic fuzzy $p$-hybrid contractions on a $b$-metric space in the following manner.

*Definition 12.* Consider a $b$-metric space $(\mho,\widehat{\mu},\widehat{\eta})$ and $\Upsilon,\Psi\colon\mho\longrightarrow(IFS)^{\mho}$ be intuitionistic fuzzy set-valued maps. Then, the pair $(\Upsilon,\Psi)$ is said to form an intuitionistic fuzzy $p$-hybrid contraction, if for all $\varsigma,\omega\in\mho$, we can find $(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)},(\breve{\alpha},\breve{\beta})_{\Psi(\omega)}\in(0,1]\times t[0,1)$ such that

$$\aleph\left([\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}},[\Psi\omega]_{(\breve{\alpha},\breve{\beta})_{\Psi(\omega)}}\right)\leq\widehat{\varphi}\mathscr{C}^p_{(\Upsilon,\Psi)}\left(\varsigma,\omega,(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)},(\breve{\alpha},\breve{\beta})_{\Psi(\omega)}\right),\tag{12}$$

where $\widehat{\varphi}\in\Omega$, $p\geq 0$, $a_i\geq 0$, $i=1,2,3,4$ with $\sum_{i=1}^4 a_i=1$ and

$$\mathscr{C}^p_{(\Upsilon,\Psi)}\left(\varsigma,\omega,(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)},(\breve{\alpha},\breve{\beta})_{\Psi(\omega)}\right)=\tag{13}$$

$$\begin{cases}\left[a_1(\widehat{\mu}(\varsigma,\omega))^p+a_2\left(\widehat{\mu}(\varsigma),[\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}}\right)^p\right.\\ \left.+a_3\left(\widehat{\mu}\left(\omega,[\Psi\omega]_{(\breve{\alpha},\breve{\beta})_{\Psi(\omega)}}\right)\right)^p+a_4\left(\dfrac{\widehat{\mu}\left(\omega,[\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}}\right)+\widehat{\mu}\left(\varsigma,[\Psi\omega]_{(\breve{\alpha},\breve{\beta})_{\Psi(\omega)}}\right)}{2\widehat{\eta}}\right)^p\right]^{1/p},\\ \left(\widehat{\mu}(\varsigma,\omega)^{a_1}\left(\widehat{\mu}\left(\varsigma,[\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}}\right)\right)^{a_2}\left(\widehat{\mu}\left(\omega,[\Psi\omega]_{(\breve{\alpha},\breve{\beta})_{\Psi(\omega)}}\right)\right)\right)^{a_3}\\ \times\left(\dfrac{\widehat{\mu}\left(\varsigma,[\Psi\omega]_{(\breve{\alpha},\breve{\beta})_{\Psi(\omega)}}\right)+\widehat{\mu}\left(\omega,[\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}}\right)}{2\widehat{\eta}}\right)^{a_4},\\ \text{for } p=0,\varsigma,\omega\in\mho\setminus\mathscr{F}_{ix}(\Upsilon,\Psi),\end{cases}\tag{14}$$

where

$$\mathscr{F}_{ix}(\Upsilon,\Psi)=\left\{\varsigma,\omega\in\mho\colon\varsigma\in[\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}},\omega\in[\Psi\omega]_{(\breve{\alpha},\breve{\beta})_{\Psi(\omega)}}\right\}.\tag{15}$$

In particular, if (12) holds for $p=0$, then we say that the pair $(\Upsilon,\Psi)$ forms an intuitionistic fuzzy 0-hybrid contraction. Our first main result is presented hereunder.

**Theorem 1.** *Let $(\mho,\widehat{\mu},\widehat{\eta})$ be a complete b-metric space and $\Upsilon,\Psi\colon\mho\longrightarrow(IFS)^{\mho}$ be intuitionistic fuzzy set-valued maps. Suppose that for each $\varsigma\in\mho$, we can find $(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}$, $(\breve{\alpha},\breve{\beta})_{\Psi(\varsigma)}\in(0,1]\times t[0,1)$ such that $[\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}}$ and $[\Psi\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Psi(\varsigma)}}$ are nonempty bounded proximal subsets of $\mho$. If the pair $(\Upsilon,\Psi)$ forms an intuitionistic fuzzy p-hybrid contraction, then $\Upsilon$ and $\Psi$ have a common intuitionistic fuzzy Fp in $\mho$.*

*Proof.* Let $\varsigma_0\in\mho$, then, by hypotheses, we can find $(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_0)}\in(0,1]\times t[0,1)$ such that $[\Upsilon\varsigma_0]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_0)}}\in\mathscr{P}^r_b(\mho)$. Take $\varsigma_1\in[\Upsilon\ \varsigma_0]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_0)}}$ such that $\widehat{\mu}(\varsigma_0,\varsigma_1)=\widehat{\mu}(\varsigma_0,\ [\Upsilon\varsigma_0]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_0)}})$. Similarly, $[\Psi\varsigma_1]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_1)}}\in\mathscr{P}^r_b(\mho)$, by hypothesis. So, we can find $\varsigma_2\in[\Psi\varsigma_1]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_1)}}$ so that by proximality of $\Psi$, $\widehat{\mu}(\varsigma_1,\varsigma_2)=\widehat{\mu}(\varsigma_1,[\Psi\varsigma_1]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_1)}})$. Continuing in this direction, we can construct a sequence $\{\varsigma_\wp\}_{\wp\in\mathbb{N}}$ of elements of $\mho$ such that

$$\varsigma_{2k+1}\in[\Upsilon\varsigma_{2k}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k})}},\varsigma_{2k+2}\in[\Psi\varsigma_{2k+1}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k+1})}}\tag{16}$$

and

$$\begin{aligned}\widehat{\mu}(\varsigma_{2k},\varsigma_{2k+1}) &= \widehat{\mu}\left(\varsigma_{2k},[\Upsilon\varsigma_{2k}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k})}}\right),\\ \widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k+2}) &= \widehat{\mu}\left(\varsigma_{2k+1},[\Psi\varsigma_{2k+1}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k+1})}}\right), k\in\mathbb{N}.\end{aligned} \tag{17}$$

By Lemma 3 and the above relations, we have

$$\widehat{\mu}(\varsigma_{2k},\varsigma_{2k+1}) \le \aleph\left([\Upsilon\varsigma_{2k}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k})}},[\Psi\varsigma_{2k-1}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k-1})}}\right), \tag{18}$$

and

$$\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k+2}) \le \aleph\left([\Upsilon\varsigma_{2k}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k})}},[\Psi\varsigma_{2k+1}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k+1})}}\right). \tag{19}$$

Suppose that $\varsigma_{2k}=\varsigma_{2k+1}$, for some $k\in\mathbb{N}$ and $p>0$. Then, from (13), we have

$$\begin{aligned}&\mathscr{C}^{p}_{(\Upsilon,\Psi)}\left(\varsigma_{2k},\varsigma_{2k+1},(\breve{\alpha},\breve{\beta})_{\Upsilon(2k)},(\breve{\alpha},\breve{\beta})_{\Psi(2k+1)}\right)\\ &=\left[\begin{array}{c} a_1(\widehat{\mu}(\varsigma_{2k},\varsigma_{2k+1}))^p + a_2\left(\widehat{\mu}\left(\varsigma_{2k},[\Upsilon\varsigma_{2k}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k})}}\right)\right)^p + a_3\left(\widehat{\mu}\left(\varsigma_{2k+1},[\Psi\varsigma_{2k+1}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k+1})}}\right)\right)^p + \\ a_4\left(\dfrac{\widehat{\mu}\left(\varsigma_{2k+1},[\Upsilon\varsigma_{2k}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k})}}\right)+\widehat{\mu}\left(\varsigma_{2k},[\Psi\varsigma_{2k+1}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k+1})}}\right)}{2\widehat{\eta}}\right)^p\end{array}\right]^{1/p}\\ &=\left[a_1(\widehat{\mu}(\varsigma_{2k},\varsigma_{2k+1}))^p + a_2(\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k+1}))^p + a_3(\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k+2}))^p + a_4\left(\frac{(\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k+1}))^p+(\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k+2}))}{2\widehat{\eta}}\right)^p\right]^{1/p}\\ &\le\left[a_3(\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k+2}))^p + a_4\left(\widehat{\eta}\left(\frac{\widehat{\mu}(\varsigma_{2k},\varsigma_{2k+1})+\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k+2})}{2\widehat{\eta}}\right)\right)^p\right]^{1/p}\\ &\le\left[a_3(\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k+2}))^p + a_4(\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k+2}))^p\right]^{1/p}\\ &=(a_3+a_4)^{1/p}\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k+2}) = \widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k+2})\, as\ p\longrightarrow\infty.\end{aligned} \tag{20}$$

Whence, using Lemma 1, we have

$$\begin{aligned}\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k+2}) &\le \aleph\left([\Upsilon\varsigma_{2k}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k})}},[\Psi\varsigma_{2k+1}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k+1})}}\right)\\ &\le\widehat{\varphi}(\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k+2}))<\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k+2}),\end{aligned} \tag{21}$$

a contradiction. It follows that for all $k\in\mathbb{N}$,

$$\varsigma_{2k}=\varsigma_{2k+1}\in[\Upsilon\varsigma_{2k}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k})}}, \tag{22}$$

$$\begin{aligned}\varsigma_{2k}=\varsigma_{2k+1}=\varsigma_{2k+2}&\in[\Psi\varsigma_{2k+1}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k+1})}}\\ &=[\Psi\varsigma_{2k}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k})}}.\end{aligned} \tag{23}$$

It follows that $\varsigma_{2k}$ is the common intuitionistic fuzzy Fp of $\Upsilon$ and $\Psi$.

Again, for $p=0$ and $\varsigma_{2k}=\varsigma_{2k+1}$, for some $k\in\mathbb{N}$, we get $\mathscr{C}^{p}_{(\Upsilon,\Psi)}(\varsigma_{2k},\varsigma_{2k+1},(\breve{\alpha},\breve{\beta})_{\Upsilon(2k)},(\breve{\alpha},\breve{\beta})_{\Psi(2k+1)})=0$, for all $k\in\mathbb{N}$. Whence, by property $(ii)$ of $\Omega$, one obtains $\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k+2})=0$, for all $k\in\mathbb{N}$; from which, on similar arguments as above, the same conclusion follows that $\varsigma_{2k}\in[\Upsilon\varsigma_{2k}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k})}}\cap[\Upsilon\varsigma_{2k}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k})}}$. Hereafter, we assume that for all $k\in\mathbb{N}$, $\varsigma_{k+1}\neq\varsigma_k$ if and only if $\widehat{\mu}(\varsigma_{k+1},\varsigma_k)>0$.

Now, in view of (13), setting $\varsigma=\varsigma_{2k}$ and $\omega=\varsigma_{2k-1}$, we have

$$\mathscr{C}^{p}_{(\Upsilon,\Psi)}\left(\varsigma_{2k},\varsigma_{2k-1},(\breve{\alpha},\breve{\beta})_{\Upsilon(2k-1)},(\breve{\alpha},\breve{\beta})_{\Psi(2k+1)}\right)$$
$$=\begin{cases}\Bigg[a_1(\widehat{\mu}(\varsigma_{2k},\varsigma_{2k-1}))^p+a_2\left(\widehat{\mu}\left(\varsigma_{2k},[\Upsilon\varsigma_{2k}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k})}}\right)\right)^p \\ +a_3\widehat{\mu}\left(\varsigma_{2k-1},[\Psi\varsigma_{2k-1}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k})}}\right)^p+a_4\left(\dfrac{\widehat{\mu}\left(\varsigma_{2k-1},[\Upsilon\varsigma_{2k}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k})}}\right)+\widehat{\mu}\left(\varsigma_{2k},[\Psi\varsigma_{2k-1}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k})}}\right)}{2\widehat{\eta}}\right)^p\Bigg]^{1/p}, \\ for\ p>0, \\ (\widehat{\mu}(\varsigma_{2k},\varsigma_{2k-1}))^{a_1}\left(\widehat{\mu}\left(\varsigma_{2k},[\Upsilon\varsigma_{2k}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k})}}\right)\right)^{a_2}\left(\widehat{\mu}\left(\varsigma_{2k-1},[\Psi\varsigma_{2k-1}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k})}}\right)\right)^{a_3} \\ \times\left(\dfrac{\widehat{\mu}\left(\varsigma_{2k},[\Psi\varsigma_{2k-1}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k})}}\right)+\widehat{\mu}\left(\varsigma_{2k-1},[\Upsilon\varsigma_{2k}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k})}}\right)}{2\widehat{\eta}}\right)^{a_4},\ \text{for } p=0.\end{cases}\tag{24}$$

That is,

$$\mathscr{C}^{p}_{(\Upsilon,\Psi)}\left(\varsigma_{2k},\varsigma_{2k-1},(\breve{\alpha},\breve{\beta})_{\Upsilon(2k)},(\breve{\alpha},\breve{\beta})_{\Psi(2k-1)}\right)=\tag{25}$$

$$\begin{cases}\Bigg[a_1(\widehat{\mu}(\varsigma_{2k},\varsigma_{2k-1}))^p+a_2(\widehat{\mu}(\varsigma_{2k},\varsigma_{2k+1}))^p \\ +a_3(\widehat{\mu}(\varsigma_{2k-1},\varsigma_{2k}))^p+a_4\left(\dfrac{\widehat{\mu}(\varsigma_{2k-1},\varsigma_{2k+1})+\widehat{\mu}(\varsigma_{2k},\varsigma_{2k})}{2\widehat{\eta}}\right)^p\Bigg]^{1/p} \\ \text{for } p>0, \\ (\widehat{\mu}(\varsigma_{2k},\varsigma_{2k-1}))^{a_1}(\widehat{\mu}(\varsigma_{2k},\varsigma_{2k+1}))^{a_2}(\widehat{\mu}(\varsigma_{2k-1},\varsigma_{2k}))^{a_3} \\ \times\left(\dfrac{\widehat{\mu}(\varsigma_{2k},\varsigma_{2k})+\widehat{\mu}(\varsigma_{2k-1},\varsigma_{2k+1})}{2\widehat{\eta}}\right)^{a_4}, \\ \text{for } p=0.\end{cases}\tag{26}$$

Now, we consider the following two cases: □

*Case 1.* $p>0$. Suppose that $\widehat{\mu}(\varsigma_{2k},\varsigma_{2k+1})\geq\widehat{\mu}(\varsigma_{2k-1},\varsigma_{2k})$, then from (25), we have

$$\begin{aligned}
&\mathscr{C}^{p}_{(\Upsilon,\Psi)}\Big(\varsigma_{2k},\varsigma_{2k-1},(\breve{\alpha},\breve{\beta})_{\Upsilon(2k)},(\breve{\alpha},\breve{\beta})_{\Psi(2k-1)}\Big)\\
&\leq\Big[a_1(\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k}))^p+a_2(\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k}))^p\\
&\quad+a_3(\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k}))^p+a_4\bigg(\widehat{\eta}\bigg(\frac{\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k})+\widehat{\mu}(\varsigma_{2k},\varsigma_{2k-1})}{2\widehat{\eta}}\bigg)\bigg)^p\Big]^{1/p}\\
&\leq\Big[a_1(\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k}))^p+a_2(\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k}))^p+a_3(\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k}))^p\\
&\quad+a_4\bigg(\widehat{\eta}\bigg(\frac{\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k})+\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k})}{2\widehat{\eta}}\bigg)^p\bigg)\Big]^{1/p}\\
&\leq\Big[a_1(\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k}))^p+a_2(\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k}))^p\\
&\quad+a_3(\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k}))^p+a_4(\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k}))^p\Big]^{1/p}\\
&=\Big[(a_1+a_2+a_3+a_4)\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k})^p\Big]^{1/p}\\
&=\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k})\bigg(\sum_{i=1}^{4}a_i\bigg)^{1/p}=\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k}).
\end{aligned}\tag{27}$$

Hence, from (12) and (27), we have

$$\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k})\leq\widehat{\varphi}(\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k})).\tag{28}$$

Given that $\widehat{\varphi}$ is a $b$-comparison function, (28) implies

$$\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k})<\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k}),\tag{29}$$

which is a contradiction. Whence, it follows that $\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k})\leq\widehat{\mu}(\varsigma_{2k},\varsigma_{2k-1})$. Thus, from (28), we obtain

$$\widehat{\mu}(\varsigma_{2k+1},\varsigma_{2k})\leq\widehat{\varphi}(\widehat{\mu}(\varsigma_{2k},\varsigma_{2k-1})).\tag{30}$$

Setting $\wp=2k\in\mathbb{N}$ in (30), yields

$$\begin{aligned}
\widehat{\mu}(\varsigma_{\wp+1},\varsigma_{\wp})&\leq\widehat{\varphi}\big(\widehat{\mu}(\varsigma_{\wp},\varsigma_{\wp-1})\big)\\
&\leq\widehat{\varphi}^2\big(\widehat{\mu}(\varsigma_{\wp-1},\varsigma_{\wp-2})\big)\\
&\leq\widehat{\varphi}^3\big(\widehat{\mu}(\varsigma_{\wp-2},\varsigma_{\wp-3})\big)\\
&\;\;\vdots\quad\vdots\\
&\leq\widehat{\varphi}^{\wp}(\widehat{\mu}(\varsigma_1,\varsigma_0)).
\end{aligned}\tag{31}$$

From (31), by triangle inequality on $(\mho,\widehat{\mu},\widehat{\eta})$, for all $k\geq 1$, we have

$$\begin{aligned}
\widehat{\mu}(\varsigma_{\wp+k},\varsigma_{\wp})&\leq\widehat{\eta}\big(\widehat{\mu}(\varsigma_{\wp+k},\varsigma_{\wp+1})+\widehat{\mu}(\varsigma_{\wp+1},\varsigma_{\wp})\big)\\
&\leq\frac{1}{\widehat{\eta}^{\wp-1}}\sum_{i=\wp}^{\wp+k-1}\widehat{\eta}^k\widehat{\mu}(\varsigma_i,\varsigma_{i+1})\\
&\leq\frac{1}{\widehat{\eta}^{\wp-1}}\sum_{i=\wp}^{\wp+k-1}\widehat{\eta}^k\widehat{\varphi}^k(\widehat{\mu}(\varsigma_1,\varsigma_0))\\
&\leq\frac{1}{\widehat{\eta}^{\wp-1}}\sum_{i=\wp}^{\infty}\widehat{\eta}^i\widehat{\varphi}^i(\widehat{\mu}(\varsigma_1,\varsigma_0)).
\end{aligned}\tag{32}$$

Letting $\wp \longrightarrow \infty$ in (32) and applying Lemma 2, we find that $\lim_{\wp\longrightarrow\infty}\widehat{\mu}(\varsigma_{\wp+k},\varsigma_{\wp}) = 0$. Whence, $\{\varsigma_{\wp}\}_{\wp\in\mathbb{N}}$ is a Cauchy sequence of points of $(\mho,\widehat{\mu},\widehat{\eta})$. The completeness of this space implies that we can find $u \in \mho$ such that

$$\lim_{\wp\longrightarrow\infty}\widehat{\mu}\left(\varsigma_{\wp},u\right) = 0. \tag{33}$$

Now, we show that $u$ is the anticipated common intuitionistic fuzzy Fp of $\Upsilon$ and $\Psi$. First, assume that $u \notin [\Upsilon u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}}$ so that $\widehat{\mu}(u,[\Upsilon u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}}) > 0$. Then, by Lemma 3 and for $p > 0$ in the contractive inequality (3.1), we have

$$\begin{aligned}
\widehat{\mu}\left(u,[\Upsilon u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}}\right) &\le \widehat{\eta}\widehat{\mu}\left(u,\varsigma_{\wp}\right) + \widehat{\eta}\widehat{\mu}\left(\varsigma_{\wp},[\Upsilon u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}}\right) \le \widehat{\eta}\widehat{\mu}\left(u,\varsigma_{\wp}\right) + \widehat{\eta}\aleph\left([\Upsilon u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}},\left[\Psi\varsigma_{\wp-1}\right]_{(\breve{\alpha},\breve{\beta})_{\Psi}\left(\varsigma_{\wp-1}\right)}\right)\\
&\le \widehat{\eta}\widehat{\mu}\left(u,\varsigma_{\wp}\right) + \widehat{\eta}\widehat{\varphi}\left(\mathscr{C}^{p}_{(\Upsilon,\Psi)}\left(u,\varsigma_{\wp-1}\right)\right) = \widehat{\eta}\widehat{\mu}\left(u,\varsigma_{\wp}\right) + \widehat{\eta}\widehat{\varphi}\left(\left[a_1\left(\widehat{\mu}\left(u,\varsigma_{\wp-1}\right)\right)^{p} + a_2\left(\widehat{\mu}\left(u,[\Upsilon u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}}\right)\right)\right]\right)^{p}\\
&\quad + a_3\left(\widehat{\mu}\left(\varsigma_{\wp-1},\left[\Psi\varsigma_{\wp-1}\right]_{(\breve{\alpha},\breve{\beta})_{\Psi}\left(\varsigma_{\wp-1}\right)}\right)\right)^{p} + a_4\left(\frac{\widehat{\mu}\left(\varsigma_{\wp-1},[\Upsilon u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}}\right) + \widehat{\mu}\left(u,\left[\Psi\varsigma_{\wp-1}\right]_{(\breve{\alpha},\breve{\beta})_{\Psi}\left(\varsigma_{\wp-1}\right)}\right)}{2\widehat{\eta}}\right)^{p}\Bigg]^{1/p}\Bigg)\\
&= \widehat{\eta}\widehat{\mu}\left(u,\varsigma_{\wp}\right) + \widehat{\eta}\widehat{\varphi}\left(a_1\left(\widehat{\mu}\left(u,\varsigma_{\wp-1}\right)\right)^{p} + a_2\left(\widehat{\mu}\left(u,[\Upsilon u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}}\right)\right)\right)^{p} + a_3\left(\widehat{\mu}\left(\varsigma_{\wp-1},\varsigma_{\wp}\right)\right)^{p}\\
&\quad + a_4\left(\frac{\widehat{\mu}\left(\varsigma_{\wp-1},[\Upsilon u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}}\right) + \widehat{\mu}\left(u,\varsigma_{\wp}\right)}{2\widehat{\eta}}\right)^{p}\Bigg]^{1/p}\Bigg).
\end{aligned} \tag{34}$$

Letting $\wp \longrightarrow \infty$ in (34), and using the properties of $\widehat{\varphi} \in \Omega$, gives

$$\widehat{\mu}\left(u,[\Upsilon u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}}\right) < \widehat{\eta}\widehat{\mu}\left(u,[\Upsilon u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}}\right)\left(a_2 + a_4\right)^{1/p}, \tag{35}$$

and as $p \longrightarrow \infty$,

$$\widehat{\mu}\left(u,[\Upsilon u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}}\right) < \widehat{\eta}\widehat{\mu}\left(u,[\Upsilon u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}}\right), \tag{36}$$

which is a contradiction for $\widehat{\eta} = 1$ . Thus, $\widehat{\mu}(u,[\Upsilon u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}}) = 0$, which further implies that $u \in [\Upsilon u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}}$. On similar steps, by assuming that $u$ is not an intuitionistic fuzzy Fp of $\Psi$, and considering

$$\begin{aligned}
\widehat{\mu}\left(u,[\Psi u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}}\right) &\le \widehat{\eta}\widehat{\mu}\left(u,\varsigma_{\wp}\right) + \widehat{\eta}\widehat{\mu}\left(\varsigma_{\wp},[\Psi u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}}\right)\\
&\le \widehat{\eta}\widehat{\mu}\left(u,\varsigma_{\wp}\right) + \widehat{\eta}\aleph\left(\left[\Upsilon\varsigma_{\wp-1}\right]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}},[\Psi u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}}\right)\\
&\le \widehat{\eta}\widehat{\mu}\left(u,\varsigma_{\wp}\right) + \widehat{\eta}\widehat{\varphi}\left(\mathscr{C}^{p}_{(\Psi,\Upsilon)}\left(\varsigma_{\wp-1},u,(\breve{\alpha},\breve{\beta})_{\Psi}\left(\varsigma_{\wp-1}\right),(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}\right)\right),
\end{aligned} \tag{37}$$

we can show that $u \in [\Psi u]_{(\breve{\alpha},\breve{\beta})_{\Psi(u)}}$. Whence, for $p > 0$, we can find $u \in \mho$ such that $u \in [\Upsilon u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}} \cap [\Psi u]_{(\breve{\alpha},\breve{\beta})_{\Psi(u)}}$.

*Case 2.* $p = 0$. Applying the inequality (25) on account of $b$-comparison of $\widehat{\varphi}$, we have

$$\begin{aligned}
\widehat{\mu}(\varsigma_{2k}, \varsigma_{2k-1}) &\leq \aleph\left( [\Upsilon\varsigma_{2k-1}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(2k-1)}}, [\Psi\varsigma_{2k-2}]_{(\breve{\alpha},\breve{\beta})_{\Psi(2k-2)}} \right) \\
&\leq \widehat{\varphi}\left( \mathscr{C}^{p}_{(\Upsilon,\Psi)}(\varsigma_{2k-1}, \varsigma_{2k-2}), (\breve{\alpha},\breve{\beta})_{\Upsilon(2k-1)}, (\breve{\alpha},\breve{\beta})_{\Psi(2k-2)} \right) \\
&< (\widehat{\mu}(\varsigma_{2k-1}, \varsigma_{2k-2}))^{a_1} \left( \varsigma_{2k-1}, [\Upsilon\varsigma_{2k-1}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(2k-1)}} \right)^{a_2} \left( \widehat{\mu}\left( \varsigma_{2k-2}, [\Psi\varsigma_{2k-2}]_{(\breve{\alpha},\breve{\beta})_{\Psi(2k-2)}} \right) \right)^{a_3} \\
&\quad \times \left( \frac{\widehat{\mu}\left( \varsigma_{2k-2}, [\Psi\varsigma_{2k-2}]_{(\breve{\alpha},\breve{\beta})_{\Psi(2k-2)}} \right) + \widehat{\mu}\left( \varsigma_{2k-2}, [\Upsilon\varsigma_{2k-1}]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(2k-1)}} \right)}{2\widehat{\eta}} \right)^{a_4} \\
&= (\widehat{\mu}(\varsigma_{2k-1}, \varsigma_{2k-2}))^{a_1} (\widehat{\mu}(\varsigma_{2k-1}, \varsigma_{2k}))^{a_2} (\widehat{\mu}(\varsigma_{2k-2}, \varsigma_{2k-1}))^{a_3} \\
&\quad \times \left( \frac{\widehat{\mu}(\varsigma_{2k-1}, \varsigma_{2k-1}) + \widehat{\mu}(\varsigma_{2k-2}, \varsigma_{2k})}{2\widehat{\eta}} \right)^{a_4} \\
&\leq (\widehat{\mu}(\varsigma_{2k-1}, \varsigma_{2k-2}))^{a_1} (\widehat{\mu}(\varsigma_{2k-1}, \varsigma_{2k}))^{a_2} (\widehat{\mu}(\varsigma_{2k-2}, \varsigma_{2k-1}))^{a_3} \\
&\quad \times \left( \frac{\widehat{\mu}(\varsigma_{2k}, \varsigma_{2k-1}) + \widehat{\mu}(\varsigma_{2k-1}, \varsigma_{2k-2})}{2} \right)^{a_4} \\
&= (\widehat{\mu}(\varsigma_{2k-1}, \varsigma_{2k-2}))^{a_1+a_3} (\widehat{\mu}(\varsigma_{2k-1}, \varsigma_{2k}))^{a_2} \\
&\quad \times \left( \frac{\widehat{\mu}(\varsigma_{2k}, \varsigma_{2k-1}) + \widehat{\mu}(\varsigma_{2k-1}, \varsigma_{2k-2})}{2} \right)^{1-a_1-a_2-a_3}.
\end{aligned} \tag{38}$$

Assume that $\widehat{\mu}(\varsigma_{2k-1}, \varsigma_{2k-2}) \leq \widehat{\mu}(\varsigma_{2k}, \varsigma_{2k-1})$, then (3.14) gives

$$\begin{aligned}
\widehat{\mu}(\varsigma_{2k}, \varsigma_{2k-1}) &\leq \widehat{\varphi}\left( \mathscr{C}^{p}_{(\Upsilon,\Psi)}\left( \varsigma_{2k-1}, \varsigma_{2k-2}, (\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{2k-1})}, (\breve{\alpha},\breve{\beta})_{\Psi(\varsigma_{2k-2})} \right) \right) \\
&< (\widehat{\mu}(\varsigma_{2k}, \varsigma_{2k-1}))^{a_1+a_2+a_3} (\widehat{\mu}(\varsigma_{2k}, \varsigma_{2k-1}))^{1-a_1-a_2-a_3} \\
&= \widehat{\mu}(\varsigma_{2k}, \varsigma_{2k-1}),
\end{aligned} \tag{39}$$

a contradiction. Whence,

$$\widehat{\mu}(\varsigma_{2k}, \varsigma_{2k-1}) \leq \widehat{\mu}(\varsigma_{2k-1}, \varsigma_{2k-2}). \tag{40}$$

Using (38) and (40), we obtain

$$\widehat{\mu}(\varsigma_{2k}, \varsigma_{2k-1}) \leq \widehat{\varphi}(\widehat{\mu}(\varsigma_{2k-1}, \varsigma_{2k-2})). \tag{41}$$

Note that, (41) is equivalent to (3.9). So, on similar steps, we infer that the sequence $\{\varsigma_{\wp}\}_{\wp\in\mathbb{N}}$ is Cauchy in $(\mho, \widehat{\mu}, \widehat{\eta})$. Thus, the completeness of this space guarantees that $\widehat{\mu}(\varsigma_{\wp}, u) \longrightarrow 0$ as $\wp \longrightarrow \infty$, for some $u \in \mho$.

To see that $u$ is a common intuitionistic fuzzy Fp of $\Psi$ and $\Upsilon$, we employ Lemma 3 and inequality (3.5) as follows:

$$\begin{aligned}\widehat{\mu}\Big(u,[\Psi u]_{(\breve{\alpha},\breve{\beta})_{\Psi(u)}}\Big) &\le \widehat{\eta}\widehat{\mu}\big(u,\varsigma_{\wp}\big)+\widehat{\eta}\widehat{\mu}\Big(\varsigma_{\wp},[\Psi u]_{(\breve{\alpha},\breve{\beta})_{\Psi(u)}}\Big)\\ &\le \widehat{\eta}\widehat{\mu}\big(u,\varsigma_{\wp}\big)+\widehat{\eta}\aleph\Big(\big[\Upsilon\varsigma_{\wp-1}\big]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{\wp-1})}},[\Psi u]_{(\breve{\alpha},\breve{\beta})_{\Psi(u)}}\Big)\\ &\le \widehat{\eta}\widehat{\mu}\big(u,\varsigma_{\wp}\big)+\widehat{\eta}\widehat{\varphi}\Big(\mathscr{C}^{p}_{(\Upsilon,\Psi)}\Big(\varsigma_{\wp-1},u,(\breve{\alpha},\breve{\beta})_{\Upsilon(u)},(\breve{\alpha},\breve{\beta})\Big)\Big),\end{aligned} \tag{42}$$

where

$$\begin{aligned}&\mathscr{C}^{p}_{(\Upsilon,\Psi)}\Big(\varsigma_{\wp-1},u,(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{\wp-1})},(\breve{\alpha},\breve{\beta})_{\Psi(u)}\Big)\\ &=\big(\widehat{\mu}(\varsigma_{\wp-1},u)\big)^{a_1}\Big(\widehat{\mu}\Big(\varsigma_{\wp-1},\big[\Upsilon\varsigma_{\wp-1}\big]_{(\breve{\alpha},\breve{\beta})_{\Upsilon_{\varsigma_{\wp-1}}}}\Big)\Big)^{a_2}\Big(\widehat{\mu}\Big(u,[\Psi u]_{(\breve{\alpha},\breve{\beta})_{\Psi(u)}}\Big)\Big)^{a_3}\\ &\times\left(\frac{\widehat{\mu}\Big(\varsigma_{\wp-1},[\Psi u]_{(\breve{\alpha},\breve{\beta})_{\Psi(u)}}\Big)+\widehat{\mu}\Big(u,\big[\Upsilon\varsigma_{\wp-1}\big]_{(\breve{\alpha},\breve{\beta})_{\Upsilon_{\varsigma_{\wp-1}}}}\Big)}{2\widehat{\eta}}\right)^{a_4}\\ &=\big(\widehat{\mu}(\varsigma_{\wp-1},u)\big)^{a_1}\big(\widehat{\mu}(\varsigma_{\wp-1},\varsigma_{\wp})\big)^{a_2}\Big(\widehat{\mu}\Big(u,[\Psi u]_{(\breve{\alpha},\breve{\beta})_{\Psi(u)}}\Big)\Big)^{a_3}\\ &\times\left(\frac{\widehat{\mu}\Big(\varsigma_{\wp-1},[\Psi u]_{(\breve{\alpha},\breve{\beta})_{\Psi(u)}}\Big)+\widehat{\mu}\big(u,\varsigma_{\wp}\big)}{2\widehat{\eta}}\right)^{a_4}.\end{aligned} \tag{43}$$

We see that $\lim_{\wp\longrightarrow\infty}\mathscr{C}^{p}_{(\Upsilon,\Psi)}(\varsigma_{\wp-1},u,(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma_{\wp-1})},(\breve{\alpha},\breve{\beta})_{\Psi(u)})=0$. Hence, under this limiting case, (3.17) becomes

$$\widehat{\mu}(u,\Psi u)\le\widehat{\eta}\widehat{\varphi}(0). \tag{44}$$

By criterion (*ii*) of $\widehat{\varphi}$, (3.18) implies that $\widehat{\mu}(u,[\Psi u]_{(\breve{\alpha},\breve{\beta})_{\Psi(u)}})=0$. Whence, $u\in[\Psi u]_{(\breve{\alpha},\breve{\beta})_{\Psi(u)}}$. On similar steps, we can show that $u\in[\Upsilon u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}}$. Whence, we can find $(\breve{\alpha},\breve{\beta})_{\Upsilon(u)},(\breve{\alpha},\breve{\beta})_{\Psi(u)}\in(0,1]\times t[0,1)$ such that $u\in[\Upsilon u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}}\cap[\Psi u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}}$.

From Case 2 in the Proof of Theorem 1, we have also proved the next result.

**Theorem 2.** *Let $(\mho,\widehat{\mu},\widehat{\eta})$ be a complete b-metric space and $\Upsilon,\Psi\colon\mho\longrightarrow(IFS)^{\mho}$ be intuitionistic fuzzy set-valued maps. Suppose that for each $\varsigma\in\mho$, we can find $(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}$, $(\breve{\alpha},\breve{\beta})_{\Psi(\varsigma)}\in(0,1]\times t[0,1)$ such that $[\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}}$ and $[\Psi\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Psi(\varsigma)}}$ are nonempty bounded proximal subsets of $\mho$. If the pair $(\Upsilon,\Psi)$ forms a 0-hybrid intuitionistic fuzzy contraction, then $\Upsilon$ and $\Psi$ have a common intuitionistic fuzzy Fp in $\mho$.*

Next, we examine the idea of intuitionistic fuzzy $p$-hybrid contractions in view of $(\widetilde{M},\widetilde{\omega})$-level set (see [13]) and $\widehat{\mu}_{(\infty,\infty)}$-distance as some consequences of Theorem 1. It is important to point out that the investigation of Fp of intuitionistic fuzzy set-valued maps in the frame of $\widehat{\mu}_{(\infty,\infty)}$-metric is of great significant in computing Hausdorff dimensions. These dimensions aid us to grasp the basis of $\varepsilon^{\infty}$-space which is of enormous significant in higher energy physics. Consistent with Azam and Tabassum [12], we give some needed auxiliary concepts in the framework of a $b$-metric space as follows. Consider a $b$-metric space $(\mho,\widehat{\mu},\widehat{\eta})$ and take $(\breve{\alpha},\breve{\beta})\in(0,1]\times t[0,1)$ such that $[\widetilde{\nabla}]_{(\breve{\alpha},\breve{\beta})},[\widetilde{\triangle}]_{(\breve{\alpha},\breve{\beta})}\in\mathscr{P}^{r}_{b}(\mho)$. Then, define

$$\begin{aligned}p_{(\breve{\alpha},\breve{\beta})}(\widetilde{\nabla},\widetilde{\triangle})&=\inf_{\varsigma\in[\widetilde{\nabla}]_{(\breve{\alpha},t\breve{\beta})},\omega\in[\widetilde{\triangle}]_{(\breve{\alpha},\breve{\beta})}}\widehat{\mu}(\varsigma,\omega),\\ D_{(\breve{\alpha},\breve{\beta})}(\widetilde{\nabla},\widetilde{\triangle})&=\aleph\Big([\widetilde{\nabla}]_{(\breve{\alpha},\breve{\beta})},[\widetilde{\triangle}]_{(\breve{\alpha},\breve{\beta})}\Big),\\ p(\widetilde{\nabla},\widetilde{\triangle})&=\sup_{(\breve{\alpha},\breve{\beta})}p_{(\breve{\alpha},\breve{\beta})}(\widetilde{\nabla},\widetilde{\triangle}),\\ \widehat{\mu}_{(\infty,\infty)}(\widetilde{\nabla},\widetilde{\triangle})&=\sup_{(\breve{\alpha},\breve{\beta})}D_{(\breve{\alpha},\breve{\beta})}(\widetilde{\nabla},\widetilde{\triangle}).\end{aligned} \tag{45}$$

Note that, $\widehat{\mu}_{(\infty,\infty)}$ is a metric on $\mathscr{P}^{r}_{b}(\mho)$ (induced by the Hausdorff metric $\aleph$) and the completeness of $(\mho,\widehat{\mu},\widehat{\eta})$

implies the completeness of the corresponding metric space $(\mathscr{K}_{\mathcal{IFS}}(\mho), \widehat{\mu}_{(\infty,\infty)})$. Moreover, $(\mho, \widehat{\mu}, \widehat{\eta}) \mapsto (\mathscr{P}_b^r(\mho), \aleph) \mapsto (\mathscr{K}_{\mathcal{IFS}}(\mho), \widehat{\mu}_{(\infty,\infty)}, \widehat{\eta})$, are isometric embeddings via the relations $\varsigma \longrightarrow \{\varsigma\}$ and $M \longrightarrow \chi_M$, respectively; where

$$\mathscr{K}_{\mathcal{IFS}}(\mho) = \left\{\widetilde{\nabla} \in (IFS)^{\mho}\colon [\widetilde{\nabla}]_{(\breve{\alpha}, t\breve{\beta})} \in \mathscr{P}_b^r(\mho), \text{for each } \breve{\alpha}, \breve{\beta} \in (0,1] \times t[0,1)\right\}, \tag{46}$$

and $\chi_M$ is the characteristic function of $M$.

**Theorem 3.** *Let $(\mho, \widehat{\mu}, \widehat{\eta})$ be a complete b-metric space and $\Upsilon, \Psi\colon \mho \longrightarrow (IFS)^{\mho}$ be intuitionistic fuzzy set-valued maps. Assume that the following criteria are obeyed:*

*(i) $[\Upsilon\varsigma]_{(\widetilde{M},\widetilde{\varpi})_{\Upsilon(\varsigma)}}$ and $[\Psi\varsigma]_{(\widetilde{M},\widetilde{\varpi})_{\Psi(\varsigma)}}$ are nonempty bounded proximal subsets of $\mho$, for each $\varsigma \in \mho$;*

*(ii) for each $\varsigma, \omega \in \mho$,*

$$\widehat{\mu}_{(\infty,\infty)}(\Upsilon(\varsigma), \Psi(\omega)) \leq \widehat{\varphi}\left(\mathscr{C}^p_{(\Upsilon,\Psi)}\varsigma, \omega, (\widetilde{M}, \widetilde{\varpi})_{\Upsilon(\varsigma)}, (\widetilde{M}, \widetilde{\varpi})_{\Psi(\varsigma)}\right), \tag{47}$$

*where $\widehat{\varphi} \in \Omega$, $p \geq 0$, $a_i \geq 0$, $i = 1, 2, 3, 4$ with $\sum_{i=1}^4 a_i = 1$ and*

$$\mathscr{C}^p_{(\Upsilon,\Psi)}\left(\varsigma, \omega, (\widetilde{M}, \widetilde{\varpi})_{\Upsilon(\varsigma)}, (\widetilde{M}, \widetilde{\varpi})_{\Psi(\omega)}\right) = \begin{cases} \left[a_1(\widehat{\mu}(\varsigma,\omega))^p + a_2\left(\widehat{\mu}(\varsigma), [\Upsilon\varsigma]_{(\widetilde{M},\widetilde{\varpi})_{\Upsilon(\varsigma)}}\right)^p \right. \\ \left. +a_3\left(\widehat{\mu}\left(\omega, [\Psi\omega]_{(\widetilde{M},\widetilde{\varpi})_{\Psi(\omega)}}\right)^p\right) + a_4\left(\dfrac{\widehat{\mu}\left(\omega, [\Upsilon\varsigma]_{(\widetilde{M},\widetilde{\varpi})_{\Upsilon\varsigma}}\right)^p + \widehat{\mu}\left(\varsigma, [\Psi\omega]_{(\widetilde{M},\widetilde{\varpi})_{\Psi(\omega)}}\right)^p}{2\widehat{\eta}}\right)^p\right]^{1/p} \\ \text{for } p > 0, \ \varsigma, \omega \in \mho, \\ \left((\widehat{\mu}(\varsigma,\omega))^{a_1}\left(\widehat{\mu}\left(\varsigma, [\Upsilon\varsigma]_{(\widetilde{M},\widetilde{\varpi})_{\Upsilon(\varsigma)}}\right)\right)\right)^{a_2}\left(\widehat{\mu}\left(\omega, [\Psi\omega]_{(\widetilde{M},\widetilde{\varpi})_{\Psi(\omega)}}\right)\right)^{a_3} \\ \times\left(\dfrac{\widehat{\mu}\left(\varsigma, [\Psi\omega]_{(\widetilde{M},\widetilde{\varpi})_{\Psi(\omega)}}\right) + \widehat{\mu}\left(\omega, [\Upsilon\varsigma]_{(\widetilde{M},\widetilde{\varpi})_{\Upsilon(\varsigma)}}\right)}{2\widehat{\eta}}\right)^{a_4} \\ \text{for } p = 0, \varsigma, \omega \in \mho \setminus \mathscr{F}_{ix}(\Upsilon, \Psi). \end{cases} \tag{48}$$

*Then, $\Upsilon$ and $\Psi$ have a common intuitionistic fuzzy Fp in $\mho$.*

*Proof.* Let $\varsigma \in \mho$. Then, by assumption, $[\Upsilon(\varsigma)]_{(M,\wp)_{\Upsilon(\varsigma)}}$ and $[\Psi(\varsigma)]_{(M,\wp)_{\Psi(\varsigma)}}$ are nonempty bounded proximal subsets of $\mho$. Hence, for each $\varsigma, \omega \in \mho$,

$$\begin{aligned} &\aleph\left([\Upsilon(\varsigma)]_{(\widetilde{M},\widetilde{\varpi})_{\Upsilon(\varsigma)}}, [\Psi(\omega)]_{(M,\widetilde{\varpi})_{\Psi(\omega)}}\right) \\ &\quad = D_{(\widetilde{M},\widetilde{\varpi})}(\Upsilon(\varsigma), \Psi(\omega)) \leq \widehat{\mu}_{(\infty,\infty)}(\Upsilon(\varsigma), \Psi(\omega)) \\ &\quad \leq \widehat{\varphi}\left(\mathscr{C}^p_{(\Upsilon,\Psi)}\left(\varsigma, \omega, (\widetilde{M}, \widetilde{\varpi})_{\Upsilon(\varsigma)}, (\widetilde{M}, \widetilde{\varpi})_{\Psi(\omega)}\right)\right). \end{aligned} \tag{49}$$

Whence, it follows from Theorem 1 that $\Upsilon$ and $\Psi$ have a common intuitionistic fuzzy Fp in $\mho$. □

**Theorem 4.** *Let $(\mho, \widehat{\mu}, \widehat{\eta})$ be a complete b-metric space and $\Upsilon, \Psi\colon \mho \longrightarrow \mathscr{K}_{\mathcal{IFS}}(\mho)$ be intuitionistic fuzzy set-valued maps such that*

$$\widehat{\mu}_{(\infty,\infty)}(\Upsilon(\varsigma), \Psi(\omega)) \leq \widehat{\varphi}\left(\mathscr{G}^p_{(\Upsilon,\Psi)}(\varsigma, \omega)\right), \tag{50}$$

*where $\widehat{\varphi} \in \Omega$, $p \geq 0$, $a_i \geq 0$, $i = 1, 2, 3, 4$ with $\sum_{i=1}^4 a_i = 1$ and*

$$\mathcal{G}^{p}_{(\Upsilon,\Psi)}(\varsigma,\omega)=\begin{cases}\left[a_1(\widehat{\mu}(\varsigma,\omega))^p+a_2(p(\varsigma,\Upsilon(\varsigma)))^p+a_3(p(\omega,\Psi(\omega)))^p+a_4\left(\dfrac{p(\omega,\Upsilon(\varsigma))+p(\varsigma,\Psi(\omega))}{2\widehat{\eta}}\right)^p\right]^{1/p}, & \text{for } p>0,\ \varsigma,\omega\in\mho,\\ (p(\varsigma,\omega))^{a_1}(p(\varsigma,\Upsilon(\varsigma)))^{a_2}(p(\omega,\Psi(\omega)))^{a_3}\left(\dfrac{p(\varsigma,\Psi(\omega))+p(\omega,\Upsilon(\varsigma))}{2\widehat{\eta}}\right)^{a_4}, & \text{for } p=0,\varsigma,\omega\in\mho\setminus\mathcal{F}^{*}_{ix}(\Upsilon,\Psi),\end{cases}\tag{51}$$

*where*

$$\mathcal{F}^{*}_{ix}(\Upsilon,\Psi)=\{\varsigma,\omega\in\mho:\ \{\varsigma\}\subset\Upsilon(\varsigma),\ \{\omega\}\subset\Psi(\omega)\}.\tag{52}$$

*Then, we can find $u\in\mho$ such that $\{u\}\subset\Upsilon(u)\cap\Psi(u)$.*

*Proof.* Choose $\varsigma\in\mho$. For each $\varsigma\in\mho$, define two functions $\breve{\alpha}_{\Upsilon},\breve{\beta}_{\Psi}\colon\mho\longrightarrow[0,1]$ by $\breve{\alpha}_{\Upsilon}(\varsigma)\colon=\breve{\alpha}(\varsigma)=1$ and $\breve{\beta}_{\Psi}(\varsigma)\colon=\breve{\beta}(\varsigma)=0$. Then, by hypothesis, $[\Upsilon\varsigma]_{(1,0)}$ and $[\Psi\varsigma]_{(1,0)}$ are nonempty bounded proximal subsets of $\mho$. Now, for all $\varsigma,\omega\in\mho$,

$$\begin{aligned}D_{(1,0)}(\Upsilon(\varsigma),\Psi(\omega))&\le\widehat{\mu}_{(\infty,\infty)}(\Upsilon(\varsigma),\Psi(\omega))\\&\le\widehat{\varphi}\left(\mathcal{G}^{p}_{(\Upsilon,\Psi)}(\varsigma,\omega)\right).\end{aligned}\tag{53}$$

Since $[\Upsilon\varsigma]_{(1,0)_{\Upsilon(\varsigma)}}\subseteq[\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}}\in\mathcal{P}^{r}_{b}(\mho)$ for each $(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}\in(0,1]\times t[0,1)$, then $\widehat{\mu}(\varsigma,[\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}})\le\widehat{\mu}(\varsigma,[\Upsilon\varsigma]_{(1,0)_{\Upsilon(\varsigma)}})$ for each $(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}\in(0,1]\times t[0,1)$. So, $p(\varsigma,\Upsilon(\varsigma))\le\widehat{\mu}(\varsigma,[\Upsilon\varsigma]_{(1,0)_{\Upsilon(\varsigma)}})$. This further implies that we can find $(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)},(\breve{\alpha},\breve{\beta})_{\Psi(\omega)}\in(0,1]\times t[0,1)$ for each $\varsigma,\omega\in\mho$ :

$$\begin{aligned}&\aleph\left([\Upsilon\varsigma]_{(1,0)_{\Upsilon(\varsigma)}},[\Psi\omega]_{(1,0)_{\Psi(\omega)}}\right)\\&\quad\le\widehat{\varphi}\left(\mathcal{C}^{p}_{(\Upsilon,\Psi)}\left(\varsigma,\omega,(1,0)_{\Upsilon(\varsigma)},(1,0)_{\Psi(\omega)}\right)\right).\end{aligned}\tag{54}$$

Hence, Theorem 1 can be applied to find $u\in\mho$ such that $u\in[\Upsilon u]_{(1,0)_{\Upsilon(u)}}\cap[\Psi u]_{(1,0)_{\Psi(u)}}$. □

*Remark 3.* By putting $\widehat{\eta}=1$, $p=1$ and $h=1-\widehat{\mu}_{\Upsilon}-\nu_{\Psi}=0$, Theorem 2 can be applied to deduce the main results of [27], Theorems 10 and 11] as special cases. Also, Theorem 2 is a proper extension of the results of [28, 29] and some references therein.

The following example is constructed to verify the hypotheses of Theorem 1.

*Example 2.* Let $\mho=[0,\infty)$ and $\widehat{\mu}(\varsigma,\omega)=|\varsigma-\omega|^2$ for all $\varsigma,\omega\in\mho$. Then, $(\mho,\widehat{\mu},\widehat{\eta}=2)$ is a complete $b$-metric space. Note that $(\mho,\widehat{\mu},\widehat{\eta}=2)$ is not a metric space, since for $\varsigma=1$, $\omega=4$ and $\xi=2$,

$$\widehat{\mu}(\varsigma,\omega)=9>5=\widehat{\mu}(\varsigma,\xi)+\widehat{\mu}(\xi,\omega).\tag{55}$$

Take $\gamma,\lambda\in(0,1]$. Then, for each $\varsigma\in\mho$, consider two intuitionistic fuzzy set-valued maps $\Upsilon,\Psi\colon\mho\longrightarrow(IFS)^{\mho}$ defined as follows:

If $\varsigma=0$,

$$\begin{aligned}\widehat{\mu}_{\Upsilon(\varsigma)}(t)=\widehat{\mu}_{\Psi(\varsigma)}(t)&=\begin{cases}\dfrac{\gamma}{6}, & \text{if } t=0,\\ 0, & \text{if } t\neq0.\end{cases}\\ \nu_{\Upsilon(\varsigma)}(t)=\nu_{\Psi(\varsigma)}(t)&=\begin{cases}0, & \text{if } t=0,\\ \dfrac{\lambda}{2}, & \text{if } t\neq0.\end{cases}\end{aligned}\tag{56}$$

If $\varsigma\in(0,1]$,

$$\begin{aligned}\widehat{\mu}_{\Upsilon(\varsigma)}(t)&=\begin{cases}\dfrac{\gamma}{4}, & \text{if } 0\le t\le\varsigma-\dfrac{\varsigma^2}{40},\\ \dfrac{\gamma}{6}, & \text{if } \varsigma-\dfrac{\varsigma^2}{40}<t\le\varsigma-\dfrac{\varsigma^2}{12},\\ 0, & \text{if } \varsigma-\dfrac{\varsigma^2}{12}<t<\infty,\end{cases}\\ \nu_{\Upsilon(\varsigma)}(t)&=\begin{cases}0, & \text{if } 0\le t\le\varsigma-\dfrac{\varsigma^2}{100},\\ \dfrac{\lambda}{4}, & \text{if } \varsigma-\dfrac{\varsigma^2}{100}<t\le\varsigma-\dfrac{\varsigma^2}{12}\\ \lambda, & \text{if } \varsigma-\dfrac{\varsigma^2}{12}<t<\infty,\end{cases}\end{aligned}\tag{57}$$

$$\begin{aligned}\widehat{\mu}_{\Psi(\varsigma)}(t)&=\begin{cases}\dfrac{\gamma}{3}, & \text{if } 0\le t\le\varsigma-\dfrac{\varsigma^2}{50},\\ \dfrac{\gamma}{6}, & \text{if } \varsigma-\dfrac{\varsigma^2}{50}<t\le\varsigma-\dfrac{\varsigma^2}{12},\\ 0, & \text{if } \varsigma-\dfrac{\varsigma^2}{12}<t<\infty,\end{cases}\\ \nu_{\Psi(\varsigma)}(t)&=\begin{cases}0, & \text{if } 0\le t\le\varsigma-\dfrac{\varsigma^2}{70},\\ \dfrac{\lambda}{4}, & \text{if } \varsigma-\dfrac{\varsigma^2}{70}<t\le\varsigma-\dfrac{\varsigma^2}{12},\\ \dfrac{\lambda}{2}, & \text{if } \varsigma-\dfrac{\varsigma^2}{12}<t<\infty.\end{cases}\end{aligned}\tag{58}$$

If $\varsigma>1$,

$$\widehat{\mu}_{\Upsilon(\varsigma)}(t)=\widehat{\mu}_{\Psi(\varsigma)}(t)=\begin{cases}\dfrac{\gamma}{6}, & \text{if } 0\leq t\leq 9,\\[2ex] 0, & \text{if } t>9,\end{cases}\tag{59}$$

$$\nu_{\Upsilon(\varsigma)}(t)=\nu_{\Psi(\varsigma)}(t)=\begin{cases}0, & \text{if } 0\leq t\leq 9,\\ \lambda, & \text{if } t>9.\end{cases}\tag{60}$$

Define the function $\widehat{\varphi}\colon \mathbb{R}_+\longrightarrow\mathbb{R}_+$ by

$$\widehat{\varphi}(t)=\begin{cases}t-\dfrac{t^2}{12}, & \text{if } 0\leq t\leq 1,\\[2ex] \dfrac{1}{12}, & \text{if } t>1.\end{cases}\tag{61}$$

Obviously, $\widehat{\varphi}(t)<t$ for all $t>0$. Suppose that $(\breve{\alpha},\breve{\beta})=(\gamma/6,\lambda/4)$. Then, clearly, we can find $(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}$, $(\breve{\alpha},\breve{\beta})_{\Psi(\varsigma)}\in(0,1]\times t[0,1)$ such that $[\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}}$ and $[\Psi\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Psi(\varsigma)}}$ are nonempty bounded proximal subsets of $\mho$ for each $\varsigma\in\mho$. Now, to check the inequality 3.1, consider the following possibilities:

Case 1. If $\varsigma=\omega=0$, $p=0$, then for all $a_i\geq 0$ $(i=1,2,3,4)$, we have $[\Upsilon\varsigma]_{(\gamma/6,\lambda/4)_{\Upsilon(\varsigma)}}=[\Psi\omega]_{(\gamma/6,\lambda/4)_{\Psi(\varsigma)}}$ and hence,

$$\begin{aligned}&\aleph\left([\Upsilon\varsigma]_{(\gamma/6,\lambda/4)_{\Upsilon(\varsigma)}},[\Psi\omega]_{(\gamma/6,\lambda/4)_{\Psi(\varsigma)}}\right)\\&\quad=0\leq\widehat{\varphi}\left(\mathscr{C}^p_{(\Upsilon,\Psi)}\left(\varsigma,\omega,(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)},(\breve{\alpha},\breve{\beta})_{\Psi(\varsigma)}\right)\right).\end{aligned}\tag{62}$$

Case 2. If $\varsigma=0,\omega\in(0,1]$, $p=1$, $a_1=1$ and $a_2=a_3=a_4=0$, we have

$$[\Upsilon 0]_{(\gamma/6,\lambda/4)_{\Upsilon(0)}}=\{0\},\ [\Psi\omega]_{(\gamma/6,\lambda/4)_{\Psi(\varsigma)}}=\left[0,\omega-\frac{\omega^2}{12}\right].\tag{63}$$

Thus,

$$\begin{aligned}&\aleph\left([\Upsilon 0]_{(\gamma/6,\lambda/4)_{\Upsilon(0)}},[\Psi\omega]_{(\gamma/6,\lambda/4)_{\Psi(\varsigma)}}\right)\\&\quad=\left|\omega-\frac{\omega^2}{12}\right|^2\\&\quad=\widehat{\varphi}\left(|\omega-0|^2\right)\\&\quad\leq\widehat{\varphi}\left(\mathscr{C}^p_{(\Upsilon,\Psi)}\left(\varsigma,\omega,\left(\frac{\gamma}{6},\frac{\lambda}{4}\right)_{\Upsilon(\varsigma)},\left(\frac{\gamma}{6},\frac{\lambda}{4}\right)_{\Psi(\omega)}\right)\right).\end{aligned}\tag{64}$$

Note that, if $\omega=0,\varsigma\in(0,1]$, $p=1$, $a_1=1$ and $a_2=a_3=a_4=0$, we obtain same conclusion as in Case 2.

Case 3. If $\varsigma,\omega\in(0,1]$, $p=1$, $a_1=1$ and $a_2=a_3=a_4=0$, we have

$$\begin{aligned}&\aleph\left([\Upsilon\varsigma]_{(\gamma/6,\lambda/4)_{\Upsilon(\varsigma)}},[\Psi\omega]_{(\gamma/6,\lambda/4)_{\Psi(\varsigma)}}\right)\\&\quad=\aleph\left(\left[0,\varsigma-\frac{\varsigma^2}{12}\right],\left[0,\omega-\frac{\omega^2}{12}\right]\right)\\&\quad=\left|\varsigma-\frac{\varsigma^2}{12}-\omega+\frac{\omega^2}{12}\right|^2\\&\quad=\left|(\varsigma-\omega)-\frac{1}{12}\left(\varsigma^2-\omega^2\right)\right|^2\\&\quad=\left|(\varsigma-\omega)\left(1-\frac{|\varsigma+\omega|}{12}\right)\right|^2\\&\quad\leq\varsigma-\omega\left|\left(1-\frac{|\varsigma+\omega|}{12}\right)\right|^2\\&\quad=\varsigma-\omega\left|-\frac{|\varsigma+\omega|}{12}\right|^2\\&\quad=\widehat{\varphi}\left(|\varsigma-\omega|^2\right)\\&\quad\leq\widehat{\varphi}\left(\mathscr{C}^p_{(\Upsilon,\Psi)}\left(\varsigma,\omega,\left(\frac{\gamma}{6},\frac{\lambda}{4}\right)_{\Upsilon(\varsigma)},\left(\frac{\gamma}{6},\frac{\lambda}{4}\right)_{\Psi(\omega)}\right)\right).\end{aligned}\tag{65}$$

Case 4. If $\varsigma,\omega\in(1,\infty)$, then for all $a_i\geq 0$ $(i=1,2,3,4)$, $p>0$, we get $[\Upsilon\varsigma]_{(\gamma/6,\lambda/4)_{\Upsilon(\varsigma)}}=[\Psi\omega]_{(\gamma/6,\lambda/4)_{\Psi(\varsigma)}}$ and

$$\begin{aligned}&\aleph\left([\Upsilon\varsigma]_{(\gamma/6,\lambda/4)_{\Upsilon(\varsigma)}},[\Psi\omega]_{(\gamma/6,\lambda/4)_{\Psi(\varsigma)}}\right)\\&\quad=0\leq\widehat{\varphi}\left(\mathscr{C}^p_{(\Upsilon,\Psi)}\left(\varsigma,\omega,\left(\frac{\gamma}{6},\frac{\lambda}{4}\right)_{\Upsilon(\varsigma)},\left(\frac{\gamma}{6},\frac{\lambda}{4}\right)_{\Psi(\omega)}\right)\right).\end{aligned}\tag{66}$$

Thus, all the assumptions of Theorem 1 are obeyed. It follows that $\Upsilon$ and $\Psi$ have a common intuitionistic fuzzy Fp in $\mho$.

In what follows, we discuss further consequences of our main results.

**Corollary 1.** *Let $(\mho,\widehat{\mu},\widehat{\eta})$ be a complete b-metric space and $\Upsilon\colon\mho\longrightarrow(IFS)^{\mho}$ be an intuitionistic fuzzy set-valued map. Suppose that for each $\varsigma\in\mho$, we can find an $(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}\in(0,1]\times t[0,1)$ such that $[\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}}$ is a nonempty bounded proximal subsets of $\mho$. If*

$$\aleph\left([\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}},[\Upsilon\omega]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\omega)}}\right)\leq\widehat{\varphi}\left(\frac{1}{4}\mathscr{C}^p_{(\Upsilon)}(\varsigma,\omega)\right),\tag{67}$$

*for all $\varsigma,\omega\in\mho$, where $\widehat{\varphi}\in\Omega$ and*

$$\begin{aligned}\mathscr{C}^p_{(\Upsilon)}=\ &\widehat{\mu}(\varsigma,\omega)+\widehat{\mu}\left(\varsigma,[\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}}\right)+\widehat{\mu}\left(\omega,[\Upsilon\omega]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\omega)}}\right)\\&+\frac{\widehat{\mu}\left(\omega,[\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}}\right)+\widehat{\mu}\left(\varsigma,[\Upsilon\omega]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\omega)}}\right)}{2\widehat{\eta}}.\end{aligned}\tag{68}$$

*then, we can find $u\in\mho$ such that $u\in[\Upsilon u]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(u)}}$.*

*Proof.* Put $\Upsilon = \Psi$, $p = 1$ and $a_1 = a_2 = a_3 = a_4 = 1/4$ in Theorem 1. □

**Corollary 2.** *Let $(\mho, \widehat{\mu}, \widehat{\eta})$ be a complete b-metric space and $\Upsilon, \Psi\colon \mho \longrightarrow (IFS)^{\mho}$ be intuitionistic fuzzy set-valued maps. Suppose that for each $\varsigma, \omega \in \mho$, we can find $(\breve{\alpha}, \breve{\beta})_{\Upsilon(\varsigma)}$, $(\breve{\alpha}, \breve{\beta})_{\Psi(\omega)} \in (0,1] \times t[0,1)$ such that $[\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}}$ and $[\Psi\omega]_{(\breve{\alpha},\breve{\beta})_{\Psi(\omega)}}$ are nonempty bounded proximal subsets of $\mho$. If we can find $\lambda \in [0,1)$ :*

$$\aleph\left([\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}}, [\Psi\omega]_{(\breve{\alpha},\breve{\beta})_{\Psi(\omega)}}\right) \leq \lambda \cdot \left(\sqrt[4]{\left(\widehat{\mu}(\varsigma,\omega)\widehat{\mu}\left(\varsigma, [\Upsilon\varsigma]_{(\breve{\alpha},t\breve{\beta})_{\Upsilon(\varsigma)}}\right)\widehat{\mu}\left(\omega, [\Psi\omega]_{(\breve{\alpha},t\breve{\beta})_{\Psi(\omega)}}\right)\right)\left(\frac{\widehat{\mu}\left(\varsigma, [\Psi\omega]_{(\breve{\alpha},t\breve{\beta})_{\Psi(\omega)}}\right) + \widehat{\mu}\left(\omega, [\Psi\omega]_{(\breve{\alpha},t\breve{\beta})_{\Psi(\omega)}}\right)}{2\widehat{\eta}}\right)}\right), \tag{69}$$

*then $\Upsilon$ and $\Psi$ have a common intuitionistic fuzzy Fp in $\mho$.*

*Proof.* Take $a_1 = a_2 = a_3 = a_4 = 1/4$, $\widehat{\varphi}(t) = \lambda t$ for all $t \geq 0$ and $p = 0$ in Theorem 1. □

**Corollary 3.** *Let $(\mho, \widehat{\mu}, \widehat{\eta})$ be a complete b-metric space and $\Upsilon, \Psi\colon \mho \longrightarrow (IFS)^{\mho}$ be intuitionistic fuzzy set-valued maps. Suppose that for each $\varsigma, \omega \in \mho$, we can find $(\breve{\alpha}, \breve{\beta})_{\Upsilon(\varsigma)}$, $(\breve{\alpha}, \breve{\beta})_{\Psi(\omega)} \in (0,1] \times t[0,1)$ such that $[\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}}$ and $[\Psi\omega]_{(\breve{\alpha},\breve{\beta})_{\Psi(\omega)}}$ are nonempty bounded proximal subsets of $\mho$. If*

$$\aleph\left([\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}}, [\Psi\omega]_{(\breve{\alpha},\breve{\beta})_{\Psi(\omega)}}\right) \leq \widehat{\varphi}\left(\max\left\{\begin{array}{c}\widehat{\mu}(\varsigma,\omega), \widehat{\mu}\left(\varsigma, [\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}}\right), \widehat{\mu}\left(\omega, [\Psi\omega]_{(\breve{\alpha},\breve{\beta})_{\Psi(\omega)}}\right) \\ \dfrac{1}{2}\left[\widehat{\mu}\left(\varsigma, [\Psi\omega]_{(\breve{\alpha},\breve{\beta})_{\Psi(\omega)}}\right) + \widehat{\mu}\left(\omega, [\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}}\right)\right]\end{array}\right\}\right), \tag{70}$$

*then, we can find $u \in \mho$ such that $u \in [\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}} \cap [\Psi\omega]_{(\breve{\alpha},\breve{\beta})_{\Psi(\omega)}}$.*

**Corollary 4.** *(Nadler-type (see [4])) Let $(\mho, \widehat{\mu}, \widehat{\eta})$ be a complete b-metric space and $\Upsilon\colon \mho \longrightarrow (IFS)^{\mho}$ be an intuitionistic fuzzy set-valued map. Suppose that for each $\varsigma \in \mho$, we can find $(\breve{\alpha}, \breve{\beta})_{\Upsilon(\varsigma)}$ such that $[\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}}$ is a nonempty bounded proximal subsets of $\mho$. If there exists $\lambda \in [0,1)$ :*

$$\aleph\left([\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}}, [\Upsilon\omega]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}}\right) \leq \lambda\widehat{\mu}(\varsigma,\omega), \tag{71}$$

*then, we can find $u \in \mho$ such that $u \in [\Upsilon\varsigma]_{(\breve{\alpha},\breve{\beta})_{\Upsilon(\varsigma)}}$.*

*Proof.* Put $\Upsilon = \Psi$, $a_1 = p = 1$, $a_2 = a_3 = a_4 = 0$ and $\widehat{\varphi}(t) = \lambda t$, $t \geq 0$ in Theorem 1. □

Consistent with the proof of Theorem 2, the next result can easily be obtained by applying Corollary 4.

**Corollary 5.** *(Heilpern-type* (see [3])) *Let $(\mho, \widehat{\mu}, \widehat{\eta})$ be a complete b-metric space and $\Upsilon\colon \mho \longrightarrow \mathscr{K}_{\mathscr{IFS}}(\mho)$ be an intuitinoistic fuzzy set-valued map. Suppose that for each $\varsigma, \omega \in \mho$, we can find $\lambda \in [0,1)$ :*

$$\widehat{\mu}_{(\infty,\infty)}(\Upsilon(\varsigma), \Upsilon(\omega)) \leq \lambda\widehat{\mu}(\varsigma,\omega). \tag{72}$$

*Then, we can find $u \in \mho$ such that $\{u\} \subset \Upsilon(u)$.*

## 4. Applications to Multivalued and Single-Valued Mappings

In this section, we apply some results from previous the section to deduce their corresponding crisp Fp results of multivalued and single-valued mappings.

**Corollary 6.** *Let $(\mho, \widehat{\mu}, \widehat{\eta})$ be a complete b-metric space and $\Theta, \Lambda\colon \mho \longrightarrow \mathscr{K}(\mho)$ be multivalued mappings. If for all $\varsigma, \omega \in \mho$,*

$$\aleph(\Theta\varsigma, \Lambda\omega) \leq \widehat{\varphi}\left(\max\left\{\begin{array}{c}\widehat{\mu}(\varsigma,\omega), \widehat{\mu}(\varsigma,\Theta\varsigma), \widehat{\mu}(\omega,\Lambda\omega), \\ \dfrac{1}{2}[\widehat{\mu}(\varsigma,\Lambda\omega) + \widehat{\mu}(\omega,\Theta\varsigma)]\end{array}\right\}\right), \tag{73}$$

*then, we can find $u \in \mho$ such that $u \in \Theta u \cap \Lambda u$.*

*Proof.* Consider the intuitionistic fuzzy set-valued maps $\Upsilon, \Psi\colon \mho \longrightarrow (IFS)^{\mho}$ defined by

$$\widehat{\mu}_{\Upsilon(\varsigma)}(t)=\begin{cases}1, & \text{if } t\in\Theta\varsigma,\\ 0, & \text{if } t\notin\Theta\varsigma,\end{cases}\quad \nu_{\Upsilon(\varsigma)}(t)=\begin{cases}0, & \text{if } t\in\Theta\varsigma,\\ 1, & \text{if } t\notin\Theta\varsigma.\end{cases} \tag{74}$$

and

$$\widehat{\mu}_{\Psi(\varsigma)}(t)=\begin{cases}1, & \text{if } t\in\Lambda\varsigma,\\ 0, & \text{if } t\notin\Lambda\varsigma,\end{cases}\quad \nu_{\Psi(\varsigma)}(t)=\begin{cases}0, & \text{if } t\in\Lambda\varsigma,\\ 1, & \text{if } t\notin\Lambda\varsigma.\end{cases} \tag{75}$$

Take $(\breve{\alpha},\breve{\beta})=(1,0)$. Then $[\Upsilon\varsigma]_{(1,0)_{\Upsilon(u)}}=\Theta\varsigma$ and $[\Psi\varsigma]_{(1,0)_{\Psi(u)}}=\Lambda\varsigma$ for each $\varsigma\in\mho$. Whence, Corollary 3.10 can be applied to find a point $u\in\mho$ such that $u\in[\Upsilon u]_{(1,0)_{\Upsilon(u)}}\cap[\Psi u]_{(1,0)_{\Psi(u)}}=\Theta u\cap\Lambda u$. □

Following the Proof of Corollary 6, we can easily derive the next result by applying Corollary 1.

**Corollary 7.** *[30] Let $(\mho,\widehat{\mu},\widehat{\eta})$ be a complete b-metric space and $\Theta\colon\mho\longrightarrow\mathscr{P}_b^r(\mho)$ be a multivalued mapping:*

$$\aleph(\Theta\varsigma,\Theta\omega)\le\widehat{\varphi}\left(\frac{1}{4}\mathscr{C}^p_{(\Theta)}(\varsigma,\omega)\right), \tag{76}$$

*for all $\varsigma,\omega\in\mho$, where $\widehat{\varphi}\in\Omega$ and*

$$\begin{aligned}\mathscr{C}^p_{(\Theta)}&=\widehat{\mu}(\varsigma,\omega)+\widehat{\mu}(\varsigma,\Theta\varsigma)+\widehat{\mu}(\omega,\Theta\omega)\\&\quad+\frac{\widehat{\mu}(\omega,\Theta\varsigma)+\widehat{\mu}(\varsigma,\Theta\omega)}{2\widehat{\eta}}.\end{aligned} \tag{77}$$

*Then, we can find $u\in\mho$ such that $u\in\Upsilon u$.*

**Corollary 8.** *[22], Theorem 1] Let $(\mho,\widehat{\mu},\widehat{\eta})$ be a complete b-metric space and $g\colon\mho\longrightarrow\mho$ be a single-valued mapping. If*

$$\widehat{\mu}(g\varsigma,g\omega)\le\widehat{\varphi}\left(\mathscr{C}^p_g(\varsigma,\omega)\right) \tag{78}$$

*for all $\varsigma,\omega\in\mho$, where $\widehat{\varphi}\in\Omega$, $p\ge 0$, $a_i\ge 0$, $i=1,2,3,4$ with $\sum_{i=1}^4 a_i=1$ and*

$$\mathscr{C}^p_g(\varsigma,\omega)=\begin{cases}\left[a_1(\widehat{\mu}(\varsigma,\omega))^p+a_2(\widehat{\mu}(\varsigma,g\varsigma))^p+a_3(\widehat{\mu}(\omega,g\omega))^p+a_4\left(\dfrac{\widehat{\mu}(\omega,g\varsigma)+\widehat{\mu}(\varsigma,g\omega)}{2\widehat{\eta}}\right)^p\right]^{1/p}, & \text{for } p>0,\ \varsigma,\omega\in\mho\\[2ex] (\widehat{\mu}(\varsigma,\omega))^{a_1}(\widehat{\mu}(\varsigma,g\varsigma))^{a_2}(\widehat{\mu}(\omega,g\omega))^{a_3}\left(\dfrac{\widehat{\mu}(\varsigma,g\omega)+\widehat{\mu}(\omega,g\varsigma)}{2\widehat{\eta}}\right)^{a_4}, & \text{for } p=0,\varsigma,\omega\in\mho\setminus\mathscr{F}_{ix}(g).\end{cases} \tag{79}$$

*where*

$$\mathscr{F}_{ix}(g)=\{\varsigma\in\mho\colon\varsigma=g\varsigma\}. \tag{80}$$

*Then, we can find $u\in\mho$ such that $u=gu$.*

*Proof.* Consider an intuitionistic fuzzy set-valued map $\Upsilon\colon\mho\longrightarrow(IFS)^{\mho}$ defined by

$$\widehat{\mu}_{\Upsilon(\varsigma)}(t)=\begin{cases}1, & \text{if } t\in\Theta\varsigma,\\ 0, & \text{if } t\notin\Theta\varsigma,\end{cases}\quad \nu_{\Upsilon(\varsigma)}(t)=\begin{cases}0, & \text{if } t=g\varsigma,\\ 1, & \text{if } t\ne g\varsigma.\end{cases} \tag{81}$$

Put $(\breve{\alpha},\breve{\beta})=(1,0)$. Then $[\Upsilon\varsigma]_{(1,0)_{\Upsilon(u)}}=\{g\varsigma\}$. Obviously, $\{g\varsigma\}\in\mathscr{P}_b^r(\mho)$, for each $\varsigma\in\mho$. Notice that in this case, for all $\varsigma,\omega\in\mho$,

$$\aleph\left([\Upsilon\varsigma]_{(1,0)_{\Upsilon(\varsigma)}},[\Upsilon\omega]_{(1,0)_{\Upsilon(\omega)}}\right)=\widehat{\mu}(g(\varsigma),g(\omega)). \tag{82}$$

Hence, by Theorem 1, we can find $u\in\mho$ such that $u\in[\Upsilon u]_{(1,0)_{\Upsilon(u)}}=\{g(u)\}$; which further implies that $g(u)=u$. □

By using the method of proving Corollary 7, we can deduce the next Fp result due to Czerwik [21] by applying Corollary 5.

(i) For $\widehat{\eta}=1$ and $h=1-\widehat{\mu}_{\Upsilon}-\nu_{\Upsilon}=0$ in Corollary 6, we deduce the result of [27], Theorem 7].

(ii) It is clear that if we let $\widehat{\eta}=1$ in all the above-given results, we can deduce their analogues in the setting of metric space. Also, by setting $h=1-\widehat{\mu}_{\Upsilon}-\nu_{\Upsilon}=0$ all our main results reduce to their crisp analogues.

**Corollary 9** [21]. *Let $(\mho,\widehat{\mu},\widehat{\eta})$ be a complete b-metric space and $g\colon\mho\longrightarrow\mho$ be a point-valued mapping. If we can find $\lambda\in(0,1)$ such that for all $\varsigma,\omega\in\mho$,*

$$\widehat{\mu}(g(\varsigma),g(\omega))\le\lambda\widehat{\mu}(\varsigma,\omega) \tag{83}$$

*then, there exists $u\in\mho$ such that $g(u)=u$.*

*Remark 4.*

## Data Availability

No data were used to support this study.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Authors' Contributions

Conceptualization was done by M. S. Shagari; Methodology was done by S. Kanwal; Formal analysis was done by A. Azam, H. Aydi; Review and editing was done by A. Azam;

Writing, review, and editing were done by M. S. Shagari. H. Aydi, Y.U. Gaba; Funding was done by Y.U. Gaba.

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