

Research Article

An Empirical Study on the Influence of Embedded Option on Interest Rate Risk Based on Fuzzy Monte Carlo Simulation

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With the gradual progress of interest rate marketization, China's interest rate fluctuates more and more frequently, and the range is also growing. As a result, more and more implicit options are embedded in commercial banks' balance sheets, which brings new challenges to commercial banks' interest rate risk management. On the basis of identifying implicit options and theoretically analyzing the mechanism, fuzzy MCS method is used to calculate C_{eff} and D_{eff} when implicit options exist, and compared with the traditional duration value and traditional convexity value when implicit options do not exist, further analyzing how implicit options affect the interest rate risk.

1. Introduction

With the development of China's market economy and the growing international demand for interest rate marketization, China has been constantly promoting the process of interest rate marketization [1, 2]. In recent years, the benchmark interest rate of deposits and loans has been adjusted more and more frequently, and the range of adjustment has become larger and larger, which makes the implicit options exist. The exercise of options has brought potential losses to commercial banks, which has become a difficult problem in interest rate risk management. The existence of implied options will have a greater impact on the borrower. The borrower of a commercial bank may prepay when the market interest rate is lowered once and then reborrow at a lower market interest rate. If commercial banks are not aware of the existence of implicit options, they will not use the more accurate interest rate risk measurement tools— C_{eff} and D_{eff} —when implicit options exist but still use the traditional duration and traditional convexity for interest rate risk management, which is bound to cause large errors and bring potential losses to banks. Although the

existence of implicit options makes a big impact, in actual operation, commercial banks ignore implicit options and give valuable implicit options to customers for free [3–6]. In addition, the existence of implied options also makes the traditional interest rate risk measurement and management methods inaccurate or even ineffective. Therefore, it is of great significance to understand the implied options and study how the implied options affect the traditional interest rate risk measurement and management.

Gup and Brooks [7] first studied embedded option. They believed that the prepayment of loans and the withdrawal of deposits in advance would change the bank's exposure to assets and liabilities, leading to interest rate risk, which is called embedded option risk. At the same time, Brooks and Gup [8] pointed out that ignoring the impact of embedded option on the interest rate of financial institutions after studying the impact of financial institutions holding embedded option positions, the shareholders' equity of various financial institutions with long-term exposure will decrease. Later, Gilkeson et al. [9] studied the pricing of embedded option, pointed out that under certain pricing assumptions of early withdrawal

options, CD was used as the research object, and put forward a testable conclusion of option pricing theory. The research on embedded option of banks in China is relatively late. Zheng and Lin [10] put forward a new view on the embedded option in bank assets and liabilities by using the basic principles of financial engineering and decomposed the embedded option. They priced the embedded option using nonarbitrage analysis and numerical calculation methods. Chuan-he and Lian [11] analyzed the identification, measurement, and control of interest rate risk of embedded option based on the general characteristics of interest rate management and concluded that embedded option is common in bank deposit business and loan business. Xia et al. [12] used the Monte Carlo simulation method to conduct numerical pricing and risk measurement on deposits and loans with embedded option after building the jump model of benchmark interest rates on deposits and loans and studied the impact of embedded option in deposits and loans on interest rate risk based on the duration-convexity gap model. Feng-qin and Xiao-fei [13] used the Monte Carlo simulation method to price the embedded option value of commercial banks based on the CIR-jump interest rate fluctuation model. On the basis of combing previous literature, this research attempts to use a new computer simulation method (fuzzy Monte Carlo simulation method) to calculate the effective duration and effective convexity based on OAS and then analyze how the original interest rate risk measurement and management of commercial banks will be affected when embedded option exists.

2. Embedded Option in Deposits and Loans

Embedded option assets in commercial banks mainly refer to time deposits that can be drawn in advance, and embedded option liabilities mainly refer to bank loans that can be repaid in advance [14].

2.1. Embedded Option in Bank Asset Business. China's housing mortgage loan mainly adopts the interest measurement method of floating interest rate, that is, the loan interest rate automatically adjusts with the adjustment of the benchmark interest rate. Therefore, when the benchmark interest rate increases, the borrower's interest expenditure will increase, which will inevitably lead to early repayment to reduce the interest burden. If the fixed rate loan is adopted for housing mortgage loan, when the market interest rate decreases, the borrower will repay in advance, so as to reborrow at a lower market interest rate to reduce the interest expense. However, in any case, the borrower's prepayment will cause the income loss of commercial banks. Specifically, the exercise of embedded option will not only lead to the reinvestment risk of commercial banks but also lead to the mismatch between asset duration and liability duration.

Take the housing mortgage loan with floating interest rate as an example, where A represents the total loan amount, r represents the interest rate, N represents the loan

term, and MP represents the monthly repayment amount; then,

$$MP = \frac{A(1+r)^n}{(1+r)^n - 1} * r. \quad (1)$$

The interest part of the payment in the k th month can be expressed as

$$R_k = MP \left(1 - \frac{1}{(1+r)^{n-k+1}} \right). \quad (2)$$

The principal part of the payment in the k th month can be expressed as

$$P_k = MP - R_k. \quad (3)$$

If prepayment occurs in the k th month, the loss of commercial banks caused by prepayment is

$$\begin{aligned} L &= a + \sum_{i=k+1}^n R_i * \frac{1}{(1+r)^i} \\ &= a + A \frac{(1+r)^{n+1-k} + (k-n-1)^r - 1}{(1+r)[(1+r)^n - 1]}. \end{aligned} \quad (4)$$

In formula (4), a is the compensation for reinvestment risk [15, 16].

2.2. Embedded Option in Bank Liability Business. As for the embedded option in the liability business of commercial banks, here we take time deposits as an example to illustrate that in China, the interest rate of time deposits does not change with the benchmark interest rate during the deposit period, that is, the interest is calculated according to the fixed interest rate. If a time deposit is withdrawn by a depositor before its maturity, the interest is calculated according to the current deposit interest rate. In fact, the current deposit interest rate is extremely low, that is, if a fixed deposit is about to mature, the early withdrawal will not occur (except for emergency needs), so the early withdrawal is likely to occur at the beginning of the fixed deposit. If the increase of the benchmark interest rate makes it more cost-effective for depositors to deposit at a new higher interest rate, early withdrawal will occur. Withdrawal of deposits in advance means the following risks for commercial banks. First, the interest burden of commercial banks increases. Second, the effect of the original interest rate risk management strategy based on duration matching will be greatly discounted because customers withdraw deposits in advance, that is, the asset duration and liability duration will no longer match, and interest rate risk exposure will appear. Third, due to the possibility and unpredictability of such early withdrawal at any time, commercial banks must prepare a portion of funds specifically for the early withdrawal of depositors, which will reduce the commercial banks' profit-making assets and thus reduce their income. Fourth, if early withdrawal becomes a widespread phenomenon, the operation of commercial banks will become chaotic, and even the entire financial system will be affected.

3. Mechanism of Embedded Option Affecting Interest Rate Risk

3.1. Impact Mechanism Analysis. When choosing duration and convexity to measure interest rate risk of commercial banks, we should consider that the calculation of duration and convexity must require a stable cash flow. However, for assets and liabilities when implicit options exist, because borrowers will repay in advance due to changes in interest rates, depositors will withdraw deposits in advance due to changes in interest rates and then transfer them, and such behavior will make the cash flow of assets and liabilities change with changes in interest rates. This means that the traditional duration and convexity of assets and the duration and convexity of liabilities can no longer meet the needs of risk measurement when implicit options exist. If commercial banks ignore implicit options and still use them to measure interest rate risk, it will inevitably produce greater errors and bring potential losses to commercial banks.

Here, it is assumed that when there is no implied option, assets are greater than liabilities, that is, net assets are positive. The impact of implied options on the duration, convexity, and net assets of assets and liabilities is shown in Figure 1.

When there is no implied option, commercial banks can well match duration and convexity, so that net assets are E^* , and the initial interest rate is assumed to be between R^* and R^{**} . When the interest rate falls below R^{**} , the interest rate price curve of liabilities does not change, while the slope of the interest rate price curve of assets becomes smaller, the duration becomes smaller, and the original positive convexity becomes negative convexity. With the decline of market interest rates, negative convexity limits the room for asset value to rise, that is, the value of assets cannot catch up with the value of liabilities. Therefore, the net assets of commercial banks have changed from E^* to E_1 , and obviously E_1 is less than E^* . In this case, the duration and convexity no longer match; the interest rate risk exposure becomes larger [17–20].

When the interest rate gradually rises from the initial interest rate to more than R^* , the interest rate price curve of assets will not change, while the interest rate price curve of liabilities has changed, and the absolute value and convexity of duration will become smaller, which slows down the reduction rate of the value of liabilities. The reduction rate of the value of liabilities cannot catch up with the reduction rate of the value of assets. The value of net assets has changed from E^* to E_2 , and obviously E_2 is less than E^* .

Here, with the help of the gap model, the net assets can be expressed as

$$dE = -(D_A - KD_L)AdR + \frac{1}{2}(C_A - KC_L)A(dR)^2. \quad (5)$$

In formula (5), the leverage coefficient can be expressed as

$$K = \frac{L}{A}. \quad (6)$$

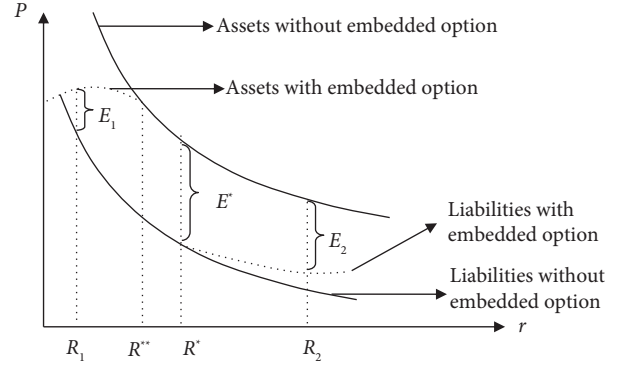


FIGURE 1: Comparison of changes in interest rate price curve of assets and liabilities.

D_{GAP} measures the effect of duration matching of assets and liabilities, which can be expressed as the following formula:

$$D_{GAP} = D_A - KD_L. \quad (7)$$

C_{GAP} measures the effect of convexity matching of assets and liabilities, which can be expressed as the following formula:

$$C_{GAP} = C_A - KC_L. \quad (8)$$

When r falls below R^{**} , D_{GAP} becomes negative, and the value of net assets of banks decreases. At the same time, C_{GAP} also changed from 0 to negative, and the change of interest rate reduced the value of net assets. When r rises above R^{**} , D_{GAP} becomes a positive number, and the value of net assets of banks decreases. At the same time, C_{GAP} also changed from 0 to negative, and changes in interest rates reduced the value of net assets.

That is to say, whether interest rates rise or fall, the existence of implied options will reduce the net asset value. Implicit options are valuable when interest rates change to a certain extent, while commercial banks often ignore such implicit options and give such option values to customers free of charge. Once customers choose to exercise, it will cause potential losses to banks from the income level, and from the interest rate risk management level, it will also cause the mismatch between the duration and convexity of assets and the duration and convexity of liabilities. In this case, measuring and managing interest rate risk requires the introduction of new tools— D_{eff} and C_{eff} [21].

3.2. New Tools: D_{eff} and C_{eff} . When implied options exist, assets and liabilities are regarded as loaded bonds, and D_{eff} and C_{eff} are expressed as

$$D_{eff} = \frac{P_+ - P_-}{2\Delta r P_0}, \quad (9)$$

$$C_{eff} = \frac{P_+ - 2P_0 + P_-}{\Delta r^2 P_0}. \quad (10)$$

In formulas (9) and (10), the expressions for P_+ and P_- are as in formulas (11) and (12):

$$P_+ = \frac{1}{m} \sum_{j=1}^m \sum_{t=1}^n \frac{CF_t^j}{\prod_{i=1}^t (1 + r_i^j + \Delta r + OAS)}, \quad (11)$$

$$P_- = \frac{1}{m} \sum_{j=1}^m \sum_{t=1}^n \frac{CF_t^j}{\prod_{i=1}^t (1 + r_i^j - \Delta r + OAS)}. \quad (12)$$

In formulas (9)–(12), Δr is expressed as a basis point spread; P_0 is the initial bond price; P_+ and P_- represent the bond prices after the downward and upward shift of the term structure of interest rates; m represents the number of interest rate change paths; j represents the j th interest rate path; t represents the generation period of cash flow; CF_t^j represents cash flow; and r_i^j denotes market interest rate [22, 23].

Here, OAS is the key to calculating D_{eff} and C_{eff} . OAS is the risk compensation, that is to say, OAS is the spread floating on the basis of the treasury rate.

$$P = \frac{1}{m} \sum_{j=1}^m \sum_{t=1}^n \frac{CF_t^j}{\prod_{i=1}^t (1 + r_i^j + OAS)}, \quad (13)$$

where P is the market price and $1/m$ is the mean.

The implementation steps of OAS are as follows: (1) construct the term structure of risk-free interest rate; (2) select an interest rate model that can better fit the real interest rate to simulate the path of interest rate change [24]; (3) select the advance payoff model or advance withdrawal model to calculate the cash flow; and (4) obtain the theoretical value of bonds by discounted and weighted cash flows, compare them with market prices, and gradually adjust the value of OAS until the value when the two are equal is the correct value.

4. Empirical Analysis Based on Fuzzy Monte Carlo Simulation

4.1. Selection of Simulation Methods. Traditional Monte Carlo simulation (MCS) is mainly used for simulation and calculation when the data are very accurate and sufficient. However, in reality, the situation is usually not so ideal, such as insufficient simulation information obtained and inaccurate measurement data obtained. In this case, it depends on the subjective speculation of experts, so traditional Monte Carlo simulation based on a single probability distribution is no longer applicable because it does not consider the subjectivity of data input and the possibility of incompleteness. In this context, this study intends to adopt a more advanced fuzzy MCS, which can adapt to various probability distributions and accommodate subjective reasoning and incomplete accuracy defects of data [25–27].

4.2. Case Selection. In order to obtain more reliable results, here we propose a more appropriate fuzzy Monte Carlo simulation method.

Take a five-year 400000-yuan housing mortgage loan of Agricultural Bank of China as an example. The contract starts at September 1, 2022. One year after the loan contract comes into effect, the customer can apply for prepayment. There is no minimum payment limit. In addition, loan interest rate is 2% higher than the 7-day interbank offered rate, and any intermediate fees in banking business are ignored. For the convenience of expression, letters are used here to replace variables in the case [19], as shown in Table 1.

4.3. Empirical Process

4.3.1. Construction of Interest Rate. The coupon stripping method is adopted here to integrate the relevant data of national debt. This paper selects the data of interest-bearing national debt on September 1, 2022, and uses the software EViews to conduct regression on the basis of data processing to form the discount factor expression as shown in the following formula:

$$\begin{aligned} d(t) &= a_0 + a_1 t + a_2 t^2 + a_3 t^3 \\ &= 1 - 0.018044t - 0.002401t^2 + 0.000098t^3. \end{aligned} \quad (14)$$

The risk-free interest rate data are plotted in the coordinate chart, as shown in Figure 2.

4.3.2. Interest Rate Model Selection and Parameter Estimation. Select the data from September 1, 2019, to September 1, 2022. First, the 7-day compounded interbank offered rate was tested by ADF, and the results are shown in Table 2.

This group of data is a stable series without unit root, which means that an interest rate model with mean reversion characteristics needs to be selected. In order to avoid negative interest rate, the CIR model is selected as shown in the following formula [28]:

$$dr = k(\mu - r)dt + \sigma\sqrt{r}dz. \quad (15)$$

In formula (15), k represents the average recovery speed, μ represents the average level, σ represents the interest rate volatility, and \sqrt{r} avoids the possibility that the interest rate is negative. However, in the fitting process, further “ARCH-LM” test is required, where the lag order $P = 3$ is taken, and the results are shown in Table 3.

Since both P values are small, it indicates that the residual sequence has “ARCH” effect, so GARCH (1, 1) should be added for correction. The final estimated result is

$$dr = 0.00991(0.95995 - r)dt + 1.00994\sigma_\varepsilon\sqrt{r}dz. \quad (16)$$

In formula (16), σ_ε meets $\sigma_\varepsilon^2 = 0.946132 + 0.355302\varepsilon_{t-1}^2 + 0.640152\sigma_{\varepsilon_{t-1}}^2$ and then it is subject to “ARCH-LM” test, and the test results are shown in Table 4.

We can see both P values are large, so the model with GARCH(1, 1) does not have the “ARCH” effect and can be used as the interest rate model here.

TABLE 1: Parameters and explanation.

Parameters	Explanation
$I_t = MB_{t-1} * i$	Interest payable
MB_{t-1}	Loan balance
$i = i_b + 2\%$	Mortgage interest rate
i_b	7-day interbank offered rate
$MP_t = MB_{t-1} [(i(1+i)^{n-t+1}) / ((1+i)^{n-t+1} - 1)]$	Repayment
$SP_t = MP_t - I_t$	Principal repayment
SMM_t	Prepayment rate
$PR_t = SMM_t (MB_t - SP_t)$	Prepayment amount

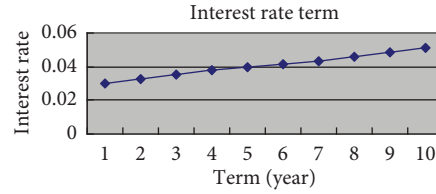


FIGURE 2: Risk-free rate.

TABLE 2: ADF test results.

ADF test statistic	-3.647328	1% critical value	-3.3895
		5% critical value	-2.8472
		10% critical value	-2.6025

TABLE 3: ARCH-LM test results.

F - statistic	2.189942	Prob.F(3, 744)	0.0905
Obs * R - squared	6.587964	Prob.Chi - Square(3)	0.0902

4.3.3. D_{eff} and C_{eff} Calculations Based on OAS. With the 7-day interbank offered rate on the initial date as the initial interest rate, select the CIR model with GARCH(1, 1) as the interest rate model, use the fuzzy MCS method, and select the PSA prepayment model to calculate the cash flow of the loan. For example, the first period cash flow is calculated as follows:

$$MB_0 = 40 \text{ (Unit: 10000 yuan),} \tag{17}$$

$$MP_1 = MB_0 \left[\frac{i(1+i)^{n-t+1}}{(1+i)^{n-t+1} - 1} \right] = 40 \cdot \frac{i(1+i)^{60}}{(1+i)^{60} - 1}, \tag{18}$$

$$I_1 = MB_0 * i = 0.1323, \tag{19}$$

$$SP_1 = MP_1 - I_1 = 0.4375, \tag{20}$$

$$PR_1 = SMM_1 (MB_0 - SP_1) = 0.05913, \tag{21}$$

$$C_1 = I_1 + SP_1 + PR_1 = 0.62893, \tag{22}$$

$$C_k = I_k + SP_k + PR_k, \dots \tag{23}$$

With the help of MATLAB, the OAS value of each period of cash flow and different prepayment rates can be calculated

according to the order of formulas (17)–(23), and then D_{eff} and C_{eff} can be calculated based on the OAS value, as shown in Table 5.

4.4. Analysis of Empirical Results. For this housing mortgage loan of Agricultural Bank of China, if prepayment is not allowed, OAS will be zero, that is, there is no compensation for implied option risk. However, when prepayment is allowed, OAS for “50% PSA” is 2.2719%, OAS for “100% PSA” is 2.3293%, and OAS for “50% PSA” is 2.3997%. That is, with the gradual improvement of prepayment rate, the compensation for implied option risk will become higher and higher.

Next, the changes of D_{eff} and C_{eff} are analyzed. If prepayment is not allowed, D_{eff} is 2.5001. However, when prepayment is allowed and prepayment occurs, that is, when the implicit option is exercised, D_{eff} gradually decreases. For example, D_{eff} is 1.91 for “50% PSA,” 1.5998 for “100% PSA,” and 1.3893 for “50% PSA.” That is, D_{eff} gradually decreases with the gradual increase of prepayment rate. In the same way, when the implicit option does not exist, C_{eff} is 138.9013, while the prepayment behavior occurs. That is, when the implicit option is exercised, C_{eff} is 53.6993 for “50% PSA,” C_{eff} is 6.7012 for “100% PSA,” and C_{eff} is -118.9774 for “50% PSA.” That is, with the gradual

TABLE 4: ARCH-LM test results after adding GARCH(1,1).

F – statistic	0.290564	Prob. F (3, 742)	0.8594
Obs * R – squared	0.74073	Prob.Chi – Square (3)	0.8582

TABLE 5: Results of D_{eff} and C_{eff} .

	50%		100%		150%		No embedded option	
Simulation results	2.21%	30.0998	2.31%	30.0501	2.31%	30.0502	1.89%	30.0803
	2.2724%	30.0000	2.3506%	30.0000	2.3901%	30.0000	1.9725%	30.0000
	2.52%	29.9976	2.51%	29.9673	2.51%	29.9542	2.12%	29.9297
OAS	2.2719%		2.3293%		2.3997%		—	
D_{eff}	1.91		1.5998		1.3893		2.5001	
C_{eff}	53.6993		6.7012		-118.9774		138.9013	

improvement of the prepayment rate, C_{eff} gradually decreases or even becomes negative.

5. Conclusion

- (1) From the perspective of the impact of embedded option on the net assets of commercial banks, when the interest rate drops to a certain extent, the lender will exercise the right to prepay, while the depositor will not be affected. That is to say, the existence of embedded option will limit the rise of the bank's asset value when the interest rate drops but will not affect the rise of the bank's responsible value, thus reducing the balance between assets and liabilities of the bank. When the interest rate rises to a certain extent, the time depositor will exercise the right to withdraw in advance, but the fixed interest rate lender will not be affected, that is, the existence of embedded option will lower the value of the bank when the interest rate rises but will not affect the value of the bank's assets, thus reducing the balance between assets and liabilities of the bank. That is to say, no matter the interest rate rises or falls, the existence of embedded option will reduce the net asset value of commercial banks.
- (2) From the perspective of interest rate risk management of commercial banks, the existence of embedded option changes the duration and convexity of bank assets and liabilities. Embedded option disrupts the asset liability items with good matching of duration and convexity. The effect of traditional interest rate risk management strategies based on duration convexity matching will be greatly reduced. Therefore, new interest rate risk measurement tools—effective duration and effective convexity—need to be introduced. Furthermore, it is necessary to match the effective duration and effective convexity of assets and liabilities, so as to truly achieve the purpose of accurate measurement and management of interest rate risk.
- (3) From the perspective of option value, embedded option, as a special option embedded in the balance sheet of commercial banks, also has a certain value. However, commercial banks often ignore the value

of this option and give it to customers for free, which brings them potential losses and inconvenience in operation and management. Therefore, commercial banks should reexamine this problem and charge such options to a certain extent or impose a certain penalty on the default of early withdrawal of deposits and prepayment of loans, the amount of which is approximately equal to the option premium.

Data Availability

The 7-day interbank lending rate data used to support the findings of this study can be accessed after applying for membership through the following website: <https://insights.ceicdata.com>.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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