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Research Article

Obtaining the Soliton Type Solutions of the Conformable Time-Fractional Complex Ginzburg–Landau Equation with Kerr Law Nonlinearity by Using Two Kinds of Kudryashov Methods

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The main idea of this study is to obtain the soliton-type solutions of the conformable time-fractional complex Ginzburg–Landau equation with Kerr law nonlinearity. For this aim, the generalized and modified Kudryashov methods are applied to the given model. The reason for using a conformable derivative is that the chain rule can be applied to this derivative. Thus, using the suitable wave transform, the given equation is converted into an ordinary differential equation. Then, the proposed methods are applied to the reduced equation. According to our results, both of the used methods are effective and powerful. Finally, 3D and contour plots are given for some results with suitable variables. Our findings in this paper are critical for explaining a wide range of scientific and physical applications. According to our knowledge, our results are new in the literature.

1. Introduction

The exact solutions of the nonlinear partial differential equations (NLPDEs) have an important place in different fields of science, such as fluid mechanics, plasma physics, solid-state physics, and optical fibers. This being the case, many methods were discovered to solve nonlinear partial differential equations, for example, the method of undetermined coefficients [1], the Riccati equation mapping approach [2], the trial equation method [3], the finite element method [4], the extended trial approach [5], the Petrov–Galerkin method [6], the unified and $\exp_a$ function methods [7], the modified extended tanh expansion method [8], the modified simple procedure [9], the exponential rational function procedure [10], the Kudryashov method [11], the ansatz method [12], and so on.

In this study, the following equation, called the conformable time-fractional complex Ginzburg–Landau equation, will be considered [13]:

$$iq_t^{\delta} + \epsilon q_{xx} + \lambda F\left(|q|^2\right)q - \left(|q|^2 q^*\right)^{-1}\left[\rho|q|^2\left(|q|^2\right)_{xx} - \sigma\left[\left(|q|^2\right)_x\right]^2\right] - \varepsilon q = 0, \tag{1}$$

where $\delta \in (0,1]$ represents the conformable derivative, $q(x,t)$ is a complex-valued function, the spatial coordinate is represented by x and the temporal coordinate is represented by t. The group velocity dispersions are represented by ϵ and λ, the perturbation effects are represented by ρ, σ, and ε. $F(|q|^2)$ is a function of $|q|^2$ and F is a real-valued algebraic function that must have the smoothness of the function $F(|q|^2)q\colon \mathbb{C} \longrightarrow \mathbb{C}$. When the complex plane $\mathbb{C}$ is assumed as 2D linear space $\mathbb{R}^2$, the $F(|q|^2)q$ is k times continuously differentiable, namely, $F(|q|^2)q \in \cup_{a,b=1}^{\infty}\mathbb{C}^k\ (-b,b)\times(-a,a);\mathbb{R}^2)$.

In literature, lots of researchers obtained the exact solutions of the given model with the different types of nonlinearity. Some researchers obtained the exact solutions of the generalized derivative of the given model for example Kudryashov applied the first integral method to the equation in [14], Das et al. applied the F-expansion to the model in [15], the modified $(G^{'}/G)$-expansion method is applied to the model by Wang et al. in [16], the modified Jacobi elliptic expansion method is applied by Hosseini et al. in [17], Hosseini et al. implemented Kudryashov and exponential methods to the model including the parabolic nonlinearity in [18]. Some researchers obtained the exact solutions of equation (1) with different kinds of fractional derivatives, for example, Tozar obtained the analytical solutions of the conformable time-fractional complex Ginzburg–Landau equation with the help of the $(1/G^{'})$ method in [19], optical solutions were discovered with the help of the generalized exponential rational function method in [20], Sulaiman et al. explored the optical solitons with the help of the extended sinh-Gordon equation expansion method in [21], the form of the space-time conformable fractional complex Ginzburg–Landau equation is handled in [22], Sadaf et al. applied the $(w(\xi)/2)$ method to the model with the different types of senses as the conformable, beta, truncated derivatives in [23].

1.1. The Conformable Derivative. In literature, fractional derivatives have an essential role, so many definitions of fractional derivatives are discovered, for example, Riemann–Liouville, Grunwald–Letnikov, the Caputo, Atangana–Baleanu, and modified Riemann–Liouville derivatives [24, 25]. In this study, the conformable derivative will be used, which is developed by Khalil et al. [26]. An important feature of this derivative is that we can apply the chain rule so we can reduce nonlinear differential equations to ordinary differential equations with the help of wave transforms. Basic definitions of the conformable derivative are given as follows:

When $\psi\colon (0,\infty) \longrightarrow \mathbb{R}$, the conformable derivative of ψ of order $\delta, 0<\delta<1$, is defined as follows [27, 28]:

$$T_{\delta}(\psi)(t) = \lim_{\varepsilon\longrightarrow 0} \frac{\psi\left(t+\epsilon t^{1-\delta}\right)-\psi(t)}{\epsilon}, \tag{2}$$

for all $t>0$. The basic properties of the conformable derivative are given as follows [29–31]:

(1) $T_{\delta}(a\psi+b\varphi) = aT_{\delta}(\psi)+bT_{\delta}(\varphi)$, for all $a,b\in\mathbb{R}$

(2) $T_{\delta}(t^{\alpha}) = \alpha t^{\alpha-\delta}$, for all $\alpha\in\mathbb{R}$

(3) $T_{\delta}(\psi\varphi) = \psi T_{\delta}(\varphi)+\varphi T_{\delta}(\psi)$

(4) $T_{\delta}(\psi/\varphi) = (\varphi T_{\delta}(\psi)-\psi T_{\delta}(\varphi)/\varphi^2)$

(5) If ψ is differentiable, then $T_{\delta}(\psi)(t) = t^{1-\delta}\mathrm{d}\psi/\mathrm{d}t$

(6) $\psi(t)=\lambda$, $T_{\delta}(\lambda)=0$, for all constant functions

(7) Chain rule: Let $\psi,\varphi\colon (0,\infty)\longrightarrow\mathbb{R}$ be a differentiable and δ– differentiable function then the chain rule is given by the following:

$$T_{\delta}(\psi\circ\varphi)(t) = t^{1-\alpha}\varphi^{'}(t)\psi^{'}(\varphi(t)). \tag{3}$$

In this paper, the conformable time-fractional complex Ginzburg–Landau equation with Kerr law was solved by two procedures, namely, the generalized Kudryashov and the modified Kudryashov procedures. For this aim, the main ideas of generalized Kudryashov and the modified Kudryashov procedures were in Section 2. Then, these procedures were applied to the given model, and 3D and contour plots of obtained solutions were given in Section 3. Finally, conclusions were given.

2. The Procedures

In this section, the used procedures will be given. We take into consideration a general nonlinear differential equation in the following form:

$$\Phi\left(q, \frac{\partial^{\delta}q}{\partial t^{\delta}}, \frac{\partial q}{\partial x}, \frac{\partial^{2\delta}q}{\partial t^{2\delta}}, \frac{\partial^2 u}{\partial x^2}, \ldots\right) = 0, \tag{4}$$

where $q=q(x,t)$ is a complex-valued function and δ represents a conformable derivative. If we apply the following wave transformation to equation (4):

$$q(x,t) = u(\zeta)e^{i\varphi}, \tag{5}$$

where $\zeta = x-vt^{\delta}/\delta$ and $\varphi = -\kappa x+\omega t^{\delta}/\delta+\eta$, the following ordinary differential equation (ODE) is obtained:

$$\phi\left(u, u^{'}, u^{''}, \ldots,\right) = 0, \tag{6}$$

here prime represents the differentiation of u with respect to ζ.

2.1. The Generalized Kudryashov Procedure. According to the method, we assume $u(\zeta)$ as follows (32, 33):

$$u(\zeta) = \frac{\sum_{n=0}^{N} a_n\psi^n(\zeta)}{\sum_{m=0}^{M} b_m\psi^m(\zeta)}, \tag{7}$$

where $a_n, b_m\,(n=0,1,\ldots N, m=0,1,\ldots M)$ are constants and they should be $a_N\neq 0$, $b_M\neq 0$ and the following ODE is satisfied by $\psi(\zeta)$:

$$\frac{\mathrm{d}\psi}{\mathrm{d}\zeta} = \psi^2(\zeta)-\psi(\zeta), \tag{8}$$

and $\psi(\zeta)$ is given as follows:

$$\psi(\xi) = \frac{1}{1+\chi e^{\zeta}}, \quad \chi \text{ is integration constant}, \tag{9}$$

N and M are calculated by the homogeneous balance principle at (6). We can calculate a polynomial of ψ by substituting equation (7) into equation (6) without ignoring equation (8). Then, all the coefficients of the polynomial ψ are set to zero. If the obtained system is solved, the values of the $a_n, b_m, \kappa, v, \omega$ are obtained. Finally, the soliton-type solutions of the given model are obtained.

2.2. The Modified Kudryashov Procedure. According to the method, the solutions of equation (6) are assumed as follows [34–36]:

$$u(\zeta) = \sum_{m=0}^{M} \varpi_m (\psi(\zeta))^m, \ \varpi_M \neq 0, \tag{10}$$

where $\varpi_m (m = 0, 1, \ldots, M)$ are constants that will be determined later, M is calculated by the homogeneous balance principle, and the function $\psi(\zeta)$ is given by the following:

$$\psi(\zeta) = \frac{1}{1 + \chi a^{\zeta}}, \tag{11}$$

where (11) satisfies the following ODE:

$$\psi'(\zeta) = \left(\psi^2(\zeta) - \psi(\zeta)\right)\ln a. \tag{12}$$

Substituting equation (10) into equation (6) without ignoring equation (12), a set of algebraic equations is obtained for $\varpi_m, a, \chi, \kappa, \nu$ and ω. Finally, solving this obtained system, the exact solutions of equation (2) are calculated.

3. The Applications

In this section, the used procedures will be applied to the given model. For this aim, the given model will be reduced to the nonlinear differential equation by the wave transformation. If we implement the wave transformation, namely, equation (5) to equation (1) then separate the real and imaginary parts, we get the following ODE:

$$-\omega u + \epsilon\left(u'' - \kappa^2 u\right) + \lambda F\left(u^2\right)u - 2(\rho - 2\sigma)\frac{\left(u'\right)^2}{u} - 2\rho u'' - \varepsilon u = 0, \tag{13}$$

$$\nu = -2\epsilon\kappa. \tag{14}$$

If we take $\rho = 2\sigma$, equation (13) reduces to the following ODE:

$$(\epsilon - 4\sigma)u'' - \left(\omega + \epsilon\kappa^2 + \varepsilon\right)u + \lambda F\left(u^2\right)u = 0. \tag{15}$$

If we take $F(u^2) = u$ for the Kerr law nonlinearity, equation (15) reduces to the following ODE:

$$(\epsilon - 4\sigma)u'' - \left(\omega + \epsilon\kappa^2 + \varepsilon\right)u + \lambda u^3 = 0. \tag{16}$$

If we balance u'' and u^3 in equation (16), the balance number is obtained as 1.

3.1. First Method. In this subsection, the generalized Kudryashov procedure will be applied to the equation (16). According to the method, we assume

$$u = \frac{a_0 + a_1\psi + a_2\psi^2}{b_0 + b_1\psi}. \tag{17}$$

If we substitute the solution (17) without ignoring the (8) in equation (16), we obtain an overdetermining equation system. If the obtained system is solved, four solution families are obtained as follows.

3.1.1. First Family. The values of the arbitrary constants are obtained as follows:

$$\begin{aligned}
a_0 &= 0,\\
a_1 &= \pm b_1\sqrt{-\frac{\epsilon - 4\sigma}{2\lambda}},\\
a_2 &= \pm \frac{b_1(\epsilon - 4\sigma)}{\lambda\sqrt{-\epsilon - 4\sigma/2\lambda}},\\
b_0 &= 0,\\
b_1 &= b_1,\\
\omega &= -\epsilon\kappa^2 - \frac{\epsilon}{2} + 2\sigma - \varepsilon.
\end{aligned} \tag{18}$$

Then, the solutions of the given model are obtained as follows:

$$q_{1,2}(x,t) = \left(\mp\frac{\left(\chi e^{\left(x - \nu\left(t^{\delta}/\delta\right)\right)} - 1\right)(\epsilon - 4\sigma)}{\sqrt{-2\epsilon + 8\sigma/\lambda}\left(1 + \chi e^{\left(x - \nu t^{\delta}/\delta\right)}\right)\lambda}\right)e^{i\left(-\kappa x + \omega t^{\delta}/\delta + \eta\right)}. \tag{19}$$

3.1.2. Second Family. The values of the arbitrary constants are obtained as follows:

$$\begin{aligned}
a_0 &= 0,\\
a_1 &= \pm b_0\sqrt{-\frac{8\epsilon - 32\sigma}{\lambda}},\\
a_2 &= \mp b_0\sqrt{-\frac{8\epsilon - 32\sigma}{\lambda}},\\
b_0 &= b_0,\\
b_1 &= -2b_0,\\
\omega &= -\epsilon\kappa^2 + \epsilon - 4\sigma - \varepsilon,
\end{aligned} \tag{20}$$

and the solutions are given by the following:

$$q_{3,4}(x,t) = \left(\pm\frac{2\chi e^{\left(x - \nu t^{\delta}/\delta\right)}\sqrt{-2\epsilon + 8\sigma/\lambda}}{\left(\chi^2 e^{2\left(x - \nu t^{\delta}/\delta\right)} - 1\right)}\right)e^{i\left(-\kappa x + \omega t^{\delta}/\delta + \eta\right)}. \tag{21}$$

3.1.3. Third Family. The values of the arbitrary constants are obtained as follows:

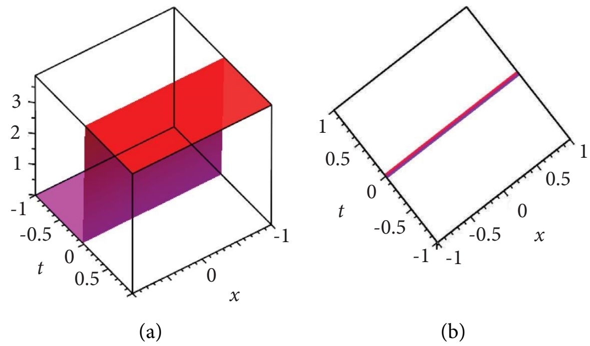


Figure 1: The plot of the $|(25)|$ for $\epsilon = 1, \lambda = 2, \sigma = 4, \kappa = -2, \eta = 1, \varepsilon = 2, \chi = 2, \delta = 0.1$: (a) 3D plot and (b) contour plot.

$$
\begin{aligned}
a_0 &= \pm \frac{b_0(\epsilon - 4\sigma)}{\lambda\sqrt{-2\epsilon - 8\sigma/\lambda}},\\
a_1 &= \mp \frac{(2b_0 - b_1)(\epsilon - 4\sigma)}{\lambda\sqrt{-2\epsilon - 8\sigma/\lambda}},\\
a_2 &= \pm b_1\sqrt{-\frac{2\epsilon - 8\sigma}{\lambda}},\\
b_0 &= b_0,\\
b_1 &= b_1,\\
\omega &= -\epsilon\kappa^2 - \frac{\epsilon}{2} + 2\sigma - \varepsilon,
\end{aligned} \tag{22}
$$

and the solutions are given as follows:

$$
q_{5,6}(x,t) = \left(\pm \frac{(\epsilon - 4\sigma)\left(\chi e^{(x - vt^{\delta}/\delta)} - 1\right)}{\lambda\left(1 + \chi e^{(x - vt^{\delta}/\delta)}\right)\sqrt{-2\epsilon - 8\sigma/\lambda}} \right) e^{i(-\kappa x + \omega t^{\delta}/\delta + \eta)}. \tag{23}
$$

3.1.4. Fourth Family. The values of the arbitrary constants are obtained as follows:

$$
\begin{aligned}
a_0 &= \mp \frac{b_1(\epsilon - 4\sigma)}{\lambda\sqrt{-2\epsilon - 8\sigma/\lambda}},\\
a_1 &= \pm \frac{2b_1(\epsilon - 4\sigma)}{\lambda\sqrt{-2\epsilon - 8\sigma/\lambda}},\\
a_2 &= \pm b_1\sqrt{-2\epsilon - 8\sigma/\lambda},\\
b_0 &= -\frac{b_1}{2},\\
b_1 &= b_1,\\
\omega &= -\epsilon\kappa^2 - 2\epsilon + 8\sigma - \varepsilon,
\end{aligned} \tag{24}
$$

and the solutions are given as follows:

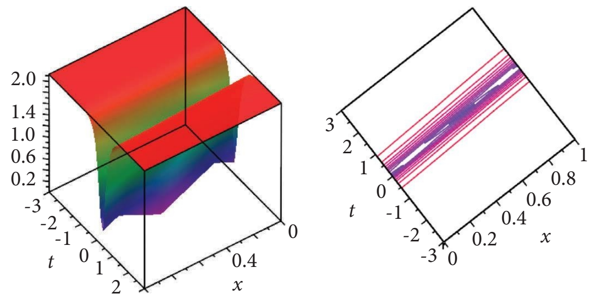
$$
q_{7,8}(x,t) = \left(\pm \frac{2(\epsilon - 4\sigma)\left(\chi^2 e^{2(x - vt^{\delta}/\delta)} + 1\right)}{\lambda\left(\chi^2 e^{2(x - vt^{\delta}/\delta)} - 1\right)\sqrt{-2\epsilon - 8\sigma/\lambda}} \right) e^{i(-\kappa x + \omega t^{\delta}/\delta + \eta)}. \tag{25}
$$

The 3D and contour plots were given for (25) in Figure 1.

3.2. Second Method. In this subsection, the modified Kudryashov procedure will be applied to the equation (16). According to the method, we assume

$$
u(\zeta) = \varpi_0 + \varpi_1\psi(\zeta). \tag{26}
$$

If we substitute the solution (26) without ignoring the (12) in equation (16) and collect the polynomial of $\psi(\zeta)$, we get an overdetermining equation system as follows:



(a)
(b)

FIGURE 2: The plot of the $|(29)|$ for $\epsilon = 1, \lambda = 2, \sigma = 4, \kappa = -2, \eta = 1, \varepsilon = 2, \chi = 2, \delta = 0.9, a = 3.$: (a) 3D plot and (b) contour plot.

$$\begin{aligned}
&\psi^3:\ 2(\ln(a))^2\varpi_1\epsilon - 8\ln(a)^2\varpi_1\sigma + \varpi_1^3\lambda,\\
&\psi^2:\ -3(\ln(a))^2\varpi_1\epsilon + 12(\ln(a))^2\varpi_1\sigma + 3\varpi_0\varpi_1^2\lambda,\\
&\psi^1:\ (\ln(a))^2\varpi_1\epsilon - 4\left(\ln(a)^2\right)\varpi_1\sigma + 3\varpi_0^2\varpi_1\lambda - \varpi_1\epsilon\kappa^2 - \varpi_1\varepsilon - \varpi_1\omega,\\
&\psi^0:\ \lambda\varpi_0^3 - \varpi_0\epsilon\kappa^2 - \varpi_0\varepsilon - \varpi_0\omega.
\end{aligned} \tag{27}$$

If the above system is solved, the values of the arbitrary constants are obtained as follows:

$$\begin{aligned}
\varpi_0 &= \pm\ln(a)\sqrt{-\frac{\epsilon - 4\sigma}{2\lambda}},\\
\varpi_1 &= \pm\frac{\ln(a)(\epsilon - 4\sigma)}{\lambda\sqrt{-\epsilon - 4\sigma/2\lambda}},\\
\omega &= -\frac{(\ln(a))^2\epsilon}{2} + 2(\ln(a))^2\sigma - \kappa^2\epsilon - \varepsilon.
\end{aligned} \tag{28}$$

Then, the exact solutions are given by

$$q_{9,10}(x,t) = \pm\frac{(4\sigma - \epsilon)\ln(a)\left(\chi a^{(x - vt^\delta/\delta)} - 1\right)}{\lambda\sqrt{-2\epsilon + 8\sigma/\lambda}\left(1 + \chi a^{(x - vt^\delta/\delta)}\right)}. \tag{29}$$

The 3D and contour plots were given for (29) in Figure 2.

4. Conclusions

In this study, the new soliton-type solutions of the conformable time-fractional complex Ginzburg–Landau equation with Kerr law nonlinearity were obtained with the help of generalized and modified Kudryashov methods. Firstly, the given model was reduced to the nonlinear differential equation with the help of the wave transformation. Then, the balance number was calculated by the balance method. We calculate the balance number for the generalized Kudryashov method in a different way than usual. The generalized Kudryashov method was applied to the given model. Four solution families were obtained. The 3D and contour plots were plotted for the latest family. Then, another method was applied to the given model. Also, the results of the modified Kudryashov method include the logarithmic solutions. The 3D and contour plots were given the obtain solutions. The Maple software program was used for all obtained results and figures. According to our knowledge, our results are new in the literature. If we can calculate the balance number, the given methods provide soliton solutions for the nonlinear partial differential equations. All obtained results were checked by Maple and they are different from each other. Our findings in this paper are critical for explaining a wide range of scientific and physical applications. Thanks to this implementation, we contributed to the physical motions of the waves and other related areas. The proposed methods are effective and powerful for finding the soliton solutions of the nonlinear differential equations.

In new studies, the given equation can be solved with a different kind of derivative and compared with our results, or the used methods can be applied to the different nonlinear partial differential equations.

Data Availability

All data generated or analyzed during this study are included in this manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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