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Research Article

On Some New Types of Fuzzy Soft Compact Spaces

S. Saleh,[1,2] **Tareq M. Al-shami**,[3,4] **and Abdelwaheb Mhemdi**[5]

[1]*Department of Computer Science, Cihan University-Erbil, Kurdistan Region, Erbil, Iraq*
[2]*Department of Mathematics, Hodeidah University, Hodeidah, Yemen*
[3]*Department of Mathematics, Sana'a University, P. O. Box 1247, Sana'a, Yemen*
[4]*Future University, New Cairo, Egypt*
[5]*Department of Mathematics, College of Sciences and Humanities in Aflaj, Prince Sattam Bin Abdulaziz University, Riyadh, Saudi Arabia*

Correspondence should be addressed to S. Saleh; s_wosabi@yahoo.com

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In this article, we apply the concept of fuzzy soft δ-open sets to define new types of compactness in fuzzy soft topological spaces, namely, δ-compactness and δ^*-compactness. We explore some of their basic properties and reveal the relationships between these types and that which are defined by other authors. Also, we show that δ^*-compactness is more general than that which is presented in other papers. To clarify the obtained results and relationships, some illustrative examples are given.

1. Introduction

Conventional mathematical tools are not sufficient to cope with many practical problems that existed in various disciplines such as social science, economics, engineering, medical science, and environment, involving vagueness/uncertainty. Zadeh [1] proposed the remarkable theory of fuzzy set (F-set) for handling these kinds of uncertainties where traditional tools fail. This theory represents a real revolution and grand paradigmatic change in mathematics. Since it is advent, it has contributed to handling numerous types of practical issues and overcoming many real-life problems. On the other hand, this theory has its inherent difficulties, which are possibly attributed to the inadequacy of the parameterization tool, as Molodtsov pointed out in [2]. So, he familiarized the idea of the soft set (S-set) as an efficient instrument for coping with uncertainties that are free from the abovementioned difficulties. Shabir and Naz [3] studied the topological structures of S sets. Then, many contributions have been conducted to soft topologies by many authors; see for example [4–6].

In recent times, the fuzzification of the S-set is progressing rapidly combining F sets and S sets, Maji et al. [7] established a novel model known as a fuzzy soft set (FS-set). The important notion of mapping FS classes was studied by Kharal and Ahmad [8]. Subsequently, the topological structure of FS sets was started by Tanay and Kandemir [9]. FS sets and their applications have been studied in different aspects, such as algebraic ([10–12]) and decision-making ([13, 14]). Mukherjee et al. [15] introduced the notions of fuzzy soft δ-open (closed) sets and fuzzy soft δ-closure (interior) with fuzzy soft δ-continuous mappings. The notion of compactness in fuzzy soft topological space (FSTS) was defined and studied in ([16, 17]) as a generalization of Chang's notion [18]. Taha [19] discussed compactness via the structure of fuzzy soft and r-Minimal spaces. Also, Saleh and Salemi [20] defined and studied a general type of fuzzy soft compactness.

In this work, we define and study new types of compactness in FSTS, namely, δ-compactness and δ^*-compactness. Some of their basic properties are studied. The relationships between these notions and that which are defined by other authors are investigated. We show that δ^*-compactness is more general than that which are presented in [16, 17, 20]. Some results and theorems related to these notions are studied with some necessary examples.

2. Basic Definitions and Results

Here, we recall the basic definitions which will be needed in this paper. X refers to the universal set, T is the set of all parameters for X, FS-refers to fuzzy soft, (X, ϑ, T) means an FSTS, and $I = [0, 1]$.

Definition 1 (see [1, 18]). An F-set$A$ of X is a mapping $A: X \longrightarrow I$ and A can be represented by the pairs $A = \{(x, A(x)): x \in X, A(x) \in I\}$. I^X refers to the set of all F sets on X. An F-pointx_λ, $\lambda \in (0, 1]$, is an F-set in X given by $x_\lambda(y) = \lambda$ at $x = y$ and $x_\lambda(y) = 0$ otherwise for all $y \in X/\{x\}$. For $\alpha \in I$, $\underline{\alpha} \in I^X$ refers to the F-constant function where $\underline{\alpha}(x) = \alpha$ for all $x \in X$.

Definition 2 (see [21]). An FS-set(f, T) over X symbolized by f_T is the set of ordered pairs $f_T = \{(t, f(t)): t \in T, f(t) \in I^X\}$. $\mathrm{FSS}(X_T)$ refers to the set of all FS sets over X. The complement of f_T symbolized by f_T^c, where $f^c: T \longrightarrow I^X$ defined by $(f(t))^c = \underline{1} - f(t) \forall t \in T$. Clearly, $(f_T^c)^c = f_T$. $\widetilde{\alpha}_T \in \mathrm{FSS}(X_T)$, $\widetilde{\alpha}_T = \{(t\, \underline{\alpha}): t \in T, \underline{\alpha} \in I^X\}$, $\underline{\alpha}: t \in T$, $\underline{\alpha} \in I^X\}$ is called an FS-constant set.

Definition 3 (see [7, 21]). Let $f_T$andg_T be two FS sets on X. Then,

(1) f_T is called a null (resp universal) FS-set, symbolized by $\widetilde{0}_T$ (resp.$\widetilde{1}_T$) if $f(t) = \underline{0}$ (resp.$f(t) = \underline{1}$) for all $t \in T$
(2) f_T is a subset of g_T if $f(t) \leq g(t) \forall t \in T$, symbolized by $f \sqsubseteq g$
(3) f_T and g_T are equal if $f_T \sqsubseteq g_T$ and $g_T \sqsubseteq f_T$. It is symbolized by $f_T = g_T$
(4) The union of f_T and g_T is an FS-seth_T defined by $h(t) = f(t) \vee g(t)$ for all $t \in T$. h_T is symbolized by $f_T \sqcup g_T$.
(5) The intersection of f_T and g_T is an FS-setl_T defined by $l(t) = f(t) \wedge g(t)$ for all $t \in T$. l_T is symbolized by $f_T \sqcap g_T$

Definition 4 (see [22–24]). An FS-pointx_α^z over X is an FS-set over X defined by, $x_\alpha^z(t') = x_\alpha$ if $t' = t$ and $x_\alpha^z(t') = \underline{0}$ if $t' \in T - \{t\}$, where x_α is the F-point in X. $\mathrm{FSP}(X_T)$ refers to the set of all FS points in X. An FS-pointx_α^z is said to belong to an FS-setf_T, symbolized by $x_\alpha^z \widetilde{\in} f_T$ if $\alpha \leq f(t)(x)$. Moreover, every non-nullFS-setf_T can be expressed as the union of all the FS points belonging to f_T.

Definition 5 (see [9, 22]). A triplet (X, ϑ, T) is FSTS, where ϑ is a family of FS sets on X which satisfies the following conditions:

(1) $\widetilde{0}_T$ and $\widetilde{1}_T$ belong to ϑ
(2) ϑ is closed under arbitrary union and finite intersection

Definition 6 (see [22, 24, 25]). The FS sets f_T and g_T are called quasi-coincident, symbolized by $f_T q g_T$ if there exist $t \in T$, $x \in X$ such that, $f(t)(x) + g(t)(x) > 1$. If f_T is not quasi-coincident with g_T, then we write $f_T \tilde{q} g_T$.

Definition 7 (see [15, 24]). An FS-setf_T of (X, ϑ, T) is called Q-neighborhood (briefly, q-nbd) of x_α^z if there exists a fuzzy soft open set (FSO-set) g_T such that $x_\alpha^z q g_T \sqsubseteq f_T$.

Definition 8 (see [8]). Let $\mathrm{FSS}(X_T)$ and $\mathrm{FSS}(Y_K)$ be two families of all FS sets over X and Y, respectively. Let $u: X \longrightarrow Y$ and $p: T \longrightarrow K$ be two maps. Then, $f_{\mathrm{up}}: \mathrm{FSS}(X_T) \longrightarrow \mathrm{FSS}(Y_K)$ is called a fuzzy soft map (FS-map), for which

(i) If $f_T \in \mathrm{FSS}(X_T)$, then the image of f_T symbolized by $f_{\mathrm{up}}(f_T)$ is an FS-set on Y given by $f_{\mathrm{up}}(f_T)(k) = \sup\{u(f(t)): t \in p^{-1}(k)\}ifp^{-1}(k) \neq \varnothing$, and $f_{\mathrm{up}}(f_T)(k) = \widetilde{0}_Y$ otherwise, for all $k \in K$
(ii) If $g_K \in \mathrm{FSS}(Y_K)$, then the preimage of g_K symbolized by $f_{\mathrm{up}}^{-1}(g_K)$ is an FS-set on X defined by, $f_{\mathrm{up}}^{-1}(g_K)(t) = u^{-1}(g(p(t)))$ for all $t \in T$

An FS-mapf_{up} is called one-to-one(onto), if u and p are one-to-one(onto).

Theorem 1 (see [8]). *Let $f_{iT} \in FSS(X_T)$and$g_{iK} \in FSS(Y_K)$for alli $\in J$, where J is an index set. Then, for an FS-mapf_{up}: $FSS(X_T) \longrightarrow FSS(YY_K)$, we have*

(1) if$f_{1T} \sqsubseteq f_{2T}$, then$f_{up}(f_{1T}) \sqsubseteq f_{up}(f_{2T})$
(2) if$g_{1K} \sqsubseteq g_{2K}$, then$f_{up}^{-1}(g_{1K}) \sqsubseteq f_{up}^{-1}(g_{2K})$
(3) $f_{up}(\sqcup_{i\in J} f_{iT}) = \sqcup_{i\in J} f_{up}(f_{iT})$
(4) $f_{up}(\sqcap_{i\in J} f_{iT}) \sqsubseteq \sqcap_{i\in J} f_{up}(f_{iT})$, the equality holds if f_{up} is one-to-one
(5) $f_{up}^{-1}(\sqcup_{i\in J} g_{iK}) = \sqcup_{i\in J} f_{up}^{-1}(g_{iK})$
(6) $f_{up}^{-1}(\sqcap_{i\in J} g_{iK}) = \sqcap_{i\in J} f_{up}^{-1}(g_{iK})$
(7) $f_{up}(\widetilde{1}_T) = \widetilde{1}_K$ if f_{up} is onto
(8) $f_{up}^{-1}(\widetilde{1}_K) = \widetilde{1}_T$ and $f_{up}^{-1}(\widetilde{0}_K) = \widetilde{0}_T$

Definition 9 (see [15, 26]). For an FS-setf_T in (X, ϑ, T), we have

(i) An FS-closure of f_T is the intersection of all FSC sets containing f_T, symbolized by $cl(f_T)$, and an FS-interior of f_T is the union of all FSO sets contained in f_T, symbolized by $\mathrm{int}(f_T)$
(ii) f_T is said to be a fuzzy soft regular open set (FSRO-set) if $f_T = \mathrm{int}(cl(f_T))$ and the complement of an FSRO-set is called a fuzzy soft-regular closed set (FSRC-set). $\mathrm{FSRO}(X_T)$ denotes the set of all FSRO sets, and $\mathrm{FSRC}(X_T)$ denotes the set of all FSRC sets

(iii) f_T is said to be an FS semiopen set if $f_T \sqsubseteq cl(\text{int}(f_T))$ and the complement of an FS semiopen set is called an FS semiclosed set

(iv) f_T is said to be an *FSδ*-neighborhood (briefly, δ-nbd) of x_α^z if and only if there exists FSRO q-nbd (FSRO q-nbd) g_T of x_α^z, such that $g_T \sqsubseteq f_T$

Definition 10 (see [15]). Let f_T be an FS-set in (X, ϑ, T). Then,

(i) An FS-pointx_α^z is called an FSδ-cluster point of f_T if and only if every FSRO q-nbdh_T of x_α^z, $h_T q f_T$. The set of all FS δ-cluster points of f_T is called the FS δ-closure of f_T, symbolized by $cl_\delta(f_T)$, that is, $cl_\delta(f_T) = \sqcap\{g_T \in \text{ FSRC}(X_T)\colon\ f_T \sqsubseteq g_T\}$.

(ii) An FS-setf_T is called an FS δ-closed set (FS δC-set, briefly) if and only if $f_T = cl_\delta(f_T)$ and the complement of an FS δ-closed set is called an FS δ-open set(FS δO-set, briefly). FS$\delta C(X_T)$ denotes to the set of all FSδC sets, and the set of all FSδO sets is symbolized by FS$\delta O(X_T)$.

(iii) The FSδ-interior of f_T is symbolized by $\text{int}_\delta(f_T)$ and defined by $\text{int}_\delta(f_T) = \widetilde{1}_T - cl_\delta(f_T^c)$, that is, $\text{int}_\delta(f_T) = \sqcup\{g_T \in \text{FSRO}(X_T)\colon\ g_T \sqsubseteq f_T\}$. Consequently, f_T is FSδO-set if and only if $f_T = \text{int}_\delta(f_T)$.

Result 1. [15] Every FSRO-set is an FSδO-set, and every FSδO-set is an FSO-set. Moreover, if f_T is an FS semiopen set in (X, ϑ, T), then $cl(f_T) = cl_\delta(f_T)$.

Result 2. [15] If f_T is an FSO-set in (X, ϑ, T), then $cl(f_T)$ is an FSRC-set, that is, $\{cl(g_T)\colon\ g_T \in \vartheta\} = \{h_T\colon\ h_T \in \text{FSRC}(X_T)\}$, and for any FS-set$f_T$ in (X, ϑ, T), $cl_\delta(f_T) = \sqcap\{cl(g_T)\colon\ f_T \sqsubseteq g_T, g_T \in \vartheta\}$.

Theorem 2 (see [15]). *For FS sets f_T and g_T in (X, ϑ, T) we have*

(1) $cl_\delta(\widetilde{0}_T) = \widetilde{0}_T$ and $cl_\delta(\widetilde{1}_T) = \widetilde{1}_T$

(2) If $f_T \sqsubseteq g_T$, then $cl(f_T) \sqsubseteq cl(g_T)$

(3) $cl_\delta(f_T)$ is an FSδC-set, that is, $cl_\delta(cl_\delta(f_T)) = cl_\delta(f_T)$

(4) If f_T, g_T are FSδC sets, then $f_T \sqcup g_T$ is an FSδC -set

Result 3. (Remark 4.15, [15]) The FS δ-closure operator on (X, ϑ, T) satisfies the Kuratowski closure axioms. So that, there exists one topology over X, this topology is defined as follows:

The set of all FSδO sets of (X, ϑ, T) forms FST, symbolized by ϑ_δ. It is called *FSδ*-topology over X, and the triplet (X, ϑ_δ, T) is called an FSδ-topological space. Moreover, $\vartheta_\delta \sqsubseteq \vartheta$.

Definition 11 (see [15]). Let $f_{\text{up}}\colon\ (X, \vartheta, T) \longrightarrow (Y, \eta, K)$ be an FS-map. Then, f_{up} is FSδ-open (FSδ-closed) if $f_{\text{up}}(f_T)$ is FSδO-set(FSδC-set) in (Y, η, K) for every FSδO-set(FSδC-set) f_T in (X, ϑ, T).

Theorem 3 (see [15]). *Let $f_{up}\colon\ (X, \vartheta, T) \longrightarrow (Y, \eta, K)$ be an FS-map. Then, the following items are equivalence:*

(1) f_{up} is FSδ-continuous

(2) $f_{up}^{-1}(g_K)$ is an FSδO-set in (X, ϑ, T) for every FSδO-set g_K in (Y, η, K)

(3) $f_{up}^{-1}(g_K)$ is an FSδC-set in (X, ϑ, T) for every FSδC-set g_K in (Y, η, K)

Definition 12 (see [22, 27]).

(i) Let (X, σ) be an FTS. Then, the collection $\vartheta_\sigma = \{f_T\colon\ f(t) = A \text{for all} t \in T \text{and} A \in \sigma\}$ defines FST on X

(ii) Let (X, ϑ, T) be an FSTS. Then, the collection $\vartheta_t = \{f(t)\colon\ f_T \in \vartheta\}$, for every $t \in T$ defines FT on X

3. New Types of Fuzzy Soft Compact Spaces

In this section, we are going to give the definitions of two types of FS-compactness, namely, FSδ-compactness and FSδ^*-compactness via FSδO sets, and study some properties of them.

Definition 13. Let $\mathscr{F}$ be a family of FSδO sets of (X, ϑ, T) and g_T be an FS-set over X. Then,

(i) $\mathscr{F}$ is said to be a fuzzy soft δ-open cover (FSδ-open cover, briefly) of g_T if $g_T \sqsubseteq \sqcup\{f_{iT}\colon\ f_{iT} \in \mathscr{F}\}$. A finite subcover is a finite subfamily of $\mathscr{F}$, which is again an FSδ-open cover

(ii) $\mathscr{F}$ is said to be a fuzzy soft δ^*-open cover (FSδ^*-open cover, briefly) of g_T if for every $x_\alpha^z \widetilde{\in} g_T$ there is $f_T \in \mathscr{F}$ satisfying $x_\alpha^z \widetilde{\in} f_T$

Remark 1. Clearly, every FSδ-open cover is an FS-open cover. The converse is not necessarily true. This follows from the fact that an FSO-set does not imply an FS δ-open set.

Remark 2. Clearly, every FSδ^*-open cover is an FSδ-open cover. The converse is not necessarily true, the next Example 1 shows it.

Definition 14. Let f_T be an FS-set in (X, ϑ, T). Then,

(i) f_T is said to be an FS δ-compact if every *FSδ*-open cover of f_T has a finite subcover. (X, ϑ, T) is said to be fuzzy soft δ-compact (FSδ-compact, briefly) if $\widetilde{1}_T$ itself is FSδ-compact

(ii) (X, ϑ, T) is said to be an FS δ^*-compact if every *FSδ**-open cover of $\widetilde{1}_T$ has a finite subcover

Example 1. Let $X = [0,1]$ and $T = \{t\}$. Then, the family $\vartheta = \{\widetilde{0}_T, \widetilde{1}_T\} \cup \{f_{iT}: i \in N\}$ is FST ϑ on X, where

$$f_{iT}(t)(x) = \begin{cases} \underline{1}, & \text{if} x = 0, \\ 1 - \dfrac{1}{i}, & \text{if} 0 < x \leq \dfrac{1}{i}, \\ \underline{1}, & \text{if} \dfrac{1}{i} < x \leq 1. \end{cases} \quad (1)$$

It is clear that, $\{cl(h_T): h_T \in \vartheta\} = \{g_T: g_T \in \text{FSRC}(X_T)\}$, and for any l_T in (X, ϑ, T), $cl_\delta(l_T) = \sqcap\{g_T \in \text{FSS}(X_T): l_T \sqsubseteq g_T, g_T \in \text{FSRC}(X_T)\}$. Since $cl(f_{iT}) = \widetilde{1}_T \forall i \in N$, $cl(\widetilde{1}_T) = \widetilde{1}_T$, and $cl(\widetilde{0}_T) = \widetilde{0}_T$, then $\text{FSRC}(X_T) = \{\widetilde{0}_T, \widetilde{1}_T\}$. So, $\text{FS}\delta C(X_T) = \{\widetilde{0}_T, \widetilde{1}_T\}$. Therefore, the only FS$\delta$-open cover of $\widetilde{1}_T$ is $\{\widetilde{1}_T\}$. So, (X, ϑ, T) is an FSδ-compact, and also is FSδ^*-compact.

Proposition 1. *Every FSδ-compact space is FSδ^*-compact.*

Proof. It follows directly from Remark 2.

The next example shows that the converse is not true in general. □

Example 2. Let X be an infinite set, $T = \{t\}$ and $\{u_{iT}: i \in N\}, \{v_{iT}: i \in N\}$ be two families of FS sets on X, given by$u_i(t)(x) = 1 - 1/i$ and $v_i(t)(x) = 1/i$ for all $x \in X$, $i \in N$. We consider FST ϑ on X which is generated by the subbase $\{u_{iT}, v_{iT}: i \in N\}$. Then, $\text{int}(cl(u_{iT})) = u_{iT}$ for all $i \in N$. So, every u_{iT} is FS δ-open. Similarly, every v_{iT} is FS δ-open. Since the family $\mathscr{F} = \{u_{iT}: i \in N\}$ is a *FSδ*-open cover of $\widetilde{1}_T$ which has no finite subcover, then (X, ϑ, T) is not *FSδ*-compact.

On the other hand, $\mathscr{F} = \{u_{iT}: i \in N\}$ is not FSδ^*-open cover of $\widetilde{1}_T$. Indeed, for $x_1^z \widetilde{\in} \widetilde{1}_T$, there is no FS$\delta$-open set u_T in $\mathscr{F}$ such that $x_1^z \widetilde{\in} u_T$, so the only FS$\delta^*$-open cover of $\widetilde{1}_T$ is the family $\{v_{iT}: i \in N\}$ which has $\{\widetilde{1}_T\}$ as a finite subcover. Therefore, (X, ϑ, T) is FSδ^*-compact.

Proposition 2. *If X and T are finite sets, then every FSTS (X, ϑ, T) is FSδ-compact (FSδ^*-compact).*

The converse is not necessarily true, as the next example shows.

Example 3. Let X be an infinite set and $\vartheta = \{\widetilde{\alpha}_T: \alpha \in I\}$. Clearly, ϑ is an FST on X. One can check that (X, ϑ, T) is FSδ-compact, while X is an infinite set.

Remark 3. For any infinite FS-setf_T in the discrete FSTS, f_T is not FSδ^*-compact (not FSδ-compact). Indeed, let (X, ϑ, T) be the discrete FS-space. Then, $\mathscr{A} = \{x_\alpha^z: x_\alpha^z \in \text{FSP}(X_T)\}$ is a family of FSO sets. Clearly, $\text{int}(cl(x_\alpha^z)) = x_\alpha^z$ for all $x_\alpha^z \in \mathscr{A}$. So, every x_α^z is FS δ-open. Since $f_T = \sqcup\{x_\alpha^z: x_\alpha^z \widetilde{\in} f_T\}$, then the family $\{x_\alpha^z: x_\alpha^z \in f_T\}$ is an FSδ^*-open cover of f_T which has no finite subcover. So, f_T is not FSδ^*-compact. When $f_T = \widetilde{1}_T$, then (X, ϑ, T) is not FSδ^*-compact.

Proposition 3. *If (X, ϑ_1, T) is FSδ-compact (FSδ^*-compact) and $\vartheta_2 \subseteq \vartheta_1$, then (X, ϑ_2, T) is FSδ-compact (FSδ^*-compact).*

Proof. Obvious. □

Theorem 4. *If (X, ϑ, T) is FSδ-compact and f_T is an FSδC-set of (X, ϑ, T), then f_T is FSδ-compact.*

Proof. Let f_T be an FS δC-set in (X, ϑ, T) and $\mathscr{F} = \{f_{iT}: i \in J\}$ be an arbitrary FSδ-open cover of f_T. Since f_T^c is FS δO-set, then $\mathscr{F} \sqcup f_T^c$ is FSδ-open cover of $\widetilde{1}_T$ and (X, ϑ, T) is FSδ-compact, then $\mathscr{F} \sqcup f_T^c$ has a finite subcover, say $\{f_{iT}: i = 1, 2, 3, \ldots, n\} \sqcup f_T^c$, that is, $\widetilde{1}_T \sqsubseteq \sqcup(\{f_{iT}: i = 1, 2, 3, \ldots, n\} \sqcup f_T^c)$. However, f_T, f_T^c are disjoint. Thus, $f_T \sqsubseteq \sqcup\{f_{iT}: i = 1, 2, 3, \ldots, n\}$ and so, the result holds. □

Corollary 1. *Every FSδ-closed subspace of FSδ-compact is FSδ-compact.*

Theorem 5. *Let f_T be an FSδ-compact and g_T be an FSδC-set in (X, ϑ, T). Then, $f_T \sqcap g_T$ is an FSδ-compact.*

Proof. A similar technique given in Theorem 4 is followed. □

Definition 15 (see [20]). Let $\mathscr{A} = \{f_{iT}: i \in J\}$ be a family of FS sets and g_T be an FS-set. Then,

(i) $\mathscr{A}$ is said to behave fuzzy soft Q-intersection (FSQ-intersection, briefly) with respect to g_T if there exists $x_\alpha^z \widetilde{\in} g_T$ with $x_\alpha^z q f_{iT} \forall i \in J$

(ii) $\mathscr{A}$ is called has a fuzzy soft finite intersection property (FSFIP briefly)*w.r.to* g_T if every finite subfamily of $\mathscr{A}$ has FSQ-intersection *w.r.to*g_T

The next theorem gives a nice characterization of FSδ^*-compact space.

Theorem 6. *(X, ϑ, T) is an FSδ^*-compact if and only if every family of FSδC sets on X, having the FSFIP w.r.to $\widetilde{1}_T$ has FSQ-intersection w.r.to $\widetilde{1}_T$.*

Proof. Let (X, ϑ, T) be FSδ^*-compact. Then $\widetilde{1}_T$ is FSδ^*-compact and let $\mathscr{A} = \{f_{iT}: i \in J\}$ be the family of FSδC sets over X, which has the FSFIP *w.r.to* $\widetilde{1}_T$. Suppose that, $\mathscr{A}$ has no FSQ-intersection *w.r.to* $\widetilde{1}_T$. Then, for all $x_\alpha^z \widetilde{\in} \widetilde{1}_T$ there is $i \in J$ such that $x_\alpha^z \widetilde{q} f_{iT}$. Thus, $\mathscr{A}^c = \{f_{iT}^c: i \in J\}$ is an FSδ^*-open cover of $\widetilde{1}_T$. Since $\widetilde{1}_T$ is FSδ^*-compact, there is a finite subcover of $\mathscr{A}^c$ say, $\{f_{iT}^c: i \in I_0 \text{ and } I_0 \text{is a finite subset of} J\}$. Hence $\{f_{iT}^c: i \in I_0 \text{ and } I_0 \text{is.a.finite.subset.of} J\}$ has no FSQ-intersection *w.r.to* $\widetilde{1}_T$. This contradicts that $\mathscr{A}$ has FSFIP *w.r.to* $\widetilde{1}_T$. Then, the result holds.

Conversely, let $\mathscr{A} = \left\{O_{x_\alpha^z}^i: i \in I\right\}$ be an FSδ^*-open cover of $\widetilde{1}_T$. Then, $\mathscr{A}^c = \left\{(O_{x_\alpha^z}^i)^c: i \in I\right\}$ has no FSQ-intersection *w.r.to* $\widetilde{1}_T$. Thus, $\mathscr{A}^c$ has no FSFIP*w.r.to*$\widetilde{1}_T$. So, there are $\{i_1, i_2, \ldots, i_n\} \subset$ J, such that $\left\{(O_{x_\alpha^z}^{i_j})^c: j = 1, 2, \ldots, n\right\}$ has no FSQ-intersection *w.r.to* $\widetilde{1}_T$. Then, $\left\{O_{x_\alpha^z}^{i_j}: j = 1, 2, \ldots, n\right\}$ is a finite subcover of $\widetilde{1}_T$. Then, $\widetilde{1}_T$ is FSδ^*-compact. The result holds. □

Definition 16 (see [16, 17]). A family $\mathscr{A} = \{f_{iT}: i \in J\}$ of FS sets is called has the FS-finite intersection property if the intersection of the members of each finite subfamily of $\mathscr{A}$ is not the null FS-set.

In the next result, we characterize an FSδ-compact space using FS δC sets.

Theorem 7. *(X, ϑ, T) is FSδ-compact if and only if every family of FSδC sets on X, which has the FS-finite intersection property has a non-null intersection.*

Proof. Let (X, ϑ, T) be FSδ-compact and let $\mathscr{A} = \{f_{iT}: i \in I\}$ be the family of FSδCsets in (X, ϑ, T) which has the FS-finite intersection property. To prove that $\{f_{iT}: i \in I\}$ is non-null, suppose otherwise, i.e., $\sqcap\{f_{iT}: i \in I\} = \widetilde{0}_T$, then $\sqcup\{f^c_{iT}: i \in I\} = \widetilde{1}_T$, that is, $\{f^c_{iT}: i \in I\}$ is an FSδ-open cover of $\widetilde{1}_T$. By compactness, $\{f^c_{iT}: i \in I\}$ has a finite subcover of $\widetilde{1}_T$ say, $\{f^c_{iT}: i = i_1, i_2, \ldots, i_n\}$. This implies that $\sqcap\{f_{iT}: i = i_1, i_2, \ldots, i_n\} = \widetilde{0}_T$. This contradicts that the family $\mathscr{A}$ has the FS-finite intersection property. So the result holds.

Conversely, let $\{g_{iT}: i \in I\}$ be an FSδ-open cover of $\widetilde{1}_T$. Then, $\{g^c_{iT}: i \in I\}$ is the family of FS δ-closed sets in (X, ϑ, T). Since $\{g_{iT}: i \in I\}$ is an FSδ-open cover of $\widetilde{1}_T$, then $\sqcap\{g^c_{iT}: i \in I\} = \widetilde{0}_T$. Thus, $\{g^c_{iT}: i \in I\}$ has no finite intersection property, that is, there is a finite subset $\{i_1, i_2, \ldots, i_n\}$ of I such that $\sqcap\{g^c_{iT}: i = i_1, i_2, \ldots, i_n\} = \widetilde{0}_T$. This implies that $\{g^c_{iT}: i = i_1, i_2, \ldots, i_n\}$ is a finite subcover of $\widetilde{1}_T$. So the result holds.

We close this section by discussing the image and preimage of FSδ-compactness under FSδ-continuous maps. ☐

Theorem 8. *If (X, ϑ_1, T) is FSδ-compact and $f_{up}: (X, \vartheta_1, T) \longrightarrow (Y, \vartheta_2, K)$ is FSδ-continuous and onto, then, (Y, ϑ_2, K) is FSδ-compact.*

Proof. Let $\{g_{iT}: i \in J\}$ be an FSδ-open cover of $\widetilde{1}_K$ in (Y, ϑ_2, K). Since f_{up} is FSδ-continuous. Then, the family $\mathscr{F} = \left\{f^{-1}_{\text{up}}(g_{iT}): i \in J\right\}$ is an FSδ-open cover of $\widetilde{1}_T$ in (X, ϑ_1, T) and (X, ϑ_1, T) is FSδ-compact, there is a finite subfamily of $\mathscr{F}$ say, $\left\{f^{-1}_{\text{up}}(g_{iT}): i = 1, 2, 3, \ldots, n\right\}$ covers $\widetilde{1}_T$, i.e., $\widetilde{1}_T \sqsubseteq \sqcup\left\{f^{-1}_{\text{up}}(g_{iT}): i = 1, 2, \ldots, n\right\}$. Thus,

$$\begin{aligned} f_{\text{up}}(\widetilde{1}_T) \sqsubseteq f_{\text{up}}\left(\sqcup\left\{f^{-1}_{\text{up}}(g_{iT}): i = 1, 2, \ldots, n\right\}\right) = \\ \sqcup\left\{f_{\text{up}}\left(f^{-1}_{\text{up}}(g_{iT})\right): i = 1, 2, \ldots, n\right\} = \\ \sqcup\{(g_{iT}): i = 1, 2, \ldots, n\}. \end{aligned} \tag{2}$$

Since f_{up} is onto, then $\widetilde{1}_K = f_{\text{up}}(\widetilde{1}_T) \sqsubseteq \sqcup\{(g_{iT}): i = 1, 2, \ldots, n\}$. Hence, (Y, ϑ_2, K) is FSδ-compact. ☐

Theorem 9. *Let $f_{up}: (X, \vartheta_1, T) \longrightarrow (Y, \vartheta_2, K)$ be FSδ-continuous. If g_T is FSδ-compact in (X, ϑ_1, T), then $f_{up}(g_T)$ is FSδ-compact in (Y, ϑ_2, K).*

Proof. Let $\{f_{iT}: i \in J\}$ be an FSδ-open cover of $f_{\text{up}}(g_T)$. Since f_{up} is δ-continuous, $f^{-1}_{\text{up}}(f_{iT})$ is FSδ-open in (X, ϑ_1, T) for all $i \in J$. Thus, the family $\mathscr{F} = \left\{f^{-1}_{\text{up}}(f_{iT}): i \in J\right\}$ is an FSδ-open cover of g_T in (X, ϑ_1, T). Since g_T is FSδ-compact, there is a finite subfamily of $\mathscr{F}$ say $\left\{f^{-1}_{\text{up}}(f_{iT}): i = 1, 2, 3, \ldots, n\right\}$ covers g_T, i.e., $g_T \sqsubseteq \sqcup\left\{f^{-1}_{\text{up}}(f_{iT}): i = 1, 2, \ldots, n\right\}$. Thus,

$$\begin{aligned} f_{\text{up}}(g_T) \sqsubseteq f_{\text{up}}\left(\sqcup\left\{f^{-1}_{\text{up}}(f_{iT}): i = 1, 2, \ldots, n\right\}\right) = \\ \sqcup\left\{f_{\text{up}}\left(f^{-1}_{\text{up}}(f_{iT})\right): i = 1, 2, \ldots, n\right\} = \\ \sqcup\{(f_{iT}): i = 1, 2, \ldots, n\}. \end{aligned} \tag{3}$$

Hence, $f_{\text{up}}(g_T)$ is *FS*δ-compact. ☐

Theorem 10. *Let $f_{up}: (X, \vartheta_1, T) \longrightarrow (Y, \vartheta_2, K)$ be an FS δ-continuous and one-to-one map. If g_T is FSδ-compact in (Y, ϑ_2, K), then $f^{-1}_{up}(g_T)$ is FSδ-compact in (X, ϑ_1, T).*

Proof. The proof is similar to that in the above theorem. ☐

4. The Relations among Different Types of Fuzzy Soft Compactness

In this section, we investigate the relations between our notions of FS-compactness in this paper and those in other papers; we make a comparison between our types of FS-compactness and that given in ([16, 17, 20]). First, we mention that Osmanoglu and Tokat [17] and Hussain [16] gave the same definition of FS-compactness as a generalization of Chang's notion [18]. Their definitions are as follows.

Definition 17 (see [16, 17]).

(i) A family $\mathscr{A}$ of FS sets in (X, ϑ, T) is a cover of FS-setg_T if $g_T \sqsubseteq \sqcup\{f_{iT}: f_{iT} \in \mathscr{A} \text{and} i \in J\}$. It is open cover if each member of $\mathscr{A}$ is an FSO-set. A subcover of $\mathscr{A}$ is a subfamily of $\mathscr{A}$ which is also a cover.

(ii) An FS-set$f_T$in(X, ϑ, T) is called a fuzzy soft compact (briefly, FS-compact) if each FS-open cover of f_T has a finite subcover. (X, ϑ, T) is called an FS-compact if every FS-open cover of $\widetilde{1}_T$ has a finite subcover.

Saleh and Salemi [20] defined another type of FS-compactness which is more general than that of the above definition. His definition is as follows.

Definition 18

(i) A family γ of FSO sets is called a fuzzy soft P-open cover (briefly, FSP-open cover) of an FS-setf_T if for every $x^z_\alpha \widetilde{\in} f_T$ there is $g_T \in \gamma$ such that $x^z_\alpha \widetilde{\in} g_T$. A finite p-subcover of γ is a finite subfamily of γ which is also an FSP-open cover.

(ii) An FS-set$f_T$in(X, ϑ, T) is called a fuzzy soft P-compact (briefly, FSP-compact) if every FSP-open cover of f_T has a finite subcover. (X, ϑ, T) is called an FSP-compact if every FSP-open cover of $\widetilde{1}_T$ has a finite p-subcover.

Remark 4. Clearly, every FSP-cover is an FS-cover, but the converse may not be true, as shown by Example (3.3) in [20].

Theorem 11. *For T finite, FS-compact $\Longrightarrow$FSδ-compact.*

Proof. Let (X, ϑ, T) be an FS-compact and $\{f_{iT}\colon i \in J\}$ be an FS δ-open cover of (X, ϑ, T). Since every FSδO-set is FS-open, thus, $\{f_{iT}\colon i \in J\}$ is an FS-open cover of (X, ϑ, T). Since $\widetilde{1}_T$ is FS-compact, there is a finite subset I_0 of J such that $\widetilde{1}_T \sqsubseteq \sqcup \{f_{iT}\colon i \in I_0\}$. Hence, (X, ϑ, T) is FSδ-compact. □

Corollary 2. *FSδ-compact $\nRightarrow$FS-compact.*

Proof. The proof follows by using the next example.

Let $X = [0, 1]$ and $T = \{t\}$. Then, the family $\vartheta = \{\widetilde{0}_T, \widetilde{1}_T\} \cup \{f_{iT}\colon i \in N\}$ is an FST on X, where f_{iT} is defined as follows:

$$f_{iT}(t)(x) = \begin{cases} \underline{1}, & \text{if} x = 0, \\ ix, & \text{if} 0 < x \leq \dfrac{1}{i}, \\ \underline{1}, & \text{if} \dfrac{1}{i} < x \leq 1. \end{cases} \tag{4}$$

Since the only FSδ-open cover of $\widetilde{1}_T$ is $\{\widetilde{1}_T\}$, then (X, ϑ, T) is FSδ-compact.

On the other hand, (X, ϑ, T) is not FS-compact. Indeed, the family $\{f_{iT}\colon i \in N\}$ is an FS-open cover of $\widetilde{1}_T$ which does not have a finite subcover.

From Theorem 11 and Proposition 1, we obtain the next result. □

Result 4. For T finite, FS-compact $\Longrightarrow$FSδ-compact $\Longrightarrow$FSδ^*-compact.

Remark 5. The converse in the implications of the above result may not be true. Corollary 2 and Example 2 show it.

Theorem 12. *For T finite, FSP-compact $\Longrightarrow$FSδ^*-compact.*

Proof. Let (X, ϑ, T) be an FSP-compact and $\{f_{iT}\colon i \in J\}$ be a family of FSδO sets of(X, ϑ, T) which is an FSδ^*-open cover of $\widetilde{1}_T$. It is clear that $\{f_{iT}\colon i \in J\}$ is also, a family of FSO sets, which is an FSP-open cover of $\widetilde{1}_T$. Since (X, ϑ, T) is an FSP-compact, then there is a finite subset I_0 of J such that $\widetilde{1}_T \sqsubseteq \sqcup \{f_{iT}\colon i \in I_0\}$. The result holds. □

Corollary 3. *FSδ^*-compact $\nRightarrow$FSP-compact.*

Proof. It follows directly from the fact that FSP-open cover need not imply FSδ^*-open cover, indeed FSO sets need not FSδO sets.

In the next, we show that the notions of FSδ-compact and FSP-compact are independent. □

Corollary 4. *FSδ -compact $\nRightarrow$FSP -compact*

Proof. Example 1, shows that (X, ϑ, T) is an FSδ-compact, but it is not FSP-compact, indeed the family $\{f_{iT}\colon i \in N\}$ is an FSP-open cover of $\widetilde{1}_T$ which has no finite p-subcover. □

Corollary 5. *FSP-compact $\nRightarrow$FSδ-compact*

Proof. Example 2, shows that (X, ϑ, T) is an FSP-compact, but it is not FSδ-compact, indeed the family $\{u_{iT}\colon i \in N\}$ is an FSδ-open cover of $\widetilde{1}_T$ which has no finite subcover. □

Proposition 4. *For T finite, FS-compact $\Longrightarrow$FSP-compact.*

Proof. It follows directly from Remark 4. □

Corollary 6. *FSP-compact $\nRightarrow$FS-compact.*

Proof. It follows by using the next example.

Let X be an infinite set and $T = \{t\}$. Then, the family $\vartheta = \{\widetilde{1}_T\} \cup \{f_{iT}\colon i \in N\}$ is an FST on X, where f_{iT} given by, $f_{iT}(t)(x) = 1 - 1/i \forall x \in X$. Since the only FSP-open cover of $\widetilde{1}_T$ is $\{\widetilde{1}_T\}$, then (X, ϑ, T) is an FSP-compact. However, (X, ϑ, T) is not FS-compact, indeed for the family $\{f_{iT}\colon i \in N\}$ is FS-open cover of $\widetilde{1}_T$ which has no finite subcover. □

Remark 6. From the previous results, we can summarize the relationships among different types of FS-compactness as in the next diagram.

$$\begin{tikzcd}
FS - compact \arrow[r, shift right] \arrow[r, leftarrow, shift left, "/" marking] \arrow[dr, shift right] \arrow[dr, leftarrow, shift left, "/" marking] & FS\delta - compact \arrow[r, shift right] \arrow[r, leftarrow, shift left, "/" marking] & FS\delta^* - compact \\
 & FSP - compact \arrow[ur, shift right] \arrow[ur, leftarrow, shift left, "/" marking] &
\end{tikzcd}$$

In the following, we study the relation between FSδ -compact spaces and some induced fuzzy compact spaces.

Theorem 13. *For finite T and an FTS (X, σ), we have, (X, ϑ_σ, T) is FSδ-compact $\Leftrightarrow (X, \sigma)$ is fuzzy δ-compact.*

Proof. Let (X, ϑ_σ, T) be an FSδ-compact and $T = \{t_1, t_2, t_3, \ldots, t_n\}$. Let $\gamma = \{A_i\colon i \in J\}$ is an arbitrary fuzzy δ-open cover of $\underline{1}$. Then, $\mathscr{F} = \{f_{iT}\colon f_i(t) = A_i \forall t \in T \text{and} A_i \in \sigma, i \in J\}$ is an FSδ-open cover of $\widetilde{1}_T$. Since $\widetilde{1}_T$ is an FSδ-compact and so, $\mathscr{F}$ has a finite subcover say, $\{f_{iT}\colon f_i(t) = A_i \forall t \in T \text{and} A_i \in \sigma, i = 1, 2, \ldots, n\}$. Thus, the family $\{A_i \in \sigma\colon i = 1, 2, \ldots, n\}$ is a finite subcover of $\underline{1}$. Hence, (X, σ) is a fuzzy δ-compact.

Conversely, let (X, σ) be a fuzzy δ-compact and $\mathscr{F} = \{f_{iT}\colon f_i(t) = A_i \forall t \in T \text{and} A_i \in \sigma, i \in J\}$ be an FSδ-open cover of $\widetilde{1}_T$ in (X, ϑ_σ, T), then the family $\gamma = \{A_i \in \sigma\colon i \in J\}$ is a fuzzy δ-open cover of $\underline{1}$. Since (X, σ) is a fuzzy δ -compact, then γ has a finite subcover say, $\{A_i \in \sigma\colon i = 1, 2, \ldots, n\}$. Thus, $\mathscr{F}$ has a finite subcover say, $\{f_{iT}\colon f_i(t) = A_i \forall t \in T \text{and} A_i \in \sigma, i = 1, 2, \ldots, n\}$. Hence, (X, ϑ_σ, T) is an FSδ-compact. □

Theorem 14. *For a finite set T with (X, ϑ, T). If (X, ϑ_t) is fuzzy δ-compact $\forall t \in T$, then (X, ϑ, T) is FSδ-compact.*

Proof. Let (X, ϑ_t) be fuzzy δ-compact $\forall t \in T$ and $T = \{t_1, t_2, t_3, \ldots, t_n\}$. Let $\mathscr{F} = \{f_{iT}\colon i \in J\}$ be an arbitrary FSδ-open cover of $\widetilde{1}_T$ in (X, ϑ, T). Then, $\gamma = \{f_i(t)\colon f_{iT} \in \vartheta, i \in J\} \forall t \in T$ is a fuzzy δ-open cover of $\underline{1}$.

Since (X, ϑ_t) is fuzzy δ-compact $\forall t \in T$, then $\underline{1}$ is fuzzy δ-compact. Thus, γ has a finite subcover, say $\{f_i(t)\colon f_{iT} \in \vartheta, i = 1, 2, 3, \ldots n\}$. Therefore, the subfamily, $\{f_{iT}\colon i = 1, 2, 3, \ldots n\}$ of $\mathscr{F}$ is a finite subcover. The proof is complete.

The converse of the above theorem may not be true in general. This fact is demonstrated by the next example. □

Example 4. Let X be an infinite set, $T = \{t_1, t_2\}$ and let the family $\vartheta = \{\tilde{1}_T\} \cup \{f_{iT}\colon f_{iT} = (t_1, A_i), A_i \in I^X \text{ and } i \in N\}$. Then, one can check that ϑ is an FST on X. Since the only FSδ-open cover of $\tilde{1}_T$ is $\{\tilde{1}_T\}$, then (X, ϑ, T) is an FSδ-compact. On the other hand, (X, ϑ_{t_1}), where $\vartheta_{t_1} = I^X$ is the discrete FT on X which is not fuzzy δ-compact, indeed the family $\{x_\alpha^i\colon x \in X, \alpha \in I \text{and} i \in N\}$ is a fuzzy open cover of $\underline{1}_T$ which has no finite subcover.

Theorem 15. *(X, ϑ_δ, T) is FSδ-compact if and only if every family of FSδC sets on X, which has the FS-finite intersection property has a non-null intersection.*

Proof. In a similar way to that in Theorem 7, from the above theorem and Theorem 7, we obtain the next result. □

Corollary 7. *(X, ϑ, T) is FSδ -compact $\Longleftrightarrow (X, \vartheta_\delta, T)$ is FS -compact.*

5. Conclusion

In this work, we have defined and studied new types of compactness in fuzzy soft topological spaces, namely, δ-compactness and δ^*-compactness. Some of their basic properties have been explored, and the relationships between these notions and that which were defined by other authors have been investigated. We have shown that δ^*-compactness is more general than that which are presented in [16, 17, 20]. Some results and theorems related to these notions have been discussed with some necessary examples. In the future work, we will study the relationships between the concepts given herein and separation axioms via fuzzy soft δ-open sets. Also, we will construct a general frame of fuzzy soft topology inspired by the class of (a, b)-Fuzzy soft sets [28]. Through this frame, it can be reintroduced all concepts of fuzzy soft topological spaces and explored their properties and characterizations.

Data Availability

No underlying data were collected or produced in this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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