Hindawi
Journal of Mathematics
Volume 2023, Article ID 5141179, 11 pages
https://doi.org/10.1155/2023/5141179

Research Article

Fuzzy Topological Characterization of qC_n Graph via Fuzzy Topological Indices

Zeeshan Saleem Mufti,[1] **Rukhshanda Anjum**,[1] **Ferdous M. O. Tawfiq**,[2] **Ali Tabraiz**,[3] **Qin Xin**,[4] **and Dilruba Akter**[5]

[1]*Department of Mathematics and Statistics, The University of Lahore, Lahore Campus, Lahore, Pakistan*
[2]*Department of Mathematics, College of Science, King Saud University, P.O. Box 22452, Riyadh 11495, Saudi Arabia*
[3]*Punjab Tianjin University of Technology, Lahore, Pakistan*
[4]*Faculty of Science and Technology, University of the Faroe Islands, Faroe Islands, Denmark*
[5]*Department of Applied Mathematics, Gono Bishwabidyalay University, Dhaka, Bangladesh*

Correspondence should be addressed to Dilruba Akter; dilruba.amath@gonouniversity.edu.bd

Received 19 August 2022; Revised 6 October 2022; Accepted 10 March 2023; Published 11 September 2023

Academic Editor: Georgios Psihoyios

Fuzzy topological indices are one of the accomplished mathematical approaches for numerous technology, engineering, and real-world problems such as telecommunications, social networking, traffic light controls, marine, neural networks, Internet routing, and wireless sensor network (Muneera et al. (2021)). This manuscript comprises the study of a particular class of graphs known as qC_n snake graphs. Some innovative results regarding fuzzy topological indices have been established. The major goal of the work is to introduce the notions of First Fuzzy Zagrab Index, Second Fuzzy Zagrab Index, Randic Fuzzy Zagrab Index, and Harmonic Fuzzy Zagrab Index of the qC_n snake graph.

1. Introduction

One cannot deny the importance of mathematics in all professions around the globe as every facet of life is vastly dependent on the use of arithmetic and numbers. Mathematics is used to develop and interpret its theories, especially chemistry, astronomy, geography, physics, etc. Learning of data entry for numerous purposes (such as computer programming, engineering, science, banking, and accounting) entirely depends upon mathematics.

Binary mathematics is the heart of computer operation which is the essential form of math used in computer science, statistics, algebra, and calculus. Creationof new generation tools and innovative and modern technological such as the Internet is one of the core necessities of modern life. Mathematics is commonly used in animation. This allows the animator to discover the unknown from a simple set of equations and extrapolate aspects of geometric figures when working with objects that move and change. The animation designer uses mathematical techniques to show how the drawings are animated and converted and also zoomed in and out. Information rising after cognition and computational perception, i.e., uncertain, partially true, vague, imprecise, or without limitations, can be resolved through fuzzy logic. Moreover, it leads to an effective resolution through better assessment and multiple criteria for options for the desired problem. Innovative computing techniques developed through fuzzy logic are used in the expansion of intelligence systems for identification, pattern recognition, control, and optimization.

Flow problems are one of the widespread applications of graph theory, which include real-life scenarios such as the scheduling of airlines or moving from one place toanother place with least possible time and by spending less money. Airlines have to fly around the world along with operating crew for each plane. Employees might belong to a particular city, so it is difficult for all the workers to be available for all flights. Graph theory is used in order to schedule the flight for crews. For this purpose, directed graphs are used where the flights are taken as the input, the destination is identified through an arrow (direction), and the cities are treated as vertices. The network flow can be seen in the resulting graph. Moreover, weights can be assigned to edges or vertices, showing the importance of a city or the need for flights to move from one city to another city so that passengers can be moved from one place to another place appropriately.

Travelling from one place to another place is a routine matter. Due to population growth and a large number of wheels on the roads, it is difficult to reach your destination within time during peak hours. Smart phones are frequently used nowadays not only for communication purposes but also they help us schedule the daily life activities, and GPRS helps us to save our time during peak hours, find the directions to our desired location, and reach our destination in the shortest possible time with the help of graph theory as the said problem lies under the category of shortest path problem, which is commonly used in graph theory. For this purpose, the current location of the user is the starting point/first vertex, and the destination is the end point/last vertex. The streets are treated as the edges and a directed graph is used for the directions, and the arrow guides the user where to go. The estimated time to reach the desired location varies with the speed of the wheels. Different colours are used to show the rush of traffic on the road, and directions can be relocated in case of any wrong turn or change of route for uncertain reasons.

It is pretty famous not only in the field of mathematics but in every field of science that graphs are simply models of relationships. A graph is a suitable way of representing information and connecting relationships between entities. The entities are characterized by vertices and relations by edges. When there is fuzziness in the description of the entities, their relationships, or both, it is natural to draw a "Fuzzy Graph."

Expansion in the field of graph theory has given us very remarkable and innovative directions such as the fuzzy graph theory, hypergraph theory, spectral graph theory, and most importantly in the field of Computer Science. Graph algorithms are a major part of this field. Some popular graph algorithms are breadth-first search, depth-first search, Dijkstra's algorithm, A^* algorithm, shortest path algorithm, minimum spanning tree algorithm, and maximum flow algorithm. Behind every algorithm, there is a deep knowledge of python coding. Due to this reason, computer languages are playing a beautiful role in improvising the networking analysis. The use of Python language in graph theory makes it beautiful and more innovative due to the diversity of the language. These days AI (artificial intelligence) has a great boom in the field of computer science and also graphs are playing a vital role in this field particularly in machine learning. The most recent use of the graph theory with machine learning is seen in the FIFA football world cup. By using machine learning and graph theory, they will monitor 19 different body parts of a player. This will help the referee to conduct a smooth game. In the same context, fuzzy graph theory is also used in social networking sites such as Facebook, Twitter, Instagram, WhatsApp, and Research gate day-by-day. Fuzzy methods and fuzzy graph theory are employed in machine learning and data mining [1].

Rosenfeld introduced Fuzzy graphs a decade later Zadeh's landmark research article "Fuzzy Sets". He established some relations regarding the properties of path graph, trees, and various graphs. The concept of fuzzy cut nodes and fuzzy bridges was established by Bhattacharya in [2]. Many properties of fuzzy graphs are similar to those of crisp graphs as fuzzy graphs are the generalization of the crisp graph. A crisp graph is a one-to one correspondence from $f\colon \mathrm{VUE} \longrightarrow N$.

In dynamical systems where the values of all state constraints are known, the fuzzy graph theory is effectively applicable, which is helpful for spotting initial behaviors in consecutive changes of assemblages of organisms. Using fuzzy relations to describe succession from the same dataset, the compositional dynamics of the species was examined to get the desired result.

One cannot deny the importance of graph theory in numerous fields of life such as to elaborate chemical structures in Chemistry and Biochemistry; in Computer Science, several algorithms are used to solve various road networks problems to solve traffic problems, to connect numerous cities through trains and air lines effectively to move great number of passengers in least possible trips in rail and flight networks, to establish the finest timetable, and to have the same teaching load for all faculty in an organization. Labeling of a graph is a practice where symbols can be assigned to elements of the graph according to our condition; sometimes, it is not essential to allocate labels to all members of the graph. Labeling of a graph has massive applications in several fields of life and science. For further study on fuzzy algebraic points of view, see [3–7].

Bibi and Devi [8] develop a new idea of fuzzy magic labeling of cycle and star graphs. Vimala and Nagarani [9] gave the generalized concept of energy for fuzzy labeling graph. Nagoor Gani et al. [10] describe various properties of fuzzy labeling of path graph and star graph. Numerous definitions and basic concepts are discussed in [2, 11, 12]. Mufti et al. [13] computed first and second fuzzy Zagrab indices of linear and multiacyclic hydrocarbons. Also one can find more details in [14–16].

In 2023, Islam and Pal [17] also discussed the Second Zagreb Index and its use in mathematical chemistry. Islam and Pal [18] presented the First Zagreb Index for fuzzy graphs and discussed its application too. In [19], a new index, the F-index, has been already investigated and improved. In 2022, Rajeshwari et al. [20] established the fuzzy topological indices for bipolar fuzzy graphs.

Barrientos [21] introduced the concept of qC_4-snake or cyclic snake as a natural extension of triangular snake graphs already defined by Rosa [22]. A graph which is connected having q isomorphic cycles of C_n, such that the block-cut point graph is a path known as qC_n snake graph. This paper deals with the First Fuzzy Zagrab Index, Second Fuzzy Zagrab Index, Randic Fuzzy Zagrab Index, and Harmonic Fuzzy Zagrab Index of qC_5 snake graph.

2. Motivation

For topological indices in crisp graphs, numerous results and applications are available. Some circumstances cannot be handled using crisp graphs in many real-life problems. Therefore, to handle such problems, new topological indices are needed to be introduced for the solution of real-world problems in the fields of medicine, engineering, technology, social sciences, telecommunication, and many more. The motivation of our paper was based on the paper [23], in which they introduce fuzzy topological indices, and we are motivated to work on the general structures of different graphs. Therefore, we have computed the fuzzy topological indices for the snake graph.

3. Preliminaries

In this section, we will define some basic definitions.

Definition 1 (see [23]). Let G be a finite, undirected fuzzy graph (without loop and parallel edges). Let $V(G)$ be a finite set of vertices and $E(G)$ be a finite set of edges. Two mappings σ and μ, where σ: $V(G) \longrightarrow [0,1]$ and μ: $V(G) \times V(G) \longrightarrow [0,1]$ such that $\mu(v_1 v_2) \leq \sigma(v_1) \wedge \sigma(v_1)$ for every pair of $v_1, v_2 \in V(G)$.

Definition 2 (see [23]). Let G be a fuzzy graph. The notion of the degree of a vertex v in the fuzzy graph is defined as the sum of all the weights of edges corresponding to vertex v, i.e., $\mathrm{d}(v) = \sum_{n=1}^{k} \mu(\mathrm{vu}), v \neq u$.

Definition 3 (see [23]). Let G be a fuzzy graph. The order of a fuzzy graph is defined as the sum of all the weights of vertices, i.e., $O(G) = \sum \sigma(v)$.

Definition 4 (see [23]). Let G be a fuzzy graph. The size of a fuzzy graph is defined as the sum of all the weights of edges, i.e., $S(G) = \sum \mu(\mathrm{vu}), v \neq u$.

Definition 5 (see [23]). Kalathian et al. [23] introduce the Fuzzy First Zagreb indices. Let G be the fuzzy graph with nonempty vertex set. The First Zagreb Index is denoted by $M(H)$ and defined as First Fuzzy Zagreb Index.

$$M(H) = \sum_{k=1}^{q} \sigma(u_k)[\mathrm{du}_k]^2. \tag{1}$$

Definition 6 (see [23]). The Second Zagreb Index is denoted by $M*(H)$ and defined as the Second Fuzzy Zagreb Index.

$$M^*(H) = \frac{1}{2} \sum_{kl \in E(G)} [\sigma(u_k)(\mathrm{du}_k)\sigma(v_l)(\mathrm{dv}_l)]. \tag{2}$$

Definition 7 (see [23]). The Randic Index of fuzzy graph H is defined as Randic Fuzzy Zagreb Index.

$$R(H) = \frac{1}{2} \sum_{kl \in E(G)} [\sigma(u_k)(\mathrm{du}_k)\sigma(v_l)(\mathrm{dv}_l)]^{-1/2}. \tag{3}$$

Definition 8 (see [23]). The Harmonic Index of G ($H(G)$) is defined as Harmonic Fuzzy Zagreb Index.

$$H(H) = \frac{1}{2} \sum_{kl \in E(G)} \left[\frac{1}{\sigma(u_k)(\mathrm{du}_k) + \sigma(v_l)(\mathrm{dv}_l)}\right]. \tag{4}$$

4. Limitations

Fuzzy Zagreb indices of qC_5 snake graph and qC_n snake graph is applicable to undirected graphs as qC_n snake graph is an undirected graph.

5. Applications of Fuzzy Graphs

Fuzzy graphs are very important in real life. The application of fuzzy graphs falls in the different fields of life such as ecology, social networks, telecommunications, link predictions, identification of human trafficking, traffic light controls, marine, neural networks, Internet routing, and many more. In [24], the authors have discussed and deployed directed fuzzy graph to identify human trafficking. They investigated the traversal of humans from one place to another place and also elaborated their properties. Fuzzy graphs are also used in decision-making analysis as an application and in applied economics for management as an optimization problem [25–30].

6. Advantages of Fuzzy Graphs

Fuzzy graphs, which combine the concepts of fuzzy sets and graphs, offer several advantages in representing and analyzing complex systems that involve uncertainty and imprecision. Here are some advantages of fuzzy graphs:

6.1. Uncertainty Modeling. Fuzzy graphs provide a framework to represent and handle uncertainty in graph-based systems. They allow for the representation of imprecise or vague information that cannot be easily captured using traditional graphs.

6.2. Flexibility in Edge Weights. Fuzzy graphs allow for the assignment of fuzzy edge weights, which represent the degree of membership or possibility of a relationship between vertices. This flexibility enables a more nuanced representation of relationships compared to traditional graphs, where edge weights are usually binary or numeric.

6.3. Reasoning under Uncertainty. Fuzzy graphs provide a basis for reasoning and decision-making in situations where there is uncertainty. By applying fuzzy logic and fuzzy reasoning techniques to fuzzy graphs, it becomes possible to perform computations and make inferences in the presence of imprecise or incomplete information.

6.4. Robustness to Noise. Fuzzy graphs are less sensitive to noise or small variations in data compared to traditional graphs. The fuzziness introduced in the graph representation can help smooth out irregularities and provide a more robust analysis in the presence of noisy or incomplete data.

6.5. MultiCriteria Analysis. Fuzzy graphs are well-suited for multicriteria decision-making. By incorporating fuzzy sets and fuzzy logic, they allow for the consideration of multiple factors or criteria simultaneously, each with its own degree of importance or membership.

6.6. Human-Like Reasoning. Fuzzy graphs align with human reasoning processes, as humans often deal with imprecise and uncertain information. By using fuzzy graphs, it becomes possible to model and mimic human-like decision-making processes, enabling more human-compatible systems and algorithms.

6.7. Applications in Various Fields. Fuzzy graphs have found applications in diverse fields such as pattern recognition, image processing, expert systems, social networks, transportation systems, control systems, and optimization. Their ability to handle uncertainty makes them useful in situations where precise information is unavailable or difficult to obtain.

It is important to note that the advantages of fuzzy graphs depend on the specific problem domain and the quality of the fuzzy modeling and reasoning techniques employed. The effectiveness of fuzzy graphs may vary depending on the complexity of the system being modeled and the availability of accurate and relevant data.

7. Zagrab Indices of qC_5

A connected graph consisting of q blocks, in which each one is isomorphic to cycle C_5, such that the block-cut vertex is a path is termed as qC_5 snake graph. A vertex in a graph G is called a cut vertex if graph without that vertex has more components than the original graph. This graph was first introduced by Barrientos [21], as a generalization of the concept of triangular snake introduced by Rosa [22].

8. Our Main Results

Theorem 9. *Let $H = qC_5$ snake graph be a fuzzy graph, then the First Fuzzy Zagreb Index of qC_5 snake graph is $M(H) = (1.726q - 0.144)$.*

Proof. qC_5 snake graph shown in Figure 1 consists of $4q + 1$ vertices and $5q$ edges.

The representation of the vertex set is given as under: Weight of the all vertices $U_i's$ is 0.6 and has a total count of $q + 1$ where $q - 1$ has weight 0.7 and other 2 have weights 0.3 and 0.4, respectively. Weight of the all vertices $W_i's$ is 0.7 and has a total count of q and all have a weight of 0.7. Weight of

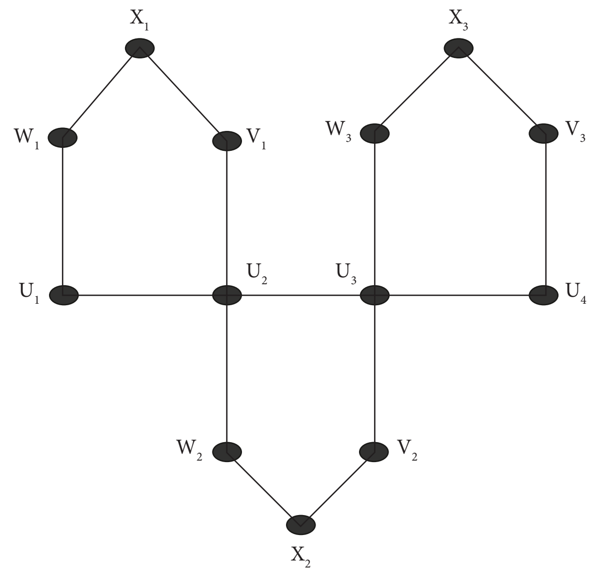

Figure 1: KC5.

the all vertices $X_i's$ is 0.8 and has a total count of q and all have a weight of 0.9. Weight of the all vertices $V_i's$ is 0.9 and has a total count of q and all have a weight of 0.7.

$$\begin{aligned} M(H) &= \sum_{k=1}^{q} \sigma(u_k)[\mathrm{du}_k]^2 \\ &= (0.9)\left[q(0.7)^2\right] + (0.7)\left[q(0.7)^2\right] + (0.8)\left[q(0.9)^2\right] \\ &\quad + (0.6)\left[1(0.3)^2 + 1(0.4)^2 + (q-1)(0.7)^2\right] \\ &= 0.441q + 0.343q + 0.648q + 0.294q - 0.144 \\ &= 1.726q - 0.144. \end{aligned} \tag{5}$$

□

Theorem 10. *Let $H = qC_5$ snake graph be a fuzzy graph, then the Second Fuzzy Zagreb index of qC_5 snake graph is $M^*(H) = (0.7266q - 0.1197)$.*

Proof. qC_5 snake graph shown in Figure 1 consists of $4q + 1$ vertices and $5q$ edges.

The representation of the edge set is given as under and also you can see in Figure 2: the edge set $E_1 = u_i, u_{i+1}$ having weights of the vertices (0.6, 0.6) has three (03) types of representations. $q - 2$ has a vertex degree (0.7, 0.7) and other 2 have (0.4, 0.7) and (0.7, 0.3), respectively. The edge set $E_2 = u_i, w_i$ having weights of the vertices (0.6, 0.7) has two (02) types of representations. $q - 1$ has a vertex degree (0.7, 0.7) and other has (0.4, 0.7). The edge set $E_3 = w_i, x_i$ has weights of the vertices (0.7, 0.8) has one (01) type of representation. All q have vertex degree (0.7, 0.9). The edge set $E_4 = x_i, v_i$ has weights of the vertices (0.8, 0.9) has one (01) type of representation. All q have a vertex degree (0.9, 0.7). The edge set $E_5 = v_i, u_{i+1}$ having weights of the vertices (0.9, 0.6) has two (02) types of representations. $q - 1$ has a vertex degree (0.7, 0.7) and other has (0.7, 0.3).

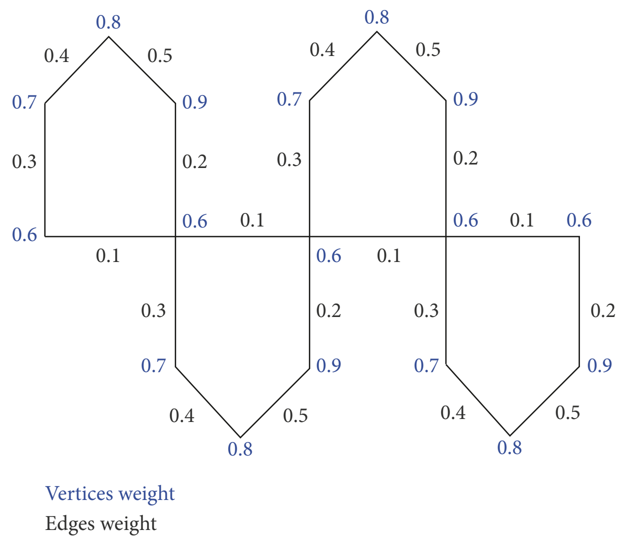

FIGURE 2: 4C5.

$$
\begin{aligned}
M^{*}(H) &= \frac{1}{2}\sum_{kl\in E(G)}\left[\sigma(u_k)(\mathrm{du}_k)\sigma(v_l)(\mathrm{dv}_l)\right]\\
&= \frac{1}{2}\left[(q-2)(0.6)(0.7)(0.7)(0.6) + 1(0.6)(0.4)(0.6)(0.7) + 1(0.6)(0.7)(0.6)(0.3)\right]\\
&\quad + \frac{1}{2}\left[(q-1)(0.6)(0.7)(0.7)(0.7) + 1(0.6)(0.4)(0.7)(0.7)\right] + \frac{1}{2}\left[(q)(0.7)(0.7)(0.8)(0.9)\right]\\
&\quad + \frac{1}{2}\left[(q-1)(0.9)(0.7)(0.6)(0.7) + 1(0.9)(0.7)(0.6)(0.3)\right] + \frac{1}{2}\left[(q)(0.8)(0.9)(0.9)(0.7)\right]\\
&= \frac{1}{2}\left[0.1764q + 0.2058q + 0.3528q + 0.4536q + 0.2646q + 0.1764 - 0.3528 + 0.1176 + 0.1134 - 0.2058 - 0.2646\right]\\
&= \frac{1}{2}\left[1.4532q - 0.4158\right]\\
&= 0.7266q - 0.2079.
\end{aligned} \tag{6}
$$

□

Theorem 11. *Let $H = qC_5$ snake graph be a fuzzy graph, then the Randic Fuzzy Zagreb Index of qC_5 snake graph is $R(H) = (4.84q + 1.89)$.*

Proof. qC_5 snake graph shown in Figure 1 consists of $4q + 1$ vertices and $5q$ edges.

The representation of the edge set is given as follows: the edge set $E_1 = u_i, u_{i+1}$ having weights of the vertices (0.6, 0.6) has three (03) types of representations. $q - 2$ has a vertex degree (0.7, 0.7) and other 2 have (0.4, 0.7) and (0.7, 0.3), respectively. The edge set $E_2 = u_i, w_i$ having weights of the vertices (0.6, 0.7) has two (02) types of representations. $q - 1$ has a vertex degree (0.7, 0.7) and other has (0.4, 0.7). The edge set $E_3 = w_i, x_i$ having weights of the vertices (0.7, 0.8) has one (01) type of representation. All q have vertex degree (0.7, 0.9). The edge set $E_4 = x_i, v_i$ having weights of the vertices (0.8, 0.9) has one (01) type of representations. All q have vertex degree (0.9, 0.7). The edge set $E_5 = v_i, u_{i+1}$ having weights of the vertices (0.9, 0.6) has two (02) types of representations. $q-1$ has a vertex degree (0.7, 0.7) and other has (0.7, 0.3).

$$\begin{aligned}
R(H) &= \frac{1}{2}\sum_{kl\in E(G)} \left[\sigma(u_k)(\mathrm{d}u_k)\sigma(v_l)(\mathrm{d}v_l)\right]^{-1/2},\\
&= \frac{1}{2}\Big[(q-2)[(0.6)(0.7)(0.7)(0.6)]^{-1/2} + 1[(0.6)(0.4)(0.6)(0.7)]^{-1/2}\\
&\quad + 1[(0.6)(0.7)(0.6)(0.3)]^{-1/2} + \frac{1}{2}[(q-1)(0.6)(0.7)(0.7)(0.7)]^{-1/2}\\
&\quad + 1[(0.6)(0.4)(0.7)(0.7)]^{-1/2} + \frac{1}{2}[(q)(0.7)(0.7)(0.8)(0.9)]^{-1/2}\\
&\quad + \frac{1}{2}[(q-1)(0.9)(0.7)(0.6)(0.7) + 1(0.9)(0.7)(0.6)(0.3)]^{-1/2}\\
&\quad + \frac{1}{2}[(q)(0.8)(0.9)(0.9)(0.7)]^{-1/2}\\
&= \frac{1}{2}[2.38q + 2.20q + 1.68q + 1.48q + 1.94q - 4.76 + 3.15 + 3.64 + 2.92 - 2.20 - 1.94 + 2.97]\\
&= \frac{1}{2}[9.68q + 3.78]\\
&= 4.84q + 1.89.
\end{aligned} \tag{7}$$

□

Theorem 12. *Let $H = qC_5$ snake graph be a fuzzy graph, then the Harmonic Fuzzy Zagreb Index of qC_5 snake graph is $R(H) = (2.405q + 0.68)$.*

Proof. qC_5 snake graph shown in Figure 1 consists of $4q+1$ vertices and $5q$ edges.

The representation of the edge set is given as follows: the edge set $E_1 = u_i, u_{i+1}$ having weights of the vertices (0.6,0.6) has three (03) types of representations. $q-2$ has a vertex degree (0.7, 0.7) and other 2 have (0.4, 0.7) and (0.7, 0.3), respectively. The edge set $E_2 = u_i, w_i$ having weights of the vertices (0.6, 0.7) has two (02) types of representations. $q-1$ has a vertex degree (0.7, 0.7) and other has (0.4, 0.7). The edge set $E_3 = w_i, x_i$ having weights of the vertices (0.7, 0.8) has one (01) type of representation. All q have a vertex degree (0.7, 0.9). The edge set $E_4 = x_i, v_i$ having weights of the vertices (0.8, 0.9) has one (01) type of representations. All q have a vertex degree (0.9, 0.7). The edge set $E_5 = v_i, u_{i+1}$ having weights of the vertices (0.9, 0.6) has two (02) types of representations. $q-1$ has a vertex degree (0.7, 0.7) and other has (0.7, 0.3).

$$\begin{aligned}
H(H) &= \frac{1}{2}\sum_{kl\in E(G)} \left[\frac{1}{\sigma(u_k)(\mathrm{d}u_k) + \sigma(v_l)(\mathrm{d}v_l)}\right]\\
&= \frac{1}{2}\Bigg[\frac{q-2}{(0.6)(0.7)+(0.7)(0.6)} + \frac{1}{(0.6)(0.4)+(0.6)(0.7)} + \frac{1}{(0.6)(0.7)+(0.6)(0.3)}\\
&\quad + \frac{q-1}{(0.6)(0.7)+(0.7)(0.7)} + \frac{1}{(0.6)(0.4)+(0.7)(0.7)} + \frac{q}{(0.7)(0.7)+(0.8)(0.9)}
\end{aligned}$$

$$+\frac{(q-1)}{(0.9)(0.7)+(0.6)(0.7)}+\frac{1}{(0.9)(0.7)+(0.6)(0.3)}+\frac{q}{(0.8)(0.9)+(0.9)(0.7)}$$
$$=\frac{1}{2}[1.19q+1.10q+0.83q+0.74q+0.95q-2.38+1.52+1.67+1.37-1.10-0.95+1.23]$$
$$=\frac{1}{2}[4.81q+1.36]$$
$$=2.405q+0.68. \tag{8}$$

□

9. Zagrab Indices of qC_n

Fuzzy Zagreb indices of qC_n snake graph has been discussed in this section (for odd n.) Barrientos [21] introduced the concept of qC_4-snake or cyclic snake as a natural extension of triangular snake graphs already defined by Rosa [22]. A graph which is connected having q isomorphic cycles of C_n, such that the block-cut point graph is a path, is known as qC_n snake graph. This paper deals with the First Fuzzy Zagrab Index, Second Fuzzy Zagrab Index, Randic Fuzzy Zagrab Index, and Harmonic Fuzzy Zagrab Index of qC_5 snake graph.

Theorem 13. *Let $H = qC_n$ snake graph be a fuzzy graph and n is odd, then the First Fuzzy Zagreb Index of qC_n snake graph is*

$$M(H)=(n+1)\times 10^{-j}\left[1\times(n\times)10^{-j}+3\times 10^{-j}+(q-1)\left[(n+4)\times 10^{-j}\right]\right]$$
$$+\left(5\times 10^{-j}\right)q+\left(5\times 10^{-j}\right)q+\cdots+\left((2n-1)\times 10^{-j}\right)q. \tag{9}$$

Proof. Let qC_n snake graph shown in Figure 3 consists of $n(q+1)$ vertices and nq edges; if $n+q\geq 10^i, q\geq 3$, and $n\geq 5$, then $j=i+1$.

The representation of the vertex set can be considered if we consider a special case for odd n. We consider qc_7 and its vertex set representation is given as $n=7$ and $q\geq 3$, so $n+q\geq 10$ implies $j=2$. Weight of the all vertices $U'_{q1}\,s$ is 0.08 and has a total count of $q+1$ where $q-1$ has weight 0.07 and other 2 have 0.03 and 0.11, respectively. Weight of the all vertices $U'_{q2}\,s$, $U'_{q3}\,s$, $U'_{q4}\,s$, $U'_{q5}\,s$, and $U'_{q6}\,s$ is 0.09, 0.10, 0.11, 0.12, 0.13, respectively, and all have a total count of q.

$$M(H)=\sum_{k=1}^{q}\sigma(u_k)\left[\mathrm{du}_k\right]^2$$
$$=0.09\left[q(0.05)^2\right]+(0.10)\left[q(0.07)^2\right]+(0.11)\left[q(0.09)^2\right]$$
$$+(0.12)\left[q(0.11)^2\right]+(0.13)\left[q(0.13)^2\right]+(0.08)\left[1(0.07)^2+1(0.03)^2+(q-1)(0.11)^2\right]$$
$$=0.006223q-0.000504. \tag{10}$$

□

Theorem 14. *Let $H = qC_n$ snake graph be a fuzzy graph and n is odd, then the Second Fuzzy Zagreb Index of qC_n snake graph is $M^*(H)=(0.7266q-0.1197)$ for $n=7$.*

Proof. qC_n snake graph shown in Figure 3 consists of $n(q+1)$ vertices and nq edges; if $n+q\geq 10^i, q\geq 3$, and $n\geq 5$, then $j=i+1$.

The representation of the vertex set can be considered if we consider a special case for odd n. We consider qc_7 and its vertex set representation is given as follows with the help of Table 1. Furthermore, $n=7$ and $q\geq 3$, so $n+q\geq 10$ implies $j=2$ the edge set $E_1=U_{11},u_{21}$ having weights of the vertices (0.08, 0.08) has three (03) types of representations. $q-2$ has a vertex degree (0.11, 0.11) and other 2 have (0.08, 0.11) and (0.11, 0.03), respectively. The edge set $E_2=U_{21},U_{12}$ having weights of the vertices (0.08, 0.09) and $E_3=U_{12},U_{13}$ having weights of the vertices (0.09,0.10) has a vertex degree $(0.05, 0.07)$, $E_4=U_{13},U_{14}$ having weights of the vertices (0.10, 0.11) have a vertex degree (0.07,0.09), $E_5=U_{14},U_{15}$ having weights of the vertices (0.11, 0.12) have a vertex degree (0.09, 0.11), $E_6=U_{15},U_{16}$ having weights of the vertices (0.12, 0.13) have a vertex degree (0.11, 0.13), and all have vertex count q. The edge set $E_7=U_{16},U_{11}$ has two (02) types of representations. $q-1$ has a vertex degree (0.13, 0.11) and other has (0.13, 0.08).

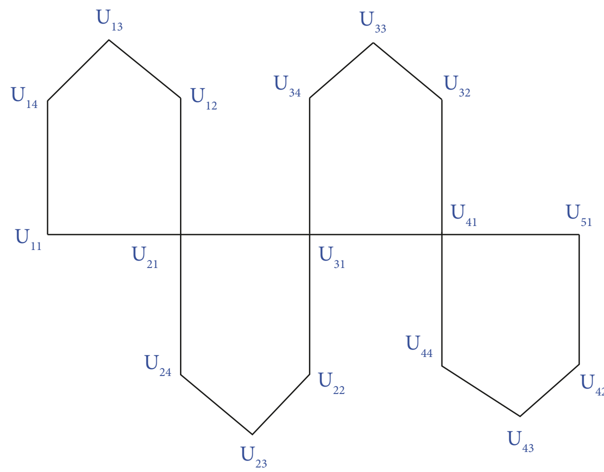

FIGURE 3: qC_n, for $q = 4$ and $n = 5$.

TABLE 1: Edge partition with respect to the vertex weight.

Vertex	Edge w.r.t. vertex weight	Vertex degree	Vertex count
(U_{11}, U_{21})	$((n+1)10^{-j}, (n+1)10^{-j})$	$((n+4)10^{-j}, (n+4)10^{-j})$	$q-2$
(U_{11}, U_{21})	$((n+1)10^{-j}, (n+1)10^{-j})$	$((n+4)10^{-j}, (n+1)10^{-j})$	1
(U_{11}, U_{21})	$((n+1)10^{-j}, (n+1)10^{-j})$	$((n+4)10^{-j}, 0.03)$	1
(U_{21}, U_{12})	$((n+1)10^{-j}, (n+2)10^{-j})$	$((n+4)10^{-j}, 0.05)$	q
(U_{12}, U_{13})	$((n+2)10^{-j}, (n+3)10^{-j})$	(0.05,0.07)	q
(U_{13}, U_{14})	$((n+3)10^{-j}, (n+4)10^{-j})$	(0.07,0.09)	q
$\vdots$	$\vdots$	$\vdots$)	$\vdots$
(U_{1n-2}, U_{1n-1})	$((2n-2)10^{-j}, (2n-1)10^{-j})$	$(2n-3, 2n-1)$	q
(U_{1n-1}, U_{11})	$((2n-1)10^{-j}, (n+1)10^{-j})$	$(2n-1, n+4)$	$q-1$
(U_{1n-1}, U_{11})	$((2n-1)10^{-j}, (n+1)10^{-j})$	$(2n-1, n+1)$	1

$$\begin{aligned}
M^*(H) &= \frac{1}{2}\sum_{kl\in E(G)}[\sigma(u_k)(\mathrm{du}_k)\sigma(v_l)(\mathrm{dv}_l)] \\
&= \frac{1}{2}[(q-2)(0.08)(0.11)(0.08)(0.11) + 1(0.08)(0.08)(0.08)(0.11) + 1(0.08)(0.11)(0.08)(0.03)] \\
&\quad + \frac{1}{2}[(q)(0.08)(0.09)(0.11)(0.05)] \\
&\quad + \frac{1}{2}[(q)(0.09)(0.10)(0.05)(0.07)] + \frac{1}{2}[(q)(0.10)(0.07)(0.11)(0.09)] \\
&\quad + \frac{1}{2}[(q)(0.11)(0.09)(0.12)(0.11)] + \frac{1}{2}[(q)(0.12)(0.11)(0.13)(0.13)] + \\
&\quad + \frac{1}{2}[(q-1)(0.13)(0.13)(0.08)(0.11) + 1(0.13)(0.13)(0.08)(0.08)] \\
&= \frac{1}{2}[0.00072044q - 0.00011805] \\
&= 0.00036022q - 0.00005903.
\end{aligned} \tag{11}$$

□

Theorem 15. *Let $H = qC_n$ snake graph be a fuzzy graph and n is odd, then the Randic Fuzzy Zagreb Index of qC_n Snake graph is $R(H) = (403.62q - 68.87)$ for $n = 7$.*

Proof. qC_n snake graph shown in Figure 3 consists of $n(q+1)$ vertices and nq edges; if $n + q \geq 10^i$, $q \geq 3$, and $n \geq 5$, then $j = i + 1$.

The representation of the vertex set can be consider if we consider a special case for odd n. We consider qc_7 and its vertex set representation is given as follows with the help of Table 1. Furthermore, $n = 7$ and $q \geq 3$, so $n + q \geq 10$ implies $j = 2$ the edge set $E_1 = U_{11}, u_{21}$ having weights of the vertices (0.08, 0.08) has three (03) types of representations. $q - 2$ has vertex degree (0.11, 0.11) and other 2 have (0.08, 0.11) and (0.11, 0.03), respectively. The edge set $E_2 = U_{21}, U_{12}$ has weights of the vertices(0.08, 0.09) and $E_3 = U_{12}, U_{13}$ has weights of the vertices (0.09, 0.10) has vertex degree $(0.05, 0.07)$, $E_4 = U_{13}, U_{14}$ has weights of the vertices (0.10, 0.11) and have a vertex degree (0.07, 0.09), $E_5 = U_{14}, U_{15}$ has weights of the vertices (0.11, 0.12) have vertex degree (0.09, 0.11), $E_6 = U_{15}, U_{16}$ has weights of the vertices (0.12, 0.13) have a vertex degree (0.11, 0.13), and all have vertex count q. The edge set $E_7 = U_{16}, U_{11}$ has two (02) types of representations. $q - 1$ has a vertex degree (0.13, 0.11) and other has (0.13, 0.08).

$$
\begin{aligned}
R(H) &= \frac{1}{2} \sum_{kl \in E(G)} \left[\sigma(u_k)(\mathrm{du}_k)\sigma(v_l)(\mathrm{dv}_l)\right]^{-1/2} \\
&= \frac{1}{2} \Big[(q-2)[(0.08)(0.11)(0.08)(0.11)]^{-1/2} + 1[(0.08)(0.08)(0.08)(0.11)]^{-1/2} \\
&\quad + 1[(0.08)(0.11)(0.08)(0.03)]^{-1/2} + \frac{1}{2}[(q)(0.08)(0.09)(0.11)(0.05)]^{-1/2}\Big] \\
&\quad + \frac{1}{2}[(q)(0.09)(0.10)(0.05)(0.07)]^{-1/2} + \frac{1}{2}[(q)(0.10)(0.07)(0.11)(0.09)]^{-1/2} \\
&\quad + \frac{1}{2}[(q)(0.11)(0.09)(0.12)(0.11)]^{-1/2} + \frac{1}{2}[(q)(0.12)(0.11)(0.13)(0.13)]^{-1/2} \\
&\quad + \frac{1}{2}[(q-1)(0.13)(0.13)(0.08)(0.11) + 1(0.13)(0.13)(0.08)(0.08)]^{-1/2} \\
&= \frac{1}{2}[807.24q - 137.73] \\
&= 403.62q - 68.87.
\end{aligned}
\tag{12}
$$

□

Theorem 16. *Let $H = qC_n$ snake graph be a fuzzy graph and n is odd, then the Harmonic Fuzzy Zagreb Index of qC_n snake graph is $R(H) = (196.78q - 54.82)$ for $n = 7$.*

Proof. qC_n snake graph shown in Figure 3 consists of $n(q + 1)$ vertices and nq edges; if $n + q \geq 10^i, q \geq 3$, and $n \geq 5$, then $j = i + 1$.

The representation of the vertex set can be consider if we consider a special case for odd n. We consider qc_7 and its vertex set representation is given as follows with the help of Table 1. Furthermore, $n = 7$ and $q \geq 3$, so $n + q \geq 10$ implies $j = 2$ the edge set $E_1 = U_{11}, u_{21}$ having weights of the vertices (0.08, 0.08) has three (03) types of representations. $q - 2$ has a vertex degree (0.11, 0.11) and other 2 have (0.08, 0.11) and (0.11, 0.03), respectively. The edge set $E_2 = U_{21}, U_{12}$ has weights of the vertices (0.08, 0.09) and $E_3 = U_{12}, U_{13}$ having weights of the vertices (0.09,0.10) has a vertex degree $(0.05, 0.07)$, $E_4 = U_{13}, U_{14}$ having weights of the vertices (0.10,0.11) have vertex degree (0.07, 0.09), $E_5 = U_{14}, U_{15}$ having weights of the vertices (0.11, 0.12) have vertex degree (0.09, 0.11), $E_6 = U_{15}, U_{16}$ has weights of the vertices (0.12, 0.13) and has a vertex degree (0.11, 0.13), and all have vertex count q. The edge set $E_7 = U_{16}, U_{11}$ has two (02) types of representations. $q - 1$ has a vertex degree (0.13, 0.11) and other has (0.13, 0.08).

$$
\begin{aligned}
H(H) &= \frac{1}{2} \sum_{kl \in E(G)} \left[\frac{1}{\sigma(u_k)(\mathrm{du}_k) + \sigma(v_l)(\mathrm{dv}_l)}\right] \\
&= \frac{1}{2}\left[\frac{q-2}{(0.08)(0.11) + (0.08)(0.11)} + \frac{1}{(0.08)(0.08) + (0.08)(0.11)} + \frac{1}{(0.08)(0.11) + (0.08)(0.03)}\right.
\end{aligned}
$$

Table 2: Graphical analysis of qC_n

Grid graph	1st Fuzzy Zagrab	2nd Fuzzy Zagrab	Fuzzy Randic	Fuzzy Harmonic
$3C_5$	5.034	2.0601	16.41	7.895
$4C_5$	6.76	2.7867	21.25	10.3
$5C_5$	8.486	3.5133	26.09	12.705
$6C_5$	10.212	4.2399	30.93	15.11

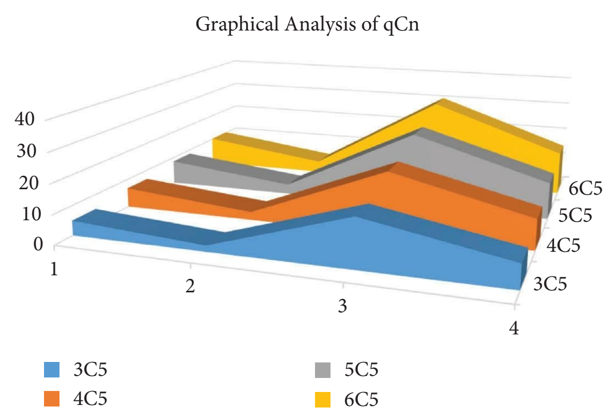

Figure 4: Graphical analysis of qC_n.

$$\begin{aligned}
&+\frac{q}{(0.08)(0.11)+(0.09)(0.05)}+\frac{q}{(0.09)(0.05)+(0.10)(0.07)}+\frac{q}{(0.07)(0.10)+(0.11)(0.09)}\\
&+\frac{q}{(0.11)(0.09)+(0.12)(0.11)}+\frac{q}{(0.12)(0.11)+(0.13)(0.13)}\\
&+\frac{(q-1)}{(0.13)(0.13)+(0.11)(0.08)}+\frac{1}{(0.13)(0.13)+(0.08)(0.08)}\\
&=\frac{1}{2}[393.56q-109.63]\\
&=196.78q-54.8.
\end{aligned}\tag{13}$$

□

10. Graphical Analysis of qC_n

10.1. Conclusion. In this paper, First Fuzzy Zagreb Index, Second Fuzzy Zagreb Index, Harmonic, Randic Fuzzy Zagreb index, and Harmonic Fuzzy Zagreb Index of qC_5 snake graph have been discussed and it has been observed that all of these indices can be calculated using formulas. We have computed the fuzzy topological indices for general structure, the comparison of $H = qC_n$ snake graph with different values of q and n has also been discussed in Table 2, and graphical analysis is done in Figure 4. Using this approach, anyone can find the abovementioned indices of any general structure. This section is closed by raising the following open problem.

10.2. Open Problem. Fuzzy Zagreb indices of qC_n snake graph for even n is still an open problem.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

Acknowledgments

This study was supported by Researchers Supporting Project number (RSP2023R440), King Saud University, Riyadh, Saudi Arabia.

References

[1] E. Hullermeier, "Fuzzy methods in machine learning and data mining: status and prospects," *Fuzzy Sets and Systems*, vol. 156, no. 3, pp. 387–406, 2005.

[2] M. Akram and W. A. Dudek, "Interval-valued fuzzy graphs," *Computers and Mathematics with Applications*, vol. 61, no. 2, pp. 289–299, 2011.

[3] M. Gulzar, D. Alghazzawi, M. H. Mateen, and N. Kausar, "A certain class of t-intuitionistic fuzzy subgroups," *IEEE Access*, vol. 8, pp. 163260–163268, 2020.

[4] M. Gulzar, M. H. Mateen, and D. Alghazzawi, "A novel applications of Complex intuitionistic fuzzy sets in group theory," *IEEE Access*, vol. 8, pp. 196075–196085, 2020.
[5] M. Gulzar, M. H. Mateen, Y. M. Chu, D. Alghazzawi, and G. Abbas, "Generalized direct product of complex intuitionistic fuzzy subrings," *International Journal of Computational Intelligence Systems*, vol. 14, no. 1, pp. 582–593, 2021.
[6] N. Kausar, M. Munir, M. Gulzar, and M. Addis, "Characterizing the ordered AG-Groupoids through the properties of their different classes of ideals," *Mathematics series*, vol. 4, no. 100, 2020.
[7] N. Kausar, M. Munir, M. Gulzar, G. M. Addis, and R. Anjum, "Anti fuzzy bi-ideals on ordered AG-groupoids," *Journal of the Indonesian Mathematical Society*, vol. 26, no. 3, pp. 299–318, 2020.
[8] K. A. Bibi and M. Devi, "Fuzzy Bi-magic labeling on cycle graph and star graph," *Global Journal of Pure and Applied Mathematics*, vol. 13, no. 6, 2017.
[9] S. Vimala and A. Nagarani, "Energy of fuzzy labeling graph [EFl(G)]- Part I," *International Journal of Scientific Engineering and Research*, vol. 7, no. 4, 2016.
[10] A. Nagoor Gani, M. Akram, and D. R. Subahashini, "Novel properties of fuzzy labeling graphs," *Journal of Mathematics*, vol. 2014, Article ID 375135, 6 pages, 2014.
[11] M. Akram, "Bipolar fuzzy graphs," *Information Sciences*, vol. 181, no. 24, pp. 5548–5564, 2011.
[12] M. Akram and W. A. Dudek, "Intuitionistic fuzzy hypergraphs with applications," *Information Sciences*, vol. 218, pp. 182–193, 2013.
[13] Z. S. Mufti, E. Fatima, R. Anjum, F. Tchier, Q. Xin, and M. M. Hossain, "Computing first and second fuzzy Zagreb indices of linear and multiacyclic hydrocarbons," *Journal of Function Spaces*, vol. 2022, Article ID 6281592, 8 pages, 2022.
[14] A. Rosenfeld, "Fuzzy graphs," in *Fuzzy Sets and Their Applications to Cognitive and Decision Processes*, pp. 77–95, Academic Press, 1975.
[15] P. Bhattacharya, "Some remarks on fuzzy graphs," *Pattern Recognition Letters*, vol. 6, no. 5, pp. 297–302, 1987.
[16] J. N. Mordeson and P. Chang-Shyh, "Operations on fuzzy graphs," *Information Sciences*, vol. 79, no. 3-4, pp. 159–170, 1994.
[17] S. R. Islam and M. Pal, "Second Zagreb index for fuzzy graphs and its application in mathematical chemistry Iran," *International Journal of Fuzzy Systems*, vol. 20, pp. 119–136, 2023.
[18] S. R. Islam and M. Pal, "First Zagreb index on a fuzzy graph and its application," *Journal of Intelligent and Fuzzy Systems*, vol. 40, no. 6, pp. 10575–10587, 2021.
[19] S. R. Islam and M. Pal, "Further development of F-index for fuzzy graph and its application in Indian railway crime," *Applied Mathematics and Computation*, vol. 69, no. 1, pp. 321–353, 2022.
[20] M. Rajeshwari, R. Murugesan, M. Kaviyarasu, and C. Subrahmanyam, "Bipolar fuzzy graph on certain topological indices," *Journal of Algebra*, vol. 13, 2022.
[21] C. Barientos, *Difference Vertex Labelings*, Ph.D. thesis, Universitat Po- litecnica De Catalunya, Barcelona, Spain, 2004.
[22] A. Rosa, "Cyclic steiner triple systems and labelings of triangular cacti," *Scientia*, vol. 1, 1998.
[23] S. Kalathian, S. Ramalingam, S. Raman, and N. Srinivasan, "Some topological indices in fuzzy graphs," *Journal of Intelligent and Fuzzy Systems*, vol. 39, no. 5, pp. 6033–6046, 2020.
[24] J. N. Mordeson and S. Mathew, "Non-deterministic flow in fuzzy networks and its application in identification of human trafficking chains," *New Mathematics and Natural Computation*, vol. 13, no. 3, pp. 231–243, 2017.
[25] V. Obraztsov and V. V. Krasnoproshin, "Management optimization problems on fuzzy graphs," *Investigaciones europeas de direccion y economia de la empresa*, vol. 1, no. 3, pp. 63–70, 1995.
[26] R. R. Yager, "Generalized orthopair fuzzy sets," *IEEE Transactions on Fuzzy Systems*, vol. 25, no. 5, pp. 1222–1230, 2017.
[27] D. Dubois, "The role of fuzzy sets in decision sciences: old techniques and new directions," *Fuzzy Sets and Systems*, vol. 184, no. 1, pp. 3–28, 2011.
[28] J. A. Gallian, "A dynamic survey of graph labeling," *Electronic Journal of Combinatorics*, vol. 1000, 2022.
[29] A. Muneera, T. N. Rao, R. V. N. S. Rao, and J. V. Rao, *Applications of Fuzzy Graph Theory Portrayed In Various Fields*, Research Square, Durham, NC, USA, 2021.
[30] N. Saeed, K. Long, T. U. Islam, Z. S. Mufti, and A. Abbas, "Topological study of zeolite socony mobil-5 via degree-based topological indices," *Journal of Chemistry*, vol. 2021, Article ID 5522800, 13 pages, 2021.