# MAGDM Model Using the Exponential Similarity Measure of Neutrosophic Confidence Cubic Sets in a Single-Valued Neutrosophic Multivalued Circumstance 

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#### Abstract

Probability estimation of small sample data is a key tool to ensure the probability that sample data fall within the confidence interval at a certain confidence level and probability distribution, which shows its advantages in practical engineering applications. Then, regarding a group decision-making (GDM) problem in the situation of indeterminacy and inconsistency, several experts/ decision makers will assign several true, false, and indeterminate fuzzy values to the evaluation values of each alternative over different attributes, and then form a single-valued neutrosophic multivalued set (SvNMVS) as their assessed information. To ensure some confidence level of the evaluation values in the circumstance of SvNMVSs and GDM reliability, this paper aims to propose a conversion technique from SvNMVS to a neutrosophic confidence cubic set (NCCS) and a GDM model using the exponential similarity measure of NCCSs in the circumstance of SvNMVSs. First, we give the definition of NCCS, which is transformed from SvNMVS in terms of average values and confidence intervals of true, false, and indeterminate fuzzy sequences subject to the conditions of the normal distribution and confidence levels. Second, we present the exponential similarity measure of NCCSs and the weighted exponential similarity measure of NCCSs and their characteristics. Third, a GDM model is developed by using the weighted exponential similarity measure of NCCSs in the circumstance of SvNMVSs. Fourth, the developed GDM model is applied to a choice case of landslide treatment schemes in the circumstance of SvNMVSs to reveal its usability and suitability in actual GDM problems. Compared with the existing GDM models, the developed GDM model indicates its superiorities in decision flexibility and credibility/reliability subject to $90 \%, 95 \%$, and $99 \%$ confidence levels.


## 1. Introduction

In indeterminate and inconsistent environments, the information expressions and decision-making approaches of neutrosophic sets, including the subsets such as simplified neutrosophic sets (SNSs) and single-valued and intervalvalued neutrosophic sets (SvNSs and IvNSs), show their merits in actual applications [1-4]. Therefore, they have been applied in many fields, such as social science, economics, and medicine [5-11]. Then, in group decision-making (GDM) issues with neutrosophic information, the multivalued neutrosophic information implies its importance and necessary in the expression of group evaluation information. For instance, multivalued neutrosophic sets (MVNSs)/ neutrosophic hesitant fuzzy sets (NHFSs) were used for the
expression of group evaluation information, and then various aggregation operators were applied to their GDM issues [12-16]. However, owing to hesitant characteristics, MVNS/ NHFS may lose some same fuzzy values in the expression of hesitant information, which shows its flaw.

On the other hand, probability MVNSs were used for the information expression of group evaluation values from a probability perspective, and then their GDM approaches were presented to solve multiattribute GDM problems with probability MVNS information [17-22]. Then, the probability GDM approaches need a lot of evaluated data to yield reasonable probability values; otherwise, it is difficult to ensure the credibility and reliability of the probability neutrosophic values in the GDM process. Therefore, it is difficult to use the probability GDM models in actual applications.

To avoid some defects of the expressions and GDM approaches of MVNSs/NHFSs and probability MVNSs, Ye et al. [23, 24] proposed single-valued neutrosophic multivalued sets (SvNMVSs) with the different and/or identical fuzzy values to ensure the complete expression of all group evaluation values in the GDM process, and then they introduced two transformation techniques from SvNMVSs to consistency SvNSs (C-SvNSs) [23] and single-valued neutrosophic enthalpy sets [24] to solve the operation issue between different fuzzy sequence lengths in SvNMVSs and developed the correlation coefficients of C-SvNSs [23] and the Einstein weighted aggregation operators of single-valued neutrosophic enthalpy values [24] for GDM issues in the scenario of SvNMVSs. However, the existing transformation techniques were based on the mean and standard deviation/ Shannon/probability entropy of true, false, and indeterminate fuzzy sequences in SvNMVSs [23, 24]. From a probability estimation perspective, these transformation techniques cannot reflect some confidence level and probability distribution of multiple fuzzy values, which show their insufficiencies.

Since the neutrosophic number (NN) $(u=a+\lambda I=$ $\left[a+\lambda I^{-}, a+\lambda I^{+}\right]$for an indeterminacy $I=\left[I^{-}, I^{+}\right]$and $a$, $\lambda \in \mathfrak{R})$ presented by Smarandache $[1,25,26]$ shows the flexible representation merit of indeterminate information subject to different indeterminate ranges of $I$. Recently, from a probability perspective, the notion of a confidence neutrosophic number (CNN) or confidence interval (CI) [27] was presented in terms of the $95 \%$ confidence level and the normal and lognormal distributions of multivalued datasets to ensure the $95 \%$ confidence level of multivalued datasets falling within the CNN/CI, and then CNN linear programming methods were introduced subject to the confidence level and normal and lognormal distributions to carry out production planning problems in indeterminate scenarios. However, CNNs/CIs are not used for GDM issues in the neutrosophic multivalued setting.

In addition, a neutrosophic cubic set (NCS) [28] is composed of the true, false, and indeterminate interval fuzzy values and the true, false, and indeterminate fuzzy values, which implies the representation merit of the mixed information. Therefore, NCSs have been applied in pattern recognition [28] and decision-making issues [29-33] in NCS circumstances. However, NCSs cannot be applied to GDM issues in the neutrosophic multivalued setting. Meanwhile, it is difficult to reflect some confidence level/reliability of finite group evaluation information in the GDM issues.

Since there are the aforementioned insufficiencies of the existing transformation techniques [23, 24], by the motivation of the CI/CNN notion with some confidence level [27], this paper aims to propose a new transformation technique from SvNMVS to the neutrosophic confidence cubic set (NCCS) that consists of CIs and average values of true, false, and indeterminate fuzzy sequences in SvNMVS and a GDM model based on the exponential similarity measure (ESM) of NCCSs to carry out GDM issues subject to some confidence levels and the normal distribution (the most common distribution in the real world) in a SvNMVS circumstance.

The remainder of this paper consists of the following parts. The second part introduces the definition of NCCS along with a conversion technique from SvNMVS to NCCS and some relationships of neutrosophic confidence cubic elements (NCCEs). The third part proposes an ESM method between NCCSs and the weighted ESM of NCCSs. The fourth part develops a GDM model using the weighted ESM of NCCSs in a SvNMVS circumstance. The fifth part applies the developed GDM model to a choice case of landslide treatment schemes (LTSs) in the scenario of SvNMVSs to reveal its usability and suitability in actual GDM problems. In the sixth part, compared to the existing related GDM models, the developed GDM model indicates its superiority in decision flexibility and credibility/reliability subject to $90 \%, 95 \%$, and $99 \%$ confidence levels. The final part remarks the conclusions and further research issues.

## 2. Neutrosophic Confidence Cubic Sets (NCCSs)

This part presents the definition of NCCS along with a conversion technique from SvNMVS to NCCS under the circumstance of SvNMVSs and some relationships of NCCEs.

To give the definition of NCCS, we first introduce the notion of SvNMVS [23, 24].

A SvNMVS NM in a nonempty set $Z=\left\{z_{1}, z_{2}, \ldots, z_{p}\right\}$ is defined as $N_{M}=\left\{z_{k}, \tau_{M}\left(z_{k}\right), \kappa_{M}\left(z_{k}\right), v_{M}\left(z_{k}\right) \mid z_{k} \in Z\right\}$, where $\tau_{M}\left(z_{k}\right), \kappa_{M}\left(z_{k}\right)$, and $v_{M}\left(z_{k}\right)$ contains multiple true, indeterminate, and false membership degrees of each element $z_{k}$ to the set $N_{M}$, denoted by the three fuzzy sequences $\tau_{M}\left(z_{k}\right)=\left(\tau_{M}^{1}\left(z_{k}\right), \tau_{M}^{2}\left(z_{k}\right), \ldots, \tau_{M}^{g_{k}}\left(z_{k}\right)\right), \kappa_{M}\left(y_{k}\right)=\left(\kappa_{M}^{1}\left(z_{k}\right)\right.$, $\left.\kappa_{M}^{2}\left(z_{k}\right), \ldots, \kappa_{M}^{g_{k}}\left(z_{k}\right)\right), \quad$ and $\quad v_{F}\left(z_{k}\right)=\left(v_{M}^{1}\left(z_{k}\right), v_{M}^{2}\left(z_{k}\right), \ldots\right.$, $\left.v_{M}^{g_{k}}\left(z_{k}\right)\right)$ with different and/or identical fuzzy values in $[0,1]$ subject to their fuzzy sequence lengths $g_{k}$ and $0 \leq \sup \tau_{M}\left(z_{k}\right)+\sup \kappa_{M}\left(z_{k}\right)+\sup v_{M}\left(z_{k}\right) \leq 3 \quad$ for $\quad z_{k} \in Z$ $(k=1,2, \ldots, p)$.

For simplicity, the $k$-th element $\left\langle z_{k}, \tau_{M}\left(z_{k}\right)\right.$, $\left.\kappa_{M}\left(z_{k}\right), v_{M}\left(z_{k}\right)\right\rangle$ in $N_{M}$ is simply represented as the singlevalued neutrosophic multivalued element (SvNMVE) $n_{M k}=$ $\left\langle\tau_{M k}, \kappa_{M k}, v_{M k}\right\rangle=\left\langle\left(\tau_{M k}^{1}, \tau_{M k}^{2}, \ldots, \tau_{M k}^{g_{k}}\right),\left(\kappa_{M k}^{1}, \kappa_{M k}^{2}, \ldots, \kappa_{M k}^{g_{k}}\right)\right.$, $\left.\left(v_{M k}^{1}, v_{M k}^{2}, \ldots, v_{M k}^{g_{k}}\right)\right\rangle$ in an increasing fuzzy sequence. Especially when $g_{k}=1(k=1,2, \ldots, p)$, the SvNMVS $N_{M}$ is reduced to SvNS.

In view of a probability estimation of small-scale sample data, a confidence level of $1-\delta$ for a level $\delta$ reflects that the $(1-\delta) \times 100 \%$ probability of sample data will fall within CI under the normal distribution condition of sample data, and then the $\delta \times 100 \%$ probability of sample data is outside CI. In light of CI subject to a confidence level of $(1-\delta) \times 100 \%$, we give the definition of NCCS based on a conversion technique from SvNMVS to NCCS. As we all know, the normal distribution is the most common distribution in the real world. Therefore, this paper only considers CIs under the normal distribution condition. In this case, we give the following definition of NCCS.

Definition 1. Let SvNMVS be $N_{M 1}=\left\{z_{k}, \tau_{M 1}\left(z_{k}\right)\right.$, $\left.\kappa_{M 1}\left(z_{k}\right), v_{M 1}\left(z_{k}\right) \mid z_{k} \in Z\right\}$ containing the true, indeterminate, and false fuzzy sequences $\tau_{M 1}\left(z_{k}\right)=\left(\tau_{M 1 k}^{1}, \tau_{M 1 k}^{2}, \ldots, \tau_{M 1 k}^{g_{k}}\right)$,
$\kappa_{M 1}\left(z_{k}\right)=\left(\kappa_{M 1 k}^{1}, \kappa_{M 1 k}^{2}, \ldots, \kappa_{M 1 k}^{g_{k}}\right)$, and $v_{M 1}\left(z_{k}\right)=\left(v_{M 1 k}^{1}\right.$, $\left.v_{M 1 k}^{2}, \ldots, v_{M 1 k}^{g_{k}}\right)(k=1,2, \ldots, p)$. Then, NCCS can be defined as the following form:

$$
N_{\delta 1}=\left\{\begin{array}{c}
\left(z_{1},\left\langle\left[\tau_{\delta 11}^{-}, \tau_{\delta 11}^{+}\right],\left[\kappa_{\delta 11}^{-}, \kappa_{\delta 11}^{+}\right],\left[v_{\delta 11}^{-}, v_{\delta 11}^{+}\right]\right\rangle,\left\langle a_{\tau 11}, a_{\kappa 11}, a_{v 11}\right\rangle\right),  \tag{1}\\
\left(z_{2},\left\langle\left[\tau_{\delta 12}^{-}, \tau_{\delta 12}^{+}\right],\left[\kappa_{\delta 12}^{-}, \kappa_{\delta 12}^{+}\right],\left[v_{\delta 12}^{-}, v_{\delta 12}^{+}\right]\right\rangle,\left\langle a_{\tau 12}, a_{\kappa 12}, a_{v 12}\right\rangle\right), \ldots, \\
\left(z_{p},\left\langle\left[\tau_{\delta 1 p}^{-}, \tau_{\delta 1 p}^{+}\right],\left[\kappa_{\delta 1 p}^{-}, \kappa_{\delta 1 p}^{+}\right],\left[v_{\delta 1 p}^{-}, v_{\delta 1 p}^{+}\right]\right\rangle,\left\langle a_{\tau 1 p}, a_{\kappa 1 p}, a_{v 1 p}\right\rangle\right) \mid \delta \in[0,1]
\end{array}\right\},
$$

whereas the $k$-th element $\quad\left(z_{k},\left\langle\left[\tau_{\delta 1 k}^{-}, \tau_{\delta 1 k}^{+}\right], \quad\left[\kappa_{\delta 1 k}^{-}\right.\right.\right.$, $\left.\left.\left.\kappa_{\delta 1 k}^{+}\right],\left[v_{\delta 12}^{-}, v_{\delta 1 k}^{+}\right]\right\rangle,\left\langle a_{\tau 1 k}, a_{\kappa 1 k}, a_{v 1 k}\right\rangle\right)$ in $N_{\delta 1}$ is called the $k$-th NCCE and simply denoted as $n_{\delta 1 k}=\left(\left\langle\left[\tau_{\delta 1 k}^{-}, \tau_{\delta 1 k}^{+}\right]\right.\right.$, $\left.\left.\left[\kappa_{\delta 1 k}^{-}, \kappa_{\delta 1 k}^{+}\right],\left[v_{\delta 12}^{-}, v_{\delta 1 k}^{+}\right]\right\rangle,\left\langle a_{\tau 1 k}, a_{\kappa 1 k}, a_{v 1 k}\right\rangle\right)$ for convenient expression, where $\left[\tau_{\delta 1 k}^{-}, \tau_{\delta 1 k}^{+}\right],\left[\kappa_{\delta 1 k}^{-}, \kappa_{\delta 1 k}^{+}\right]$, and $\left[v_{\delta 1 k}^{-}, v_{\delta 1 k}^{+}\right]$are the true, indeterminate, and false CIs of the corresponding fuzzy sequences, and $a_{\tau 1 k}, a_{\kappa 1 k}$, and $a_{v 1 k}$ are the true, indeterminate, and false average values of the corresponding fuzzy sequences, which are given by the following formulae:

$$
\begin{align*}
{\left[\tau_{\delta 1 k}^{-}, \tau_{\delta 1 k}^{+}\right] } & =\left[a_{\tau 1 k}-\frac{\rho_{\tau 1 k}}{\sqrt{g_{k}}} s_{\delta / 2}, a_{\tau 1 k}+\frac{\rho_{\tau 1 k}}{\sqrt{g_{k}}} s_{\delta / 2}\right],  \tag{2}\\
{\left[\kappa_{\delta 1 k}^{-}, \kappa_{\delta 1 k}^{+}\right] } & =\left[a_{\kappa 1 k}-\frac{\rho_{\kappa 1 k}}{\sqrt{g_{k}}} s_{\delta / 2}, a_{\kappa 1 k}+\frac{\rho_{\kappa 1 k}}{\sqrt{g_{k}}} s_{\delta / 2}\right],  \tag{3}\\
{\left[v_{\delta 1 k}^{-}, v_{\delta 1 k}^{+}\right] } & =\left[a_{v 1 k}-\frac{\rho_{v 1 k}}{\sqrt{g_{k}}} s_{\delta / 2}, a_{v 1 k}+\frac{\rho_{v 1 k}}{\sqrt{g_{k}}} s_{\delta / 2}\right],  \tag{4}\\
a_{\tau 1 k} & =\frac{1}{g_{k}} \sum_{j=1}^{g_{k}} \tau_{M 1 k}^{j}  \tag{5}\\
a_{\kappa 1 k} & =\frac{1}{g_{k}} \sum_{j=1}^{g_{k}} \kappa_{M 1 k}^{j}  \tag{6}\\
a_{v 1 k} & =\frac{1}{g_{k}} \sum_{j=1}^{g_{k}} v_{M 1 k}^{j} \tag{7}
\end{align*}
$$

$\rho_{\tau 1 k}=\sqrt{\frac{1}{g_{k}-1} \sum_{j=1}^{g_{k}}\left(\tau_{M 1 k}^{j}-a_{\tau 1 k}\right)^{2}}$ (True standard deviation),
$\rho_{\kappa 1 k}=\sqrt{\frac{1}{g_{k}-1} \sum_{j=1}^{g_{k}}\left(\kappa_{M 1 k}^{j}-a_{\kappa 1 k}\right)^{2}}$ (Indeterminate standard deviation),
$\rho_{v 1 k}=\sqrt{\frac{1}{g_{k}-1} \sum_{j=1}^{g_{k}}\left(v_{M 1 k}^{j}-a_{v 1 k}\right)^{2}}$ (False standard deviation).

Then, $s_{\delta / 2}$ in equations (2)-(4) is a specific value related to a confidence level of $(1-\delta) \times 100 \%$, which is constructed as twosided CIs for the confidence level of $(1-\delta) \times 100 \%$ in the normal distribution situation of fuzzy data. In actual applications, the specific values of $s_{\delta / 2}$ are usually specified as $1.645,1.960$, and 2.576 [27] subject to the confidence levels of $90 \%, 95 \%$, and $99 \%$ under the normal distribution condition of fuzzy data.

Example 1. Suppose that there is the SvNMVS $N_{M 1}=\left\{<z_{1}\right.$, ( $0.6,0.7,0.8,0.8$ ), ( $0.1,0.2,0.2,0.3$ ), ( $0.2,0.3,0.3,0.4)\rangle,\left\langle z_{2}\right.$, ( $0.5,0.7,0.8),(0.3,0.3,0.4),(0.2,0.2,0.2)>\}$ in the twoelement set $Z=\left\{z_{1}, z_{2}\right\}$. Using equations (1)-(10) at the confidence level of $95 \%$ with $s_{\delta / 2}=1.96$, the SvNMVS $N_{M 1}$ can be converted to the NCCS $N_{\delta 1}$ in the normal distribution situation by the following calculation process.

First, using equations (5)-(10), the average values and standard deviations of the fuzzy sequences in $N_{M 1}$ are yielded as follows:

$$
\begin{align*}
& a_{\tau 11}=0.725, a_{\kappa 11}=0.2, a_{v 11}=0.3, a_{\tau 12}=0.6667, a_{\kappa 12}=0.3333, \text { and } a_{v 12}=0.2 ; \\
& \rho_{\tau 11}=0.0957, \rho_{\kappa 11}=0.0816, \rho_{v 11}=0.0816, \rho_{\tau 12}=0.1528, \rho_{\kappa 12}=0.0577, \text { and } \rho_{v 12}=0 . \tag{11}
\end{align*}
$$

Then, using equations (2)-(4), the true, false and indeterminate CIs are obtained by the following calculations:

$$
\begin{align*}
{\left[\tau_{\delta 11}^{-}, \tau_{\delta 11}^{+}\right] } & =\left[0.725-\frac{0.0957}{\sqrt{4}} \times 1.96,0.725+\frac{0.0957}{\sqrt{4}} \times 1.96\right] \\
& =[0.6312,0.8188], \\
{\left[\kappa_{\delta 11}^{-}, \kappa_{\delta 11}^{+}\right] } & =\left[0.2-\frac{0.0816}{\sqrt{4}} \times 1.96,0.2+\frac{0.0816}{\sqrt{4}} \times 1.96\right] \\
& =[0.12,0.28], \\
{\left[v_{\delta 11}^{-}, v_{\delta 11}^{+}\right] } & =\left[0.3-\frac{0.0816}{\sqrt{4}} \times 1.96,0.3+\frac{0.0816}{\sqrt{4}} \times 1.96\right] \\
& =[0.22,0.38], \\
{\left[\tau_{\delta 12}^{-}, \tau_{\delta 12}^{+}\right] } & =\left[0.6667-\frac{0.1528}{\sqrt{3}} \times 1.96,0.6667+\frac{0.1528}{\sqrt{3}} \times 1.96\right]  \tag{12}\\
& =[0.4938,0.8395], \\
{\left[\kappa_{\delta 12}^{-}, \kappa_{\delta 12}^{+}\right] } & =\left[0.3333-\frac{0.0577}{\sqrt{3}} \times 1.96,0.3333+\frac{0.0577}{\sqrt{3}} \times 1.96\right] \\
& =[0.268,0.3987], \\
{\left[v_{\delta 12}^{-}, v_{\delta 12}^{+}\right] } & =\left[0.2-\frac{0}{\sqrt{3}} \times 1.96,0.2+\frac{0}{\sqrt{3}} \times 1.96\right] \\
& =[0.2,0.2],
\end{align*}
$$

Lastly, using equation (1), the NCCS $N_{\delta 1}$ for $\delta=0.05$ is obtained in the following:

$$
\begin{align*}
N_{\delta 1}= & \left\{\left(z_{1},\langle[0.6312,0.8188],[0.12,0.28],[0.22,0.38]\rangle,\langle 0.725,0.2,0.3\rangle\right),\left(z_{2},\right.\right. \\
& <[0.4938,0.8395],[0.268,0.3987],[0.2,0.2]\rangle,\langle 0.6667,0.3333,0.2\rangle) \mid \delta=0.05\} . \tag{13}
\end{align*}
$$

For two NCCEs $\quad n_{\delta 1 k}=\left(\left\langle\left[\tau_{\delta 1 k}^{-}, \tau_{\delta 1 k}^{+}\right],\left[\kappa_{\delta 1 k}^{-}, \kappa_{\delta 1 k}^{+}\right], \quad\left[\kappa_{\delta 2 k}^{-}, \kappa_{\delta 2 k}^{+}\right],\left[v_{\delta 2 k}^{-}, v_{\delta 2 k}^{+}\right]\right\rangle,\left\langle a_{\tau 2 k}, a_{\kappa 2 k}, a_{v 2 k}\right\rangle\right)$, their relationships $\left.\left.\left[v_{\delta 1 k}^{-}, v_{\delta 1 k}^{+}\right]\right\rangle,\left\langle a_{\tau 1 k}, a_{\kappa 1 k}, a_{v 1 k}\right\rangle\right)$ and $n_{\delta 2 k}=\left(\left\langle\left[\tau_{\delta 2 k}^{-}, \tau_{\delta 2 k}^{+}\right]\right.\right.$, are given in the following.

Definition 2. Set two NCCEs as $n_{\delta 1 k}=\left(\left\langle\left[\tau_{\delta 1 k}^{-}\right.\right.\right.$, $\left.\left.\left.\tau_{\delta 1 k}^{+}\right],\left[\kappa_{\delta 1 k}^{-}, \kappa_{\delta 1 k}^{+}\right],\left[v_{\delta 1 k}^{-}, v_{\delta 1 k}^{+}\right]\right\rangle,\left\langle a_{\tau 1 k}, a_{\kappa 1 k}, a_{v 1 k}\right\rangle\right)$ and $n_{\delta 2 k}=$ $\left(\left\langle\left[\tau_{\delta 2 k}^{-}, \tau_{\delta 2 k}^{+}\right],\left[\kappa_{\delta 2 k}^{-}, \quad \kappa_{\delta 2 k}^{+}\right],\left[v_{\delta 2 k}^{-}, v_{\delta 2 k}^{+}\right]\right\rangle,\left\langle a_{\tau 2 k}, a_{\kappa 2 k}, a_{v 2 k}\right\rangle\right)$. Then, their relationships are given in the following:
(1) $n_{\delta 1 k} \subseteq n_{\delta 2 k} \Leftrightarrow\left[\tau_{\delta 1 k}^{-}, \tau_{\delta 1 k}^{+}\right] \subseteq\left[\tau_{\delta 2 k}^{-}, \tau_{\delta 2 k}^{+}\right], \quad\left[\kappa_{\delta 1 k}^{-}, \kappa_{\delta 1 k}^{+}\right] \supseteq$ $\left[\kappa_{\delta 2 k}^{-}, \kappa_{\delta 2 k}^{+}\right], \quad\left[v_{\delta 1 k}^{-}, v_{\delta 1 k}^{+}\right] \supseteq\left[v_{\delta 2 k}^{-}, v_{\delta 2 k}^{+}\right], \quad a_{\tau 1 k} \leq a_{\tau 2 k}$, $a_{\kappa 1 k} \geq a_{\kappa 2 k}$, and $a_{v 1 k} \geq a_{v 2 k}$;
(2) $n_{\delta 1 k}=n_{\delta 2 k} \Leftrightarrow n_{\delta 1 k} \subseteq n_{\delta 2 k}$ and $n_{\delta 1 k} \supseteq n_{\delta 2 k}$, i.e., $\tau_{\delta 1 k}^{-}=\tau_{\delta 2 k}^{-}, \quad \tau_{\delta 1 k}^{+}=\tau_{\delta 2 k}^{+}, \quad \kappa_{\delta 1 k}^{-}=\kappa_{\delta 2 k}^{-}, \quad \kappa_{\delta 1 k}^{+}=\kappa_{\delta 2 k}^{+}$, $v_{\delta 1 k}^{-}=v_{\delta 2 k}^{-}, v_{\delta 1 k}^{+}=v_{\delta 2 k}^{+}, a_{\tau 1 k}=a_{\tau 2 k}, a_{\kappa 1 k}=a_{\kappa 2 k}$, and $a_{v 1 k}=a_{v 2 k}$;
(3) $n_{\delta 1 k} \cup n_{\delta 2 k}=$
(4) $n_{\delta 1 k} \cap n_{\delta 2 k}=$
(5) $n_{\delta 1 k}^{c}=\left(\left\langle\left[v_{\delta 1 k}^{-}, v_{\delta 1 k}^{+}\right],\left[1-\kappa_{\delta 1 k}^{+}, 1-\kappa_{\delta 1 k}^{-}\right],\left[\tau_{\delta 1 k}^{-}, \tau_{\delta 1 k}^{+}\right]\right\rangle\right.$, $\left.\left\langle a_{v 1 k}, 1-a_{k 1 k}, a_{\tau 1 k}\right\rangle\right)$ (Complement of $\left.n_{\delta 1 k}\right)$.

## 3. ESM between NCCSs

This part proposes ESM between NCCSs in a SvNMVS circumstance.

Definition 3. Set $N_{\delta 1}=\left\{n_{\delta 11}, n_{\delta 12}, \ldots, n_{\delta 1 p}\right\}$ and $N_{\delta 2}=\left\{n_{\delta 21}\right.$, $\left.n_{\delta 22}, \ldots, n_{\delta 2 p}\right\}$ as two NCCSs, where $n_{\delta 1 k}=\left(\left\langle\left[\tau_{\delta 1 k}^{-}, \tau_{\delta 1 k}^{+}\right]\right.\right.$, $\left.\left.\left[\kappa_{\delta 1 k}^{-}, \kappa_{\delta 1 k}^{+}\right],\left[v_{\delta 1 k}^{-}, v_{\delta 1 k}^{+}\right]\right\rangle,\left\langle a_{\tau 1 k}, a_{\kappa 1 k}, a_{v 1 k}\right\rangle\right) \quad$ and $\quad n_{\delta 2 k}=$ $\left(\left\langle\left[\tau_{\delta 2 k}^{-}, \quad \tau_{\delta 2 k}^{+}\right],\left[\kappa_{\delta 2 k}^{-}, \kappa_{\delta 2 k}^{+}\right],\left[v_{\delta 2 k}^{-}, v_{\delta 2 k}^{+}\right]\right\rangle,\left\langle a_{\tau 2 k}, a_{\kappa 2 k}, a_{v 2 k}\right\rangle\right)$ ( $k=1,2, \ldots, p$ ) are two groups of NCCEs. Thus, ESM of two NCCSs $N_{\delta 1}$ and $N_{\delta 2}$ is given as follows:

$$
E_{\delta}\left(N_{\delta 1}, N_{\delta 2}\right)=\frac{1}{p} \sum_{k=1}^{p}\left\{\exp \left(-\left(\begin{array}{c}
\left(\tau_{\delta 1 k}^{-}-\tau_{\delta 2 k}^{-}\right)^{2}+\left(\tau_{\delta 1 k}^{+}-\tau_{\delta 2 k}^{+}\right)^{2}+\left(\kappa_{\delta 1 k}^{-}-\kappa_{\delta 2 k}^{-}\right)^{2}  \tag{14}\\
+\left(\kappa_{\delta 1 k}^{+}-\kappa_{\delta 2 k}^{+}\right)^{2}+\left(v_{\delta 1 k}^{-}-v_{\delta 2 k}^{-}\right)^{2}+\left(v_{\delta 1 k}^{+}-v_{\delta 2 k}^{+}\right)^{2} \\
+\left(a_{\tau 1 k}-a_{\tau 2 k}\right)^{2}+\left(a_{\kappa 1 k}-a_{\kappa 2 k}\right)^{2}+\left(a_{v 1 k}-a_{v 2 k}\right)^{2}
\end{array}\right)\right)-\exp (-9)\right\} /\{1-\exp (-9)\}
$$

Proposition 1. The ESM $E_{\delta}\left(N_{\delta 1}, N_{\delta 2}\right)$ reflects the following characteristics:
(a) $E_{\delta}\left(N_{\delta 1}, N_{\delta 2}\right)=E_{\delta}\left(N_{\delta 2}, N_{\delta 1}\right)$;
(b) $0 \leq E_{\delta}\left(N_{\delta 1}, N_{\delta 2}\right) \leq 1$;
(c) $E_{\delta}\left(N_{\delta 1}, N_{\delta 2}\right)=1$ iff $N_{\delta 1}=N_{\delta 2}$;
(d) If $N_{\delta 1} \subseteq N_{\delta 2} \subseteq N_{\delta 3}$ for three NCCSs $N_{\delta 1}, N_{\delta 2}$, and $N_{\delta 3}$, then $E_{\delta}\left(N_{\delta 1}, N_{\delta 2}\right) \geq E_{\delta}\left(N_{\delta 1}, N_{\delta 3}\right)$ and $E_{\delta}\left(N_{\delta 2}, N_{\delta 3}\right) \geq$ $E_{\delta}\left(N_{\delta 1}, N_{\delta 3}\right)$ exist.

Proof
(a) This characteristic is clear.
(b) Since there are the following inequalities:

$$
\begin{align*}
& 0 \leq\left(\tau_{\delta 1 k}^{-}-\tau_{\delta 2 k}^{-}\right)^{2}+\left(\kappa_{\delta 1 k}^{-}-\kappa_{\delta 2 k}^{-}\right)^{2}+\left(v_{\delta 1 k}^{-}-v_{\delta 2 k}^{-}\right)^{2} \leq 3 \\
& 0 \leq\left(\tau_{\delta 1 k}^{+}-\tau_{\delta 2 k}^{+}\right)^{2}+\left(\kappa_{\delta 1 k}^{+}-\kappa_{\delta 2 k}^{+}\right)^{2}+\left(v_{\delta 1 k}^{+}-v_{\delta 2 k}^{+}\right)^{2} \leq 3  \tag{15}\\
& 0 \leq\left(a_{\tau 1 k}-a_{\tau 2 k}\right)^{2}+\left(a_{\kappa 1 k}-a_{\kappa 2 k}\right)^{2}+\left(a_{v 1 k}-a_{v 2 k}\right)^{2} \leq 3
\end{align*}
$$

Thus, $\quad \exp (0)=1 \leq \exp \left(-\left(a_{\kappa 1 k}-a_{\kappa 2 k}\right)^{2}+\right.$ $\left.\left.\left(a_{v 1 k}-a_{v 2 k}\right)^{2}\right)\right) \leq \exp (-9)$ can hold. Therefore, the
value of equation (14) also belongs to [0, 1], i.e., $0 \leq E_{\delta}\left(N_{\delta 1}, N_{\delta 2}\right) \leq 1$.
(c) If $N_{\delta 1}=N_{\delta 2}$, then $n_{\delta 1 k}=n_{\delta 2 k}(k=1,2, \ldots, p)$. Thus, there are $\tau_{\delta 1 k}^{-}=\tau_{\delta 2 k}^{-}, \quad \tau_{\delta 1 k}^{+}=\tau_{\delta 2 k}^{+}, \quad \kappa_{\delta 1 k}^{-}=\kappa_{\delta 2 k}^{-}$, $\kappa_{\delta 1 k}^{+}=\kappa_{\delta 2 k}^{+}, \quad v_{\delta 1 k}^{-}=v_{\delta 2 k}^{-}, \quad v_{\delta 1 k}^{+}=v_{\delta 2 k}^{+}, \quad a_{\tau 1 k}=a_{\tau 2 k}$, $a_{\kappa 1 k}=a_{\kappa 2 k}$, and $a_{v 1 k}=a_{v 2 k}(k=1,2, \ldots, p)$. In this case, the value of $\exp (0)$ in equation (14) is equal to 1 , and then $E_{\delta}\left(N_{\delta 1}, N_{\delta 2}\right)=1$ exists.
If $E_{\delta}\left(N_{\delta 1}, N_{\delta 2}\right)=1$, then the value of $\exp (0)$ in equation (14) is equal to 1 . Thus, $\tau_{\delta 1 k}^{-}=\tau_{\delta 2 k}^{-}$, $\tau_{\delta 1 k}^{+}=\tau_{\delta 2 k}^{+}, \quad \kappa_{\delta 1 k}^{-}=\kappa_{\delta 2 k}^{-}, \quad \kappa_{\delta 1 k}^{+}=\kappa_{\delta 2 k}^{+}, \quad v_{\delta 1 k}^{-}=v_{\delta 2 k}^{-}$, $v_{\delta 1 k}^{+}=v_{\delta 2 k}^{+}, a_{\tau 1 k}=a_{\tau 2 k}, a_{\kappa 1 k}=a_{\kappa 2 k}$, and $a_{v 1 k}=a_{v 2 k}$ exist. Therefore, there are $n_{\delta 1 k}=n_{\delta 2 k}(k=1,2, \ldots, p)$, and then $N_{\delta 1},=N_{\delta 2}$.
(d) If $N_{\delta 1} \subseteq N_{\delta 2} \subseteq N_{\delta 3}$, there are $n_{\delta 1 k} \subseteq n_{\delta 2 k} \subseteq n_{\delta 3 k}$, and then $\left[\tau_{\delta 1 k}^{-}, \tau_{\delta 1 k}^{+}\right] \subseteq\left[\tau_{\delta 2 k}^{-}, \tau_{\delta 2 k}^{+}\right] \subseteq\left[\tau_{\delta 3 k}^{-}, \tau_{\delta k k}^{+}\right],\left[\kappa_{\delta 1 k}^{-}, \kappa_{\delta 1 k}^{+}\right]$ $\supseteq\left[\kappa_{\delta 2 k}^{-}, \kappa_{\delta 2 k}^{+}\right] \supseteq\left[\kappa_{\delta 3 k}^{-}, \kappa_{\delta 3 k}^{+}\right], \quad\left[v_{\delta 1 k}^{-}, v_{\delta 1 k}^{+}\right] \supseteq\left[v_{\delta 2 k}^{-}, v_{\delta 2 k}^{+}\right] \supseteq$ $\left[v_{\delta 3 k}^{-}, v_{\delta 3 k}^{+}\right], a_{\tau 1 k} \leq a_{\tau 2 k} \leq a_{\tau 3 k}, a_{\kappa 1 k} \geq a_{\kappa 2 k} \geq a_{\kappa 3 k}$, and $a_{v 1 k} \geq a_{v 2 k} \geq a_{v 3 k}(k=1,2, \ldots, p)$. Thus, they have the following inequalities:

$$
\begin{align*}
& \left(\tau_{\delta 1 k}^{-}-\tau_{\delta 2 k}^{-}\right)^{2} \leq\left(\tau_{\delta 1 k}^{-}-\tau_{\delta 3 k}^{-}\right)^{2}, \\
& \left(\tau_{\delta 1 k}^{+}-\tau_{\delta 2 k}^{+}\right)^{2} \leq\left(\tau_{\delta 1 k}^{+}-\tau_{\delta 3 k}^{+}\right)^{2}, \\
& \left(\tau_{\delta 2 k}^{-}-\tau_{\delta j k}^{-}\right)^{2} \leq\left(\tau_{\delta 1 k}^{-}-\tau_{\delta j k}^{-}\right)^{2}, \\
& \left(\tau_{\delta 2 k}^{+}-\tau_{\delta 3 k}^{+}\right)^{2} \leq\left(\tau_{\delta 1 k}^{+}-\tau_{\delta 3 k}^{+}\right)^{2}, \\
& \left(\kappa_{\delta 1 k}^{-}-\kappa_{\delta 2 k}^{-}\right)^{2} \leq\left(\kappa_{\delta 1 k}^{-}-\kappa_{\delta j k}^{-}\right)^{2}, \\
& \left(\kappa_{\delta 1 k}^{+}-\kappa_{\delta 2 k}^{+}\right)^{2} \leq\left(\kappa_{\delta 1 k}^{+}-\kappa_{\delta j k}^{+}\right)^{2}, \\
& \left(\kappa_{\delta 2 k}^{-}-\kappa_{\delta j k}^{-}\right)^{2} \leq\left(\kappa_{\delta 1 k}^{-}-\kappa_{\delta j k}^{-}\right)^{2}, \\
& \left(\kappa_{\delta 2 k}^{+}-\kappa_{\delta j k}^{+}\right)^{2} \leq\left(\kappa_{\delta 1 k}^{+}-\kappa_{\delta j k}^{+}\right)^{2}, \\
& \left(v_{\delta 1 k}^{-}-v_{\delta 2 k}^{-}\right)^{2} \leq\left(v_{\delta 1 k}^{-}-v_{\delta j k}^{-}\right)^{2},  \tag{16}\\
& \left(v_{\delta 1 k}^{+}-v_{\delta 2 k}^{+}\right)^{2} \leq\left(v_{\delta 1 k}^{+}-v_{\delta j k}^{+}\right)^{2} \text {, } \\
& \left(v_{\delta 2 k}^{-}-v_{\delta j k}^{-}\right)^{2} \leq\left(v_{\delta 1 k}^{-}-v_{\delta j k}^{-}\right)^{2}, \\
& \left(v_{\delta 2 k}^{+}-v_{\delta j k}^{+}\right)^{2} \leq\left(v_{\delta 1 k}^{+}-v_{\delta j k}^{+}\right)^{2}, \\
& \left(a_{\tau 1 k}-a_{\tau 2 k}\right)^{2} \leq\left(a_{\tau 1 k}-a_{\tau 3 k}\right)^{2}, \\
& \left(a_{\tau 2 k}-a_{\tau 3 k}\right)^{2} \leq\left(a_{\tau 1 k}-a_{\tau 3 k}\right)^{2} \\
& \left(a_{\kappa 1 k}-a_{\kappa 2 k}\right)^{2} \leq\left(a_{\kappa 1 k}-a_{\kappa 3 k}\right)^{2}, \\
& \left(a_{\kappa 2 k}-a_{\kappa 3 k}\right)^{2} \leq\left(a_{\kappa 1 k}-a_{\kappa 3 k}\right)^{2}, \\
& \left(a_{v 1 k}-a_{v 2 k}\right)^{2} \leq\left(a_{v 1 k}-a_{v 3 k}\right)^{2}, \\
& \left(a_{v 2 k}-a_{v 3 k}\right)^{2} \leq\left(a_{v 1 k}-a_{v 3 k}\right)^{2} .
\end{align*}
$$

$$
E_{W \delta}\left(N_{\delta 1}, N_{\delta 2}\right)=\sum_{k=1}^{p} \varphi_{k}\left\{\exp \left(-\left(\begin{array}{c}
\left(\tau_{\delta 1 k}^{-}-\tau_{\delta 2 k}^{-}\right)^{2}+\left(\tau_{\delta 1 k}^{+}-\tau_{\delta 2 k}^{+}\right)^{2}+\mid\left(\kappa_{\delta 1 k}^{-}-\kappa_{\delta 2 k}^{-}\right)^{2}  \tag{17}\\
+\left(\kappa_{\delta 1 k}^{+}-\kappa_{\delta 2 k}^{+}\right)^{2}+\left(v_{\delta 1 k}^{-}-v_{\delta \delta k}^{-}\right)^{2}+\left(v_{\delta 1 k}^{+}-v_{\delta 2 k}^{+}\right)^{2} \\
+\left(a_{\tau 1 k}-a_{\tau 2 k}\right)^{2}+\left(a_{\kappa 1 k}-a_{\kappa 2 k}\right)^{2}+\left(a_{v 1 k}-a_{v 2 k}\right)^{2}
\end{array}\right)\right)-\exp (-9)\right\} /\{1-\exp (-9)\} .
$$

Proposition 2. The weighted ESM of $E_{W \delta}\left(N_{\delta 1}, N_{\delta 2}\right)$ also implies the following characteristics:
(a) $E_{W \delta}\left(N_{\delta 1}, N_{\delta 2}\right)=E_{W \delta}\left(N_{\delta 2}, N_{\delta 1}\right)$;
(b) $0 \leq E_{W \delta}\left(N_{\delta 1}, N_{\delta 2}\right) \leq 1$;
(c) $E_{W \delta}\left(N_{\delta 1}, N_{\delta 2}\right)=1$ iff $N_{\delta 1}=N_{\delta 2}$;
(d) If $N_{\delta 1} \subseteq N_{\delta 2} \subseteq N_{\delta 3}$ for three NCCSs $N_{\delta 1}, N_{\delta 2}$, and $N_{\delta 3}$, then $E_{W \delta}\left(N_{\delta 1}, N_{\delta 2}\right) \geq E_{W \delta}\left(N_{\delta 1}, N_{\delta 3}\right)$ and $E_{W \delta}\left(N_{\delta 2}\right.$, $\left.N_{\delta 3}\right) \geq E_{W \delta}\left(N_{\delta 1}, N_{\delta 3}\right)$ exist.

In view of the same proof process of Proposition 1, we can easily verify Proposition 2 (omitted).

## 4. GDM Model Using ESM of NCCSs

In multiple attribute GDM problems, there are usually a set of potential alternatives $P a=\left\{P a_{1}, P a_{2}, \ldots, P a_{q}\right\}$ and a set of several important assessment attributes $A c=\left\{A c_{1}, A c_{2}, \ldots\right.$, $\left.A c_{p}\right\}$. Taking into account the importance of various

Since $\exp (-z)$ for $z \geq 0$ is a decreasing function, $E_{\delta}\left(N_{\delta 1}\right.$, $\left.N_{\delta 2}\right) \geq E_{\delta}\left(N_{\delta 1}, N_{\delta 3}\right)$, and $E_{\delta}\left(N_{\delta 2}, N_{\delta 3}\right) \geq E_{\delta}\left(N_{\delta 1}, N_{\delta 3}\right)$ can hold.

Considering the weight value of $n_{\delta i k}(k=1,2, \ldots, p ; i=1$, $2)$, it is assigned by $\varphi_{k} \in[0,1]$ for $\sum_{k=1}^{p} \varphi_{k}=1$. Thus, the weighted ESM of NCCSs is presented by the measure equation:
attributes, the weight vector of $A c$ is assigned by $\varphi=\left(\varphi_{1}, \varphi_{2}\right.$, $\ldots, \varphi_{p}$ ). In the assessment process, a team of experts/decision makers can be invited to provide their evaluation values of each alternative with respect to the attributes by true, false, and indeterminate fuzzy values and to form SvNMVS.

In the GDM problem, the GDM model can be established and described by the following decision procedures.

Step 1: A team of experts/decision makers gives their evaluation values of each alternative $P a_{i}$ with respect to the attributes $A c_{k}$, which are expressed by the true, indeterminate, and false fuzzy sequences $\tau_{M i k}=\left(\tau_{M i k}^{1}, \tau_{M i k}^{2}, \ldots, \tau_{M i k}^{g_{k}}\right), \quad \kappa_{M i k}=\left(\kappa_{M i k}^{1}, \kappa_{M i k}^{2}, \ldots\right.$, $\left.\kappa_{M i k}^{g_{k}}\right)$, and $v_{M i k}=\left(v_{M i k}^{1}, v_{M i k}^{2}, \ldots, v_{M i k}^{g_{k}}\right)(k=1,2, \ldots, p$; $i=1,2, \ldots, q$ ) and constructed as the SvNMVS $N_{M i}=$ $\left\{\begin{array}{llll}n_{M i 1} & n_{M i 2} & \left.\ldots, n_{M i p}\right\} \text { containing SvNMVEs }\end{array}\right.$ $n_{M i k}=\left\langle\tau_{M i k}, \kappa_{M i k}, v_{M i k}\right\rangle=\left\langle\left(\tau_{M i k}^{1}, \tau_{M i k}^{2}, \ldots, \tau_{M i k}^{g_{k}}\right)\right.$,

Table 1: Decision results subject to the $90 \%, 95 \%$, and $99 \%$ confidence levels.

| $\delta$ | $s_{\delta / 2}$ | $E_{W \delta}\left(N_{\delta i}, N^{*}\right)$ | Sorting order | Optimal LTS |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 0.1 | 1.645 | $0.6237,0.6526,0.6268,0.6033,0.5286$, and 0.5685 | $P a_{2}>P a_{3}>P a_{1}>P a_{4}>P a_{6}>P a_{5}$ | $P a_{2}$ |
| 0.05 | 1.96 | $0.6209,0.6462,0.6222,0.5993,0.5234$, and 0.5603 | $P a_{2}>P a_{3}>P a_{1}>P a_{4}>P a_{6}>P a_{5}$ | $P a_{2}$ |
| 0.01 | 2.576 | $0.6142,0.6310,0.6112,0.5896,0.5105$, and 0.5408 | $P a_{2}>P a_{1}>P a_{3}>P a_{4}>P a_{6}>P a_{5}$ | $P a_{2}$ |

$\left.\left(\kappa_{M i k}^{1}, \kappa_{M i k}^{2}, \ldots, \kappa_{M i k}^{g_{k}}\right),\left(v_{M i k}^{1}, v_{M i k}^{2}, \ldots, v_{M i k}^{g_{k}}\right)\right\rangle$, and then establishes the decision matrix $M_{N}=\left(n_{M i k}\right)_{q \times p}$.
Step 2: Using equations (2)-(10) corresponding to some confidence levels of $(1-\delta) 100 \%$ with the corresponding values of $s_{\delta / 2}$, the SvNMVSs $N_{M i}$ can be converted to the NCCSs $N_{\delta i}=\left\{n_{\delta i 1}, n_{\delta i 2}, \ldots, n_{\delta i p}\right\}$ containing the NCCEs $n_{\delta i k}=\left(\left\langle\left[\tau_{\delta i k}^{-}, \tau_{\delta i k}^{+}\right]\right.\right.$, $\left.\left.\left[\kappa_{\delta i k}^{-}, \kappa_{\delta i k}^{+}\right],\left[v_{\delta i k}^{-}, v_{\delta i k}^{+}\right]\right\rangle,\left\langle a_{\tau i k}, a_{\kappa i k}, a_{v i k}\right\rangle\right)(i=1,2, \ldots, q ;$ $k=1,2, \ldots, p$ ) for some levels of $\delta$. Thus, their decision matrix is denoted as $M_{\delta}=\left(n_{\delta i k}\right)_{q \times p}$.

Step 3: Since the maximum interval-valued neutrosophic number and the maximum single-valued neutrosophic number are $<[1,1],[0,0],[0,0]>,<1$, $0,0>$, respectively, we can consider the most ideal solution (the maximum NCCS) as $N^{*}=\left\{\left(z_{1},<[1,1],[0\right.\right.$, $0],[0,0]\rangle,\langle 1,0,0\rangle),\left(z_{2},<[1,1],[0,0],[0,0]\right\rangle,\langle 1,0$, $\left.0>), \ldots,\left(z_{p},<[1,1],[0,0],[0,0]>,<1,0,0>\right)\right\}$, then the weighted ESM values of $E_{W \delta}\left(N_{\delta i}, N^{*}\right)$ are given by the following equation:

$$
\begin{equation*}
E_{W \delta}\left(N_{\delta i}, N^{*}\right)=\sum_{k=1}^{p} \varphi_{k}\left\{\exp \left(-\binom{\left(\tau_{\delta i k}^{-}-1\right)^{2}+\left(\tau_{\delta i k}^{+}-1\right)^{2}+\left(\kappa_{\delta i k}^{-}\right)^{2}+\left(\kappa_{\delta i k}^{+}\right)^{2}}{+\left(v_{\delta i k}^{-}\right)^{2}+\left(v_{\delta i k}^{+}\right)^{2}+\left(a_{\tau i k}-1\right)^{2}+a_{\kappa i k}^{2}+a_{v i k}^{2}}\right)-\exp (-9)\right\} /\{1-\exp (-9)\} . \tag{18}
\end{equation*}
$$

Step 4: The sorting order of the alternatives and the best choice are given in terms of the weighted ESM values. Step 5: End.

## 5. Actual GDM Example

This section provides an actual GDM example, which is a choice case of LTSs in the circumstance of SvNMVSs, to reflect the feasibility and rationality of the proposed GDM model subject to the $90 \%, 95 \%$, and $99 \%$ confidence levels.

A construction company wants to select the optimal LTS from six potential LTSs for Shaoxing City in China, which are denoted as a set of alternatives $P a=\left\{P a_{1}, P a_{2}, P a_{3}, P a_{4}\right.$, $\left.P a_{5}, P a_{6}\right\}$. Then, the six potential LTSs are detailed in the following:
$P a_{1}$ : Mortar rubble masonry pavements, retaining walls, and surface water treatment;
$P a_{2}$ : Surface-drainage works, grid beams, and monitoring measures;
$P a_{3}$ : Cut-off drains treatment, anchor antislide pile, and monitoring measures;
$P a_{4}$ : Cantilever piles, anchor antislide piles, and slope protection;
$P a_{5}$ : Retaining walls, antislide piles, and cut-off drain treatment;
$P a_{6}$ : Antislide piles, reduce-loading works, and surfacedrainage works.
Regarding the assessment of the six alternatives, they must meet four important attributes: $A c_{1}$ (construction cost), $A c_{2}$ (technique condition), $A c_{3}$ (treatment risk), and $A c_{4}$ (environment situation). Then, the weight vector $\varphi=(0.3,0.22,0.25,0.23)$ is assigned to a set of the four attributes $A c=\left\{A c_{1}, A c_{2}, A c_{3}, A c_{4}\right\}$.

Based on the choice case of LTSs, the proposed GDM model can be used for the GDM problem and addressed by the following decision procedures.

Step 1: The three experts/decision makers invited by the technical department give their evaluation values of each alternative $P a_{j}(j=1,2, \ldots, 6)$ with respect to the attributes $A c_{k}(k=1,2,3,4)$, which are constructed as the SvNMVS decision matrix:

$$
\begin{aligned}
{\left[\begin{array}{l}
\langle(0.6,0.6,0.6),(0.1,0.2,0.2),(0.1,0.15,0.15)\rangle \\
\langle(0.7,0.7,0.75),(0.1,0.2,0.25),(0.1,0.1,0.15)\rangle \\
\langle(0.6,0.6,0.65),(0.1,0.15,0.2),(0.15,0.15,0.2)\rangle \\
M_{N}=
\end{array}\right.} & \begin{array}{l}
\langle(0.6,0.7,0.7),(0.1,0.2,0.25),(0.1,0.25,0.35)\rangle \\
\langle(0.6,0.7,0.75),(0.15,0.2,0.3),(0.15,0.2,0.3)\rangle \\
\langle(0.65,0.7,0.7),(0.1,0.2,0.25),(0.1,0.15,0.3)\rangle \\
\\
\langle(0.7,0.8,0.85),(0.15,0.2,0.3),(0.1,0.15,0.2)\rangle \\
\\
\langle(0.7,0.75,0.8),(0.2,0.2,0.25),(0.1,0.25,0.25)\rangle \\
\\
\langle(0.75,0.8,0.8),(0.1,0.2,0.25),(0.1,0.2,0.25)\rangle \\
\\
\langle(0.6,0.7,0.7),(0.15,0.2,0.25),(0.1,0.15,0.15)\rangle \\
\\
\langle(0.6,0.7,0.7),(0.1,0.2,0.25),(0.2,0.25,0.3)\rangle \\
\end{array}\langle(0.65,0.7,0.7),(0.2,0.3,0.35),(0.15,0.2,0.2)\rangle
\end{aligned}
$$

Step 2：Using equations（2）－（10）corresponding to the confidence levels of $90 \%, 95 \%$ ，and $99 \%$ with the specified values $s_{\delta / 2}=1.645,1.96$ ，and 2.576 for $\delta=0.1$ ，
$\langle(0.6,0.65,0.65),(0.2,0.2,0.25),(0.1,0.1,0.15)\rangle$
$\langle(0.75,0.8,0.8),(0.1,0.2,0.25),(0.15,0.2,0.25)\rangle$
$\langle(0.7,0.7,0.8),(0.1,0.2,0.25),(0.1,0.15,0.2)\rangle$
$\langle(0.75,0.75,0.8),(0.2,0.2,0.25),(0.1,0.2,0.25)\rangle^{\prime}$
$\langle(0.7,0.7,0.75),(0.2,0.3,0.3),(0.15,0.2,0.25)\rangle$
$\langle(0.6,0.7,0.75),(0.1,0.2,0.25),(0.15,0.2,0.25)\rangle$
$\langle(0.8,0.8,0.8),(0.15,0.15,0.2),(0.15,0.2,0.2)\rangle$
$\langle(0.75,0.8,0.8),(0.1,0.2,0.3),(0.1,0.25,0.35)\rangle$
$\langle(0.65,0.7,0.8),(0.15,0.2,0.2),(0.15,0.2,0.25)\rangle$
$\langle(0.65,0.7,0.75),(0.15,0.2,0.2),(0.15,0.2,0.25)\rangle$
$\langle(0.5,0.65,0.7),(0.1,0.2,0.35),(0.2,0.2,0.25)\rangle$
$\langle(0.6,0.8,0.8),(0.1,0.2,0.4),(0.15,0.15,0.2)\rangle$
0.05 ，and 0.01 ，the SvNMVS decision matrix $M_{N}$ can be converted to the NCCS decision matrix $M_{\delta}$ ：
（〈［0．6059，0．6607］，［0．1893，0．2441］，［0．0892，0．1441］$\rangle,\langle 0.6333,0.2167,0.1167\rangle)$ $(\langle[0.7559,0.8107],[0.1108,0.2559],[0.1525,0.2475]\rangle,\langle 0.7833,0.1833,0.2000\rangle)$ $(\langle[0.6785,0.7882],[0.1108,0.2559],[0.1025,0.1975]\rangle,\langle 0.7333,0.1833,0.1500\rangle)$ $(\langle[0.7392,0.7941],[0.1893,0.2441],[0.1118,0.2215]\rangle,\langle 0.7667,0.2167,0.1667\rangle)$ $(\langle[0.6893,0.7441],[0.2118,0.3215],[0.1525,0.2475]\rangle,\langle 0.7167,0.2667,0.2000\rangle)$ （ $\langle[0.6108,0.7559],[0.1108,0.2559],[0.1525,0.2475]\rangle,\langle 0.6833,0.1833,0.2000\rangle)$ $(\langle[0.8000,0.8000],[0.1392,0.1941],[0.1559,0.2108]\rangle,\langle 0.8000,0.1667,0.1833\rangle)]$ （ $\langle[0.7559,0.8107],[0.1050,0.2950],[0.1138,0.3528]\rangle,\langle 0.7833,0.2000,0.2333\rangle)$ （ $\langle[0.6441,0.7892],[0.1559,0.2108],[0.1763,0.2475]\rangle,\langle 0.7167,0.1833,0.2000\rangle)$ $(\langle[0.6525,0.7475],[0.1559,0.2108],[0.1525,0.2475]\rangle,\langle 0.7000,0.1833,0.2000\rangle)$ $(\langle[0.5178,0.7155],[0.0972,0.3362],[0.1893,0.2441]\rangle,\langle 0.6167,0.2167,0.2167\rangle)$ $(\langle[0.6237,0.8430],[0.0883,0.3784],[0.1392,0.1941]\rangle,\langle 0.7333,0.2333,0.1667\rangle)]$ $(\langle[0.6007,0.6660],[0.1840,0.2493],[0.0840,0.1493]\rangle,\langle 0.6333,0.2167,0.1167\rangle)$ $(\langle[0.7507,0.8160],[0.0969,0.2698],[0.1434,0.2566]\rangle,\langle 0.7833,0.1833,0.2000\rangle)$ （〈［0．6680，0．7987］，［0．0969，0．2698］，［0．0934，0．2066］〉，〈0．7333，0．1833，0．1500〉） $(\langle[0.7340,0.7993],[0.1840,0.2493],[0.1013,0.2320]\rangle,\langle 0.7667,0.2167,0.1667\rangle)$ $(\langle[0.6840,0.7493],[0.2013,0.3320],[0.1434,0.2566]\rangle,\langle 0.7167,0.2667,0.2000\rangle)$ （ $\langle[0.5969,0.7698],[0.0969,0.2698],[0.1434,0.2566]\rangle,\langle 0.6833,0.1833,0.2000\rangle)$ $(\langle[0.8000,0.8000],[0.1340,0.1993],[0.1507,0.2160]\rangle,\langle 0.8000,0.1667,0.1833\rangle)$ （〈［0．7507，0．8160］，［0．0868，0．3132］，［0．0909，0．3757］〉，〈0．7833，0．2000，0．2333＞） $(\langle[0.6302,0.8031],[0.1507,0.2160],[0.1717,0.2566]\rangle,\langle 0.7167,0.1833,0.2000\rangle)$ （＜［0．6434，0．7566］，［0．1507，0．2160］，［0．1434，0．2566］$\rangle,\langle 0.7000,0.1833,0.2000\rangle)$ （ $\langle[0.4989,0.7344],[0.0743,0.3591],[0.1840,0.2493]\rangle,\langle 0.6167,0.2167,0.2167\rangle)$ $(\langle[0.6027,0.8640],[0.0605,0.4062],[0.1340,0.1993]\rangle,\langle 0.7333,0.2333,0.1667\rangle)]$ $(\langle[0.5904,0.6763],[0.1737,0.2596],[0.0737,0.1596]\rangle,\langle 0.6333,0.2167,0.1167\rangle)$ $(\langle[0.7404,0.8263],[0.0697,0.2969],[0.1256,0.2744]\rangle,\langle 0.7833,0.1833,0.2000\rangle)$ $(\langle[0.6475,0.8192],[0.0697,0.2969],[0.0756,0.2244]\rangle,\langle 0.7333,0.1833,0.1500\rangle)$ $(\langle[0.7237,0.8096],[0.1737,0.2596],[0.0808,0.2525]\rangle,\langle 0.7667,0.2167,0.1667\rangle)$ $(\langle[0.6737,0.7596],[0.1808,0.3525],[0.1256,0.2744]\rangle,\langle 0.7167,0.2667,0.2000\rangle)$ $(\langle[0.5697,0.7969],[0.0697,0.2969],[0.1256,0.2744]\rangle,\langle 0.6833,0.1833,0.2000\rangle)$ $(\langle[0.8000,0.8000],[0.1237,0.2096],[0.1404,0.2263]\rangle,\langle 0.8000,0.1667,0.1833\rangle)]$ $(\langle[0.7404,0.8263],[0.0513,0.3487],[0.0462,0.4205]\rangle,\langle 0.7833,0.2000,0.2333\rangle)$ $(\langle[0.6031,0.8303],[0.1404,0.2263],[0.1628,0.2744]\rangle,\langle 0.7167,0.1833,0.2000\rangle)$ （＜［0．6256，0．7744］，［0．1404，0．2263］，［0．1256，0．2744］〉，〈0．7000，0．1833，0．2000＞） （＜［0．4619，0．7715］，［0．0295，0．4038］，［0．1737，0．2596］〉，〈0．6167，0．2167，0．2167＞） $(\langle[0.5616,0.9051],[0.0062,0.4605],[0.1237,0.2096]\rangle,\langle 0.7333,0.2333,0.1667\rangle)]$

Table 2: Standard deviations of all the measure values and the sorting results of the six alternatives.

| Measure method | Measure value | Standard deviation of <br> measure values | Sorting result |
| :--- | :--- | :---: | :---: | :---: |
| $E_{W \delta}\left(N_{\delta i}, N^{*}\right)$ for $\delta=0.1$ | $0.6237,0.6526,0.6268,0.6033,0.5286$, and 0.5685 | 0.0450 | $P a_{2}>P a_{3}>P a_{1}>P a_{4}>P a_{6}>P a_{5}$ |
| $E_{W \delta}\left(N_{\delta i}, N^{*}\right)$ for $\delta=0.05$ | $0.6209,0.6462,0.6222,0.5993,0.5234$, and 0.5603 | 0.0456 | $P a_{2}>P a_{3}>P a_{1}>P a_{4}>P a_{6}>P a_{5}$ |
| $E_{W \delta}\left(N_{\delta i}, N^{*}\right)$ for $\delta=0.01$ | $0.6142,0.6310,0.612,0.5896,0.5105$, and 0.5408 | 0.0472 | $P a_{2}>P a_{1}>P a_{3}>P a_{4}>P a_{6}>P a_{5}$ |
| $R_{w}\left(N_{c 1}, N^{*}\right)[23]$ | $0.9818,0.9842,0.9830,0.9814,0.9756$, and 0.9793 | 0.0031 | $P a_{2}>P a_{3}>P a_{1}>P a_{4}>P a_{6}>P a_{5}$ |

Step 3: Using equation (18) for the confidence levels of $90 \%, 95 \%$, and $99 \%$, the weighted ESM values of $E_{W \delta}$ $\left(N_{\delta i}, N^{*}\right)$ are given in Table 1.
Step 4: The sorting orders of the alternatives and the optimal LTS are given in terms of the weighted ESM values, which are also shown in Table 1.

In view of the decision results in Table 1, different confidence levels can make the sorting orders changeable. Therefore, it is clear that the sorting results reveal some sensitivity and flexibility to different confidence levels. Then, the optimal LTS is $\mathrm{Pa}_{2}$.

## 6. Comparative Investigation

To conveniently compare the proposed GDM model with the related GDM model [23] in the circumstance of SvNMVSs, we first introduced the notion of C-SvNS $N_{c i}=\left\{n_{c i 1}, n_{c i 2}, \ldots, n_{c i p}\right\}$ including the $p$ consistency single-valued neutrosophic elements $n_{c i k}=\left(\left\langle a_{\tau i k}, a_{k i k}, a_{v i k}\right\rangle,\left\langle c_{\tau i k}, c_{\text {kik }}, c_{v i k}\right\rangle\right)$, which are transformed from SvNMVSs by the average values of equations (5)-(7) and the following consistency degrees of $c_{\tau i k}, c_{\kappa i k}$, and $c_{v i k}$ [23]:

$$
\begin{align*}
& c_{\tau i k}=1-\rho_{\tau i k}=1-\sqrt{\frac{1}{g_{k}-1} \sum_{j=1}^{g_{k}}\left(\tau_{M i k}^{j}-a_{\tau i k}\right)^{2}} \text { (True consistency degree), }  \tag{21}\\
& c_{\kappa i k}=1-\rho_{\kappa i k}=1-\sqrt{\frac{1}{g_{k}-1} \sum_{j=1}^{g_{k}}\left(\kappa_{M i k}^{j}-a_{\kappa i k}\right)^{2}} \text { (Indeterminate consistency degree), }  \tag{22}\\
& c_{v i k}=1-\rho_{v i k}=\sqrt{\frac{1}{g_{k}-1} \sum_{j=1}^{g_{k}}\left(v_{M i k}^{j}-a_{v i k}\right)^{2}} \text { (False consistency degree). } \tag{23}
\end{align*}
$$

Since the maximum single-valued neutrosophic number and the most ideal consistency single-valued neutrosophic number (the complete consistency of group arguments) are $<1,0,0\rangle$ and $\langle 1,1,1\rangle$, respectively, we can consider the most ideal solution/C-SvNS as $N^{*}=\left\{\left(z_{1},\langle 1,0,0\rangle,\langle 1,1\right.\right.$, $\left.\left.\left.\left.1>),\left(z_{2},<1,0,0\right\rangle,<1,1,1\right\rangle\right), \ldots,\left(z_{p},<1,0,0\right\rangle,\langle 1,1,1\rangle\right)\right\}$, then the weighted correlation coefficient of C-SvNSs is introduced in the following [23]:

$$
\begin{equation*}
R_{w}\left(N_{c i}, N^{*}\right)=\frac{\sum_{k=1}^{p} \varphi_{k} a_{\tau i k}+\sum_{k=1}^{p} \varphi_{k}\left(c_{\tau i k}+c_{k i k}+c_{v i k}\right)}{2 \sqrt{\sum_{k=1}^{p} \varphi_{k}\left(a_{\tau i k}^{2}+a_{\tau i k}^{2}+a_{\tau i k}^{2}+c_{\tau i k}^{2}+c_{\tau i k}^{2}+c_{\tau i k}^{2}\right)}} \tag{24}
\end{equation*}
$$

In view of the existing GDM model using the correlation coefficient of $\mathrm{C}-\mathrm{SvNSs}$ in the scenario of SvNMVSs [23], we can utilize it in the above GDM example.

First, using equations (5)-(7) and (21)-(23), SvNMVSs are transformed into C-SvNSs, which are constructed as the following C-SvNS matrix:

$$
\begin{gathered}
{\left[\begin{array}{l}
\langle(0.6000,1.0000),(0.1667,0.9423),(0.1333,0.9711)\rangle \\
\langle(0.7167,0.9711),(0.1833,0.9236),(0.1167,0.9711)\rangle \\
\langle(0.6167,0.9711),(0.1500,0.9500),(0.1667,0.9711)\rangle \\
\langle(0.6667,0.9423),(0.1833,0.9236),(0.2000,0.9134)\rangle \\
\langle(0.6833,0.9236),(0.2167,0.9236),(0.2167,0.9236)\rangle \\
\langle(0.6833,0.9711),(0.1833,0.9236),(0.1833,0.8959)\rangle \\
\langle(0.7833,0.9236),(0.2167,0.9236),(0.1500,0.9500)\rangle \\
\langle(0.7500,0.9500),(0.2167,0.9711),(0.2000,0.9134)\rangle \\
\langle(0.7833,0.9711),(0.1833,0.9236),(0.1833,0.9236)\rangle \\
\langle(0.6667,0.9423),(0.2000,0.9500),(0.1333,0.9711)\rangle \\
\langle(0.6667,0.9423),(0.1833,0.9236),(0.2500,0.9500)\rangle \\
\langle(0.6833,0.9711),(0.2833,0.9236),(0.1833,0.9711)\rangle
\end{array} .\right.}
\end{gathered}
$$

Then, using equation (24), we give the weighted correlation coefficient values

$$
\begin{align*}
& R_{w}\left(N_{c 1}, N^{*}\right)=0.9818, \\
& R_{w}\left(N_{c 2}, N^{*}\right)=0.9842, \\
& R_{w}\left(N_{c 3}, N^{*}\right)=0.9830,  \tag{26}\\
& R_{w}\left(N_{c 4}, N^{*}\right)=0.9814, \\
& R_{w}\left(N_{c 5}, N^{*}\right)=0.9756, \\
& R_{w}\left(N_{c 6}, N^{*}\right)=0.9793 .
\end{align*}
$$

Finally, the sorting order of the six alternatives is $P a_{2}>P a_{3}>P a_{1}>P a_{4}>P a_{6}>P a_{5}$ and the optimal LTS is $P a_{2}$.

For the convenient comparison of the decision results in light of the situations of C-SvNSs and NCCSs under the SvNMVS circumstance, we give the standard deviations of the measure values corresponding to the proposed GDM model and the existing GDM model [23] to indicate the difference degree of the measure values, and then the standard deviations and the sorting results of the six alternatives are shown in Table 2.

In view of the sorting results of the six alternatives in Table 2, the proposed GDM model using the weighted ESM for $\delta=0.1,0.05$ and the existing GDM model using the weighted correlation coefficient [23] reflect their same sorting results, but the sorting result of the proposed GDM model using the weighted ESM for $\delta=0.01$ is different from that of the existing GDM model [23]. Since the proposed GDM model contains different sorting orders depending on different confidence levels of the true, indeterminate, and false fuzzy sequences in the SvNMVS setting, it reveals the decision flexibility and credibility/reliability in the situation of NCCSs from a probabilistic estimation perspective; while

```
\(\langle(0.6333,0.9711),(0.2167,0.9711),(0.1167,0.9711)\rangle\)
\(\langle(0.7833,0.9711),(0.1833,0.9236),(0.2000,0.9500)\rangle\)
\(\langle(0.7333,0.9423),(0.1833,0.9236),(0.1500,0.9500)\rangle\)
\(\langle(0.7667,0.9711),(0.2167,0.9711),(0.1667,0.9423)\rangle{ }^{\prime}\)
\(\langle(0.7167,0.9711),(0.2667,0.9423),(0.2000,0.9500)\rangle\)
\(\langle(0.6833,0.9236),(0.1833,0.9236),(0.2000,0.9500)\rangle\)
\(\langle(0.8000,1.0000),(0.1667,0.9711),(0.1833,0.9711)\rangle]\)
\(\langle(0.7167,0.9236),(0.1833,0.9711),(0.2000,0.9500)\rangle\)
\(\langle(0.7167,0.9236),(0.1833,0.9711),(0.2000,0.9500)\rangle\)
\(\langle(0.7000,0.9500),(0.1833,0.9711),(0.2000,0.9500)\rangle\)
\(\langle(0.6167,0.8959),(0.2167,0.8742),(0.2167,0.9711)\rangle\)
\(\langle(0.7333,0.8845),(0.2333,0.8472),(0.1667,0.9711)\rangle\) ]
```

the existing GDM model [23] cannot reflect the decision flexibility and credibility in the situation of C-SvNSs although it contains consistency levels of the true, indeterminate, and false fuzzy sequences in the SvNMVS setting. Therefore, the proposed GDM model reveals obvious superiority over the existing GDM model [23] in GDM flexibility and credibility.

In view of the standard deviations of the different measure methods in Table 2, it is obvious that the proposed ESM method for NCCSs reveals larger standard deviations of the measure values than the weighted correlation coefficient of C-SvNSs [23]. In light of the concept of the standard deviation, the proposed weighted ESM method of NCCSs has better discriminative ability than the weighted correlation coefficient of C-SvNSs since the measure values of the weighted correlation coefficient imply smaller difference. Therefore, the proposed ESM of NCCSs is superior to the weighted correlation coefficient of C-SvNSs in the SvNMVS setting.

Furthermore, existing probabilistic neutrosophic GDM models [17-21] are difficultly applied to the above GDM example with small-scale group arguments because they lack some confidence levels of the neutrosophic evaluation values, so as to difficultly ensure the credibility and rationality of the probability neutrosophic values and the decision results in the small-scale GDM process, while the proposed GDM model can guarantee some confidence levels of the neutrosophic evaluation values from a probability estimation perspective in the circumstance of SvNMVSs and reveal its usability and suitability in small-scale GDM problems. Therefore, the proposed GDM model is obviously superior to the probabilistic neutrosophic GDM models [17-21] with respect to the credibility and rationality of GDM in a SvNMVS circumstance.

## 7. Conclusion

From a probability estimation perspective in a circumstance of SvNMVSs, this article presented the definition of NCCS, which was obtained by a conversion technique from SvNMVS to NCCS based on the average values and CIs of true, false, and indeterminate fuzzy sequences in SvNMVS. Since NCCS is composed of the average values and CIs, this information representation can ensure that the true, false, and indeterminate fuzzy values in SvNMVSs fall within their CIs subject to some confidence levels. Then, we proposed the ESM of NCCs and its GDM model corresponding to some confidence levels of $(1-\delta) \times 100 \%$ (usually using $90 \%, 95 \%$, and $99 \%$ confidence levels). Moreover, the proposed GDM model was applied to a choice problem of LTSs in a circumstance of SvMVSs as an actual case in Shaoxing City, China, to reveal its usability and suitability in actual GDM problems. Compared with the related GDM models in the setting of SvNMVSs [17-23], the proposed GDM model indicated the following superiorities over existing methods:
(a) NCCS implies obvious merits in the conversion method based on the average values and CIs corresponding to some confidence levels under the circumstance of SvNMVSs because it makes the information expression more confident and reasonable in light of the probability estimation of fuzzy data (small-scale sample data with normal distribution).
(b) The proposed weighted ESM method of NCCSs has better discriminative ability than the weighted correlation coefficient method of C-SvNSs in the identification process of their measure values.
(c) The proposed GDM model is more credible and more reasonable subject to some confidence levels of $(1-\delta) \times 100 \%$ than the existing GDM models under the circumstance of SvNMVSs.
(d) The proposed GDM model reveals the decision flexibility and credibility in the GDM application of the LTS selection problem.
In this study, the proposed conversion technique and GDM model are only suitable for GDM problems in the normal distribution situation of the fuzzy data in SvNMVSs, but cannot solve GDM problems in other distribution situations of the fuzzy data in SvNMVSs, which shows the limitations/insufficiencies of this paper. In future research, we shall further extend the proposed GDM model to GDM models corresponding to lognormal, logarithmic, and exponential distributions of fuzzy data and their applications in the situation of SvNMVSs.

## Data Availability

No data were used to support the findings of this study.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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