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Research Article

A Note on Approximation of Blending Type Bernstein–Schurer–Kantorovich Operators with Shape Parameter α

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The objective of this paper is to construct univariate and bivariate blending type α-Schurer–Kantorovich operators depending on two parameters $\alpha \in [0,1]$ and $\rho > 0$ to approximate a class of measurable functions on $[0, 1+q]$, $q > 0$. We present some auxiliary results and obtain the rate of convergence of these operators. Next, we study the global and local approximation properties in terms of first- and second-order modulus of smoothness, weight functions, and by Peetre's K-functional in different function spaces. Moreover, we present some study on numerical and graphical analysis for our operators.

1. Introduction

In 1912, Bernstein [1] introduced a polynomial as

$$\mathbb{B}_l(h;\nu) = \sum_{q=0}^{l} P_{l,q}(\nu) h\left(\frac{q}{l}\right), \quad l \in \mathbb{N}, \tag{1}$$

where $P_{l,q}(\nu) = \binom{l}{q} \nu^q (1-\nu)^{q-l}$. For the operators given by (1), he showed that $\mathbb{B}(.;.)$ converges to g uniformly where $g \in C[0,1]$. In 1962, Schurer [2] modified operators given in (1) as: for $q > 0$, a real number

$$\mathbf{B}_{l,q}(g;\nu) = \sum_{q=0}^{l+q} \binom{l+q}{q} \nu^q (1-\nu)^{q+q-l} g\left(\frac{q}{l}\right), \quad \nu \in [0, 1+q], \tag{2}$$

where $g \in C[0, 1+q]$. One can note that for $q = 0$, the polynomials presented in (2) reduces to polynomials given by (1). The operators are introduced in (1) and (2) are restricted for continuous functions only and are different in respect to the domain of function f. Additionally, several researchers, e.g., Mursaleen et al. [3], Acar et al. [4, 5], Mohiuddine et al. [6], Acu et al. [7], İçöz and Çekim [8, 9], and Kajla and Micláus [10, 11] constructed new sequences of linear positive operators to investigate the rapidity of convergence and order of approximation in different functional spaces in terms of several generating functions. Some other researchers developed many other useful operators [6, 12–30] in the same field. In the recent past, for $g \in [0,1], m \in \mathbb{N}$ and $\alpha \in [-1,1]$, Chen et al. [31] constructed a sequence of new linear positive operators as

$$T_{m,\alpha}(g;y) = \sum_{i=0}^{m} g\left(\frac{i}{m}\right) p_{m,i}^{\alpha}(y), \quad (y \in [0,1]), \tag{3}$$

where $p_{1,0}^{(\alpha)} = 1 - y$, $p_{1,1}^{(\alpha)} = y$, and

$$p_{m,i}^{\alpha}(y) = \left[(1-\alpha)y\binom{m-2}{i} + (1-\alpha)(1-y)\binom{m-2}{i-2} + \alpha y(1-y)\binom{m}{i}\right] y^{i-1}(1-y)^{m-i-1}, \quad (m \geq 2). \tag{4}$$

The operators defined in (3) are named as α-Berstein operator of order m.

Remark 1. One can note that for $\alpha = 1$, the relation (3) reduces to classical Bernstein operators [1].

Later, Aral and Erbay [32] introduced a parametric extension of Baskakov operators. Recently, Özger et al. [33] constructed a sequence α-Bernstein–Schurer operators to approximate a class of continuous functions as: For every $g \in C_B[0,\infty)$ where $C_B[0,\infty)$ stands for the continuous and bounded function as

$$\Psi_{m+q,\alpha}(g;y) = \sum_{i=0}^{m+q} g_i p_{m+q,i}^{(\alpha)}(y), \tag{5}$$

where

$$\begin{aligned} g_i &= g\left(\frac{i}{m}\right), \\ p_{m+q,i}^{(\alpha)}(y) &= \left[(1-\alpha)y\binom{m+q-2}{i} + (1-\alpha)(1-y)\binom{m+q-2}{i-2} + \alpha y(1-y)\binom{m+q}{i}\right] y^{i-1}(1-y)^{m+q-i-1}, \quad (m \geq 2), \end{aligned} \tag{6}$$

and $\rho > 0$.

Now, we construct the α-Bernstein–Schurer–Kantorovich operators and their moments.

$$\Psi_{m+q,\alpha}^{*}(g;y) = (m+1)\sum_{i=0}^{m+q} p_{m+q,i}^{(\alpha)}(y)\int_{i/m+1}^{i+1/m+1} g(s)\mathrm{d}s. \tag{7}$$

Motivated by the above development, we introduce positive linear operators to discuss the approximation properties in Lebesgue measurable space as

$$K_{m,\alpha}^{\rho}(f;y) = \sum_{i=0}^{m+q} p_{m+q,i}^{\alpha}(y)\int_0^1 f\left(\frac{i+t^{\rho}}{m+1}\right)\mathrm{d}s. \tag{8}$$

In the remaining part, we calculate basic Lemmas and order of approximation. Moreover, results of global and local approximation in terms of continuity modulus, weight functions, Lipschitz class and Lipschitz maximal function, Peetre's K-functional, and second-order smoothness modulus were analyzed. Furthermore, α-bivariate Schurer–Kantorovich operators are constructed and their pointwise and uniform approximation results are investigated.

2. Basic Estimates

Lemma 2 (see [33]). *Let $e_t(s) = s^t$, $t \in \{0,1,2\}$. For the operator defined in (5), one has*

$$\begin{aligned} \Psi_{m,q,\alpha}(e_0;u) &= 1, \\ \Psi_{m,q,\alpha}(e_1;u) &= u + \frac{q}{m}u, \\ \Psi_{m,q,\alpha}(e_2;u) &= u^2 + \frac{(m+q+2(1-\alpha))(u-u^2)}{m^2} + \frac{q(q+2m)u^2}{m^2}. \end{aligned} \tag{9}$$

Lemma 3. *For the operators discussed in (7), we obtain*

$$\begin{aligned}
\Psi^{*}_{m,q,\alpha}(e_0;u) &= 1,\\
\Psi^{*}_{m,q,\alpha}(e_1;u) &= \left(\frac{m+q}{m+1}\right)u + \frac{1}{2(m+1)},\\
\Psi^{*}_{m,q,\alpha}(e_2;u) &= \left[\frac{m^2}{(m+1)^2} - \frac{m+q+2(1-\alpha)}{(m+1)^2} + \frac{q(q+2m)}{(m+1)^2}\right]u^2 + \left[\frac{m+q+2(1-\alpha)}{(m+1)^2} + \frac{m+q}{(m+1)^2}\right]u + \frac{1}{3(m+1)^2}.
\end{aligned} \tag{10}$$

Lemma 4. *For the operator given by (8), we obtain*

$$\begin{aligned}
K^{\rho}_{m,\alpha}(e_0;u) &= 1,\\
K^{\rho}_{m,\alpha}(e_1;u) &= \frac{m+2(\alpha-1)}{m+1}u + \frac{(\alpha+1)(\rho+1)+1}{2(\rho+1)(m+1)},\\
K^{\rho}_{m,\alpha}(e_2;u) &= \left(1+\frac{4\alpha-3}{m}\right)\frac{m^2u^2}{(m+1)^2} + \frac{[(\rho+1)(m(2\alpha+3)+(\alpha-1)(2\alpha+7))]+4(\alpha-1)}{(\rho+1)(m+1)^2}u\\
&\quad + \frac{2m(2\rho+1)+(\alpha+1)(2\rho+1)((\alpha+2)(\rho+1)+2)+\rho+1}{(2\rho+1)(\rho+1)(m+1)^2}.
\end{aligned} \tag{11}$$

Proof. By Lemma 3, it easily demonstrated Lemma 4. □

Lemma 5. *Let $e_t(s) = (e_1(s)-u)^t = \psi_u^t(s)$, $t \in \mathbb{N}$, represent the central moments of $K^{\rho}_{m,\alpha}(.;.)$ introduced in (8). Then, we have*

$$\begin{aligned}
K^{\rho}_{m,\alpha}((e_1(s)-u);u) &= \frac{2\alpha-3}{m+1}u + \frac{(\alpha+1)(\rho+1)+1}{(\rho+1)(m+1)},\\
K^{\rho}_{m,\alpha}\left((e_1(s)-u)^2;u\right) &= \left[\left(1+\frac{4\alpha-3}{m}\right)\frac{m^2}{(m+1)^2} - \frac{2m+4\alpha-1}{m+1} + 1\right]u^2\\
&\quad + \frac{[(\rho+1)(m(2\alpha+3)+(\alpha-1)(2\alpha+7)-2(\alpha+1))]+\alpha-6}{(\rho+1)(m+1)^2}u\\
&\quad + \frac{2m(2\rho+1)+(\alpha+1)(2\rho+1)((\alpha+2)(\rho+1)+2)+\rho+1}{(2\rho+1)(\rho+1)(m+1)^2}.
\end{aligned} \tag{12}$$

Proof. Using Lemmas 3 and 4, it can be easily be proved. □

3. Convergence Behaviour of $K^{\rho}_{m,\alpha}(.;.)$

Definition 6 (see [6]). For $g \in C[0,1+p]$, $p>0$, the modulus of continuity for a uniformly continuous function g is defined as

$$\omega(g;\delta) = \sup_{|r_1-r_2|\le\delta} |g(r_1)-g(r_2)|, \quad r_1, r_2 \in [0,1+q], p>0. \tag{13}$$

In $C[0,1+p]$, let g represent continuous function that is uniformaly, $p>0$ and $\delta>0$. Then, one get

$$|g(r_1)-g(r_2)|\le\left(1+\frac{(r_1-r_2)^2}{\delta^2}\right)\omega(g;\delta). \tag{14}$$

Theorem 7. *Let $K_{m,\alpha}^{\rho}(.;.)$ represent the set of operators provided by (8). On each bounded subset of $[0,1+p]$, $p>0$; then, $K_{m,\alpha}^{\rho}(.;.)$ converges uniformly to f where $f\in C[0,1+p]$, $p>0\cap\{f\colon u\ge 0, f(v)/1+v^2$ converges as $u\longrightarrow\infty\}$.*

Proof. For the proof of this result, it is enough to that

$$K_{m,\alpha}^{\rho}(e_j;v)\longrightarrow e_j(v), \text{for } j\in\{0,1,2\}. \tag{15}$$

By Lemma 3, it is obvious that $K_{m,\alpha}^{\rho}(e_j;v)\longrightarrow e_i(u)$ for $j=0,1,2$ as $m\longrightarrow\infty$. The proof of Theorem 7 is complete. □

Example 1. As one can see, for the following set of parameters $p=5$, $\rho=0.1$, and $\alpha=0.5$, the operator $K_{m,\alpha}^{\rho}(f;y)$ converges uniformly to the function $f(y)=y^3-5y^2+6y+2$ as m increases which is illustrated in Figure 1.

Figure 2 and Table 1 also demonstrated our analytical results.

Theorem 8 (see [34]). *Let $L\colon C([c,d])\longrightarrow B([c,d])$ be a linear and positive operator and let φ_y be the function defined by*

$$\varphi_x(s)=|s-x|,\ (y,s)\in[c,d]\times[c,d]. \tag{16}$$

If $f\in C_B([c,d])$ for any $y\in[c,d]$ and any $\eta>0$, the operator L verifies

$$|(Lf)(y)-f(y)|\le|f(y)||(Le_0)(y)-1|+\left\{(Le_0)(y)+\eta^{-1}\sqrt{(Le_0)(y)\left(L\varphi_y^2\right)(y)}\right\}\omega_f(\eta). \tag{17}$$

Theorem 9. *Let the sequence of operators $K_{m,\alpha}^{\rho}(.;.)$ introduced by (8) and $f\in C_B[0,1+p]$, $q>0$, we obtain*

$$\left|K_{m,\alpha}^{\rho}(f;u)-f(u)\right|\le 2\omega(f;\eta), \tag{18}$$

where $\eta=\sqrt{K_{m,\alpha}^{\rho}(\psi_u^2;u)}$.

Proof. In the light of Theorem 8 and Lemmas 3 and 4, we have

$$\left|K_{m,\alpha}^{\rho}(f;v)-f(v)\right|\le\left\{1+\delta^{-1}\sqrt{K_{m,\alpha}^{\rho}(f;v)\left(\psi_v^2;u\right)}\right\}\omega(f;\eta). \tag{19}$$

On choosing, $\eta=\sqrt{K_{m,\alpha}^{\rho}(\psi_v^2;v)}$.

Hence, it completes the proof of this result. □

4. Pointwise Approximation Results

Here, we consider the Lipschitz type space [35] as

$$\mathrm{Lip}_M^{k_1,k_2}(\Gamma)\coloneqq\left\{f\in C_B[0,1+p], q>0\colon |f(t)-f(v)|\le M\frac{|t-v|^{\Gamma}}{\left(t+k_1v+k_2v^2\right)^{\Gamma/2}}\colon v,t\in(0,\infty)\right\}, \tag{20}$$

where $M\ge 0$ is a real valued constant number, $k_1,k_2>0$, $\rho>0$, and $\Gamma\in(0,1]$.

Theorem 10. *For $f\in Lip_M^{k_1,k_2}(\Gamma)$, one yield*

$$\left|K_{m,\alpha}^{\rho}(f;v)-f(v)\right|\le M\left(\frac{\eta_m^*(u)}{k_1v+k_2v^2}\right)^{\Gamma/2}, \tag{21}$$

where $v>0$ and $\eta_m^*(v)=K_{m,\alpha}^{\rho}(\psi_u^2;u)$.

Proof. For $\Gamma=1$, one has

$$\left|K_{m,\alpha}^{\rho}(f;v)-f(v)\right|\le K_{m,\alpha}^{\Gamma}(|f(t)-f(v)|)(v)\le MK_{m,\alpha}^{\rho}\left(\frac{|t-u|}{\left(t+k_1u+k_2u^2\right)^{1/2}};u\right). \tag{22}$$

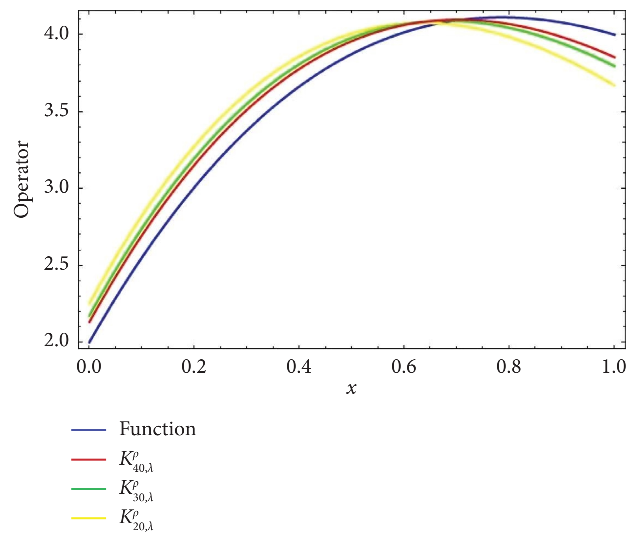

Figure 1: Approximation by operator $K^{\rho}_{m,\alpha}(;,;)$.

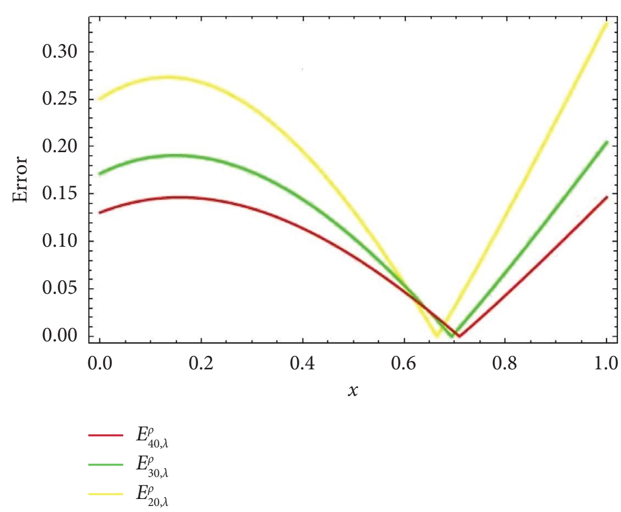

Figure 2: Error of the operators $K^{\rho}_{m,\alpha}(;,;)$ for various values of m.

Table 1: Error estimation table.

x	$E^{\rho}_{20,\alpha}(f;x)$	$E^{\rho}_{30,\alpha}(f;x)$	$E^{\rho}_{40,\alpha}(f;x)$
0.1	0.2717372121	0.1887446733	0.1445360958
0.2	0.2677254718	0.1886482134	0.1455017073
0.3	0.2412358918	0.1732429202	0.1348677403
0.4	0.1951644878	0.1444179547	0.1140349291
0.5	0.1324072752	0.1040624783	0.0844040078
0.6	0.0558602697	0.0540656519	0.0473757106
0.7	0.0315805132	0.0036833631	0.0043507716
0.8	0.1270190580	0.0672954058	0.0432700748
0.9	0.2275593491	0.134881315	0.0940860947
1	0.3303053711	0.2045519293	0.1466965539

Since $1/t + k_1 v + k_2 v^2 < 1/k_1 v + k_2 v^2$ for all $t, u \in (0, \infty)$, we get

$$\left|K^{\rho}_{m,\alpha}(f; v) - f(v)\right| \leq \frac{M}{\left(k_1 v + k_2 v^2\right)^{1/2}} \left(K^{\rho}_{m,\alpha}\left((s - v)^2; v\right)\right)^{1/2} \leq M\left(\frac{\eta^*_m(v)}{k_1 v + k_2 v^2}\right)^{1/2}. \tag{23}$$

For $\Gamma = 1$, this outcome is valid and $\Gamma \in (0, 1)$ by Hölder's inequality with $r_1 = 2/\Gamma$ and $r_2 = 2/2 - \Gamma$, we get

$$\left|K^{\rho}_{m,\alpha}(f; v) - f(v)\right| \leq \left(K^{\rho}_{m,\alpha}\left((|f(t) - f(v)|)^{2/\Gamma}; v\right)\right)^{\Gamma/2} \leq M\left(K^{\rho}_{m,\alpha}\left(\frac{|s - v|^2}{(s + k_1 v + k_2 v^2)}; v\right)\right)^{\Gamma/2}. \tag{24}$$

Since $1/t + k_1 v + k_2 v^2 < 1/k_1 v + k_2 v^2$ for all $t, v \in (0, \infty)$, we obtain

$$\left|K^{\rho}_{m,\alpha}(f; v) - f(u)\right| \leq M\left(\frac{\mathscr{P}^{\mu,q}_m\left(|t - v|^2; v\right)}{k_1 v + k_2 v^2}\right)^{\Gamma/2} \leq M\left(\frac{\eta^*_m(v)}{k_1 v + k_2 v^2}\right)^{\Gamma/2}. \tag{25}$$

Hence, completes the proof. □

5. Global Approximation

From [36], we recall some notation to prove the global approximation results.

In terms of the weight function $1 + u^2$ and $0 \leq u < \infty$, we have

$B_{1+v^2}[0, 1 + p], q > 0 = \{f(v): |f(v)| \leq M_f(1 + v^2),$ the constant Mf depends on $f\}$.

$C_{1+v^2}[0, 1 + p], q > 0 \subset B_{1+v^2}[0, 1 + p], q > 0$ endowed with the norm space of continuous functions $\|f\|_{1+v^2} = \sup_{v \in [0,1+p], q>0} |f|/1 + v^2$ and

$$C^k_{1+v^2}[0, 1 + p], q > 0 = \left\{f \in C_{1+v^2}: \lim_{v \longrightarrow \infty} \frac{f(v)}{1 + v^2} = k, \text{where } k \text{ is } a \text{ constant}\right\}. \tag{26}$$

Theorem 11. *Let the $K^{\rho}_{m,\alpha}(.;.)$ be the operators given by (8) and $K^{\rho}_{m,\alpha}(.;.)$: $C^k_{1+v^2}[0, 1 + p], q > 0 \longrightarrow B_{1+v^2}[0, 1 + p]$, $q > 0$. Then, we have*

$$\lim_{m \longrightarrow \infty} \left\|K^{\rho}_{m,\alpha}(f; v) - f\right\|_{1+v^2} = 0, \tag{27}$$

where $f \in C^k_{1+v^2}[0, 1 + p], q > 0$.

Proof. To prove this result, it is sufficient to prove that

$$\lim_{m \longrightarrow \infty} \left\|K^{\rho}_{m,\alpha}\left(e_j; v\right) - v^i\right\|_{1+v^2} = 0, \quad j = 0, 1, 2. \tag{28}$$

From Lemma 3, we get

$$\left\|K^{\rho}_{m,\alpha}(e_0; v) - v^0\right\|_{1+v^2} = \sup_{v \in [0,1+p], q>0} \frac{\left|K^{\rho}_{m,\alpha}(1; v) - 1\right|}{1 + v^2} = 0 \quad \text{for } i = 0. \tag{29}$$

For $i = 1$,

$$\|K^{\rho}_{m,\alpha}(e_1;v) - v^1\|_{1+v^2} = \sup_{v\in[0,1+p],q>0} \frac{m + 2(\alpha-1)/m + 1u + (\alpha+1)(\rho+1) + 1/2(\rho+1)(m+1)}{1+v^2}$$
$$= \left(\frac{m+2(\alpha-1)}{m+1} - 1\right) \sup_{v\in[0,1+p],q>0} \frac{v}{1+v^2} + \frac{(\alpha+1)(\rho+1)+1}{2(\rho+1)(m+1)} \sup_{v\in[0,1+p],q>0} \frac{1}{1+v^2}, \tag{30}$$

which implies that $\|K^{\rho}_{m,\alpha}(e_1;v) - v^1\|_{1+v^2} \longrightarrow 0$ as $m \longrightarrow \infty$.

Similarly, we see that $\|K^{\rho}_{m,\alpha}(e_2;v) - v^2\|_{1+u^2} \longrightarrow 0$ as $m \longrightarrow \infty$. □

6. Bivariate Case of Operators $K^{\rho}_{n,\alpha}$ and Their Approximation Behaviour

Take $\mathcal{I}^2 = \{(y_1,y_2)\colon 0 \le y_1 < 1+q_1, 0 \le y_2 < 1+q_2\}$ and $C(\mathcal{I}^2)$ is the class of all continuous functions on $\mathcal{I}^2$ equipped with the norm $\|g\|_{C(\mathcal{I}^2)} = \sup_{(y_1,y_2)\in\mathcal{I}^2}|g(y_1,y_2)|$.

Then, for all $h \in C(\mathcal{I}^2)$ and $m_1, m_2 \in \mathbb{N}$, we construct a bivariate sequence. Bivariate generalized baskakov operators are as follows:

$$K^{\rho,\alpha_1,\alpha_2}_{m_1,m_2}(f;y_1,y_2) = \sum_{i_1=0}^{m_1+q_1} p^{\alpha_1}_{m_1,i_1}(y_1) \sum_{i_2=0}^{m_2+q_2} p^{\alpha_2}_{m_2,i_2}(y_2) \cdot \int_0^1\int_0^1 g\left(\frac{i_1+t_1^{\rho}}{m_1+1}, \frac{i_2+t_2^{\rho}}{m_2+1}\right) dt_1 dt_2, \tag{31}$$

where

$$p^{\alpha_k}_{m_k,i_k}(y_k) = \left[(1-\alpha)y_k\binom{m_k+q_k-2}{i_k} + (1-\alpha_k)(1-y_k)\binom{m_k+q_k-2}{i_k-2} + \alpha y_k(1-y_k)\binom{m_k+q_k}{i_k}\right] \cdot y_k^{i_k-1}(1-y_k)^{m_k+q_k-i_k-1}, \quad (m_1,m_2 \ge 2). \tag{32}$$

Lemma 12. *Let $e_{j,k} = y_1^j y_2^k$ represent the moments. Then, for the operators (equation (31)), one has*

$$\begin{aligned}
K^{\rho,\alpha_1,\alpha_2}_{m_1,m_2}(e_{0,0};y_i,y_2) &= 1,\\
K^{\rho,\alpha_1,\alpha_2}_{m_1,m_2}(e_{1,0};y_i,y_2) &= \frac{m_1+2(\alpha_1-1)}{m_1+1}y_1 + \frac{(\alpha_1+1)(\rho+1)+1}{2(\rho+1)(m_1+1)},\\
K^{\rho,\alpha_1,\alpha_2}_{m_1,m_2}(e_{0,1};y_i,y_2) &= \frac{m_1+2(\alpha_1-1)}{m_2+1}y_2 + \frac{(\alpha_2+1)(\rho+1)+1}{2(\rho+1)(m_2+1)},\\
K^{\rho,\alpha_1,\alpha_2}_{m_1,m_2}(e_{1,1};y_i,y_2) &= \frac{m_1+2(\alpha_1-1)}{m_1+1}y_1 + \frac{(\alpha_1+1)(\rho+1)+1}{2(\rho+1)(m_1+1)} \times \frac{m_2+2(\alpha_2-1)}{m_2+1}y_2 + \frac{(\alpha_2+1)(\rho+1)+1}{2(\rho+1)(m_2+1)},\\
K^{\rho,\alpha_1,\alpha_2}_{m_1,m_2}(e_{2,0};y_i,y_2) &= \left(1+\frac{4\alpha_1-3}{m_1}\right)\frac{m_1^2y_1^2}{(m_1+1)^2}\\
&\quad+ \frac{[(\rho+1)(m_1(2\alpha_1+3)+(\alpha_1-1)(2\alpha_1+7))]+4(\alpha_1-1)}{(\rho+1)(m_1+1)^2}y_1\\
&\quad+ \frac{2m_1(2\rho+1)+(\alpha_1+1)(2\rho+1)((\alpha_1+2)(\rho+1)+2)+\rho+1}{(2\rho+1)(\rho+1)(m_1+1)^2},\\
K^{\rho,\alpha_1,\alpha_2}_{m_1,m_2}(e_{0,2};y_i,y_2) &= \left(1+\frac{4\alpha_2-3}{m_2}\right)\frac{m_2^2y_2^2}{(m_2+1)^2}\\
&\quad+ \frac{[(\rho+1)(m_2(2\alpha_2+3)+(\alpha_2-1)(2\alpha_2+7))]+4(\alpha_2-1)}{(\rho+1)(m_2+1)^2}y_2\\
&\quad+ \frac{2m_2(2\rho+1)+(\alpha_2+1)(2\rho+1)((\alpha_2+2)(\rho+1)+2)+\rho+1}{(2\rho+1)(\rho+1)(m_2+1)^2}.
\end{aligned} \tag{33}$$

Proof. From Lemma 3 and linearity property, we have

$$\begin{aligned}
K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(e_{0,0}; y_i, y_2\right) &= K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(e_0; y_i, y_2\right)K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(e_0; y_i, y_2\right),\\
K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(e_{1,0}; y_i, y_2\right) &= K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(e_1; y_i, y_2\right)K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(e_0; y_i, y_2\right),\\
K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(e_{0,1}; y_i, y_2\right) &= K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(e_0; y_i, y_2\right)K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(e_1; y_i, y_2\right),\\
K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(e_{1,1}; y_i, y_2\right) &= K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(e_1; y_i, y_2\right)K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(e_1; y_i, y_2\right),\\
K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(e_{2,0}; y_i, y_2\right) &= K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(e_2; y_i, y_2\right)K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(e_0; y_i, y_2\right),\\
K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(e_{0,2}; y_i, y_2\right) &= K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(e_0; y_i, y_2\right)K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(e_2; y_i, y_2\right),
\end{aligned} \tag{34}$$

which proves Lemma 12. □

Lemma 13. *Let* $\Psi_{y_1,y_2}^{j,k}(s,t) = \eta_{j,k}(s,t) = (t-y_1)^j (t-y_2)^k$, $j,k \in \{0,1,2\}$ *represent central moments functions. Then, the operators* $K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(.;.)$ *defined by (Section 6.1) satisfies the following identities:*

$$\begin{aligned}
K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(\eta_{0,0}; y_i, y_2\right) &= 1,\\
K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(\eta_{1,0}; y_i, y_2\right) &= \frac{2\alpha_1-3}{m_1+1}y_1 + \frac{(\alpha_1+1)(\rho+1)+1}{(\rho+1)(m_1+1)},\\
K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(\eta_{0,1}; y_i, y_2\right) &= \frac{2\alpha_1-3}{m_2+1}y_2 + \frac{(\alpha_2+1)(\rho+1)+1}{(\rho+1)(m_2+1)},\\
K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(\eta_{1,1}; y_i, y_2\right) &= \frac{2\alpha_1-3}{m_1+1}y_1 + \frac{(\alpha_1+1)(\rho+1)+1}{(\rho+1)(m_1+1)} \times \frac{2\alpha_1-3}{m_2+1}y_2 + \frac{(\alpha_2+1)(\rho+1)+1}{(\rho+1)(m_2+1)},\\
K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(\eta_{2,0}; y_i, y_2\right) &= \left[\left(1+\frac{4\alpha_1-3}{m_1}\right)\frac{m_1^2}{(m_1+1)^2} - \frac{2m_1+4\alpha_1-1}{m_1+1} + 1\right]y_1^2\\
&\quad + \frac{[(\rho+1)(m_1(2\alpha_1+3)+(\alpha_1-1)(2\alpha_1+7)-2(\alpha_1+1))]+\alpha_1-6}{(\rho+1)(m_1+1)^2}y_1\\
&\quad + \frac{2m_1(2\rho+1)+(\alpha_1+1)(2\rho+1)((\alpha_1+2)(\rho+1)+2)+\rho+1}{(2\rho+1)(\rho+1)(m_1+1)^2},\\
K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(\eta_{0,2}; y_i, y_2\right) &= \left[\left(1+\frac{4\alpha_2-3}{m_2}\right)\frac{m_2^2}{(m_2+1)^2} - \frac{2m_2+4\alpha_2-1}{m_2+1} + 1\right]y_2^2\\
&\quad + \frac{[(\rho+1)(m_1(2\alpha_2+3)+(\alpha_2-1)(2\alpha_2+7)-2(\alpha_2+1))]+\alpha_2-6}{(\rho+1)(m_2+1)^2}y_2\\
&\quad + \frac{2m_2(2\rho+1)+(\alpha_2+1)(2\rho+1)((\alpha_2+2)(\rho+1)+2)+\rho+1}{(2\rho+1)(\rho+1)(m_2+1)^2}.
\end{aligned} \tag{35}$$

Proof. From Lemma 12 and linearity property, we have

$$\begin{aligned}
K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(\eta_{0,0}; y_i, y_2) &= K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(\eta_0; y_i, y_2) K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(\eta_0; y_i, y_2),\\
K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(\eta_{1,0}; y_i, y_2) &= K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(\eta_1; y_i, y_2) K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(\eta_0; y_i, y_2),\\
K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(\eta_{0,1}; y_i, y_2) &= K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(\eta_0; y_i, y_2) K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(\eta_1; y_i, y_2),\\
K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(\eta_{1,1}; y_i, y_2) &= K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(\eta_1; y_i, y_2) K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(\eta_1; y_i, y_2),\\
K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(\eta_{2,0}; y_i, y_2) &= K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(\eta_2; y_i, y_2) K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(\eta_0; y_i, y_2),\\
K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(\eta_{0,2}; y_i, y_2) &= K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(\eta_0; y_i, y_2) K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(\eta_2; y_i, y_2),
\end{aligned} \tag{36}$$

which proves Lemma 13. □

7. Degree of Convergence

For any $g \in C(\mathcal{I}^2)$ and $\eta > 0$ modulus of continuity of the second order is given by

$$\omega(g; \delta_{n_1}, \eta_{n_2}) = \sup\{|g(t,s) - g(y_1, y_2)|:\ (t,s), (y_1, y_2) \in \mathcal{I}^2\}, \tag{37}$$

with $|t - y_1| \le \eta_{n_1}, |s - y_2| \le \eta_{n_2}$ defined by the partial modulus of continuity as

$$\begin{aligned}
\omega_1(g;\eta) &= \sup_{0\le y_2 \le \infty} \sup_{|x_1 - x_2| \le \eta} \{|g(x_1, y_2) - g(x_2, y_2)|\},\\
\omega_2(g;\eta) &= \sup_{0\le y_1 \le \infty} \sup_{|y_1 - y_2| \le \eta} \{|g(y_1, y_1) - g(y_1, y_2)|\}.
\end{aligned} \tag{38}$$

Theorem 14. *For any $g \in C(\mathcal{I}^2)$, we have*

$$\left|K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(g; y_1, y_2) - g(y_1, y_2)\right| \le 2\left(\omega_1(g; \delta_{y_1,n_1}) + \omega_2(g; \delta_{n_2,y_2})\right). \tag{39}$$

Proof. For the proof of Theorem 14, generally, we use the well-known Cauchy–Schwarz inequality. Thus, we see that

$$\begin{aligned}
K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}|(g; y_1, y_2) - g(y_1, y_2)| &\le K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(|g(t,s) - g(y_1, y_2)|; y_1, y_2)\\
&\le K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(|g(t,s) - g(y_1, s)|; y_1, y_2)\\
&\quad + K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(|g(y_1,s) - g(y_1, y_2)|; y_1, y_2)\\
&\le K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(\omega_1(g; |t - y_1|); y_1, y_2)\\
&\quad + K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(\omega_2(g; |s - y_2|); y_1, y_2)\\
&\le \omega_1(g; \delta_{n_1})\left(1 + \delta_{n_1}^{-1} K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(|t - y_1|; y_1, y_2)\right)\\
&\quad + \omega_2(g; \delta_{n_2})\left(1 + \delta_{n_2}^{-1} K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(|s - y_2|; y_1, y_2)\right)\\
&\le \omega_1(g; \delta_{n_1})\left(1 + \frac{1}{\delta_{n_1}}\sqrt{K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}((t - y_1)^2; y_1, y_2)}\right)\\
&\quad + \omega_2(g; \delta_{n_2})\left(1 + \frac{1}{\delta_{n_2}}\sqrt{K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}((s - y_2)^2; y_1, y_2)}\right).
\end{aligned} \tag{40}$$

If we choose $\delta_{n_1}^2 = \delta_{n_1,y_1}^2 = K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}((t-y_1)^2; y_1, y_2)$ and $\delta_{n_2}^2 = \delta_{n_2,y_2}^2 = K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}((s-y_2)^2; y_1, y_2)$, then we can simply achieve our objectives.

Here, we analyze convergence in terms of the Lipschitz class for bivariate functions. For $M>0$ and $\tau, \tau \in [0,1+p], q>0$, maximal Lipschitz function space on $E \times E \subset \mathcal{I}^2$ is given by

$$\mathcal{L}_{\tau,\tau}(E\times E) = \left\{ g\colon \sup (1+t)^{\tau}(1+s)^{\tau}\left(g_{\tau,\tau}(t,s) - g_{\tau,\tau}(y_1,y_2)\right) \le M \frac{1}{(1+y_1)^{\tau}} \frac{1}{(1+y_2)^{\tau}} \right\}, \tag{41}$$

where g is continuous and bounded on $\mathcal{I}^2$, and

$$g_{\tau,\tau}(t,s) - g_{\tau,\tau}(y_1,y_2) = \frac{\left|g(t,s) - g(y_1,y_2)\right|}{|t-y_1|^{\tau}\,|s-y_2|^{\tau}}; \quad (t,s), (y_1,y_2) \in \mathcal{I}^2. \tag{42}$$

□

Theorem 15. *Let $g \in \mathcal{L}_{\tau,\tau}(E\times E)$, then for any $\tau, \tau \in [0,1+p], q>0$, there exists $M>0$ such that*

$$\left|K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(g; y_1, y_2) - g(y_1,y_2)\right| \le M\left\{\left((d(y_1,E))^{\tau} + \left(\delta_{n_1,y_1}^2\right)^{\tau/2}\right) \times \left((d(y_2,E))^{\tau} + \left(\delta_{n_2,y_2}^2\right)^{\tau/2}\right) + (d(y_1,E))^{\tau}(d(y_2,E))^{\tau}\right\}, \tag{43}$$

where δ_{n_1,y_1} and δ_{n_2,y_2} are defined by Theorem 14.

Proof. Take $|y_1 - x_0| = d(y_1,E)$ and $|y_2 - y_0| = d(y_2,E)$. For any $(y_1,y_2) \in \mathcal{I}^2$, and $(x_0,y_0) \in E\times E$, we let $d(y_1,E) = \inf\{|y_1 - y_2|\colon y_2 \in E\}$. Thus, we can write here

$$\left|g(t,s) - g(y_1,y_2)\right| \le M\left|g(t,s) - g(x_0,y_0)\right| + \left|g(x_0,y_0) - g(y_1,y_2)\right|. \tag{44}$$

Apply $K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}$, we obtain

$$\begin{aligned}
\left|K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(g; y_1, y_2) - g(y_1,y_2)\right| &\le K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(\left|g(y_1,y_2) - g(x_0,y_0)\right| + \left|g(x_0,y_0) - g(y_1,y_2)\right|\right)\\
&\le M K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(|t-x_0|^{\tau}|s-y_0|^{\tau}; y_1, y_2\right)\\
&\quad + M|y_1 - x_0|^{\tau}|y_2 - y_0|^{\tau}.
\end{aligned} \tag{45}$$

For all, $A, B \ge 0$ and $\tau \in [0,1+p], q>0$, the inequality $(A+B)^{\tau} \le A^{\tau} + B^{\tau}$, thus

$$\begin{aligned}
|t-x_0|^{\tau} &\le |t-y_1|^{\tau} + |y_1 - x_0|^{\tau},\\
|s-y_0|^{\tau} &\le |s-y_2|^{\tau} + |y_2 - y_0|^{\tau}.
\end{aligned} \tag{46}$$

Therefore,

$$\begin{aligned}
\left|K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}(g; y_1, y_2) - g(y_1,y_2)\right| &\le M K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(|t-y_1|^{\tau}|s-y_2|^{\tau}; y_1, y_2\right)\\
&\quad + M|y_1 - x_0|^{\tau} K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(|s-y_2|^{\tau}; y_1, y_2\right)\\
&\quad + M|y_2 - y_0|^{\tau} K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(|t-y_1|^{\tau}; y_1, y_2\right)\\
&\quad + 2M|y_1 - x_0|^{\tau}|y_2 - y_0|^{\tau} K_{m_1,m_2}^{\rho,\alpha_1,\alpha_2}\left(\mu_{0,0}; y_1, y_2\right).
\end{aligned} \tag{47}$$

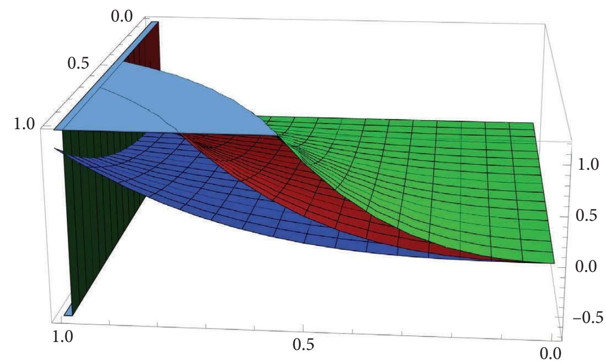

Figure 3: $K^{\rho,\alpha_1,\alpha_2}_{m_1,m_2}(,;,)$ converges to $f(x) = y_1^3 y_2^2$.

On applying Hölder inequality on $K^{\rho,\alpha_1,\alpha_2}_{m_1,m_2}$, we get

$$\begin{aligned} K^{\rho,\alpha_1,\alpha_2}_{m_1,m_2}\left(|t-y_1|^\tau |s-y_2|^\tau; y_1,y_2\right) &= \mathscr{U}^{\alpha_1}_{n_1,k}\left(|t-y_1|^\tau; y_1,y_2\right) \\ &\quad \times \mathscr{V}^{\alpha_2}_{n_2,l}\left(|s-y_2|^\tau; y_1,y_2\right) \\ &\le \left(K^{\rho,\alpha_1,\alpha_2}_{m_1,m_2}\left(|t-y_1|^2; y_1,y_2\right)\right)^{\tau/2} \\ &\quad \times\left(K^{\rho,\alpha_1,\alpha_2}_{m_1,m_2}\left(\mu_{0,0}; y_1,y_2\right)\right)^{2-\tau/2} \\ &\quad \times\left(K^{\rho,\alpha_1,\alpha_2}_{m_1,m_2}\left(|s-y_2|^2; y_1,y_2\right)\right)^{\tau/2} \\ &\quad \times\left(K^{\rho,\alpha_1,\alpha_2}_{m_1,m_2}\left(\mu_{0,0}; y_1,y_2\right)\right)^{2-\tau/2}. \end{aligned} \tag{48}$$

Thus, we can obtain

$$\begin{aligned} \left|K^{\rho,\alpha_1,\alpha_2}_{m_1,m_2}(g;y_1,y_2) - g(y_1,y_2)\right| &\le M\left(\delta^2_{n_1,y_1}\right)^{\tau/2}\left(\delta^2_{n_2,y_2}\right)^{\tau/2} \\ &\quad + 2M\left(d(y_1,E)\right)^\tau \left(d(y_2,E)\right)^\tau \\ &\quad + M\left(d(y_1,E)\right)^\tau\left(\delta^2_{n_2,y_2}\right)^{\tau/2} + L\left(d(y_2,E)\right)^\tau\left(\delta^2_{n_1,y_1}\right)^{\tau/2}. \end{aligned} \tag{49}$$

We have completed the proof. □

Example 2. It is observed in this example that for the different set of parameters $q = 5$, $\rho = 0.9$, and $\alpha = 0.5$, the operator $K^{\rho,\alpha_1,\alpha_2}_{m_1,m_2}(,;,)$ converges uniformly to the function $f(y) = y_1^3 y_2^2$ (blue) as $m_1 = m_2 = 10$ (green) and $m_1 = m_2 = 20$ (red) increases which is shown in Figure 3.

8. Conclusion

In this paper, we construct the univariate and bivariate blending type α-Schurer–Kantorovich operators depending on two parameters $\alpha \in [0,1]$ and $\rho > 0$ to approximate a class of measurable functions on $[0, 1+q], q > 0$. Along with the auxiliary results, we obtain the rate of convergence of these operators. Also, we study the global and local approximation properties in terms of first- and second-order modulus of smoothness, weight functions, and by Peetre's K-functional in different function spaces. Additionally, we give some examples on numerical and graphical analysis for our operators.

Data Availability

Data sharing is not applicable to this article as no data sets were generated or analyzed during the current study.

Disclosure

A preprint Rao et al. [37] has previously been published.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors have equal contributions.

References

[1] S. N. Berntein, "Démonstration du théorème de Weierstrass fondée sur le calcul de probabilités," *Communications of the Kharkov Mathematical Society*, vol. 13, no. 2, pp. 1-2, 1912.

[2] F. Schurer, *Linear Positive Operators in Approximation Theory*, doctoral thesis, Technical University, Delf Report, Delft, Netherlands, 1962.

[3] M. Mursaleen, K. J. Ansari, and A. Khan, "Some approximation results by p,q-analogue of Bernstein–Stancu operators," *Applied Mathematics and Computation*, vol. 264, pp. 392–402, 2015.

[4] T. Acar, A. Aral, and S. A. Mohiuddine, "Approximation by bivariate (p, q)-Bernstein–Kantorovich operators," *Iranian Journal of Science and Technology Transaction A-Science*, vol. 42, no. 2, pp. 655–662, 2018.

[5] T. Acar and A. Kajla, "Degree of Approximation for bivariate generalized Bernstein type operators," *Results in Mathematics*, vol. 73, no. 2, p. 79, 2018.

[6] S. A. Mohiuddine, T. Acar, and A. Alotaibi, "Construction of a new family of Bernstein-Kantorovich operators," *Mathematical Methods in the Applied Sciences*, vol. 40, no. 18, pp. 7749–7759, 2017.

[7] A. M. Acu, T. Acar, and V. A. Radu, "Approximation by modified U n ρ operators," *Revista de la Real Academia de Ciencias Exactas, Fisicas y Naturales- Serie A: Matematicas*, vol. 113, no. 3, pp. 2715–2729, 2019.

[8] G. İçöz and B. Çekim, "Dunkl generalization of Szász operators via q-calculus," *Journal of Inequalities and Applications*, vol. 2015, no. 1, p. 284, 2015.

[9] G. İçöz and B. Çekim, "Stancu-type generalization of Dunkl analogue of Szász-Kantorovich operators," *Mathematical Methods in the Applied Sciences*, vol. 39, no. 7, pp. 1803–1810, 2016.

[10] A. Kajla and D. Micláus, "Blending type approximation by GBS operators of generalized Bernstein-Durrmeyer type," *Results in Mathematics*, vol. 73, p. 1, 2018.

[11] A. Kajla and T. Acar, "Blending type approximation by generalized Bernstein Durrmeyer type operators," *Miskolc Mathematical Notes*, vol. 19, no. 1, pp. 319–336, 2018.

[12] A. D. Gadjiev and C. Orhan, "Some approximation theorems via statistical convergence, Rocky Mountain," *Journal of Mathematics*, vol. 32, no. 1, pp. 129–138, 2007.

[13] J. Bustamante, "Directs estimates and a Voronovskaja-type formula for Mihesan operators," *Constructive Mathematical Analysis*, vol. 5, no. 4, pp. 202–213, 2022.

[14] L. Angelonı, N. Çetin, D. Costarellı, A. R. Sambucını, and G. Vıntı, "Multivariate sampling Kantorovich operators: quantitative estimates in Orlicz spaces," *Constructive Mathematical Analysis*, vol. 4, no. 2, pp. 229–241, 2021.

[15] K. Bozkurt, F. Özsaraç, and A. Aral, "Bivariate Bernstein polynomials that reproduce exponential functions," *Communications Faculty Of Science University of Ankara Series A1Mathematics and Statistics*, vol. 70, no. 1, pp. 541–554, 2021.

[16] R. Paltanea, "Durrmeyer type operators on a simplex," *Constructive Mathematical Analysis*, vol. 4, no. 2, pp. 215–228, 2021.

[17] R. Aslan, "On a Stancu form Szász-Mirakjan-Kantorovich operators based on shape parameter λ," *Advanced Studies: Euro-Tbilisi Mathematical Journal*, vol. 15, no. 1, pp. 151–166, 2022.

[18] R. Aslan, "Approximation by Szasz-Mirakjan-Durrmeyer operators based on shape parameter λ," *Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics*, vol. 71, no. 2, pp. 407–421, 2022.

[19] R. Aslan, "Approximation properties of univariate and bivariate new class λ-Bernstein–Kantorovich operators and its associated GBS operators," *Computational and Applied Mathematics*, vol. 42, no. 1, p. 34, 2023.

[20] F. Özger, E. Aljimi, and M. Temizer Ersoy, "Rate of weighted statistical convergence for generalized blending-type bernstein-kantorovich operators," *Mathematics*, vol. 10, no. 12, p. 2027, 2022.

[21] K. J. Ansari, F. Özger, and Z. Ödemiş Özger, "Numerical and theoretical approximation results for Schurer–Stancu operators with shape parameter λ," *Computational and Applied Mathematics*, vol. 41, no. 4, p. 181, 2022.

[22] A. Kajla, S. A. Mohiuddine, and A. Alotaibi, "Blending-type approximation by Lupas Durrmeyer-type operators involving Polya distribution," *Mathematical Methods in the Applied Sciences*, vol. 44, no. 11, pp. 9407–9418, 2021.

[23] S. A. Mohiuddine, "Approximation by bivariate generalized Bernstein–Schurer operators and associated GBS operators," *Advances in Difference Equations*, vol. 2020, no. 1, p. 676, 2020.

[24] M. Raiz, R. S. Rajawat, and V. N. Mishra, "α-Schurer Durrmeyer operators and their approximation properties," *Annals of the University of Craiova- Mathematics and Computer Science Series*, vol. 50, no. 1, pp. 189–204, 2023.

[25] M. Nasiruzzaman, M. A. O. Tom, S. Serra-Capizzano, N. Rao, and M. Ayman-Mursaleen, "Approximation results for Beta Jakimovski-Leviatan type operators via q-analogue," *Filomat*, vol. 37, pp. 8389–8404, 2023.

[26] M. Ayman-Mursaleen and S. Serra-Capizzano, "Statistical convergence via q-calculus and a Korovkin's type approximation theorem," *Axioms*, vol. 11, no. 2, p. 70, 2022.

[27] A. A. H. A. Al-Abied, A. Mursaleen, and M. Mursaleen, "Szász type operators involving Charlier polynomials and approximation properties," *Filomat*, vol. 35, no. 15, pp. 5149–5159, 2021.

[28] M. Qasim, M. S. Mansoori, A. Khan, Z. Abbas, and M. Mursaleen, "Convergence of modified Szász-Mirakyan-Durrmeyer operators depending on certain parameters," *Mathematical Foundations of Computing*, vol. 5, no. 3, pp. 187–196, 2022.

[29] M. Nasiruzzaman, A. Kilicman, and M. Ayman-Mursaleen, "Construction of q-baskakov operators by wavelets and approximation properties," *Iranian Journal of Science and Technology Transaction A-Science*, vol. 46, no. 5, pp. 1495–1503, 2022.

[30] M. Heshamuddin, N. Rao, B. P. Lamichhane, A. Kiliçman, and M. Ayman-Mursaleen, "On one- and two-dimensional α–stancu–schurer–kantorovich operators and their approximation properties," *Mathematics*, vol. 10, no. 18, p. 3227, 2022.

[31] X. Chen, J. Tan, Z. Liu, and J. Xie, "Approximation of functions by a new family of generalized Bernstein operators," *Journal of Mathematical Analysis and Applications*, vol. 450, no. 1, pp. 244–261, 2017.

[32] A. Aral and H. Erbay, "Parametric generalization of Baskakov operators," *Mathematical Communications*, vol. 24, pp. 119–131, 2019.

[33] F. Özger, H. M. Srivastava, and S. A. Mohiuddine, "Approximation of functions by a new class of generalized Bernstein–Schurer operators," *Revista de la Real Academia de Ciencias Exactas, Fisicas y Naturales- Serie A: Matematicas*, vol. 114, no. 4, p. 173, 2020.

[34] O. Shisha and B. Mond, "The degree of convergence of linear positive operators," *Proceedings of the National Academy of Sciences*, vol. 60, no. 4, pp. 1196–1200, 1968.

[35] A. Ozarslan and H. Aktuğlu, "Local approximation properties for certain King type operators," *Filomat*, vol. 27, no. 1, pp. 173–181, 2013.

[36] A. D. Gadziev, "Theorems of the type of PP Korovkin's theorems," *Mathematical Notes*, vol. 20, no. 5-6, pp. 995–998, 1976.

[37] N. Rao, M. Rani, A. Kilicman, P. Malik, and M. Ayman-Mursaleen, "Blending type Approximations by Kantorovich variant of α-Schurer operators," 2022, https://arxiv.org/abs/2208.12740.