Hindawi
Journal of Mathematics
Volume 2023, Article ID 5522746, 7 pages
https://doi.org/10.1155/2023/5522746

Research Article

Stochastic Energy Performance Evaluation Using a Bayesian Approach

Erol Terzi, Serpil Gumustekin Aydin, and Mehmet Ali Cengiz

Ondokuz Mayıs University, Faculty of Science, Department of Statistics, Samsun, Turkey

Correspondence should be addressed to Serpil Gumustekin Aydin; serpil.gumustekin@omu.edu.tr

Received 7 February 2023; Revised 20 September 2023; Accepted 11 October 2023; Published 25 October 2023

Academic Editor: Ali Sajid

Copyright © 2023 Erol Terzi et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In the past two decades, stochastic frontier analysis (SFA) has been extensively employed to assess energy efficiency. However, the use of the Bayesian approach in SFA for energy performance evaluation has not received significant attention. This study aims to address this gap by measuring the energy-based development performance of 29 OECD countries using stochastic frontier analysis with a Bayesian approach. In the existing literature, there is no apparent method for selecting the distribution of the inefficiency term, which represents the unexplained deviation from the production frontier. To address this issue, we propose different models with various inefficiency components, namely, the half normal, truncated normal, exponential distribution, and gamma distribution. Our analysis utilizes a panel dataset covering the period from 2004 to 2010. The Bayesian implementation of the proposed models is conducted using the WinBUGS package, employing the Markov chain Monte Carlo (MCMC) method. The primary objective of our study is to compare these models, each assuming a different distribution for the inefficiency term, using the deviance information criterion (DIC). The DIC serves as a reliable measure for model comparison and enables us to identify the most suitable model that accurately captures the energy efficiency scores of the countries. Based on the comparison of models with different distributional assumptions using the DIC, we find that the model with a half-normal inefficiency distribution yields the lowest DIC score. Consequently, this model is employed to rank the energy efficiency scores of the countries. In summary, our study fills a research gap by applying the Bayesian approach to SFA in the context of energy efficiency analysis. By proposing and comparing models with different inefficiency components, we contribute to the literature and offer insights into the relative energy efficiency performance of 29 OECD countries. The findings of our study not only inform the selection of an appropriate model but also facilitate the ranking of countries based on their energy efficiency using the identified best model.

1. Introduction

The theoretical definition of the production function describing the maximum amount of output that can be obtained from a given amount of input has been accepted for years. The estimation of frontier production functions and the results of theoretical and empirical studies converge. Stochastic frontier analysis (SFA), which was first introduced by Aigner et al. [1] and Meeusen and van den Broeck [2], has been used extensively in the determination of energy efficiency over the last two decades (see [3–9]). Some significant shortcomings of previous studies in this field can be overcome by the fact that the error term in estimating the frontier production function consists of two components. SFA is frequently used as a parametric method to estimate the boundary functions and measure production efficiency. It establishes a functional relationship between output variables such as cost, profit, and production and input variables such as explosive and environmental factors.

In recent years, there has been a remarkable increase in the use of SFA in the energy sector. Some studies in the literature regarding this situation are discussed. First, Huntington [10] described the relationship between energy efficiency and productivity using SFA. In addition to the standard randomly distributed error term, the econometric approach specifies a second error term with a skewed distribution to allow decision-making units to be above the limit rather than below it. Estimated productivity varies according to the assumed essential production function's form and researchers' deterministic or stochastic approach.

Buck and Young [11] discussed a parametric approach to estimate a stochastic boundary function for energy use in Canadian commercial plants. The stochastic frontier approach explicitly acknowledges that not all decision-making units will use energy effectively, given the building activities and the level of technology. Boyd [12] used a similar methodology in moist maize milling plants, while Filippini and Hunt [3] used panel stochastic boundary analysis to calculate the energy efficiency of 29 countries. When panel data are used with the stochastic frontier model, the estimated measurement error productivity is considered, and a more reliable measure of productivity is calculated. The maximum likelihood method, one of the most popular estimation methods, is frequently used in deriving parameter estimates of SFA. In recent years, there has been an increase in the use of the Bayesian approach for SFA parameter estimation. The Bayesian approach expresses the results in terms of probability density functions and provides a direct probability interpretation of unknown parameters. The significant advantage of using a Bayesian approach is the preliminary distribution, which allows one to express uncertainty about unknown parameters before considering some evidence.

Because of its many advantages, several studies on stochastic frontier analysis with a Bayesian approach exist. The Bayesian approach to SFA was first introduced by Van den Broeck et al. [13]. They reconsidered the error model on productivity with different sampling distributions. According to them, given a particular model, all efficiencies are assumed to be derived from the same distribution, but which one is unknown. Mixing different distributions for each efficiency in the sampling model is generally avoided, making analysis easier. Instead, simple models are mixed at the final stage. If we want to choose a particular distribution to calculate efficiency, it may be an excellent alternative to use Bayes factors as a criterion for model selection. Koop et al. [14] described the Markov chain Monte Carlo (MCMC) as a numeric integration method in a stochastic frontier frame. The advantage of Bayesian methods is that they provide precise finite sample results for any feature of interest and that parameter uncertainty is fully considered. The posterior features were evaluated using Monte Carlo integration. They explained how Gibbs sampling methods used the Bayesian approach to significantly reduce the computational burdens of stochastic boundary models. In addition, current developments in Bayesian stochastic frontier analysis (see [15–19]) can also be viewed. More recently, Griffin and Steel [20] described MCMC methods for Bayesian analysis of stochastic frontier models using the WinBUGS package software. Tsionas and Papadakis [21] provided a Bayesian approach to the problem organized around simulation techniques. Tabak and Tacles [22] used a Bayesian stochastic frontier for the cost and profit efficiency of the Indian banking industry. Tonini [23] estimated the total factor productivity growth in agriculture for the European Union and candidate countries using SFA with a Bayesian approach. Feng and Zhang [24] compared the efficiency of large and community banks in the United States from 1997 to 2006 using a Bayesian approach. Assaf and Josiassen [25] estimated the efficiency of healthcare food service operations with Bayesian SFA. Assaf et al. [26] analyzed the efficiency of Turkish banks from 2002 to 2010 using a Bayesian stochastic frontier approach. Barros [27] studied airport efficiency in Mozambique, estimating a cost function with random and fixed-effects stochastic frontier models with a Bayesian stochastic frontier model.

Despite its rapid growth across several disciplines, the use of the Bayesian approach to measuring energy performance has yet to gain strong attention in energy research. The authors in [1, 2] used exponential and half-normal distributions, respectively. Gamma distributions were used in [28], and log-normal distributions were studied by Migon and Medrano [29]. Griffin and Steel [20] described a semiparametric modeling technique to estimate the inefficiency distribution. Alghalith [30] described an alternative method for specifying the distribution of the inefficiency term. Each of these inefficiency terms can cause different behaviors in the distribution of technical efficiencies [31]. There are no apparent reasons for selecting one distributional form over the other; each has its pros and cons [32].

Here, we propose different models with different inefficiency components as exponential, half normal, truncated normal, and gamma in a formal Bayesian framework. Bayesian methods appear suitable for stochastic frontier models because they provide precise small-sample results (inference of efficiencies), allow prior knowledge and regularities conditions to be incorporated during the estimation, and more accurately represent parameter uncertainties through kernel densities. Stochastic frontier models require numeric integration methods because they are so complex; the most appropriate method is MCMC. Efficiency measurement with stochastic frontier models is troublesome in many situations because decomposing the overall error term into a two-sided and a one-sided disturbance term may be problematic. The reason is that when the noise-to-signal ratio is relatively high, the overall error term would appear to be approximately symmetric, in which case identification of the efficiency component would be problematic [19]. The main aim is to compare these models for different distributions of the inefficiency term using the MCMC method and to rank countries according to their technical efficiency with the best model. A comparison of models with different distributional assumptions was performed using the DIC.

While there are numerous advantages to employing the Bayesian approach, its application in the context of energy efficiency has been limited. Our research seeks to fill this gap by conducting a comprehensive analysis of 29 OECD countries' energy-based development performance using SFA with a Bayesian approach. One specific aspect that has received insufficient attention in the literature is the selection of the distribution for the inefficiency term. The inefficiency term represents the unexplained deviation from the production frontier and plays a crucial role in accurately measuring energy efficiency. Despite its importance, no apparent method for selecting the distribution of the inefficiency term has been established in the literature. To address this gap, we propose and compare different models with various inefficiency components, including the half normal, truncated normal, exponential distribution, and gamma distribution. By considering these alternative models, we aim to explore the impact of different distributional assumptions on the measurement of energy efficiency. To conduct our analysis, we utilized a panel dataset spanning from 2004 to 2010. The Bayesian implementation of the proposed models is performed using the WinBUGS package, employing the Markov chain Monte Carlo (MCMC) method.

The primary objective of our study is to compare the performance of these models, each assuming a different distribution for the inefficiency term, using the deviance information criterion (DIC). The DIC provides a robust basis for model comparison, enabling us to identify the model that best fits the data and captures the true energy efficiency scores of the countries. By addressing this research gap and employing a Bayesian SFA approach, we contribute to the existing body of knowledge on energy efficiency analysis. Our findings will not only shed light on the most appropriate model for measuring energy efficiency but also allow us to rank countries based on their technical efficiency using the identified best model.

The paper proceeds as follows: Section 2 describes the Bayesian stochastic frontier model. Section 3 shows different models with different inefficiency components, such as the half normal, truncated normal, exponential distribution, and gamma distribution. Section 4 presents the result from the model estimation. Section 5 provides further discussion of the results, and the conclusion summarises the results and provides directions for future research.

2. Bayesian Stochastic Frontier Model

The SFA approach [1] can be illustrated as the following equation:

$$\begin{aligned} y_i &= X_i\beta + \varepsilon_i, \\ \varepsilon_i &= v_i - u_i, \\ u_i &\geq 0, \end{aligned} \tag{1}$$

where y_i is the log of output for DMU i ($i = 1, 2, \ldots, N$), X_i is a vector of input variables, β is the vector of coefficients, v_i is a symmetric disturbance capturing measurement error in the stochastic frontier, the error term is independent and identically distributed (IID), and u_i is a nonnegative disturbance capturing the level of DMU inefficiency ($u_i \geq 0$). The error term $\varepsilon_i = v_i - u_i$ has a symmetric distribution. There needs to be more clarity about the inefficiency of term distribution. A particular distributional assumption on u is needed. In the literature on efficiency estimation, four distributional assumptions have been proposed, namely, an exponential distribution [2], a half-normal distribution [1], a half-truncated normal distribution [24], and a gamma distribution [19]. The posterior distribution is shown in the following equation:

$$P\left(\beta, \sigma^2, u, \frac{\theta}{y}\right) \propto P\left(\frac{y}{X}, \beta, u, \sigma^2\right) \prod_{i=1}^{N} P\left(\frac{u_i}{\theta}\right) P(\beta) P\left(\sigma^2\right) P(\theta), \tag{2}$$

where β is the set of coefficients in the production function, θ is the set of parameters in the prior distribution, and X is the matrix with logarithms of the input variables. The complete conditional distributions are given by

$$\begin{aligned} P\left(\frac{\beta}{y}, \theta, \sigma^2, u\right) &\propto P\left(\frac{y}{X}, \beta, u, \sigma^2\right) P(\beta), \\ P\left(\frac{u_i}{y}, \beta, \theta, \sigma^2\right) &\propto P\left(\frac{y_i}{x_i}, \beta, \sigma^2\right) P\left(\frac{u_i}{\theta}\right), \\ P\left(\frac{\sigma^2}{y}, \beta, \theta, u,\right) &\propto P\left(\frac{y}{X}, \beta, u, \sigma^2\right) P\left(\sigma^2\right), \\ \text{the } P\left(\frac{\theta}{y}, \beta, \sigma^2, u\right) &\propto \prod_{i=1}^{N} P\left(\frac{u_i}{\theta}\right) P(\theta). \end{aligned} \tag{3}$$

3. Models

In this study, we adopt a preferred aggregate energy demand model for a panel of OECD countries, which was previously utilized in [21]. The model, as depicted in equation (5), relies on an unbalanced dataset encompassing a sample of 29 OECD countries from 2004 to 2010. The dataset is sourced from the International Energy Agency (IEA) database and the OECD database. We use energy consumption (EC) as the dependent variable and gross domestic product (GDP), the accurate price of energy for households and industry (RPE), the area size of a country measured in squared km (ASC), the share of value added of the industrial sector (SVAIS), and the share of value added for the service sector (SVASS) as independent variables. We focus on comparing these production functions and different distributions of the efficiency term. We adopt a Bayesian approach and use Markov chain Monte Carlo (MCMC) simulation to estimate parameters and compare models. Therefore, we propose different models with different inefficiency components as the half normal, truncated normal, exponential distribution, and gamma distribution. The proposed model is specified as the following equations:

$$u_i \overset{i.i.d.}{\sim} \mathrm{Exp}(\lambda), \tag{4a}$$

$$u_i \overset{i.i.d.}{\sim} N^{+}(0, \lambda), \tag{4b}$$

$$u_i \overset{i.i.d.}{\sim} N^{+}(\psi, \lambda), \tag{4c}$$

$$u_i \overset{i.i.d.}{\sim} Ga(\varphi, \lambda), \tag{4d}$$

$$\begin{aligned} Y_{it} = {} & \beta_0 + \beta_1 X1_{it} + \beta_2 X2_{it} + \beta_3 X3_{it} \\ & + \beta_4 X4_{it} + \beta_5 X5_{it} + \beta_6 t + v_{it} - u_{it}, \end{aligned} \tag{5}$$

where for ith country in tth year, Y_{it} is the logarithm of energy consumption (EC), $X1_{it}$ is the logarithm of GDP, $X2_{it}$ is the logarithm of the accurate price of energy for households and industry (RPE) (2005 = 100), $X3_{it}$ is the logarithm of the area size of a country measured in squared km (ASC), $X4_{it}$ is the share of value added of the industrial sector (SVAIS), $X5_{it}$ is the share of value added for the service sector (SVASS), t is a time trend, v_{it} is a symmetric disturbance representing the effect of noise, and u_{it} is a term for inefficient energy use. Descriptive statistics for the variables used in the model are presented in Table 1.

Table 1: Descriptive statistics for the variables.

Variable descriptions	Name	Mean	Std. dev.	Minimum	Maximum
Energy consumption (ktoe)	EC	132213.4	294020.13	2537.987	1789290
GDP (billion US 2005$PPP)	GDP	894.762	1716.2709	9.576	12617.02
Real price of energy (2005 = 100)	RPE	109.615	18.2262	59.9872	190.736
Area size in km^2	ASC	1396835	3092748.6	2900.8	11182830
Share of industrial sector in % of GDP	SVAIS	34.507	5.7942	17.696	50.176
Share of service sector in % of GDP	SVASS	70.279	7.5036	49.392	93.968

Appropriate prior specifications for the parameters need to be included. Various suggestions for prior choices have been made in the literature (e.g. [33, 34]). We used the same priors proposed by Griffin and Steel [20]. We define the prior distribution for the parameters in θ that all parameters are independent and $\beta_j \sim \mathrm{N}(0, \sigma_\beta^2)$. We assigned prior $\lambda \sim \mathrm{Exp}(-\log r^*)$, $(r^* \in (0, 1))$ for a half-normal distribution supposing that $u_i \sim \mathrm{Exp}(\lambda)$. We also used a Gamma prior for the precision; that is, $\lambda^{-1} \sim Ga(c, d)$, a truncated normal distribution is assumed. We enclosed a normal prior for the location, that is, $\varepsilon' \sim N(0, \varepsilon'^2)$, and prefer the same prior for λ as in $\lambda^{-1} \sim Ga(c, d)$. Our model used the WinBUGS package program to implement the Bayesian implementation. For all our applications, the MCMC algorithm involved 32,001 MCMC iterations where the first 15,000 were discarded in a burn-in phase. We used the deviance information criterion (DIC), which was introduced in [35] and commonly used in Bayesian analysis, to evaluate the models defining the deviance of a model with parameters θ as follows:

$$D(\theta) = -2 \log p\left(\frac{y}{\theta}\right), \qquad (6)$$

then the DIC is

$$\mathrm{DIC} = D + pD, \qquad (7)$$

where D is the expected deviance and pD is a complex term such that

$$pD = D - D(\overline{\theta}), \qquad (8)$$

where $\overline{\theta}$ is the mean of the posterior parameter distribution. The DIC can be evaluated automatically within the WinBUGS setup, and a good description of its use in stochastic frontier models can be seen in the study by Griffin and Steel [34]. Before using the Bayesian approach results, it is necessary to check the convergence assessment which involves checking whether the chain is converged. This study considers several statistical diagnostic tests for Markov chain convergence, such as Gelman, Rubin, Geweke, and Raftery–Lewis. The diagnostic statistics indicate that the Markov chain has reached convergence for each parameter for all models using different convergence methods such as Geweke, Gelman, Rubin, and Raftery–Lewis diagnostics.

4. Results and Discussion

Posterior summaries and densities for the frontier model in equation (4a), after running the MCMC algorithm for 47,000 iterations and discarding the initials 15,000, are shown in Table 1. It presents the posterior mean, standard deviation (SD), and the 95% prediction intervals of the parameters β's in model 1. We also obtain the MC (Monte Carlo) error to see if the convergence is satisfied and simulate that the MC error for each parameter is less than 5% of the sample SD. One way to assess the accuracy of the posterior estimates is by calculating the MC error for each parameter. This is an estimate of the difference between the mean of the sampled values and the true posterior mean. As seen in Table 2, the MC error for each parameter is less than 5% of the sample SD.

If the prediction interval passes through zero, one can conclude that the parameter is not significant.

From the table, it is clear that the only four significant coefficients are the ones associated with the gross domestic product (β_1), the accurate price of energy for households and industry (β_2), the area size of a country measured in squared km (β_3), and time trend (β_6), confirming the previous results by Table 3 which compares the posterior results from all models.

The average annual efficiency of countries in terms of distributions with the TIME variant is presented in Table 4. It shows a summary of the posterior distribution for the countries in the sample (high rank corresponds to high efficiencies). The posterior distribution clearly demonstrates a large spread of the rankings. From these, it is observed that the mean efficiency values are in the range of 0.482–0.921 for the exponential distribution, 0.489–0.868 for the gamma distribution, 0.493–0.914 for half-normal distribution, and 0.372–0.901 for the truncated normal distribution. The average technical efficiency scores imply that on average, the countries were producing about 49.6%, 72.0%, 74.5%, and 66.2% of the outputs that could be produced using the observed input quantities using exponential, gamma, half-normal, and truncated normal distributions, respectively. The half-normal distribution gave higher technical efficiency estimates than the other distributions.

In this study, 29 OECD countries' energy-based development performances were measured using stochastic frontier analysis with a Bayesian approach. To do this, four different models have been considered. According to these models, the energy efficiency scores of the countries have been estimated, and according to these scores, rankings of efficiency have been done. In Table 4, "*" indicates countries with different average energy efficiency rankings for different distributions. Table 5 examines the correlation among efficiency rankings for all the models using Spearman's rank correlation coefficient.

Table 2: Bayesian estimated parameters of the stochastic production frontier.

Nodes	Mean	Sd	MC error	2.5%	Median	97.5%	Start	Sample
Constant*	8.40900	1.397000	0.04763	5.75000	8.38000	11.24000	15000	32001
GDP*	0.00076	0.000100	$1.10E-07$	0.00051	0.00063	0.00073	15000	32001
RPE*	−0.00231	0.000200	$4.25E-04$	−0.02690	−0.00245	−0.00087	15000	32001
ASC*	0.00003	0.000003	$9.91E-10$	0.00002	0.00003	0.00005	15000	32001
SVAIS	0.00340	0.002630	$2.65E-04$	−0.01274	0.00334	0.01725	15000	32001
SVASS	0.00160	0.001300	$1.23E-04$	−0.00699	0.00170	0.00760	15000	32001
TIME*	0.00770	0.000710	$2.19E-06$	0.00699	0.00740	0.00960	15000	32001
Lambda	1.37250	0.012300	0.0000023	0.98100	1.24100	1.48100	15000	32001

"*"Statistically significant parameters.

Table 3: Summary of posterior results from all models.

Distributions		Constant	GDP	RPE	ASC	SVAIS	SVASS	TIME
	0.025	4.79	0.00041	−0.0217	0.00012	−0.0161	−0.0027	0.0016
Half normal	Median	7.93	0.00054	−0.0017	0.00037	0.0012	0.0021	0.0071
	0.975	10.83	0.00069	−0.0006	0.00051	0.0165	0.0093	0.0103
	0.025	5.43	0.00051	−0.0269	0.00034	−0.0159	−0.0085	0.0048
Truncated normal	Median	8.11	0.00063	−0.0025	0.00036	0.0012	0.0063	0.0068
	0.975	10.93	0.00073	−0.0009	0.00097	0.0230	0.0098	0.0091
	0.025	5.75	0.00051	−0.0269	0.00001	−0.0127	−0.0069	0.0069
Exponential	Median	8.38	0.00063	−0.0025	0.00003	0.0033	0.0017	0.0074
	97.50%	11.24	0.00073	−0.0009	0.00005	0.0173	0.0076	0.0096
	2.50%	5.69	0.00051	−0.0269	0.00023	−0.0187	−0.0032	0.0021
Gamma	Median	8.21	0.00063	−0.0025	0.00032	0.0086	0.0068	0.0042
	97.50%	11.03	0.00081	−0.0007	0.00059	0.0181	0.0099	0.0079

Table 4: Comparison of the average energy efficiency score and rankings for all models of the whole period.

Countries	Exponential	Rank	Gamma	Rank	Half normal	Rank	Truncated normal	Rank
Australia	0.786	18	0.720	18	0.742	18	0.648	18
Austria	0.794	17	0.723	17	0.745	17	0.650	17
Belgium	0.904	3	0.843	3	0.886	3	0.853	3
Canada*	0.855	8	0.792	6	0.831	7	0.766	7
Czech Republic	0.892	4	0.832	4	0.877	4	0.844	4
Denmark	0.666	25	0.622	25	0.629	25	0.517	25
Finland	0.921	1	0.868	1	0.914	1	0.901	1
France	0.846	9	0.773	9	0.802	9	0.729	9
Germany*	0.823	12	0.749	13	0.775	13	0.688	13
Greece*	0.799	16	0.748	14	0.768	15	0.683	14
Hungary*	0.799	15	0.733	16	0.753	16	0.663	16
Ireland	0.650	26	0.609	26	0.618	26	0.503	26
Italy	0.724	22	0.668	22	0.677	23	0.571	22
Japan*	0.816	13	0.741	15	0.769	14	0.682	15
Korea, Rep.	0.909	2	0.848	2	0.896	2	0.873	2
Luxembourg	0.827	11	0.757	11	0.788	11	0.710	11
Mexico*	0.482	29	0.489	29	0.495	28	0.372	29
Netherlands*	0.858	7	0.786	8	0.819	8	0.750	8
New Zealand*	0.860	6	0.791	7	0.833	6	0.779	6
Norway*	0.709	23	0.657	23	0.678	22	0.568	23
Poland	0.776	19	0.712	19	0.731	19	0.636	19
Portugal	0.699	24	0.651	24	0.658	24	0.553	24
Slovak Republic	0.831	10	0.765	10	0.799	10	0.726	10
Spain	0.749	20	0.688	20	0.702	20	0.601	20
Sweden	0.884	5	0.815	5	0.855	5	0.798	5
Switzerland	0.620	27	0.594	27	0.611	27	0.487	27
Turkey*	0.486	28	0.490	28	0.493	29	0.373	28
United Kingdom	0.735	21	0.675	21	0.689	21	0.588	21
United States*	0.815	14	0.749	12	0.781	12	0.699	12

Table 5: Spearman rank correlation coefficient.

	Exponential	Gamma	Half normal
Gamma	0.995		
Half normal	0.997	0.998	
Truncated normal	0.996	1.000	0.999

Table 6: Comparison of models with different distributional assumptions using the DIC.

Error distribution	Inefficiency distribution	DIC
Normal	Exponential	947.83
	Gamma	801.77
	Half normal	687.56
	Truncated normal	1245.15

Correlation coefficients between the efficiency rankings for all models are high and positive (minimum 0.995). In this case, there is a strong positive relationship between the orders of all models, and this relationship is statistically significant at even a 1% significance level.

Table 6 compares the DIC scores for the error and the different inefficiency distributions. The DIC is an attractive alternative to the Bayes factor; it is highly reliable and can handle complicated models (see [35, 36]). A lower value of DIC indicates a better fitting model. Overall, the results favour the half-normal distribution. Therefore, we use this model for the final decision to rank countries' energy efficiency scores for the whole period.

All models generally gave similar efficiency scores and orders of efficiency except for some minor changes, such as Canada and Greece shared values with similar events. Based on technical efficiency, the most influential country was Finland and the lowest effective was Mexico.

5. Conclusion

In our study, we aimed to address a notable gap in the existing literature on energy efficiency analysis. Over the past two decades, stochastic frontier analysis (SFA) has been widely used to assess energy efficiency. While there are numerous advantages to employing the Bayesian approach, its application in the context of energy efficiency has been limited. Our research seeks to fill this gap by conducting a comprehensive analysis of 29 OECD countries' energy-based development performance using SFA with a Bayesian approach. One specific aspect that has received insufficient attention in the literature is the selection of the distribution for the inefficiency term. The inefficiency term represents the unexplained deviation from the production frontier and plays a crucial role in accurately measuring energy efficiency. Despite its importance, no apparent method for selecting the distribution of the inefficiency term has been established in the literature. To address this gap, we proposed and compared different models with various inefficiency components, including the half normal, truncated normal, exponential distribution, and gamma distribution. By considering these alternative models, we explored the impact of different distributional assumptions on the measurement of energy efficiency. We utilized a panel dataset spanning from 2004 to 2010. The Bayesian implementation of the proposed models is performed using the WinBUGS package, employing the Markov chain Monte Carlo (MCMC) method.

We compared the performance of these models, each assuming a different distribution for the inefficiency term, using the deviance information criterion (DIC). The DIC provides a robust basis for model comparison, enabling us to identify the model that best fits the data and captures the true energy efficiency scores of the countries. By addressing this research gap and employing a Bayesian SFA approach, we contributed to the existing body of knowledge on energy efficiency analysis. Our findings not only shed light on the most appropriate model for measuring energy efficiency but also allow us to rank countries based on their technical efficiency using the identified best model.

In recent literature, several attempts have been made to overcome the main weaknesses of the field by evolving more specification and estimation procedures. The use of Bayesian techniques endows the researcher with the tools to use more flexible models, and it is not needed to impose a priori distributional assumptions on the efficiency term in the framework of stochastic frontier approaches.

In summary, our study extended the application of Bayesian SFA in energy efficiency analysis, addressed the gap in the selection of the inefficiency term distribution, and provided valuable insights into the relative energy efficiency performance of 29 OECD countries.

According to the convergence criteria such as Gelman, Rubin, Geweke, and Raftery–Lewis, the convergence of all parameters of each model was granted. For all models, GDP, RPE, ASC, and TIME were found as statistically significant parameters, while the others were found to be insignificant. The model with half-normal inefficiency distribution gave the most minimum DIC score. Therefore, we use this model to rank countries' energy efficiency scores. According to the model with half-normal inefficiency, based on technical efficiency, the most influential country was Finland and the lowest effective was Mexico. Since Mexico and Turkey are the lowest efficient countries, they should reconsider their energy policy and take precautions to improve energy efficiency. For instance, energy intensity and losses in the industry should be reduced. Energy should be used effectively and efficiently in the public sector. Future studies may consider assessing the productivity of more countries over a broader period and identify future strategies for improvement. This study covers only OECD countries. A more comprehensive study can be conducted using a larger dataset, or more detailed research can be conducted by considering the leading countries in the field of energy. In summary, the results of this study provide a reference for managers and policymakers in the energy-based development performance and show the way forward for future strategies and investments.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] D. Aigner, C. Lovell, and P. Schmidt, "Formulation and estimation of stochastic Frontier production function models," *Journal of Econometrics*, vol. 6, no. 1, pp. 21–37, 1977.

[2] W. Meeusen and J. van Den Broeck, "Efficiency estimation from cobb-douglas production functions with composed error," *International Economic Review*, vol. 18, no. 2, pp. 435–444, 1977.

[3] M. Filippini and L. C. Hunt, "Energy demand and energy efficiency in the OECD countries: a stochastic demand Frontier approach," *Energy Journal*, vol. 32, no. 2, pp. 59–80, 2011.

[4] P. Zhou, B. W. Ang, and H. Wang, "Energy and CO2 emission performance in electricity generation: a non-radial directional distance function approach," *European Journal of Operational Research*, vol. 221, no. 3, pp. 625–635, 2012.

[5] M. Filippini, L. C. Hunt, and J. Zorić, "Impact of energy policy instruments on the estimated level of underlying energy efficiency in the EU residential sector," *Energy Policy*, vol. 69, pp. 73–81, 2014.

[6] G. Marin and A. Palma, "Technology invention and adoption in residential energy consumption: a stochastic Frontier approach," *Energy Economics*, vol. 66, pp. 85–98, 2017.

[7] S. Honma and J. L. Hu, "Industry-level total-factor energy efficiency in developed countries: a Japan-centered analysis," *Applied Energy*, vol. 119, pp. 67–78, 2014.

[8] P. K. Adom, P. A. Kwakwa, and A. Amankwaa, "The long-run effects of economic, demographic, and political indices on actual and potential CO2 emissions," *Journal of Environmental Management*, vol. 218, pp. 516–526, 2018.

[9] B. K. Edziah, H. Sun, E. Anyigbah, and L. Li, "Human capital and energy efficiency: evidence from developing countries," *American Journal of Industrial and Business Management*, vol. 11, no. 06, pp. 599–610, 2021.

[10] H. G. Huntington, "Been top down so long it looks like bottom up to me," *Energy Policy*, vol. 22, no. 10, pp. 833–839, 1994.

[11] J. Buck and D. Young, "The potential for energy efficiency gains in the Canadian commercial building sector: a stochastic frontier study," *Energy*, vol. 32, no. 9, pp. 1769–1780, 2007.

[12] G. A. Boyd, "Estimating plant level manufacturing energy efficiency with stochastic frontier regression," *Energy Journal*, vol. 29, no. 2, pp. 23–44, 2008.

[13] J. van den Broeck, G. Koop, J. Osiewalski, and M. F. J. Steel, "Stochastic frontier models: a bayesian perspective," *Journal of Econometrics*, vol. 61, no. 2, pp. 273–303, 1994.

[14] G. Koop, M. F. Steel, and J. Osiewalski, *Posterior Analysis of Stochastic Frontier Models Using Gibbs Sampling*, Universidad Carlos III de Madrid, Getafe, Spain, 1992.

[15] Y. Kim and P. Schmidt, "A review and empirical comparison of bayesian and classical approaches to inference on efficiency levels in stochastic frontier models with panel data," *Journal of Productivity Analysis*, vol. 14, no. 2, pp. 91–118, 2000.

[16] L. A. Kurkalova and A. Carriquiry, "An analysis of grain production decline during the early transition in Ukraine: a Bayesian inference," *American Journal of Agricultural Economics*, vol. 84, no. 5, pp. 1256–1263, 2002.

[17] K. C. Ennsfellner, D. Lewis, and R. I. Anderson, "Production efficiency in the Austrian insurance industry: a Bayesian examination," *Journal of Risk & Insurance*, vol. 71, no. 1, pp. 135–159, 2004.

[18] D. Bernstein, C. Parmeter, and M. Tsionas, "On the performance of the United States nuclear power sector: a bayesian approach," *Energy Economics*, vol. 125, Article ID 106884, 2023.

[19] E. G. Tsionas, "Combining DEA and stochastic Frontier models: an empirical Bayes approach," *European Journal of Operational Research*, vol. 147, no. 3, pp. 499–510, 2003.

[20] J. E. Griffin and M. Steel, "Bayesian stochastic Frontier analysis using WinBUGS," *Journal of Productivity Analysis*, vol. 27, no. 3, pp. 163–176, 2007.

[21] E. G. Tsionas and E. N. Papadakis, "A Bayesian approach to statistical inference in stochastic DEA," *Omega*, vol. 38, no. 5, pp. 309–314, 2010.

[22] B. M. Tabak and P. Langsch Tecles, "Estimating a Bayesian stochastic Frontier for the Indian banking system," *International Journal of Production Economics*, vol. 125, no. 1, pp. 96–110, 2010.

[23] A. Tonini, "A Bayesian stochastic Frontier: an application to agricultural productivity growth in European countries," *Economic Change and Restructuring*, vol. 45, no. 4, pp. 247–269, 2012.

[24] G. Feng and X. Zhang, "Productivity and efficiency at large and community banks in the US: a Bayesian true random effects stochastic distance Frontier analysis," *Journal of Banking & Finance*, vol. 36, no. 7, pp. 1883–1895, 2012.

[25] A. G. Assaf and A. Josiassen, "Time-varying production efficiency in the health care food service industry: a Bayesian method," *Journal of Business Research*, vol. 65, no. 5, pp. 617–625, 2012.

[26] A. George Assaf, R. Matousek, and E. G. Tsionas, "Turkish bank efficiency: Bayesian estimation with undesirable outputs," *Journal of Banking & Finance*, vol. 37, no. 2, pp. 506–517, 2013.

[27] C. P. Barros, "Airports and tourism in Mozambique," *Tourism Management*, vol. 41, pp. 76–82, 2014.

[28] W. H. Greene, "A gamma-distributed stochastic Frontier model," *Journal of Econometrics*, vol. 46, no. 1-2, pp. 141–163, 1990.

[29] H. S. Migon and L. A. T. Medrano, "Deviance-based criteria for comparing Bayesian stochastic production Frontier models," Technical Report 176, Universidade Federal do Rio de Janeiro, Rio de Janeiro, Brazil, 2004.

[30] M. Alghalith, "An alternative method of stochastic optimization: the portfolio model," *Applied Mathematics*, vol. 02, no. 07, pp. 912-913, 2011.

[31] R. S. Ehlers, "Comparison of Bayesian models for production efficiency," *Journal of Applied Statistics*, vol. 38, no. 11, pp. 2433–2443, 2011.

[32] T. J. Coelli, D. S. P. Rao, and G. E. Battese, *An Introduction to Efficiency and Productivity Analysis*, Kluwer Academic Publishers, Boston, MA, USA, 1998.

[33] E. G. Tsionas, "Full likelihood inference in normal-gamma stochastic Frontier models," *Journal of Productivity Analysis*, vol. 13, no. 3, pp. 183–205, 2000.

[34] J. E. Griffin and M. F. Steel, "Semiparametric Bayesian inference for stochastic Frontier models," *Journal of Econometrics*, vol. 123, no. 1, pp. 121–152, 2004b.

[35] S. Y. Lee and X. Y. Song, "Bayesian analysis of structural equation models with dichotomous variables," *Statistics in Medicine*, vol. 22, no. 19, pp. 3073–3088, 2003.

[36] D. J. Spiegelhalter, N. G. Best, B. P. Carlin, and A. van der Linde, "Bayesian measures of model complexity and fit," *Journal of the Royal Statistical Society Series B: Statistical Methodology*, vol. 64, no. 4, pp. 583–639, 2002.