Hindawi
Journal of Mathematics
Volume 2023, Article ID 5862652, 12 pages
https://doi.org/10.1155/2023/5862652

Research Article

Mathematical Modeling of Feelings in Viewpoint of Analysis of Olvido Poetry with Fractional Operators

Mustafa Ali Dokuyucu,[1] **Burak Armağan**,[2] **Ülkü Eliuz**,[3] **Ahmet Ocak Akdemir**,[1] **and Mehmet Emir Köksal**[4]

[1]*Department of Mathematics, Ağrı İbrahim Çeçen University, Ağrı, Turkey*
[2]*Department of Turkish Language and Literature, Ağrı İbrahim Çeçen University, Ağrı, Turkey*
[3]*Department of Turkish Language and Literature, Karadeniz Technical University, Trabzon, Turkey*
[4]*Department of Mathematics, Ondokuz Mayis University, Atakum, Samsun 55139, Turkey*

Correspondence should be addressed to Mehmet Emir Köksal; emirkoksal@gmail.com

Received 17 February 2023; Revised 7 April 2023; Accepted 21 April 2023; Published 10 May 2023

Academic Editor: Yusuf Gurefe

The main motivation point of the article and the main factor that provides originality are an analysis of the modeling of feelings in the perspective of the analysis of Olvido's poetry with the help of two fractional operators that offer the most effective solutions to real-world problems. In the study, first of all, the effect and contribution of fractional analysis to be applied and theoretical sciences were examined. A mathematical model of the feelings described in Olvido's poem has been created to establish an important connection between fractional operators, which are an effective tool in the explanation of many physical and applied phenomena, and literary works that try to describe human life and feelings. The solutions of this model are investigated with the help of Caputo and Atangana–Baleanu fractional derivative operators. The findings were supported by various graphics in the light of special values of the parameters of the operators. The study focuses on generalizing the effects of literature research on human life to a general framework, not only providing the realization of the outputs that the literature field brings to human life and literature with the help of a mathematical model. The study has a rare content in the literature in terms of bringing together poetry and mathematics, which deal with human life with different methodologies.

1. Introduction

Fractional analysis has led to the emergence of very effective processes in the solutions of mathematically modeled problems in recent years, and it has brought a new momentum to mathematics and applied sciences. Fractional analysis offers more effective solutions than classical analysis methods in terms of the process effect or the problems that classical analysis is insufficient to explain. It has become a field that is intensively studied today with the acquisition of new derivative and integral operators. In [1], the authors have investigated a fractional tumor-immune interaction model and analyzed the solutions by using some well-known solution methods.

While mathematics and applied sciences show this development towards solving problems related to nature, human life with a complex structure has been tried to be explained by describing processes and situations in social sciences. Literature, with its various subdisciplines, has contributed to science with research aimed at describing human life in a cumulative and systematic way since the existence of human beings. In particular, the studies based on events and human feelings aimed to explain the phenomenon in the universe and make predictions by analyzing the human being and universe which have a complex structure. One of the genres in which feelings and thoughts are expressed most effectively is poetry. Poems are a genre that progresses under the influence of all environmental factors of the period, and it has survived to this day by hosting all aspects of life.

In our study, we will continue by referring to the renewal period, which has an important place in the development stages of Turkish poetry, especially considering the efforts of Turkish poetry in this direction. There are pure poetry efforts in the middle of the 20th century in the poetry branch among the artists who produce works on the axis of various understandings in the Turkish literature during the renewal period. Poets, who aim to reach pure poetry by staying out of all kinds of ideological deviations, manage to stay out of all kinds of schooling tendencies. They exalt the language within the framework of the idea of reaching the peak of poetic discourse with the aim of writing well and beautifully [2]. Ahmet Muhip Dıranas is among the artists who produce works in line with pure poetry. Dıranas, who can affect both his own generation and the next generations with his lyrical poetry and deserves the title of poet of every generation [3], published his only poetry book, Poems, in 1974. The poet, who managed to sprout in the lines of Ahmet Hamdi Tanpınar and Yahya Kemal Beyatlı and reach the original poetry line, puts his signature under simple, comfortable, and impressive utterances in the transfer of events and feelings in the Poems. Among them, Olvido is one of his most qualified poems that have a place in the memories. Let us remember the poem "Olvido" as follows and then the analysis of the poem in the viewpoint of feelings:

Olvido
These evenings are always vulgar
Once the day is gone with its splendor
Filling up everywhere with our loneliness
In a scream of colors from our garden
A hand starts to take out from our pack
The sorrows that have the smell of lavender
These evenings are always vulgar
Regrets, attacking in waves
Force that bronze door of oblivion
And the soul, full of holes with the arrows shot
Here, all of a sudden, you are in the old house you were born in
The lamp and the stairs are waiting for you
The cradle is creaking with silenced lullabies
And all the lost, defeated, crestfallen
It is with the beauty of unspoken love
The poems left incomplete on papers
One, towards a morning that has the smell of rain
Remembers one day that he opened a window
A cloud holding still, a bird flying
A stone that he knelt down and ate cheese and bread on
All of these are with the beauty of love
Love must have flown away with summer
Like girls dancing the halay holding arm in arm
How about you? The skirts of the past times
Drifting away like the moonlight
From the hidden gardens with old trees
Skirts that swing with whispers, coyness
Leaving the weary men into the night
Waiting for the return of the eternal lover
The flowers, the witnesses of false oaths
In springs that will no longer be there
Oh, deception, the most beautiful song of life
Be deceived, though the hopeless winter has arrived
All lonely footsteps are covered with snow
The flowers, sprinkled by the lover who hasn't returned
And you! oh you! Among the blowing branches
Twinkling like a sparkle
What do you ask of me in this evening hour
The woman with no smiles ever seen
How immortal you are in the mirror of love
In this time of waking up of memories
It is always you, you, among the blowing branches
Oh oblivion! close your window now
The sea already dragged me into its depth
That world will no longer come out of water
A smoke seems to be rising from sorrows
From those things whose adventure is already over
Spread all over me with your deadly night
Oh oblivion! Save me from all these griefs
Ahmet Muhip Dıranas

Olvido is the poem of remembering, although its name is oblivion. "In his famous poem Olvido, which contains the features of the Dıranas poem, a symbolic fairy tale language was used, the poet took refuge in an imaginary and unrequited love in his childhood, which was known to be very troublesome, and the poem ended with a desire for renunciation/forgetting and even unconscious death" [4]. A bitter smile appears in the poem, in which the intensity of feelings experienced by an aging person is reflected in his mind with the memories of his youth. All the memories expressed while shaping the conflict of remembering and forgetting the fiction are evaluated from the perspective of old age. The subject of the poet, who has reached the last stage of his life, makes an accounting of his life with longing, starting from childhood.

In the first stanza of Olvido, a time expression in the form of afternoon is seen as the driving force that revives the memories. The person, who cannot find the opportunity to listen to himself in the noise of the day, finds the opportunity to turn to his inner self in the afternoon, which is identified with silence. This section also refers to the last period of life. As a sign of the end of the day, the evening also evokes the end of life. The poet's evaluation of the afternoon as rough means that he accepts old age in the same way. The coarseness of the evening obviously turns into the coarseness of old age and includes the complaint of old age. The reason for this is shown as the loneliness in the third verse. Afternoon is a time when the scream of the day ceases and silence begins, and at this moment the subject of the poem realizes that he is alone. In the gradual system of correlations in relation to each other, the next step is for the events in the subconscious to come to light. Memories slowly come to life in the mind, and it smells of lavender flowers, even if it is sad. Because the recollection of memories, even if it is painful, is preferable to old age knitted with loneliness. Thus, although the revival of memories seems to be rejected in the first paragraph, it is actually requested within a naive system. Dıranas has created this section with a very meticulous workmanship because, in real life, he portrayed the situation of showing a moody reaction against a situation that the elderly who want attention, which can be seen intensely, with words. The subject of the poem knows that he can escape the loneliness and old age he complains about through memories, but he still considers the evening that reminds him of these to be rough. Word preferences such as afternoon, loneliness, and grief are black; day, color cry, garden, bundle, lavender flower expressions refer to light, brightness, and variety of colors. Thus, the conflict rings are also made evident on the color plane. Words that evoke black emphasize old age, while other colors emphasize youth. In this case, it can be said that a comparison is made between old age and youth, even by looking at the first body, and that old age is complained about and longed for youth.

The second paragraph covers the memories that are revived in the mind belonging to the childhood stage. This is the only part of the poem that both tells about childhood and gives great place to negativities. The subject of the poem, who finds himself in the house where he was born, lists all the units that he feels pain to remember and that hurt his soul. A childhood full of expectations is knitted with losses, defeats, and sorrows. This paragraph is suitable for two different interpretations. It can be said that the subject of the poem had a childhood spent with waiting and sadness, or, on the contrary, it can be said that the childhood spent with happiness cannot be regained and those stages are felt to be lost. At the same time, it can be thought that the poet, by being subjected to a psychoanalytic analysis, refers to Ahmet Muhip's return to his own childhood and watching the path of a second person, his mother/father. But in any case, the perception of this period, in which childhood is told, on the aging poetic subject is shaped within the framework of the word regret stated in the first verse and contains negative images.

The third stanza indicates a stage from childhood to youth. Love emerges as a special situation that starts here and is reflected in the fourth and fifth paragraphs. Negative feelings about childhood are interpreted with the love experienced in youth and all life conditions are turned into positive. The beautifying effect of love, which is expressed in the first verse, is revealed in a quality that gives life to an ordinary and still life and gives meaning to it. Even though love, which nurtures freedom and hope, is presented to the reader with the function of abandonment in the fourth paragraph, the phrase moonlight hidden between the lines refers to both romance and a hope that transcends darkness, and still the positive image of love is preserved. When it comes to the fifth body, waiting and deception are sanctified. Expectation and deception in youth correspond to keeping the dream alive. Although their transience and non-necessity have been noticed in old age, they have been shown as an indispensable element of the youth period, as it supports dreaming and being fit because the loss of life or hope means the loss of life expectancy and brings about a colorless, tasteless, and aimless life. Yahya Kemal Beyatlı, one of the pinnacle poets of modern Turkish poetry, expresses this situation at the end of his poem, "Walk! Until the last point where the free blue ends/Man lives in the world as long as he dreams" [5]. In this context, it can be said that waiting and deception are phenomena that seem negative but have positive qualities in terms of both youth and old age.

The sixth paragraph is the section of the general youth judgments drawn in the third, fourth, and fifth paragraphs, which are reduced to the subject of the poem. The subject, who remembers his own love, explains here that the one who repressed the feeling of loneliness expressed in the first paragraph is a woman he lost. While emphasizing the negativity of love, he makes a small reproach about remembering. Because his goal is not to complain about memories, but to make you feel the bitterness of not being able to reach the woman he loves, of being alone under the spell of unrequited love. At this point, as Eliuz and Seferoğlu mentioned, there is an inability to reach the goal, lack of satisfaction [6], and incompleteness due to not being able to reach the external instinct object.

In the seventh and last paragraph, a desperation is read as much as possible. The subject, who seeks help from forgetting, which can be considered as an undesirable situation, wants to end the act of remembering about youth, which is impossible to return. He is aware of the autumn he is living in and sees his current position as the seabed, the submerged world. He is aware that there is no possibility of getting out of this black world he has entered and mixing with colors. He turns to oblivion, which he obligedly affirms, takes advantage of its power, and demands that memories not be recalled from it. Because as he remembers, his loneliness in old age will become more evident and it is imperative that he forgets himself in order to save himself from loneliness. Even though the memories give pleasure, he knows that he is approaching the eternal darkness and that the colors he left behind and can no longer reach will not be a cure for the darkness. The subject, who cannot prevent the reminder games of his mind, wants to escape from the deadlock in this way. It is about accepting the loss and giving in.

The work is organized as follows: in Chapter 2, the development of the model is provided based on the poem, and the existence and uniqueness of the solution of the model obtained for the Caputo and Atangana–Baleanu fractional derivatives are analyzed separately. The solutions of the model developed in Chapter 3 are discussed with the help of simulations for the special parameter values of fractional operators in the context of the Olvido poem. In the last part, a general evaluation of the study was made.

2. Mathematical Model

Although fractional derivative and integral operators do not have a history as old as classical analysis, they have been an important subbranch of mathematics in the literature. Caputo introduced the fractional derivative operator in 1967. It has become one of the well-known fractional derivative operators in the literature. Then, Atangana and Baleanu introduced the fractional derivative and integral operator with a nonsingular and nonlocal kernel using the generalized Mittag–Leffler function in 2016. Detailed information about the definitions, theorems, and lemmas can be found in [7–9] studies.

In the literature, the mathematical model of some literary works has been created and analyzed. A new mathematical model has been proposed for the Olvido poem written by Ahmet Muhip Dıranas, which has not been mathematically modeled before. In analyzing the poem, the life of the subject of the poem was studied in three different stages. These stages are childhood, youth, and old age, respectively. The mathematical model created in the framework of this situation is as follows:

$$\begin{aligned}
\frac{dC}{dt} &= \alpha C(t),\\
\frac{dY}{dt} &= -\alpha C(t)Y(t) - \beta Y(t),\\
\frac{dO}{dt} &= \gamma C(t)O(t) + \delta Y(t)O(t) + \sigma O(t).
\end{aligned} \tag{1}$$

In the proposed model, α, γ, and σ are negative constants and β and δ are positive constants. Also, C, Y, and O are defined as childhood stage, youth stage, and old age stage, respectively. There is an important point that can be understood when the analysis of the poem is made. Ahmet Muhip Dıranas evaluates these three stages from the perspective of old age. α has been taken into account as negative feelings that affect his view of childhood in old age. These negative feelings were taken as waiting, losses, defeats, and sorrows. β has been taken into account as positive feelings that affect his view of youth in old age. These feelings were taken as love, freedom, togetherness, and expectations. γ has been taken into account as negative feelings in the view of childhood in old age. These negative feelings were taken as regrets, losses, defeats, sadness, and pain. δ has been taken into account as positive feelings in the youth period in old age. These positive feelings were taken as love, freedom, togetherness, and meanings. σ has been taken into account as negative feelings in the view of old age. These negative feelings were taken as loneliness, forgetting, remembering, longing, and sadness. The initial values will be taken as follows:

$$C_0(t) = C(0) > 0, Y_0(t) = Y(0) > 0, O_0(t) = O(0) > 0. \tag{2}$$

2.1. Existence and Uniqueness of Solution for Mathematical Model with Caputo Derivative. In the previous section, we proposed a mathematical model with the analysis of the Olvido poem. In this subsection, the mathematical model is extended to the Caputo fractional derivative. Then, the existence and uniqueness of the solutions are examined. When equation (1) is extended to the Caputo fractional derivative, the following system is obtained:

$$\begin{aligned}
{}_0^C D_t^\zeta C(t) &= \alpha C(t),\\
{}_0^C D_t^\zeta Y(t) &= -\alpha C(t)Y(t) - \beta Y(t),\\
{}_0^C D_t^\zeta O(t) &= \gamma C(t)O(t) + \delta Y(t)O(t) + \sigma O(t).
\end{aligned} \tag{3}$$

Let us present every continuous functions $G = C[a,b]$ in the Banach space defined in the closed set $[a,b]$ and consider $Z = \{C, Y, O \in G, C(t) \geq 0, Y(t) \geq 0 \text{ and } O(t) \geq 0, a \leq t \leq b\}$.

Definition 1. Let X be a Banach space with a cone H. H initiates a restricted order $\leq$ in E in the succeeding approach [10].

$$y \geq x \Longrightarrow y - x \in H. \tag{4}$$

Now applying the fractional integral in equation (4), the following system is obtained:

$$\begin{aligned}
C(t) - C(0) &= \frac{1}{\Gamma(\zeta)} \int_0^t (t-r)^{\zeta-1} [\alpha C(r)] \mathrm{d}r,\\
Y(t) - Y(0) &= \frac{1}{\Gamma(\zeta)} \int_0^t (t-r)^{\zeta-1} [-\alpha C(r)Y(r) - \beta Y(r)] \mathrm{d}r,\\
O(t) - O(0) &= \frac{1}{\Gamma(\zeta)} \int_0^t (t-r)^{\zeta-1} [\gamma C(r)O(r) + \delta Y(r)O(r) + \sigma O(r)] \mathrm{d}r.
\end{aligned} \tag{5}$$

We need to define an operator to show the existence of the solution of equation (5). Let $P: M \longrightarrow M$.

$$\begin{aligned}
\mathrm{PC}(t) &= \frac{1}{\Gamma(\zeta)}\int_0^t (t-r)^{\zeta-1} s(r, C(t,r))\mathrm{d}r,\\
\mathrm{PY}(t) &= \frac{1}{\Gamma(\zeta)}\int_0^t (t-r)^{\zeta-1} s(r, Y(t,r))\mathrm{d}r,\\
\mathrm{PO}(t) &= \frac{1}{\Gamma(\zeta)}\int_0^t (t-r)^{\zeta-1} s(r, O(t,r))\mathrm{d}r.
\end{aligned} \tag{6}$$

To be dealt with more easily, let us consider as follows:

$$\begin{aligned}
&s(t, C(t,r)) = \alpha C(t),\\
&s(t, Y(t,r)) = -\alpha C(t)Y(t) - \beta Y(t),\\
&s(t, O(t,r)) = \gamma C(t)O(t) + \delta Y(t)O(t) + \sigma O(t).
\end{aligned} \tag{7}$$

Lemma 1. *The mapping $P\colon M \longrightarrow M$ is completely continuous.*

Proof. Let $: K \subset M$ be bounded. There exists constants $x, y, z > 0$ such that $\|C\| < x, \|Y\| < y, \|O\| < z$. Let $\forall C, Y, O \in K$,

$$\begin{aligned}
N_1 &= \max_{\substack{0\le t\le 1\\ 0\le C\le x}} s(t, C(t)),\\
N_2 &= \max_{\substack{0\le t\le 1\\ 0\le Y\le y}} s(t, Y(t)),\\
N_3 &= \max_{\substack{0\le t\le 1\\ 0\le O\le z}} s(t, O(t)).
\end{aligned} \tag{8}$$

Firstly,

$$\|\mathrm{PC}(t)\| \le \frac{1}{\Gamma(\zeta)}\int_0^t (t-r)^{\zeta-1}\|s(r, C(t,r))\|\mathrm{d}r \le \frac{N_1}{\Gamma(\zeta)}\int_0^t (t-r)^{\zeta-1} dr = \frac{N_1}{\Gamma(\zeta+1)} t^{\zeta}. \tag{9}$$

In that case,

$$\|\mathrm{PC}\| \le \frac{N_1}{\Gamma(\zeta+1)}. \tag{10}$$

Secondly,

$$\|\mathrm{PY}(t)\| \le \frac{1}{\Gamma(\zeta)}\int_0^t (t-r)^{\zeta-1}\|s(r, Y(t,r))\|\mathrm{d}r \le \frac{N_2}{\Gamma(\zeta)}\int_0^t (t-r)^{\zeta-1}\mathrm{d}r = \frac{N_2}{\Gamma(\zeta+1)} t^{\zeta}. \tag{11}$$

In that case,

$$\|\mathrm{PY}\| \le \frac{N_2}{\Gamma(\zeta+1)}. \tag{12}$$

Finally,

$$\|\mathrm{PO}(t)\| \le \frac{1}{\Gamma(\zeta)}\int_0^t (t-r)^{\zeta-1}\|s(r, O(t,r))\|d \le \frac{N_3}{\Gamma(\zeta)}\int_0^t (t-r)^{\zeta-1}\mathrm{d}r = \frac{N_3}{\Gamma(\zeta+1)} t^{\zeta}. \tag{13}$$

In that case,

$$\|\mathrm{TO}\| \leq \frac{N_3}{\Gamma(\zeta+1)}. \tag{14}$$

Hence, $P(K)$ is bounded.

Now in the following part, we will consider $t_1 < t_2$ and $C(t), Y(t), O(t) \in M$ and then for a given $\epsilon > 0$ if $|t_2 - t_1| < \delta$, we have

$$\begin{aligned}
\left\|\mathrm{PC}(t_2) - \mathrm{PC}(t_1)\right\| &= \left\|\frac{1}{\Gamma(\zeta)}\int_0^{t_2}(t_2 - r)^{\zeta-1}[s(r, C(r))]\mathrm{d}r - \frac{1}{\Gamma(\zeta)}\int_0^{t_1}(t_1 - r)^{\zeta-1}[s(r, C(r))\mathrm{d}r]\right\| \\
&= \frac{1}{\Gamma(\zeta)}\int_0^{t_2}(t_2 - r)^{\zeta-1}\|s(r, C(r))\|\mathrm{d}r - \frac{1}{\Gamma(\zeta)}\int_0^{t_2}(t_1 - r)^{\zeta-1}\|s(r, C(r))\|\mathrm{d}r \\
&\quad - \frac{1}{\Gamma(\zeta)}\int_{t_1}^{t_2}(t_1 - r)^{\zeta-1}\|s(r, C(r))\|\mathrm{d}r \\
&\leq \frac{1}{\Gamma(\zeta)}\int_0^{t_2}\left\|(t_1 - r)^{\zeta-1}\right\|\|s(r, C(r))\|\mathrm{d}r \\
&\quad + \frac{1}{\Gamma(\zeta)}\int_{t_1}^{t_2}\left\|(t_1 - r)^{\zeta-1}\right\|\|s(r, C(r))\|\mathrm{d}r \\
&\leq \frac{N_1}{\Gamma(\zeta)}\int_0^{t_2}\left((t_2 - r)^{\zeta-1} - (t_1 - r)^{\zeta-1}\right)\mathrm{d}r + \frac{N_1}{\Gamma(\zeta)}\int_{t_1}^{t_2}(t_1 - r)^{\zeta-1}\mathrm{d}r \\
&= \frac{N_1}{\Gamma(\zeta)}\left(\int_0^{t_2}(t_2 - r)^{\zeta-1}\mathrm{d}r - \int_0^{t_2}(t_1 - r)^{\zeta-1}\mathrm{d}r + \int_{t_1}^{t_2}(t_1 - r)^{\zeta-1}\mathrm{d}r\right) \\
&= \frac{N_1}{\Gamma(1+\zeta)}\left(t_2^{\zeta} + (t_1 - t_2)^{\zeta} - t_1^{\zeta} + (t_1 - t_2)^{\zeta}\right) \\
&\leq \frac{2N_1}{\Gamma(1+\zeta)}(t_1 - t_2)^{\zeta} + \frac{N_1}{\Gamma(1+\zeta)}(t_1 - t_1)^{\zeta} \\
&= \frac{2N_1}{\Gamma(1+\zeta)}(t_1 - t_2)^{\zeta} \\
&< \frac{2N_1}{\Gamma(1+\zeta)}\delta^{\zeta} \\
&= \epsilon.
\end{aligned} \tag{15}$$

If the same steps are applied for $Y(t)$ and $O(t)$, the following equations can be easily obtained. As a result,

$$\begin{aligned}
\left\|\mathrm{PC}(t_2) - \mathrm{PC}(t_1)\right\| &\leq \epsilon, \\
\left\|\mathrm{PY}(t_2) - \mathrm{PY}(t_1)\right\| &\leq \epsilon, \\
\left\|\mathrm{PO}(t_2) - \mathrm{PO}(t_1)\right\| &\leq \epsilon,
\end{aligned} \tag{16}$$

where $\delta = (\epsilon\Gamma(1 + \zeta/2N_i))^{1/\zeta}$ and $i = 1, 2,$ and 3. Therefore, $P(K)$ is equicontinuous, so that $\overline{P(K)}$ is compact via the Arzela–Ascoli theorem.

Theorem 1. *Let* $\mathfrak{S}$: $[C_1, C_2] \times [0, \infty] \longrightarrow [0, \infty]$, *then* $\mathfrak{S}(t, .)$ *is nondecreasing for each* t *in* $[C_1, C_2]$. *There exist positive constants* ν_1 *and* ν_2 *such that* $\mathfrak{B}(n)\nu_1 \leq \mathfrak{S}(t, \nu_1)$, $\mathfrak{B}(n)\nu_2 \geq \mathfrak{S}(t, \nu_2)$, *and* $0 \leq \nu_1(t) \leq \nu_2(t), C_1 \leq t \leq C_2$. *This means that the new equation has a positive solution.*

Proof. If the fixed point theorem is used for the P operator, the operator P: $F \longrightarrow F$ is completely continuous, considering Lemma 1. Let us take two arbitrary C_1 and C_2,

$$PC_1(t) = \frac{1}{\Gamma(\zeta)}\int_0^t (t - r)^{\zeta-1}s(r, C_1(r))\mathrm{d}r \leq \frac{1}{\Gamma(\zeta)}\int_0^t (t - r)^{\zeta-1}s(r, C_2(r))\mathrm{d}r = PC_2(t). \tag{17}$$

Therefore, P is a nondecreasing operator. For this reason, the operator $P: \langle \nu_1, \nu_2 \rangle \longrightarrow \langle \nu_1, \nu_2 \rangle$ is compact and continuous via Lemma 3.1. In that case, F is a normal cone of P.

Let us now examine the uniqueness of the solution for the system of equation (23). So the uniqueness of the solution is presented as follows:

Firstly,

$$\begin{aligned}\|PC_1(x,t) - PC_2(x,t)\| &= \left\|\frac{1}{\Gamma(\zeta)}\int_0^t (t-r)^{\zeta-1}\left[s(r,C_1(r)) - s(r,C_2(r))\right]\mathrm{d}r\right\| \\ &\leq \frac{1}{\Gamma(\zeta)}\mathfrak{A}_1\int_0^t (t-r)^{\zeta-1}\|C_1(r) - C_2(r)\|\mathrm{d}r.\end{aligned} \tag{18}$$

So that,

$$\|PC_1(t) - PC_2(t)\| \leq \left(\frac{\mathfrak{A}_1 t^{\zeta}}{\Gamma(\zeta+1)}\right)\|C_1(r) - C_2(r)\|. \tag{19}$$

The following inequalities can also be easily obtained using the same method.

$$\begin{aligned}\|PY_1(t) - PY_2(t)\| &\leq \left(\frac{\mathfrak{A}_2 t^{\zeta}}{\Gamma(\zeta+1)}\right)\|Y_1(r) - Y_2(r)\|, \\ \|PO_1(t) - PO_2(t)\| &\leq \left(\frac{\mathfrak{A}_3 t^{\zeta}}{\Gamma(\zeta+1)}\right)\|O_1(r) - O_2(r)\|.\end{aligned} \tag{20}$$

Therefore, if the following conditions hold,

$$\begin{aligned}&\left(\frac{\mathfrak{A}_1 t^{\zeta}}{\Gamma(\zeta+1)}\right) < 1, \\ &\left(\frac{\mathfrak{A}_2 t^{\zeta}}{\Gamma(\zeta+1)}\right) < 1, \\ &\left(\frac{\mathfrak{A}_3 t^{\zeta}}{\Gamma(\zeta+1)}\right) < 1.\end{aligned} \tag{21}$$

Then, mapping P is a contraction, which implies fixed point, and thus the model has a unique positive solution.

2.2. Existence and Uniqueness of Solution for Mathematical Model with Atangana–Baleanu Fractional Derivative in the Sense Caputo Derivative. In the previous section, we examined existence and uniqueness solutions with Caputo fractional operator. We now show the existence and uniqueness solutions with Atangana–Baleanu fractional operator. When equation (1) is extended to the Atangana–Baleanu fractional derivative, the system is obtained:

$$\begin{aligned}{}_0^{ABC}D_t^{\zeta}C(t) &= \alpha C(t), \\ {}_0^{ABC}D_t^{\zeta}Y(t) &= -\alpha C(t)Y(t) - \beta Y(t), \\ {}_0^{ABC}D_t^{\zeta}O(t) &= \gamma C(t)O(t) + \delta Y(t)O(t) + \sigma O(t).\end{aligned} \tag{22}$$

When the definition of AB integral operator is applied to equation (22), the system of equations of the fractional integral form is obtained:

$$\begin{aligned}C(t) &= C(0) + {}_t^{ABC}I_0^{\zeta}[\alpha C(t)], \\ Y(t) &= Y(0) + {}_t^{ABC}I_0^{\zeta}[-\alpha C(t)Y(t) - \beta Y(t)], \\ O(t) &= O(0) + {}_t^{ABC}I_0^{\zeta}[\gamma C(t)O(t) + \delta Y(t)O(t) + \sigma O(t)].\end{aligned} \tag{23}$$

When equation (23) extended to Atangana–Baleanu integral form is rewritten as follows, the equation is obtained:

$$\begin{aligned}C(t) - C(0) &= \frac{1-\zeta}{B(\zeta)}\{\alpha C(t)\} + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-\omega)^{(\zeta-1)}\{\alpha C(\omega)\}d(\omega), \\ Y(t) - Y(0) &= \frac{1-\zeta}{B(\zeta)}\{-\alpha C(t)Y(t) - \beta Y(t)\} + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-\omega)^{(\zeta-1)}\{-\alpha C(\omega)Y(\omega) - \beta Y(\omega)d(\omega)\}, \\ O(t) - O(0) &= \frac{1-\zeta}{B(\zeta)}\{\gamma C(t)O(t) + \delta Y(t)O(t) + \sigma O(t)\} \\ &\quad + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-\omega)^{(\zeta-1)}\{\gamma C(\omega)O(\omega) + \delta Y(\omega)O(\omega) + \sigma O(\omega)\}d(\omega).\end{aligned} \tag{24}$$

The initial values of the extended mathematical model are as follows:

$$C_0(t) = C(0), Y_0(t) = Y(0), O_0(t) = O(0). \tag{25}$$

For convenience, the kernels will be taken as follows:

$$\begin{aligned}
&\Psi_1(t, C) = \alpha C(t),\\
&\Psi_2(t, Y) = -\alpha C(t)Y(t) - \beta Y(t),\\
&\Psi_3(t, O) = \gamma C(t)O(t) + \delta Y(t)O(t) + \sigma O(t),
\end{aligned} \tag{26}$$

and

$$\begin{aligned}
\Phi_1 &= \alpha,\\
\Phi_2 &= -\alpha b_1 - \beta,\\
\Phi_3 &= \gamma b_1 + \delta b_2 + \sigma.
\end{aligned} \tag{27}$$

For proving our results, we assume the following assumption: (H) for the following continuous functions $C(t), Y(t), O(t), C_1(t), Y_1(t),$ *and* $O_1(t) \in L[0, 1]$, such that $\|C(t)\| \leq b_1$, $\|Y(t)\| \leq b_2$, and $\|O(t)\| \leq b_3$.

Theorem 2. *The kernels $\Psi_i, i = 1, 2, 3$ satisfy the Lipschitz condition if the assumption H is true, and they are contractions, provided that $\Phi_i < 1$ for $\forall \in i = 1, 2, 3$.*

Proof. First of all, we begin to proof that $\Psi_1(t, C)$ satisfies the Lipschitz condition. Let $C(t)$ and $C_1(t)$ be two functions, then

$$\left\|\Psi_1(t, C) - \Psi_1\left(t, C_1\right)\right\| = \left\|\alpha C(t) - \alpha C_1(t)\right\| \leq (\alpha)\left\|C(t) - C_1(t)\right\| = \Phi_1\left\|C(t) - C_1(t)\right\|. \tag{28}$$

Then, we prove that $\Psi_2(t, Y)$ satisfies the Lipschitz condition. Let $Y(t)$ and $Y_1(t)$ be two functions, then

$$\begin{aligned}
\left\|\Psi_2(t, Y) - \Psi_2\left(t, Y_1\right)\right\| &= \left\|(-\alpha C(t)Y(t) - \beta Y(t)) - \left(-\alpha C(t)Y_1(t) - \beta Y_1(t)\right)\right\|\\
&\leq (-\alpha C(t) - \beta)\left\|Y(t) - Y_1(t)\right\|\\
&\leq \left(-\alpha b_1 - \beta\right)\left\|Y(t) - Y_1(t)\right\|\\
&= \Phi_2\left\|Y(t) - Y_1(t)\right\|.
\end{aligned} \tag{29}$$

Then, we prove that $\Psi_3(t, O)$ satisfies the Lipschitz condition. Let $O(t)$ and $O_1(t)$ be two functions, then

$$\begin{aligned}
\left\|\Psi_3(t, O) - \Psi_3\left(t, O_1\right)\right\| &= \left\|(\gamma C(t)O(t) + \delta Y(t)O(t) + \sigma O(t)) - \left(\gamma C(t)O_1(t) + \delta Y(t)O_1(t) + \sigma O_1(t)\right)\right\|\\
&\leq (\gamma C(t) + \delta Y(t) + \sigma)\left\|O(t) - O_1(t)\right\|\\
&\leq \left(\gamma b_1 + \delta b_2 + \sigma\right)\left\|O(t) - O_1(t)\right\|\\
&= \Phi_3\left\|O(t) - O_1(t)\right\|.
\end{aligned} \tag{30}$$

All kernels which $\Psi_i, i = 1, 2, 3$ satisfy the conditions, so that they are contractions with $\Phi_i < 1, i \in 1, 2, 3$. This completes the proof.

Now, let us rewrite equation (24) with all the initial values as zero.

$$
\begin{aligned}
C(t) &= \frac{1-\zeta}{B(\zeta)}(\alpha C(t)) + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-\omega)^{(\zeta-1)}(\alpha C(\omega))d(\omega),\\
Y(t) &= \frac{1-\zeta}{B(\zeta)}(-\alpha C(t)Y(t) - \beta Y(t)) + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-\omega)^{(\zeta-1)}(-\alpha C(\omega)Y(\omega) - \beta Y(\omega))d(\omega),\\
O(t) &= \frac{1-\zeta}{B(\zeta)}(\gamma C(t)O(t) + \delta Y(t)O(t) + \sigma O(t)) + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-\omega)^{(\zeta-1)}(\gamma C(\omega)O(\omega) + \delta Y(\omega)O(\omega) + \sigma O(\omega))d(\omega).
\end{aligned}
\tag{31}
$$

Then, with the help of a recursive formula, the following system can be defined as

$$
\begin{aligned}
C_n(t) &= \frac{1-\zeta}{B(\zeta)}\left(\alpha C(t)_{n-1}\right) + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-\omega)^{(\zeta-1)}(\alpha C_{n-1}(\omega))d(\omega),\\
Y_n(t) &= \frac{1-\zeta}{B(\zeta)}\left(-\alpha C(t)Y_{n-1}(t) - \beta Y_{n-1}(t)\right) + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-\omega)^{(\zeta-1)} - \alpha C(\omega)Y_{n-1}(\omega) - \beta Y_{n-1}(\omega)d(\omega),\\
O_n(t) &= \frac{1-\zeta}{B(\zeta)}\left(\gamma C(t)O_{n-1}(t) + \delta Y(t)O_{n-1}(t) + \sigma O_{n-1}(t)\right)\\
&\quad + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-\omega)^{(\zeta-1)}\left(\gamma C(\omega)O_{n-1}(\omega) + \delta Y(\omega)O_{n-1}(\omega) + \sigma O_{n-1}(\omega)\right)d(\omega).
\end{aligned}
\tag{32}
$$

Therefore, the differences of each equation can be written by taking the norm of both sides as follows:

$$
\begin{aligned}
\left\|(C_{n+1} - C_n)(t)\right\| &= \frac{1-\zeta}{B(\zeta)}\left\|\left[\left(\alpha C(t)_n\right) - \left(\alpha C(t)_{n-1}\right)\right]\right\| + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-\omega)^{(\zeta-1)}\left\|\left[\left(\alpha C_n(\omega)\right) - \left(\alpha C_{n-1}(\omega)\right)\right]\right\|d(\omega)\\
\left\|(Y_{n+1} - Y_n)(t)\right\| &= \frac{1-\zeta}{B(\zeta)}\left\|\left[\left(-\alpha C(t)Y_n(t) - \beta Y_n(t)\right) - \left(-\alpha C(t)Y_{n-1}(t) - \beta Y_{n-1}(t)\right)\right]\right\|\\
&\quad + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-\omega)^{(\zeta-1)}\left\|\left(-\alpha C(\omega)Y_n(\omega) - \beta Y_n(\omega)\right) - \left(-\alpha C(\omega)Y_n(\omega) - \beta Y_n(\omega)\right)\right\|d(\omega),\\
\left\|(O_{n+1} - O_n)(t)\right\| &= \frac{1-\zeta}{B(\zeta)}\left(\left(\gamma C(t)O_n(t) + \delta Y(t)O_n(t) + \sigma O_n(t)\right) - \left(\gamma C(t)O_{n-1}(t) + \delta Y(t)O_{n-1}(t) + \sigma O_{n-1}(t)\right)\right.\\
&\quad + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-\omega)^{(\zeta-1)} \times\left\|\left(\gamma C(\omega)O_n(\omega) + \delta Y(\omega)O_n(\omega) + \sigma O_n(\omega)\right)\right.\\
&\quad \left. - \left(\gamma C(\omega)O_{n-1}(\omega) + \delta Y(\omega)O_{n-1}(\omega) + \sigma O_{n-1}(\omega)\right)\right\|d(\omega).
\end{aligned}
\tag{33}
$$

Theorem 3. *The mathematical model has a solution if the following inequality is achieved:*

$$
\Upsilon_i = \max\{\Phi_i\} < 1, i = 1, 2, 3. \tag{34}
$$

Proof. Let us consider the following equations, $\mathfrak{B}_{1n}(t) = C_{n+1}(t) - C_n(t), \mathfrak{B}_{2n}(t) = Y_{n+1}(t) - Y_n(t)$, and $\mathfrak{B}_{3n}(t) = O_{n+1}(t) - O_n(t)$.

First, we start with $\mathfrak{B}_{1n}(t)$ for the $C(t)$ function,

$$\begin{aligned}\|\mathfrak{B}_{1n}(t)\| &\le \frac{1-\zeta}{B(\zeta)}\|\Psi_1(t, C_n(t)) - \Psi_1(t, C(t))\| + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-\omega)^{(\zeta-1)}\|\Psi_1(\omega, C_n(\omega)) - \Psi_1(\omega, C(\omega))\|d\omega \\ &\le \left(\frac{1-\zeta}{B(\zeta)} + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}t^\zeta\right)\Phi_1\|C_n - C\| \\ &\le \left(\frac{1-\zeta}{B(\zeta)} + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}t^\zeta\right)^n \Upsilon_1^n\|C - C_1\|.\end{aligned} \tag{35}$$

Similarly, it can be easily represented by the following inequalities:

$$\begin{aligned}\|\mathfrak{B}_{2n}(t)\| &\le \left(\frac{1-\zeta}{B(\zeta)} + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}t^\zeta\right)^n \Upsilon_1^n\|Y - Y_1\|, \\ \|\mathfrak{B}_{3n}(t)\| &\le \left(\frac{1-\zeta}{B(\zeta)} + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}t^\zeta\right)^n \Upsilon_1^n\|O - O_1\|.\end{aligned} \tag{36}$$

So that, it can be said that, we can find $\mathfrak{B}_{in}(t) \longrightarrow 0, i = 1, 2, 3,$ as $n \longrightarrow \infty$. Thus, the proof is completed.

Now, the uniqueness of the solution of the mathematical model can be shown.

Theorem 4. *The mathematical model shown in equation (21) will have a unique solution if the following inequality holds true:*

$$\left(\frac{1-\zeta}{B(\zeta)} + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}t^\zeta\right)\Phi_i \le 1, i = 1, 2, 3, 4. \tag{37}$$

Proof. Let us assume that equation (1) has solutions $C(t), Y(t)$, and $O(t)$, as well as $\widetilde{C}(t), \widetilde{Y}(t)$, *and* $\widetilde{O}(t)$. So that, the following system can be written as

$$\begin{aligned}\widetilde{C}(t) &= \frac{1-\zeta}{B(\zeta)}(\alpha\widetilde{C}(t)) + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-\omega)^{(\zeta-1)}(\alpha\widetilde{C}\omega)d(\omega), \\ \widetilde{Y}(t) &= \frac{1-\zeta}{B(\zeta)}(-\alpha C(t)\widetilde{Y}(t) - \beta\widetilde{Y}(t)) + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-\omega)^{(\zeta-1)}(-\alpha C(\omega)\widetilde{Y}(\omega) - \beta\widetilde{Y}(\omega))d(\omega), \\ \widetilde{O}(t) &= \frac{1-\zeta}{B(\zeta)}(\gamma C(t)\widetilde{O}(t) + \delta Y(t)\widetilde{O}(t) + \sigma\widetilde{O}(t)) + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-\omega)^{(\zeta-1)}(\gamma C(\omega)\widetilde{O}(\omega) + \delta Y(\omega)\widetilde{O}(\omega) + \sigma\widetilde{O}(\omega))d(\omega).\end{aligned} \tag{38}$$

Taking the norm from both sides of the system of equation (38), the following equation is obtained:

$$\begin{aligned}\|C(t) - \widetilde{C}(t)\| &\le \frac{1-\zeta}{B(\zeta)}||(\alpha C(t)) - (\alpha\widetilde{C}(t))|| + \frac{\zeta}{B(\zeta)\Gamma(\zeta)}\int_0^t (t-\omega)^{(\zeta-1)}||(\alpha C\omega) - (\alpha\widetilde{C}\omega)||d\omega \\ &\le \frac{1-\zeta}{B(\zeta)}\Phi_1\|C - \widetilde{C}\| + \frac{\zeta\Phi_1}{B(\zeta)\Gamma(\zeta)}t^\zeta\|C - \widetilde{C}\|.\end{aligned} \tag{39}$$

The inequality can be written as

$$\left(\frac{1-\zeta}{B(\zeta)}\Phi_1 + \frac{\zeta\Phi_1}{B(\zeta)\Gamma(\zeta)}t^\zeta\right)\|C - \widetilde{C}\| \ge 0. \tag{40}$$

Thus, $\|C - \widetilde{C}\| = 0$. This implies $C(t) = \widetilde{C}(t)$. When the same method is applied then $Y(t) = \widetilde{Y}(t), O(t) = \widetilde{O}(t)$. According to these results, the model has a unique solution.

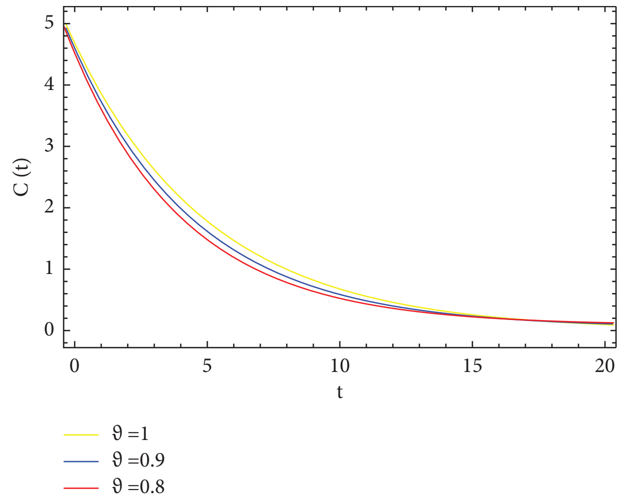

Figure 1: View from old age to childhood.

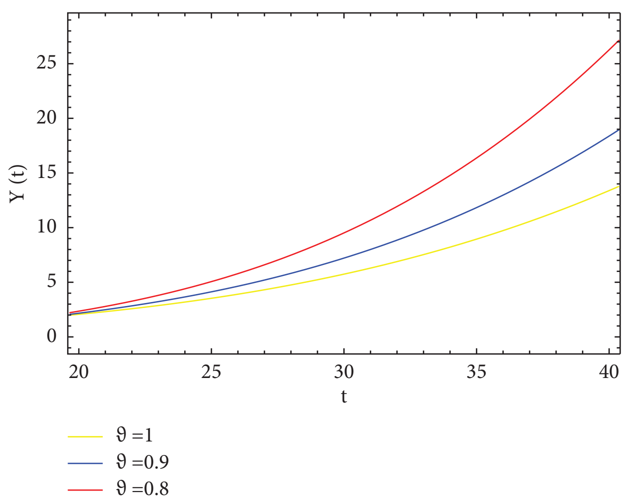

Figure 2: View from old age to youth period.

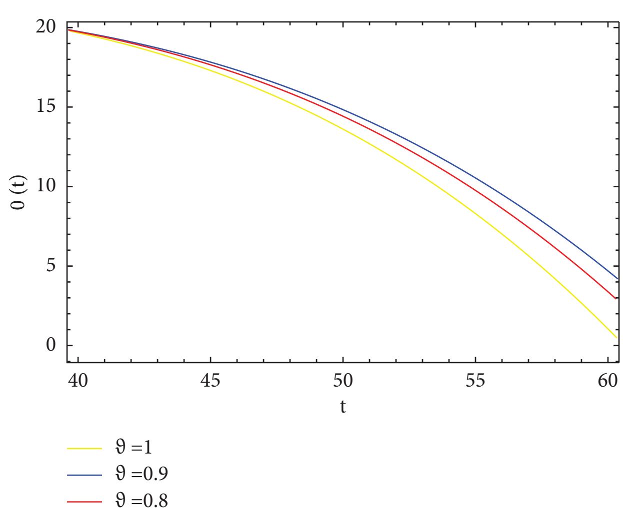

Figure 3: View from old age to old age.

3. Numerical Simulation

In this section, the proposed model analyze with the Adams–Bashforth numerical approach via Atangana–Baleanu–Caputo fractional operator [11]. Simulations of the model are shown and interpreted. Hyers Ulam [12] stability analysis of the mathematical model can also be investigated. The mathematical model has an important temporal distinction. This distinction is divided into three different parts. $0 < t < 20$ is divided into childhood, $20 < t < 40$ is youth period, and $40 < t < 60$ is old age.

Figure 1 shows the perspective of the poetic character from old age to childhood. Since α is considered as negative feelings (waiting, losses, defeats, and sorrows) that affect childhood vision in old age, it is an inevitable fact that there will be a decrease over time. In addition, three different fractional derivative values according to the Atangana–Baleanu–Caputo fractional operator are discussed in the graph. In addition, simulations were performed for fractional derivative values $\zeta = 1, \zeta = 0.9$, and $\zeta = 0.8$. As can be seen from the graph, the curves progressed significantly and regularly for each fractional derivative value.

Figure 2 shows the poetic character's point of view from the old age to the youth period. While α was a parameter containing negative feelings, and β was considered as positive feelings affecting his view of youth in old age. As it is understood from the analysis of the poem, the positive thoughts in the youth period are superior to the negative thoughts. Therefore, a rising graph is expected here. Here, the difference between the curves continues to increase for different fractional derivative values. This is a proof that the emotions felt during that period are faster.

Figure 3 shows the poetic character's point of view from old age to old age. γ was considered as negative feelings in childhood in old age, δ was considered as positive feelings in old age, and σ was considered as negative feelings in old age. This shows the fact that negative feelings are more effective than positive feelings in this period. For these reasons, a decrease in the graph is expected. It is seen that the changes between different fractional derivative values in old age are few. This situation is similar to the childhood period because the emotions are less felt than the youth period.

4. Conclusion

While the study brings innovation to the literature by including an original approach in the context of establishing a link between poetry and mathematical modeling, it also serves as an interesting interdisciplinary bridge in terms of modeling human feelings on the basis of analysis of poetry. In this sense, it is clear that the study has new and motivating aspects for literary researchers, also it is clear that the data revealed through mathematical modeling overlap with literary analysis and analyzes the poem. The study, which is based on the explanation of human feelings with a mathematical model, has revealed quite striking findings, thanks to the use of the advantages of the two basic concepts of fractional analysis, known as the Caputo fractional derivative and Atangana–Baleanu fractional derivative. Researchers on the subject can focus on the idea of mathematical model development using effective concepts of fractional analysis and studies involving feelings description, especially in literature, psychology, and sociology.

Data Availability

No underlying data were collected or produced in this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Authors' Contributions

Burak Armağan and Ülkü Eliuz have made a literary analysis of Olvido's poetry. They have also written the introduction of the article and contributed to the writing and interpretation of other sections. Mustafa Ali Dokuyucu and Ahmet Ocak Akdemir have contributed to the establishment of the mathematical model, existence, and uniqueness solutions. Mustafa Ali Dokuyucu and Mehmet Emir Köksal have contributed to the making of numerical solutions and drawing simulations. Also, Mehmet Emir Köksal has supervised the editing and conceptualization processes. All the authors have read and approved the final form of the manuscript.

References

[1] M. A. Dokuyucu and H. Dutta, "Analysis of a fractional tumor-immune interaction model with exponential kernel," *Filomat*, vol. 35, no. 6, pp. 2023–2042, 2021.

[2] R. Korkmaz and T. Özcan, *Turkish Poetry in the Republican Period. New Turkish Literature Handbook*, Graphics Publications, Ankara, Turkey, 2013.

[3] O. Yivli, "Form essays in Ahmet Muhip Dıranas' poetry," *Journal of Turcology of Ankara University Faculty of Language, History and Geography*, vol. 18, no. 2, pp. 137–147, 2011.

[4] F. Atlı, "Olvido by Ahmet Muhip Diranas," *Kesit Akademi*, vol. 25, no. 25, pp. 328–339, 2020.

[5] Y. K. Beyatlı, *Our Own Heavenly Dome*, Istanbul: Society of Istanbul Fetih, Istanbul, Turkey, 2007.

[6] Ü. Eliuz and E. Seferoğlu, "Masculinity on the threshold: a psychoanalytical interpretation of masculinity in ÇC," *Turkish Studies*, vol. 11, no. 4, pp. 329–346, 2016.

[7] A. Atangana and D. Baleanu, "New fractional derivatives with nonlocal and non-singular kernel: theory and application to heat transfer model," *Thermal Science*, vol. 20, no. 2, p. 763, 2016.

[8] M. Caputo, "Linear models of dissipation whose Q is almost frequency independent, II," *Geophysical Journal International*, vol. 13, no. 5, pp. 529–539, 1967.

[9] I. Podlubny, *Fractional Differential Equations, Volume 198 of Mathematics in Science and Engineering*, Academic Press, San Diego, CA, USA, 1999.

[10] T. Yamamoto and X. Chen, "An existence and nonexistence theorem for solutions of nonlinear systems and its application to algebraic equations," *Journal of Computational and Applied Mathematics*, vol. 30, no. 1, pp. 87–97, 1990.

[11] A. Atangana and K. M. Owolabi, "New numerical approach for fractional differential equations," *Mathematical Modelling of Natural Phenomena*, vol. 13, no. 1, p. 3, 2018.

[12] D. H. Hyers Ulam, "The stability of homomorphisms and related topics," *Global Analysis-Analysis on Manifolds*, vol. 75, pp. 140–153, 1983.