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Research Article

Exact Solutions of an Extended Jimbo-Miwa Equation by Three Distinct Methods

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In this article, we focus on exact traveling wave solutions to an extended Jimbo-Miwa equation, which is an extension of the Jimbo-Miwa equation. First, an improved (G'/G)-expansion method, extended (G'/G)-expansion method, and improved two variable $((\varphi'/\varphi), (1/\varphi))$ expansion method are introduced. Second, with these introduced methods, many new exact traveling wave solutions of EJM equation are constructed, including hyperbolic function solutions, trigonometric function solutions, and rational function solutions which contain many different parameters. Finally, we depict the physical explanation of the extracted solutions with the free choice of the different parameters by plotting some 3D and 2D illustrations. To the best of our knowledge, the received results have not been reported in other studies on the new extended JM equations. We hope that our results can help enrich the study of this new equation.

1. Introduction

Exact solutions of nonlinear evolution equations (NLEEs) have their significant importance in disclosing the internal mechanism of the complex physical phenomena. Therefore, searching for exact solutions to NLEEs is a crucial concern for research studies and scientists. Here, we study the exact solutions of a new extended Jimbo-Miwa equation.

A Jimbo-Miwa equation is a classical mathematical physics equation and studying its exact solutions attracts much attention of many scholars. The $(3+1)$-dimensional Jimbo-Miwa (JM) equation is as follows:

$$u_{xxxy} + 3u_x u_{xy} + 3u_y u_{xx} + 2u_{yt} - 3u_{xz} = 0, \tag{1}$$

where $u = u(x, y, z, t)$, which comes from the second member of a KP hierarchy, is used to describe certain interesting $(3+1)$-dimensional waves in physics [1]. Although the Jimbo-Miwa equation (1) is non-integrable, exact solutions for the equation have been studied by many researchers. By applying the extended homogeneous balance method, an iterative formula of finding exact solutions is given and a lot of solutions are obtained [2]. In [3], lump-type and interaction solutions are studied. Using the generalized three-wave method, exact three-wave solutions including periodic cross-kink wave solutions, doubly periodic solitary wave solutions, and breather type of two-solitary wave solutions for the $(3+1)$-dimensional Jimbo-Miwa equation are obtained [4]. By employing the Lie symmetry method, closed-form invariant solutions and their dynamics are discussed [5]. Using exp-function algorithm, two- and three-wave solutions and traveling wave solutions are constructed [6]. A lot of lump-type solutions and interaction solutions are obtained by symbolic calculation [7, 8]. Wazwaz employed the Hirota bilinear method to derive multiple-front solutions for these equations [9]. Their exact solutions and other properties were extensively studied in a series of papers.

As a generalization of equation (1), Wazwaz introduced the two extended Jimbo-Miwa equations as follows and discussed their multiple-soliton solutions [10].

$$u_{xxxy} + 3u_x u_{xy} + 3u_y u_{xx} + 2u_{yt} - 3(u_{xz} + u_{yz} + u_{zz}) = 0, \quad (2)$$

$$u_{xxxy} + 3u_x u_{xy} + 3u_y u_{xx} + 2(u_{xt} + u_{yt} + u_{zt}) - 3u_{xz} = 0. \quad (3)$$

Lump and lump-kink solutions were obtained for the Jimbo-Miwa equation (1) and two extended Jimbo-Miwa equations (2) and (3) were obtained by the Maple computer algebra system [11]. Many researchers give more and more attention to these Jimbo-Miwa equations. In [12], four kinds of localized waves, solitons, breathers, lumps, and rogue waves of the extended $(3+1)$-dimensional Jimbo-Miwa equation are constructed by the Hirota bilinear method. In [13], explicit rational solutions for the Jimbo-Miwa equation have been presented in the Grammian form. In [14], an extended $(3+1)$-dimensional Jimbo-Miwa equation with time-dependent coefficients is investigated, and bilinear form, Bäcklund transformation, Lax pair, and infinitely-many conservation laws are derived via the binary Bell polynomials and symbolic computation. Yin et al. constructed the exact solutions to these three Jimbo-Miwa equations including lump solutions, lump-kink solutions [15]. Manafian retrieves new periodic solitary wave solutions for the $(3+1)$-dimensional extended Jimbo-Miwa equations, based on the the Hirota bilinear method [16]. Liu studied the equation (2) by the Bell polynomial and a class of new type rogue waves solutions are found [17].

In [18], Cheng et al. introduce a new extended Jimbo-Miwa equation,

$$u_{xxxy} + \chi(u_x u_y)_x + \rho_1 u_{xy} + \rho_2 u_{xz} + \rho_3 u_{yt} + \rho_4 u_{yy} = 0, \quad (4)$$

where $\chi \neq 0$ and $\rho_i, 1 \leq i \leq 4$ are all arbitrary real constants. The constants ρ_2 and ρ_3 satisfy $\rho_2\rho_3 \neq 0$. When $\chi = 3, \rho_2 = -3, \rho_3 = 2$ and the other $\rho_i = 0$, the nonlinear evolution equation (4) becomes the Jimbo-Miwa equation (1). Taking $\chi = -3.\rho_2 = -3, \rho_3 = -1, \rho_1 = \rho_4 = 0$, the equation (4) reduces to $(3+1)$-dimensional generalized BKP equation [6].

$$u_{ty} - u_{xxxy} + 3(u_x u_y)_x + 3u_{xz} = 0. \quad (5)$$

Therefore, the study on new extended Jimbo-Miwa equations (4) is meaningful. In [18], two-wave and complexiton solutions of (4) are developed through symbolic computations with Maple.

So far, mathematicians and physicists have established several effective methods, such as F-expansion method, [19–21] the first integral method, [22] dynamical system method, [23, 24] improved Kudryashov method, [25–27] Hirota bilinear approach, [28–31] $\tan(\Theta/2)$ expansion approach, [32] $\exp(-\phi(\xi))$-expansion method, [33] generalized exponential rational function method [34], and other methods [35]. The (G'/G)-expansion method proposed by Wang et al. [36] is one of the most effective direct methods to obtain travelling wave solutions of a large number of nonlinear evolution equations, such as the KdV equation, the mKdV equation, the variant Boussinesq equations, the Hirota-Satsuma equations, and so on. Later, the further developed methods named the generalized (G'/G)-expansion method, the modified (G'/G)-expansion method, the extended (G'/G)-expansion method, and the improved (G'/G)-expansion method have been proposed in Refs. [37–40], respectively. The aim of this paper is to make some improvements on (G'/G)-expansion method and derive new traveling wave solutions of the extended Jimbo-Miwa (4) equation by improved methods.

2. Description of Methods

2.1. Improved (G'/G)-Expansion Method

Step 1. Consider a general nonlinear PDE in the form

$$F(u, u_x, u_y, u_z, u_t, u_{xx}, u_{xt}, \ldots) = 0. \quad (6)$$

Using $u(x, y, z, t) = U(\xi), \xi = x + y + z - \omega t$, we can rewrite (6) as the following nonlinear ODE:

$$F(U, U', U'', \cdots) = 0, \quad (7)$$

where the prime denotes differentiation with respect to ξ.

Step 2. Suppose that the solution of ODE (7) can be written as follows: [41].

$$U(\xi) = \sum_{i=-n}^{n} a_i \left(\frac{G'}{G + \sigma G'} + M \right)^i, \quad (8)$$

where $M, \sigma, a_i (i = -n, -n+1, \ldots)$ are constants to be determined later, n is a positive integer, and $G = G(\xi)$ satisfies the following second order linear ordinary differential equation:

$$GG'' - a(G')^2 - bGG' - cG^2 = 0, \quad (9)$$

where a, b, c are real constants. The general solutions of (9) can be listed as follows.

When $\triangle = b^2 - 4(a-1)c > 0$, we obtain the hyperbolic function solution of (9)

$$\frac{G'}{G} = \frac{b}{2(1-a)} + \frac{\sqrt{\triangle}}{2(1-a)} \frac{C_1 \sinh((1/2)\sqrt{\triangle}\,\xi) + C_2 \cosh((1/2)\sqrt{\triangle}\,\xi)}{C_1 \cosh((1/2)\sqrt{\triangle}\,\xi) + C_2 \sinh((1/2)\sqrt{\triangle}\,\xi)}. \quad (10)$$

When $\triangle = b^2 - 4(a-1)c < 0$, we obtain the trigonometric function solution of (9)

$$\frac{G'}{G} = \frac{b}{2(1-a)} + \frac{\sqrt{-\triangle}}{2(1-a)} \frac{-C_1 \sin((1/2)\sqrt{-\triangle}\,\xi) + C_2 \cos((1/2)\sqrt{-\triangle}\,\xi)}{C_1 \cos((1/2)\sqrt{-\triangle}\,\xi) + C_2 \sin((1/2)\sqrt{-\triangle}\,\xi)}. \tag{11}$$

When $\triangle = b^2 - 4(a-1)c = 0$, we obtain the rational function solution of (9)

$$\frac{G'}{G} = \frac{1}{1-a}\left(\frac{C_2}{C_1 + C_2\xi} + \frac{b}{2}\right), \tag{12}$$

where C_1 and C_2 are arbitrary constants.

Step 3. Determine the positive integer n by balancing the highest order derivatives and nonlinear terms in (7).

Step 4. Substituting (8) along with (9) into (7) and then setting all the coefficients of $(G'/G + \sigma G')^k$ $(k = 1, 2, \ldots)$ of the resulting system's numerator to zero, yields a set of overdetermined nonlinear algebraic equations for ω and $a_i\,(i = -n, -n+1, \ldots)$.

Step 5. Assuming that the constants ω and $a_i\,(i = -n, -n+1, \ldots)$ can be obtained by solving the algebraic equations in Step 4, then substituting these constants and the known general solutions of (9) into (7), we can obtain the explicit solutions of (6) immediately.

2.2. The Extended (G'/G)-Expansion Method. In the extended form of this method, [39] the solution $U(\xi)$ of (7) can be expressed as

$$U(\xi) = \sum_{i=-n}^{n} a_i\left(\frac{G'}{G} + M\right)^i + \sum_{i=1}^{n} b_i\left(\frac{G'}{G}\right)^{i-1}\sqrt{\sigma\left(1 + \frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}, \tag{13}$$

where $a_i, b_j\,(i = -n, \ldots 0, \ldots n, j = 1 \ldots n)$, $\sigma = \pm 1$ and M are constants to be determined later, $\sigma = \pm 1$, n is a positive integer, and $G = G(\xi)$ satisfies the following second order linear ODE

$$G'' + \mu G = 0, \tag{14}$$

where μ is a constant. Substituting (13) into (7) and using (14) and collecting all terms with the same order of $(G'/G)^k$ and $(G'/G)^k\sqrt{\sigma(1 + (1/\mu)(G'/G)^2)}$ together, and then equating each coefficient of the resulting polynomial to zero yield a set of algebraic equations for $\omega, a_0, a_i, b_i\,(i = 1, \ldots, n)$. On solving these algebraic equations, we obtain the values of the constants $\omega, a_0, a_i, b_i\,(i = 1, \ldots, n)$ and then substituting these constants and the known general solutions of (14) into (13), we obtain the explicit solutions of nonlinear differential (7).

2.3. Improved Two Variable $((G'/G), (1/G))$ Expansion Method. We suppose the solution of (7) owns the following form: [41].

$$u = \sum_{i=-n}^{n} a_i\phi^i + \sum_{j=1}^{n} b_i\phi^{j-1}\varphi, \tag{15}$$

where $\phi = (G'/G)$, $\varphi = (1/G)$, $a_i, b_j\,(i = -n, -n+1, \ldots, 0, 1, 2, \ldots, n;\ j = 1, 2, \ldots, n)$ are constants and $a_n b_n \neq 0$. The positive number n can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (7). The function $G = G(\xi)$ satisfies the second order linear ODE in the form

$$G'' + \lambda G = \mu, \tag{16}$$

where λ and μ are constants. We have

$$\phi' = -\lambda + \mu\varphi - \phi^2, \varphi' = -\phi\varphi. \tag{17}$$

Equation (16) has three types of general solution with double arbitrary parameters as follows: [42–44].

$$G(\xi) = \begin{cases} A_1\sinh(\sqrt{-\lambda}\,\xi) + A_2\cosh(\sqrt{-\lambda}\,\xi) + \dfrac{\mu}{\lambda}, & \text{when} \quad \lambda < 0, \\ A_1\sin(\sqrt{\lambda}\,\xi) + A_2\cos(\sqrt{\lambda}\,\xi) + \dfrac{\mu}{\lambda}, & \text{when} \quad \lambda > 0, \\ \dfrac{\mu}{2}\xi^2 + A_1\xi + A_2, & \text{when} \quad \lambda = 0. \end{cases} \tag{18}$$

and

$$\psi^2 = \begin{cases} -\dfrac{\lambda\left(-2\mu\varphi + \phi^2 + \lambda\right)}{\lambda^2\left(A_1^2 - A_2^2\right) + \mu^2}, & \text{when} \quad \lambda < 0, \\ \dfrac{\lambda\left(-2\mu\varphi + \phi^2 + \lambda\right)}{\lambda^2\left(A_1^2 + A_2^2\right) - \mu^2}, & \text{when} \quad \lambda > 0, \\ \dfrac{-2\mu\varphi + \phi^2}{-2\mu A_2 + A_1^2}, & \text{when} \quad \lambda = 0. \end{cases} \tag{19}$$

where A_1, A_2 are arbitrary constants.

By substituting (15) into (7) and using (17) and (19), collecting all terms with the same order of ϕ^i and $\phi^i\varphi$ together, the left-hand side of (7) is converted into another polynomial in ϕ^i and $\phi^i\varphi$. Equating each coefficient of this different power terms to zero yields a set of algebraic equations for a_i, b_j, λ, μ and ω. The other steps are the same as in the previous subsection. It should be pointed out that we add in the negative power in (15) which can obtain more solutions. So we call this method as improved two variable $((G'/G), (1/G))$ expansion method. Obviously, taking $b_i = 0$, it becomes into G'/G expansion method.

After the brief description of the methods, we now apply these methods for solving new extended Jimbo-Miwa equation.

3. Exact Solutions of a New Extended Jimbo-Miwa Equation

Let $\xi = x + y + z - \omega t, \omega \neq 0$, where ω is the wave speed, equation (4) can be reduced to the following ordinary differential equation (ODE)

$$u^{(4)} + 2\chi u' u'' + \left(-c\rho_3 + \rho_1 + \rho_2 + \rho_4\right)u'' = 0, \tag{20}$$

Integrating (20) once with respect to ξ and setting the constant of integration to zero, we have

$$u''' + \chi\left(u'\right)^2 + \left(-c\rho_3 + \rho_1 + \rho_2 + \rho_4\right)u' = 0. \tag{21}$$

Balancing u''' with $(u')^2$ in (10) we find $n + 3 = 2(n + 1)$, one has $n = 1$. In order to find traveling wave solutions of (4), we would apply three methods to (21).

3.1. Application of the Improved (G'/G)-Expansion Method. Suppose that (21) owns the solutions in the form

$$u(\xi) = a_0 + a_1\left(\frac{G'}{G + \sigma G'} + M\right) + b_1\left(\frac{G'}{G + \sigma G'} + M\right)^{-1}. \tag{22}$$

Substituting (22) along with (9) into (21) and then setting all the coefficients of $(G + \sigma G')^i$ of the resulting system's numerator to zero, yields a set of overdetermined nonlinear algebraic equations about $a_0, a_1, b_1, M, \omega, \chi$. Solving the overdetermined algebraic equations, we can obtain the following results.

Case 1.

$$\begin{aligned} a_1 &= 0, \omega = -\frac{4ac - b^2 - 4c - \rho_1 - \rho_2 - \rho_4}{\rho_3}, \\ \chi &= \frac{6M^2c\sigma^2 - 6M^2b\sigma + 6M^2a + 12Mc\sigma - 6M^2 - 6Mb + 6c}{b_1}, \end{aligned} \tag{23}$$

where $b_1\rho_3 \neq 0$.

Case 2.

$$\begin{aligned} a_1 &= -\frac{b_1\left(4ac - b^2 - 4c\right)}{4c^2}, \\ M &= 0, \\ \omega &= -\frac{16ac - 4b^2 - 16c - \rho_1 - \rho_2 - \rho_4}{\rho_3}, \\ \sigma &= \frac{b}{2c}, \\ \chi &= \frac{6c}{b_1}, \end{aligned} \tag{24}$$

where $b_1c\rho_3 \neq 0$.

Case 3.

$$\begin{aligned} b_1 &= 0, \\ \omega &= -\frac{4ac - b^2 - 4c - \rho_1 - \rho_2 - \rho_4}{\rho_3}, \\ \chi &= -\frac{6\left(c\sigma^2 - b\sigma + a - 1\right)}{a_1}, \end{aligned} \tag{25}$$

where $a_1\rho_3 \neq 0$.

Case 4.

$$b_1 = -\frac{a_1\left(4ac - b^2 - 4c\right)}{4\left(c\sigma^2 - b\sigma + a - 1\right)^2},$$
$$M = \frac{-2c\sigma + b}{2c\sigma^2 - 2b\sigma + 2a - 2},$$
$$\omega = -\frac{16ac - 4b^2 - 16c - \rho_1 - \rho_2 - \rho_4}{\rho_3},$$
$$\chi = -\frac{6\left(c\sigma^2 - b\sigma + a - 1\right)}{a_1}, \quad (26)$$

where $a_1\rho_3 \neq 0$.

Substituting (23)–(26) into (22), we can find the following formal solutions of Jimbo-Miwa equation (4), where $F = (G'/G + \sigma G') = ((G'/G)/1 + \sigma(G'/G))$.

$$u_1(\xi) = a_0 + \frac{b_1}{F + M}, \quad (27)$$

where $\xi = x + y + z - \omega t$.

$$u_2(\xi) = a_0 - \frac{b_1\left(4ac - b^2 - 4c\right)F}{4c^2} + \frac{b_1}{F}, \quad (28)$$

where $\xi = x + y + z + (16\mathrm{ac} - 4b^2 - 16c - \rho_1 - \rho_2 - \rho_4/\rho_3)t$.

$$u_3(\xi) = a_0 + a_1(F + M), \quad (29)$$

where $\xi = x + y + z + (4ac - b^2 - 4c - \rho_1 - \rho_2 - \rho_4/\rho_3)t$.

$$u_4(\xi) = a_0 + a_1\left(F + \frac{-2c\sigma + b}{2c\sigma^2 - 2b\sigma + 2a - 2}\right) - \frac{a_1\left(4ac - b^2 - 4c\right)}{4\left(c\sigma^2 - b\sigma + a - 1\right)^2\left(F + \left(-2c\sigma + b/2c\sigma^2 - 2b\sigma + 2a - 2\right)\right)}, \quad (30)$$

where $\xi = x + y + z + (16\mathrm{ac} - 4b^2 - 16c - \rho_1 - \rho_2 - \rho_4/\rho_3)t$.

Using the solutions F of (9), that is (10)–(12), exact traveling solutions of Jimbo-Miwa (4) can be obtained. Here, we take (27) for example.

When $\triangle = b^2 - 4(a-1)c > 0$, we obtain the hyperbolic function solution of (4)

$$u_{1-1}(\xi) = a_0 + b_1\left(\frac{G'}{G + \sigma G'} + M\right)^{-1} = a_0 + b_1\left(\frac{G'/G}{1 + \sigma G'/G} + M\right)^{-1}, \quad (31)$$

where $G'/G = b/2(1-a) + (\sqrt{\triangle}/2(1-a))(C_1\sinh(1/2\sqrt{\triangle}\,\xi) + C_2\cosh((1/2)\sqrt{\triangle}\,\xi)/C_1\cosh(1/2\sqrt{\triangle}\,\xi) + C_2\sinh(1/2\sqrt{\triangle}\,\xi))$, $\xi = x + y + z + (16\mathrm{ac} - 4b^2 - 16c - \rho_1 - \rho_2 - \rho_4/\rho_3)t$.

When $\triangle = b^2 - 4(a-1)c < 0$, we obtain the trigonometric function solution of (4)

$$u_{1-2}(\xi) = a_0 + b_1\left(\frac{G'}{G + \sigma G'} + M\right)^{-1} = a_0 + b_1\left(\frac{G'/G}{1 + \sigma(G'/G)} + M\right)^{-1}, \quad (32)$$

where $G'/G = (b/2(1-a)) + (\sqrt{\triangle}/2(1-a)) - C_1\sin(1/2\sqrt{-\triangle}\,\xi) + C_2\cos(1/2\sqrt{-\triangle}\,\xi)/C_1\cos(1/2\sqrt{-\triangle}\,\xi) + C_2\sin(1/2\sqrt{-\triangle}\,\xi)$, $\xi = x + y + z + (16\mathrm{ac} - 4b^2 - 16c - \rho_1 - \rho_2 - \rho_4/\rho_3)t$.

When $\triangle = b^2 - 4(a-1)c = 0$, we obtain the rational function solution of (4)

$$u_{1-3}(\xi) = a_0 + b_1\left(\frac{G'}{G + \sigma G'} + M\right)^{-1} = a_0 + b_1\left(\frac{G'/G}{1 + \sigma G'/G} + M\right)^{-1}, \quad (33)$$

where $G'/G = (1/1-a)((C_2/C_1 + C_2\xi) + (b/2))$, $\xi = x + y + z + (\rho_1 + \rho_2 + \rho_4/\rho_3)t$.

3.2. Application of the Extended G'/G-Expansion Method. Suppose that (21) owns the solutions in the form

$$u(\xi) = a_0 + a_1\left(\frac{G'}{G} + M\right) + b_1\left(\frac{G'}{G} + M\right)^{-1} + b_2\sqrt{\sigma\left(1 + \frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}, \quad (34)$$

where a_0, a_1, b_1, b_2, M are constants to be determined later, $\sigma = \pm 1$, and $G = G(\xi)$ satisfies the second order linear ODE (14).

Substituting (34) along with (14) into (21) and then setting all the coefficients of $(G'/G)^k$ and $(G'/G)^k \sqrt{\sigma(1+(1/\mu)(G'/G)^2)}$ $(k = 0, 1, \ldots)$ of the resulting system to zero, yields a set of overdetermined nonlinear algebraic equations about $a_0, a_1, b_1, b_2, \omega, M, \chi$. Solving the overdetermined algebraic equations, we can obtain the following results.

Case 5.

$$\begin{aligned} b_1 &= -\mu a_1, \\ b_2 &= 0, \\ M &= 0, \\ \omega &= -\frac{16\mu - \rho_4 - \rho_2 - \rho_1}{\rho_3}, \\ \chi &= \frac{6}{a_1}. \end{aligned} \tag{35}$$

Case 6.

$$\begin{aligned} a_1 &= \pm \sqrt{\frac{\sigma}{\mu}} b_2, b_1 = 0, \\ M &= 0, \\ \omega &= -\frac{\mu - \rho_4 - \rho_2 - \rho_1}{\rho_3}, \\ \chi &= \pm \frac{3\mu\sqrt{\sigma/\mu}}{b_2 \sigma}. \end{aligned} \tag{36}$$

Case 7.

$$\begin{aligned} a_1 &= 0, \\ b_2 &= 0, \\ \omega &= -\frac{4\mu - \rho_4 - \rho_2 - \rho_1}{\rho_3}, \\ \chi &= -\frac{6(M^2 + \mu)}{b_1}. \end{aligned} \tag{37}$$

Case 8.

$$\begin{aligned} b_1 &= 0, \\ b_2 &= 0, \\ \omega &= -\frac{4\mu - \rho_4 - \rho_2 - \rho_1}{\rho_3}, \\ \chi &= \frac{6}{a_1}. \end{aligned} \tag{38}$$

Case 9.

$$\begin{aligned} a_1 &= \frac{3}{\chi}, \\ b_1 &= 0, \\ b_2 &= \frac{3\sqrt{\sigma/\mu}}{\chi}, \\ \omega &= -\frac{\mu - \rho_4 - \rho_2 - \rho_1}{\rho_3}. \end{aligned} \tag{39}$$

Substituting (35)–(39) into (34), we can find the following formal solutions of Jimbo-Miwa equation (4).

$$u_4(\xi) = a_0 + a_1 \frac{G'}{G} - \mu a_1 \left(\frac{G'}{G}\right)^{-1}, \tag{40}$$

where $\xi = x + y + z + (\mu - \rho_4 - \rho_2 - \rho_1/\rho_3)t$.

$$u_5(\xi) = a_0 + b_2 \sqrt{\frac{\sigma}{\mu}} \frac{G'}{G} \pm b_2 \sqrt{\sigma\left(1 + \frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}, \tag{41}$$

where $\xi = x + y + z + (\mu - \rho_4 - \rho_2 - \rho_1/\rho_3)t$.

$$u_6(\xi) = a_0 + b_1 \left(M + \frac{G'}{G}\right)^{-1}, \tag{42}$$

where $\xi = x + y + z + (4\mu - \rho_4 - \rho_2 - \rho_1/\rho_3)t$.

$$u_7(\xi) = a_0 + a_1 \left(\frac{G'}{G} + M\right), \tag{43}$$

where $\xi = x + y + z + (4\mu - \rho_4 - \rho_2 - \rho_1/\rho_3)t$.

$$u_8(\xi) = a_0 + \frac{3}{\chi}\left(\frac{G'}{G} + M\right) \pm \frac{3}{\chi}\sqrt{\frac{\sigma}{\mu}} \sqrt{\sigma\left(1 + \frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}, \tag{44}$$

where $\xi = x + y + z + (\mu - \rho_4 - \rho_2 - \rho_1/\rho_3)t$.

Using the solutions F of (14), exact traveling solutions of Jimbo-Miwa equation (4) can be obtained. Here, we take (41) for example.

When $\mu < 0$, we have the hyperbolic function solution as

$$u_{5-1}(\xi) = a_0 + b_2\sqrt{-\sigma}\,\frac{A_1\sinh(\sqrt{-\mu}\,\xi) + A_2\cosh(\sqrt{-\mu}\,\xi)}{A_1\cosh(\sqrt{-\mu}\,\xi) + A_2\sinh(\sqrt{-\mu}\,\xi)} \pm b_2\sqrt{\sigma\left(1 - \left(\frac{A_1\sinh(\sqrt{-\mu}\,\xi) + A_2\cosh(\sqrt{-\mu}\,\xi)}{A_1\cosh(\sqrt{-\mu}\,\xi) + A_2\sinh(\sqrt{-\mu}\,\xi)}\right)^2\right)}. \quad (45)$$

where $\xi = x + y + z + (\mu - \rho_4 - \rho_2 - \rho_1/\rho_3)t$.

In particular, setting $A_1 = 0, A_2 \neq 0$, then equation (45) can be written as

$$u(\xi) = a_0 + b_2\sqrt{-\sigma}\quad\coth(\sqrt{-\mu}\,\xi) \pm b_2\sqrt{\sigma\left(1 - (\coth(\sqrt{-\mu}\,\xi))^2\right)}. \quad (46)$$

Setting $A_1 \neq 0, A_2 = 0$, then equation (45) can be written as

$$u(\xi) = a_0 - b_2\sqrt{-\sigma}\quad\tanh(\sqrt{-\mu}\,\xi) \pm b_2\sqrt{\sigma\left(1 - (\tanh(\sqrt{-\mu}\,\xi))^2\right)}. \quad (47)$$

where $\xi = x + y + z + (\mu - \rho_4 - \rho_2 - \rho_1/\rho_3)t$.

When $\mu > 0$, we have the trigonometric function solution as

$$u_{5-2}(\xi) = a_0 + b_2\sqrt{\sigma}\,\frac{-A_1\sin(\sqrt{\mu}\,\xi) + A_2\cos(\sqrt{\mu}\,\xi)}{A_1\cos(\sqrt{\mu}\,\xi) + A_2\sin(\sqrt{\mu}\,\xi)} \pm b_2\sqrt{\sigma\left(1 + \left(\frac{-A_1\sin(\sqrt{\mu}\,\xi) + A_2\cos(\sqrt{\mu}\,\xi)}{A_1\cos(\sqrt{\mu}\,\xi) + A_2\sin(\sqrt{\mu}\,\xi)}\right)^2\right)}. \quad (48)$$

where $\xi = x + y + z + (\mu - \rho_4 - \rho_2 - \rho_1/\rho_3)t$.

In particular, setting $A_1 = 0, A_2 \neq 0$, then equation (48) can be written as

$$u(\xi) = a_0 + b_2\sqrt{\sigma}\cot(\sqrt{\mu}\,\xi) \pm b_2\sqrt{\sigma\left(1 + (\cot(\sqrt{\mu}\,\xi))^2\right)}. \quad (49)$$

Setting $A_1 \neq 0, A_2 = 0$, then equation (48) can be written as

$$u(\xi) = a_0 - b_2\sqrt{\sigma}\tan(\sqrt{\mu}\,\xi) \pm b_2\sqrt{\sigma\left(1 + (\tan(\sqrt{\mu}\,\xi))^2\right)}. \quad (50)$$

where $\xi = x + y + z + (\mu - \rho_4 - \rho_2 - \rho_1/\rho_3)t$.

3.3. *Application of the Improved Two Variable $((G'/G), (1/G))$ Expansion Method.*

Suppose that (21) owns the solutions in the form

$$u(\xi) = a_0 + a_1\phi + a_2\phi^{-1} + b_1\varphi, \quad (51)$$

where $\phi = (G'/G)$, $\varphi = (1/G)$, a_0, a_1, a_2, b_1, are constants to be determined later, and $G = G(\xi)$ satisfies the second order linear ODE 16.

Substituting (51) along with (17) and (19) into (21) and then setting all the coefficients of ϕ^i and $\phi^i\varphi$ of the resulting system to zero, yields a set of overdetermined nonlinear algebraic equations about $a_0, a_1, a_2, b_1, \omega, \chi$. Solving the overdetermined algebraic equations, we can obtain the following results.

Case 10. When $\lambda > 0$

$$
\begin{aligned}
a_1 &= \pm\sqrt{\frac{\lambda}{\lambda^2\sigma-\mu^2}}b_1,\\
a_2 &= 0,\\
\omega &= -\frac{\lambda-\rho_4-\rho_2-\rho_1}{\rho_3},\\
\chi &= \frac{3}{\sqrt{\lambda/\lambda^2\sigma-\mu^2}b_1}.
\end{aligned} \tag{52}
$$

$$
\begin{aligned}
a_1 &= 0,\\
b_1 &= 0,\\
\omega &= -\frac{4\lambda-\rho_4-\rho_2-\rho_1}{\rho_3},\\
\mu &= 0,\\
\chi &= -\frac{6\lambda}{a_2}.
\end{aligned} \tag{53}
$$

$$
\begin{aligned}
a_2 &= -\lambda a_1,\\
b_1 &= 0,\\
\omega &= -\frac{16\lambda-\rho_4-\rho_2-\rho_1}{\rho_3},\\
\mu &= 0,\\
\chi &= \frac{6}{a_1}.
\end{aligned} \tag{54}
$$

$$
\begin{aligned}
a_1 &= \frac{6}{\chi},\\
a_2 &= 0,\\
b_1 &= 0,\\
\omega &= -\frac{4\lambda-\rho_4-\rho_2-\rho_1}{\rho_3},\\
\mu &= 0.
\end{aligned} \tag{55}
$$

$$
\begin{aligned}
a_1 &= \frac{3}{\chi},\\
a_2 &= 0,\\
b_1 &= \pm\frac{3\sqrt{\lambda\sigma}}{\chi},\\
\omega &= -\frac{\lambda-\rho_4-\rho_2-\rho_1}{\rho_3},\\
\mu &= 0.
\end{aligned} \tag{56}
$$

where $\sigma = A_1^2 + A_2^2$. Substituting (52)–(56) into (51), we can find the following formal solutions of Jimbo-Miwa equation (4).

$$
u_9(\xi) = a_0 \pm \sqrt{\frac{\lambda}{\lambda^2\left(A_1^2+A_2^2\right)-\mu^2}}b_1\phi(\xi) + b_1\varphi(\xi), \tag{57}
$$

where $\xi = x + y + z + (\lambda - \rho_4 - \rho_2 - \rho_1/\rho_3)t$, a_0, b_1, μ are arbitrary constants.

$$
u_{10}(\xi) = a_0 + \frac{a_2}{\phi(\xi)}, \tag{58}
$$

where $\xi = x + y + z + (4\lambda - \rho_4 - \rho_2 - \rho_1/\rho_3)t$, a_0, a_2 are arbitrary constants.

$$
u_{11}(\xi) = a_0 + a_1\phi(\xi) - \frac{\lambda a_1}{\phi(\xi)}, \tag{59}
$$

where $\xi = x + y + z + (16\lambda - \rho_4 - \rho_2 - \rho_1/\rho_3)t$, a_0, a_1 are arbitrary constants.

$$
u_{12}(\xi) = a_0 + \frac{6\phi(\xi)}{\chi}, \tag{60}
$$

where $\xi = x + y + z + (16\lambda - \rho_4 - \rho_2 - \rho_1/\rho_3)t$, a_0, χ are arbitrary constants.

$$
u_{13}(\xi) = a_0 + \frac{3\phi(\xi)}{\chi} \pm \frac{3\sqrt{\lambda\left(A_1^2+A_2^2\right)}\varphi(\xi)}{\chi}, \tag{61}
$$

where $\xi = x + y + z + (16\lambda - \rho_4 - \rho_2 - \rho_1/\rho_3)t$, a_0, χ, A_1, A_2 are arbitrary constants.

Using the solutions $G(\xi)$ of (16), exact traveling solutions of Jimbo-Miwa equation (4) can be obtained. Here, we take (61) for example.

We have the trigonometric function solution as

$$
\begin{aligned}
u_{13-1}(\xi) = a_0 &+ \frac{3\sqrt{\lambda}\left(A_1\cos(\sqrt{\lambda}\,\xi) - A_2\sin(\sqrt{\lambda}\,\xi)\right)}{\chi\left(A_1\sin(\sqrt{\lambda}\,\xi) + A_2\cos(\sqrt{\lambda}\,\xi)\right)}\\
&\pm \frac{3\sqrt{\lambda\left(A_1^2+A_2^2\right)}}{\chi\left(A_1\sin(\sqrt{\lambda}\,\xi) + A_2\cos(\sqrt{\lambda}\,\xi)\right)},
\end{aligned} \tag{62}
$$

where $\xi = x + y + z + (\mu - \rho_4 - \rho_2 - \rho_1/\rho_3)t$, a_0, χ, A_1, A_2 are arbitrary constants, $\lambda > 0$.

Case 11. When $\lambda < 0$

$$a_1 = \pm\sqrt{-\frac{\lambda}{\lambda^2\sigma+\mu^2}}b_1,\quad a_2 = 0,\quad \omega = -\frac{\lambda-\rho_4-\rho_2-\rho_1}{\rho_3},\quad \chi = \pm\frac{3}{\sqrt{-\lambda/\lambda^2\sigma+\mu^2}b_1}, \tag{63}$$

$$a_1 = 0,\quad b_1 = 0,\quad \omega = -\frac{4\lambda-\rho_4-\rho_2-\rho_1}{\rho_3},\quad \mu = 0,\quad \chi = -\frac{6\lambda}{a_2}, \tag{64}$$

$$a_2 = -\lambda a_1,\quad b_1 = 0,\quad \omega = -\frac{16\lambda-\rho_4-\rho_2-\rho_1}{\rho_3},\quad \mu = 0,\quad \chi = \frac{6}{a_1}, \tag{65}$$

$$a_1 = \frac{6}{\chi},\quad a_2 = 0,\quad b_1 = 0,\quad \omega = -\frac{4\lambda-\rho_4-\rho_2-\rho_1}{\rho_3},\quad \mu = 0, \tag{66}$$

$$a_0 = a_0,\quad a_1 = \frac{3}{\chi},\quad a_2 = 0,\quad b_1 = \pm\frac{3\sqrt{-\lambda\sigma}}{\chi},\quad \omega = -\frac{\lambda-\rho_4-\rho_2-\rho_1}{\rho_3},\quad \mu = 0, \tag{67}$$

where $\sigma = A_1^2 - A_2^2$. Substituting (63)–(67) into (51), we can find the following formal solutions of Jimbo-Miwa equation (4).

$$u_{14}(\xi) = a_0 \pm \sqrt{\frac{-\lambda}{\lambda^2\left(A_1^2-A_2^2\right)-\mu^2}}b_1\phi(\xi) + b_1\varphi(\xi), \tag{68}$$

where $\xi = x+y+z+(\lambda-\rho_4-\rho_2-\rho_1/\rho_3)t$, a_0, b_1 are arbitrary constants.

$$u_{15}(\xi) = a_0 + \frac{a_2}{\phi(\xi)}, \tag{69}$$

where $\xi = x+y+z+(4\lambda-\rho_4-\rho_2-\rho_1/\rho_3)t$, a_0, a_2 are arbitrary constants.

$$u_{16}(\xi) = a_0 + a_1\phi(\xi) - \frac{\lambda a_1}{\phi(\xi)}, \tag{70}$$

where $\xi = x+y+z+(16\lambda-\rho_4-\rho_2-\rho_1/\rho_3)t$, a_0, a_1 are arbitrary constants.

$$u_{17}(\xi) = a_0 + \frac{6\phi(\xi)}{\chi}, \tag{71}$$

where $\xi = x+y+z+(16\lambda-\rho_4-\rho_2-\rho_1/\rho_3)t$, a_0, χ are arbitrary constants.

$$u_{18}(\xi) = a_0 + \frac{3\phi(\xi)}{\chi} \pm \frac{3\sqrt{-\lambda\left(A_1^2-A_2^2\right)}\varphi(\xi)}{\chi}, \tag{72}$$

where $\xi = x+y+z+(16\lambda-\rho_4-\rho_2-\rho_1/\rho_3)t$, a_0, χ, A_1, A_2 are arbitrary constants.

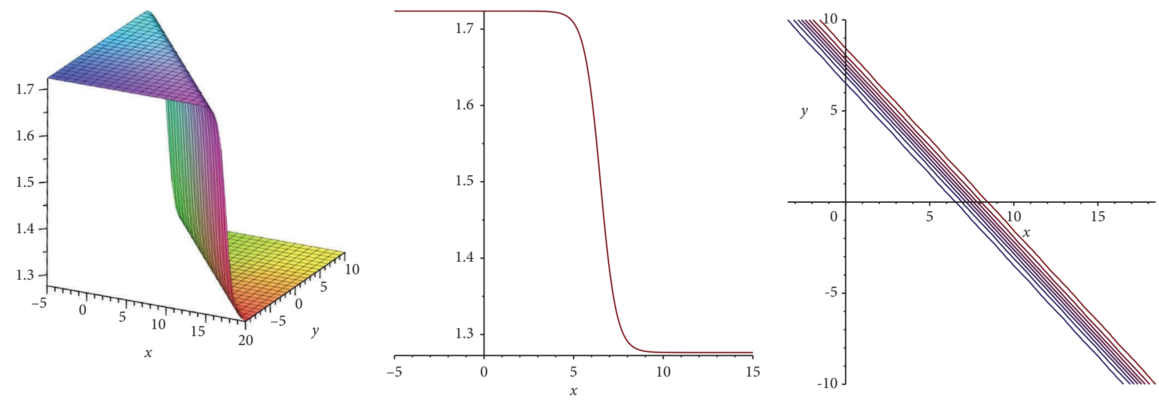
Using the solutions $G(\xi)$ of (16), exact traveling solutions of Jimbo-Miwa equation (4) can be obtained. Here, we take (72) for example.

We have the hyperbolic function solution as

$$u_{18-1}(\xi) = a_0 + \frac{3\sqrt{-\lambda}\left(A_1\cosh(\sqrt{-\lambda}\,\xi) + A_2\sinh(\sqrt{-\lambda}\,\xi)\right)}{\chi\left(A_1\sinh(\sqrt{-\lambda}\,\xi) + A_2\cosh(\sqrt{-\lambda}\,\xi)\right)} \pm \frac{3\sqrt{\lambda\left(A_1^2-A_2^2\right)}}{\chi\left(A_1\sinh(\sqrt{-\lambda}\,\xi) + A_2\cosh(\sqrt{-\lambda}\,\xi)\right)}, \tag{73}$$

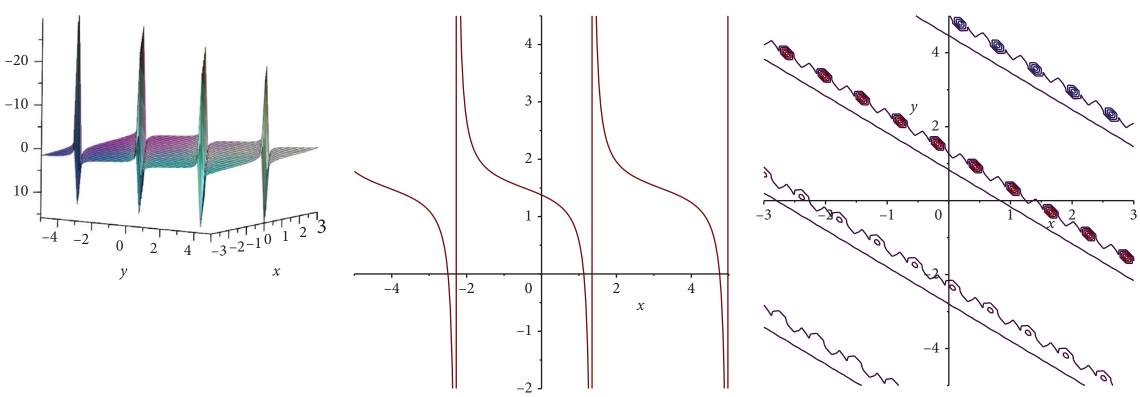
where $\xi = x+y+z+(\mu-\rho_4-\rho_2-\rho_1/\rho_3)t$, a_0, χ, A_1, A_2 are arbitrary constants, $\lambda < 0$.

Case 12. when $\lambda = 0$



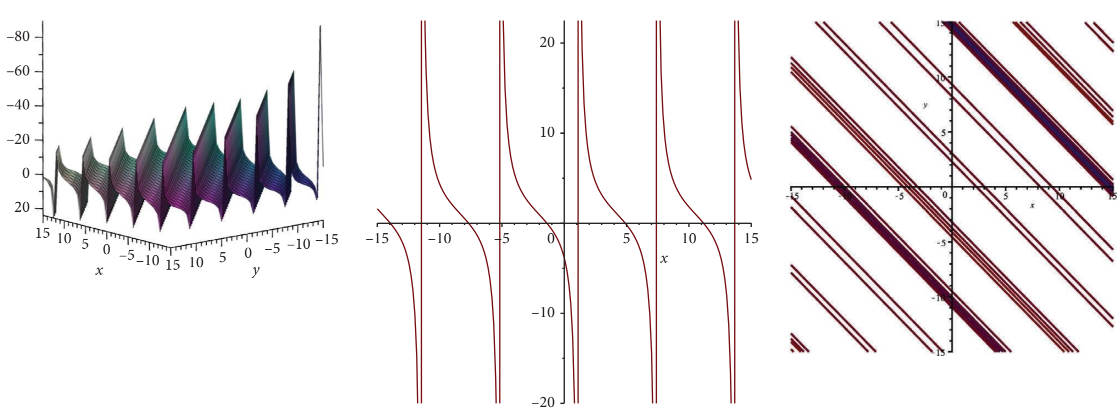
(a) (b) (c)

Figure 1: Figures on (x, y) of solution (31) with $a_0 = 1, b_1 = 1, a = 2, b = 1, c = -1, C_1 = 1, C_2 = -2, \rho_1 = 1, \rho_2 = 1, \rho_3 = 1, \rho_4 = 1, M = 1, \sigma = 1$. (a) Three-dimensional plot. (b) Two-dimensional plot with $y = 1$. (c) Contour plot.



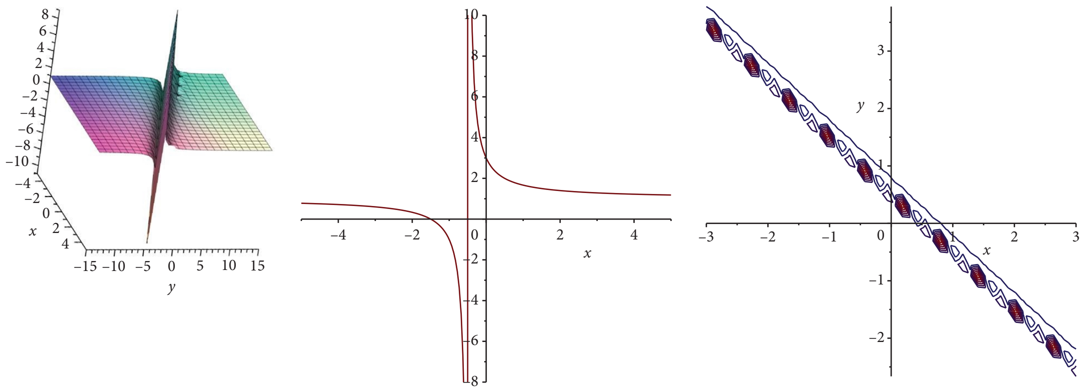
(a) (b) (c)

Figure 2: Figures on (x, y) of solution (32) with $a_0 = 1, b_1 = 1, a = 2, b = 1, c = 1, C_1 = 1, C_2 = -2, \rho_1 = 1, \rho_2 = 1, \rho_3 = 1, \rho_4 = 1, M = 1, \sigma = 1$. (a) Three-dimensional plot. (b) Two-dimensional plot with $y = 1$. (c) Contour plot.



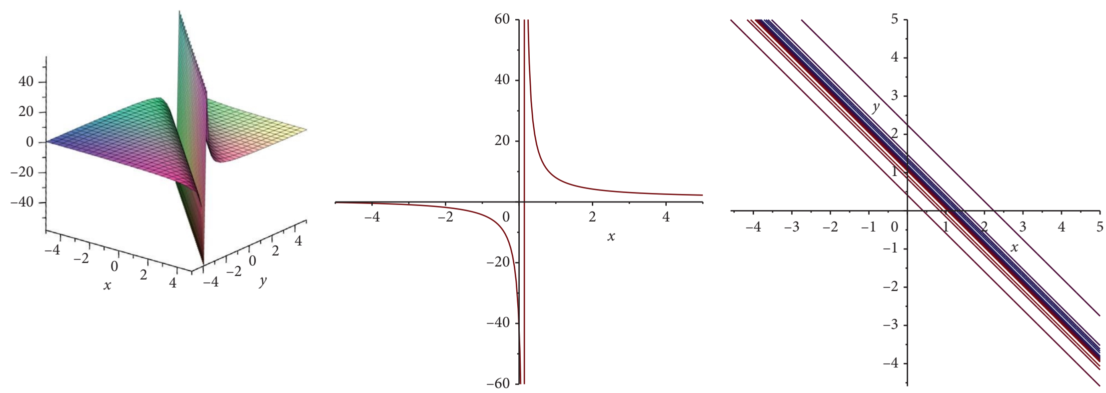
(a) (b) (c)

Figure 3: Figures on (x, y) of solution (62) with $a_0 = 1, \lambda = 1, A_1 = 1, A_2 = -2, \rho_1 = 1, \rho_2 = 1, \rho_3 = 1, \rho_4 = 1, \chi = 1$. (a) Three-dimensional plot. (b) Two-dimensional plot with $y = 1$. (c) Contour plot.



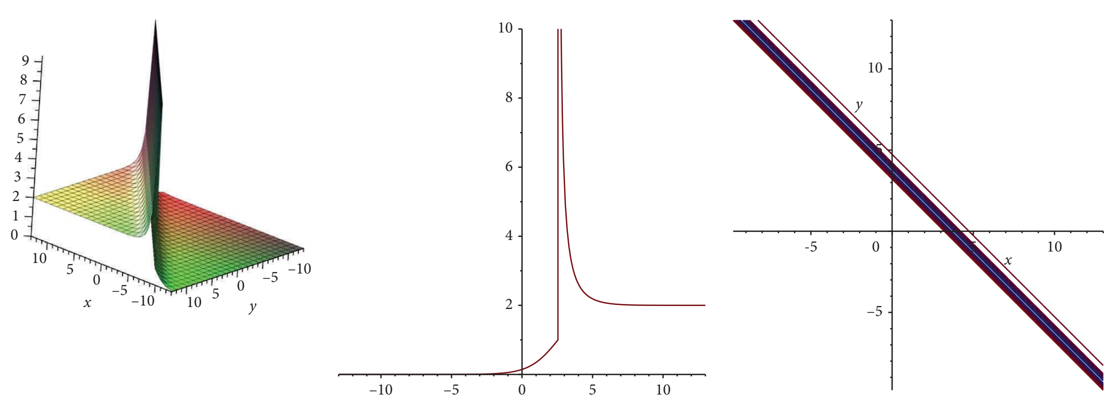
(a) (b) (c)

Figure 4: Figures on (x, y) of solution (33) with $a_0 = 1, b_1 = 1, a = 2, b = 2, C_1 = 1, C_2 = -2, \rho_1 = 1, \rho_2 = 1, \rho_3 = 1, \rho_4 = 1, M = 1, \sigma = 1$. (a) Three-dimensional plot. (b) Two-dimensional plot with $y = 1$. (c) Contour plot.



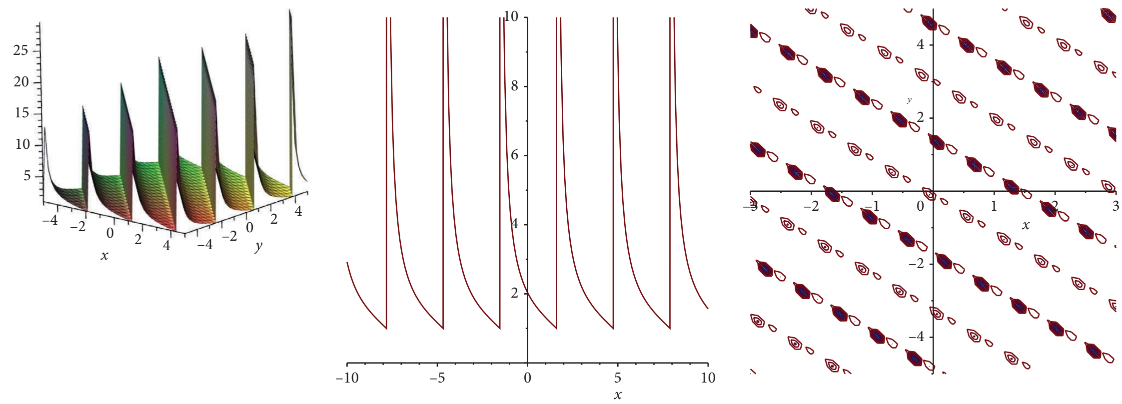
(a) (b) (c)

Figure 5: Figures on (x, y) of solution (80) with $a_0 = 1, b_1 = 1, A_1 = 2, A_2 = 1, \rho_1 = 1, \rho_2 = 1, \rho_3 = 1, \rho_4 = 1, \chi = 1$. (a) Three-dimensional plot. (b) Two-dimensional plot with $y = 1$. (c) Contour plot.



(a) (b) (c)

Figure 6: Figures on (x, y) of solution (45) with $a_0 = 1, b_2 = 1, \mu = -1, A_1 = 1, A_2 = -2, \rho_1 = 1, \rho_2 = 1, \rho_3 = 1, \rho_4 = 1, M = 1, \sigma = -1$. (a) Three-dimensional plot. (b) Two-dimensional plot with $y = 1$. (c) Contour plot.



(a) (b) (c)

Figure 7: Figures on (x, y) of solution (48) with $a_0 = 1, b_2 = 1, \mu = 1, A_1 = 1, A_2 = -2, \rho_1 = 1, \rho_2 = 1, \rho_3 = 1, \rho_4 = 1, M = 1, \sigma = 1$. (a) Three-dimensional plot. (b) Two-dimensional plot with $y = 1$. (c) Contour plot.

$$\begin{aligned} a_1 &= 0, \\ b_1 &= 0, \\ a_2 &= \frac{\rho_3\omega - \rho_1 - \rho_2 - \rho_4}{\chi}, \\ \mu &= 0, \end{aligned} \tag{74}$$

$$\begin{aligned} a_1 &= \frac{6}{\chi}, \\ b_1 &= 0, \\ a_2 &= 0, \\ \omega &= \frac{\rho_4 + \rho_2 + \rho_1}{\rho_3}, \\ \mu &= 0, \end{aligned} \tag{75}$$

$$\begin{aligned} a_1 &= \frac{3}{\chi}, \\ a_2 &= 0, \\ \omega &= \frac{\rho_4 + \rho_2 + \rho_1}{\rho_3}, \\ \mu &= -\frac{\chi^2 b_1^2 - 9A_1^2}{18A_2}. \end{aligned} \tag{76}$$

Substituting (74)–(76) into (51), we can find the following formal solutions of Jimbo-Miwa equation (4).

$$u_{19}(\xi) = a_0 + \frac{\rho_3\omega - \rho_1 - \rho_2 - \rho_4}{\chi\phi(\xi)}, \tag{77}$$

where $\xi = x + y + z - \omega t$, a_0, ω are arbitrary constants.

$$u_{20}(\xi) = a_0 + \frac{6\phi(\xi)}{\chi}, \tag{78}$$

where $\xi = x + y + z - (\rho_4 + \rho_2 + \rho_1/\rho_3)t$, a_0 is a arbitrary constants..

$$u_{21}(\xi) = a_0 + \frac{6\phi(\xi)}{\chi} + b_1\psi(\xi), \tag{79}$$

where $\xi = x + y + z - (\rho_4 + \rho_2 + \rho_1/\rho_3)t$, a_0, b_1 are arbitrary constants.

Using the solutions $G(\xi)$ of (16), exact traveling solutions of Jimbo-Miwa equation (4) can be obtained. Here, we take (79) for example.

We have the rational function solution as

$$u_{21-1}(\xi) = a_0 + \frac{6(\mu\xi + A_1)}{\chi\left(\left(\mu\xi^2/2\right) + A_1\xi + A_2\right)} + \frac{b_1}{(\mu/2)\xi^2 + A_1\xi + A_2}, \tag{80}$$

where $\xi = x + y + z - (\rho_4 + \rho_2 + \rho_1/\rho_3)t$, a_0, b_1, A_1, A_2 are arbitrary constants, $\mu = -(\chi^2 b_1^2 - 9A_1^2/18A_2)$.

4. Figures of Some Exact Solutions

In order to better understand the physical meaning of the solutions, we represented a few typical wave figures of new extended Jimbo-Miwa equation (4).Obviously, solutions obtained in this paper including kink wave solution (Figure 1), periodic wave solution (Figures 2 and 3), solitary wave solution (Figures 4 and 5), sharp wave solution (Figure 6), and periodic sharp wave solution (Figure 7).

5. Conclusions

Exact solutions of Jimbo-Miwa equation (1) and extended Jimbo-Miwa equations (2) and (3) are studied by a lot of researchers and fruitful results are obtained. However, studies on new extended Jimbo-Miwa equation (4) are few. In [18], Cheng et al. discussed two-wave solutions and nonsingular complexiton solutions. In the present work, improved (G'/G)-expansion method, extended (G'/G)-expansion method, and improved two variable $((\phi'/\phi), (1/\phi))$ expansion method are introduced by adding in negative power and constant M, by which more exact solutions can be obtained. Using these three methods, a new extended Jimbo-Miwa equation in the research and development process of nonlinear physical phenomena is discussed. Many traveling wave solutions of this equation are constructed, including hyperbolic function solutions, the trigonometric function solutions, and the rational functions solutions. The results we obtained are quite different from those obtained in reference [18]. To the best of our knowledge, the received results have not been reported in other studies on the new extended JM equations. The results of the new extended Jimbo-Miwa equation (4) have been enriched.

Data Availability

The data used to support the findings of this study are available from the corresponding author on reasonable request.

Conflicts of Interest

The author declares that there are no conflicts of interest.

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