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Research Article

Dual Hesitant q-Rung Orthopair Fuzzy Interaction Partitioned Bonferroni Mean Operators and Their Applications

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The purpose of this paper is to introduce interaction partitioned Bonferroni mean operators under dual hesitant q-rung orthopair fuzzy environment. Motivated by the idea of q-rung orthopair fuzzy interaction operational laws, partitioned Bonferroni mean, and dual hesitant q-rung orthopair fuzzy sets, for dual hesitant q-rung orthopair fuzzy numbers, we present dual hesitant q-rung orthopair fuzzy interaction operational rules and propose several dual hesitant q-rung orthopair fuzzy interaction partitioned Bonferroni mean aggregation operators, including the interaction partitioned Bonferroni mean operator for dual hesitant q-rung orthopair fuzzy numbers, the weighted interaction partitioned Bonferroni mean operator for dual hesitant q-rung orthopair fuzzy numbers, the interaction partitioned geometric Bonferroni mean operator for dual hesitant q-rung orthopair fuzzy numbers, and the weighted interaction partitioned geometric Bonferroni mean operator for dual hesitant q-rung orthopair fuzzy numbers. Moreover, some properties and special cases associated with these proposed operators are also analyzed. For dual hesitant q-rung orthopair fuzzy numbers, based on the proposed operators, a multicriteria group decision-making method is proposed. Finally, an example for missile purchase problem is illustrated to demonstrate the superiority and feasibility by comparing with other existing multicriteria group decision-making methods.

1. Introduction

In real life, many problems, from the purchase of commodities to the formulation of national policies, all reflect the widespread application of decision making (DM) [1] ideas, making the final decision results satisfactory, which involves the importance of correct DM approach. Multicriteria group decision making (MCGDM) is one of the DM methods which can address various uncertain problems. Due to the uncertainty factors and increase of complexity gradually in the actual decision-making process, it has been extensively studied. Intuitionistic fuzzy sets (IFSs) presented by Atanassov [2] are better than fuzzy sets (FSs) introduced by Zadeh [3], which can portray the uncertainty information more completely and accurately from the aspects of hesitancy degree (HD), nonmembership degree (NMD), and membership degree (MD). After that, considering that the evaluation value may exceed the application scope of IFSs, with the limits of the sum of the squares of MD and NMD to 1, Yager [4] proposed the Pythagorean fuzzy sets (PFSs), which are more effective than IFSs in dealing with MCGDM problems. For instance, the MD and NMD are 0.9 and 0.5, and we can find that only PFSs can express such data information. In addition, to meet the requirements of increasingly complex data, with the limits of the sum of the q–th power of MD and NMD to 1, Yager [5] presented q-rung orthopair fuzzy set (q-ROFS). Thus, q-ROFS is more effective and comprehensive in processing and expressing uncertainty data compared with IFSs and PFSs. For more research on hesitant fuzzy sets, see [6].

However, considering the hesitation from decision-makers in providing specific evaluation values in certain situations. For this, Torra [7] proposed hesitant fuzzy sets (HFSs) to more reasonably express the epistemic uncertainty of DMs by giving the possible MD (PMD). In 2012, Zhu et al.

[8] incorporated the idea of IFS into HFS and proposed a new dual hesitant fuzzy set (DHFS) to express the mental cognitive state of DMs more clearly by increasing the possible NMD (PNMD). So far, many uncertainty theories have been studied, such as TOPSIS [9] and fuzzy rough set [10], making it more flexible and comprehensive in describing the cognitive uncertainty aspects of DMs. For more research on decision-making problems, see [11].

In this era of big data, information fusion plays an indispensable role, which extracts required information by integrating various kinds of information in a certain way. As a method of information fusion, information aggregation operators (AOs) have attracted more and more attention and have become a general tool of modern information processing. In general, the AOs are studied from two aspects, namely, functions and operations, represented as below:

(i) *From the perspective of the AO functions*: Some traditional AOs have been proposed for data aggregation such as the Heronian mean (HM) [12] operator and the Bonferroni mean (BM) [13] operator, which consider from the perspective of the interrelationships between aggregating arguments. Xu et al. [14] presented dual hesitant q-rung orthopair fuzzy sets (DHq-ROFSs) and proposed some HM operators for dual hesitant q-rung orthopair fuzzy numbers (DHq-ROFNs). For all that, the BM operator is more widely used. Therefore, many studies with respect to the BM operator have emerged and remarkable results have been achieved in dealing with fuzzy problems. The Hesitant fuzzy geometric BM (GBM) operators were proposed by Zhu et al. [15]. Further, Jamil and Rashid [16] proposed dual hesitant fuzzy GBM (DHFGBM) operator for DHFNs. Furthermore, in certain situations, it should be taken into account that all attributes may not be interconnected, or certain attributes may exhibit a correlation with each other. For this, Dutta and Guha [17] first presented the partitioned Bonferroni mean (PBM) operator. Moreover, Saha et al. [18] proposed q-rung orthopair fuzzy weighted fairly aggregation operators in 2021.

(ii) *From the perspective of the AO operations*: The above AOs do not take into account the interaction between the NMD and MD of the evaluated value but merely use traditional operations to independently aggregate the MD and NMD, respectively. Therefore, these theories cannot deal with the MD or NMD with zero values. To solve this issue, He et al. [19] proposed the interaction operation and some related intuitionistic fuzzy geometric interaction averaging (IFGIA) operators. Xing et al. [20] proposed some interaction Hamy mean operators in 2017. Xu et al. [21] defined interaction AOs under dual hesitant fuzzy environment. In recent years, there are increasing research studies regarding AO operations. Lin et al. [22, 23] proposed linguistic q-rung orthopair fuzzy interactional partitioned Heronian mean aggregation operators and picture fuzzy interactional partitioned Heronian mean aggregation operators to further address hesitant problems.

The conclusion we can draw from the above is that the combination of the functions and operations of the AOs can better solve practical problems. Meanwhile, it has great advantages to combine the interaction operational laws with the PBM operator. Yang et al. [24] proposed Pythagorean fuzzy interaction PBM (PFIPBM) operators, and Liu et al. [25] proposed intuitionistic fuzzy interaction PBM (IFIPBM) operator. However, it is obvious that these existing operators have the following disadvantages:

(i) Although the interaction operation has been considered in [19, 23], the DHq-ROFSs and interaction operator of q-ROFSs have not been combined yet. So, it is not sufficient to process more problems for MCGDM.

(ii) Although the PBM operator has been combined with the interaction operations of IFSs and PFSs to process MCGDM problems, the PBM operator cannot get reasonable values by using interaction operational for DHq-ROFNs, which limits its application scope.

(iii) In view of the complexity of group decision making when handling MCGDM, a matrix should first be integrated by using the correlation between attributes of BM operator, and then the correlation between partitioned attributes of PBM operator should be used for integrated calculation to obtain the optimal result.

For handling these shortcomings and improving the effectiveness of existing methods in multicriteria decision-making problems, we will simultaneously use the following tools.

The DHq-ROFSs can reasonably describe hesitation attitude of DMs when giving the evaluation value. The interaction operations of q-ROFSs can more effectively depict uncertain problems by adjusting the parameter q and considering the interaction between NMD and MD. In addition, by considering the relationship between partial attributes, the PBM operator can reduce the loss of fuzzy information.

According to the aforementioned analysis, motivated by the characteristic idea of PBM and q-rung orthopair fuzzy interaction operations, this paper should achieve the following supreme goals:

(i) To propose a MCGDM method relying on the denoted interaction BM and PBM operators according to the information aggregation situation in the actual case.

(ii) To better solve the problem of extreme situation when the DMs give the evaluation value.

(iii) To propose some novel interaction PBM operators for DHq-ROFNs.

In this paper, we propose the dual hesitant q-rung orthopair fuzzy weighted interaction PGBM (DHq-

ROFWIPGBM) operator, the dual hesitant q-rung orthopair fuzzy interaction partitioned geometric Bonferroni mean (PGBM) (DHq-ROFIPGBM) operators, the dual hesitant q-rung orthopair fuzzy weighted interaction PBM (DHq-ROFWIPBM) operator, and the dual hesitant q-rung orthopair fuzzy interaction PBM (DHq-ROFIPBM) operator and give the several properties of these AOs. Moreover, we also propose some certain situations of these AOs. Furthermore, we present a MCGDM approach with DHq-ROFNs. Finally, we compare the results and characteristics of the proposed method with the methods in [24–26]. The results show that the method proposed in this paper is more effective than the methods in [24–26] in handling MCGDM problems.

This paper is organized as follows. In Section 2, some theories of DHq-ROFS, the interaction operational rules of q-ROFSs, and the BM, GBM, and PBM operators are introduced. In Section 3, we propose the DHq-ROFIPBM, DHq-ROFWIPBM, DHq-ROFIPGBM, and DHq-ROFWIPGBM operators for DHq-ROFNs. We also discuss the properties and special forms associated with AOs. In Section 4, based on the proposed AOs, we introduce a MCGDM method based on the novel BM and PBM operators. In Section 5, the feasibility and advantages of the proposed approach are verified by giving an example and comparing it with prevailing approaches. In Section 6, some conclusions and future research are given.

2. Preliminaries

In the following, we briefly introduce the theories of the BM, GBM, and PBM operators, q-ROFSs, DHq-ROFSs, and the interaction operational laws of q-ROFSs.

Definition 1 (see [5]). For a set X, a q-ROFS R in X is

$$R=\left\{\langle x,\varphi_R(x),\delta_R(x)\rangle \mid x\in X\right\}, \tag{1}$$

where $\varphi_R\colon X\longrightarrow[0,1]$ is MD, $\delta_R\colon X\longrightarrow[0,1]$ is NMD s.t. $0\le(\varphi_R(x))^q+(\delta_R(x))^q\le 1\,(q\ge 1)$. Besides, $\pi_R(x)=(1-(\varphi_R(x))^q-(\delta_R(x))^q)^{1/q}$ is HD.

In this paper, $\alpha=(\varphi,\delta)$ is called q-ROFN for convenience [5].

Definition 2 (see [20]). For q-ROFNs $\alpha=(\varphi,\delta)$, $\alpha_1=(\varphi_1,\delta_1)$, and $\alpha_2=(\varphi_2,\delta_2)$, the interaction operational laws are as follows:

$$\begin{aligned}
&(1)\ \alpha_1\oplus\alpha_2=\left\langle\left(1-\prod_{i=1}^{2}(1-\varphi_i^q)\right)^{1/q},\left(\prod_{i=1}^{2}(1-\varphi_i^q)-\prod_{i=1}^{2}(1-\varphi_i^q-\delta_i^q)\right)^{1/q}\right\rangle,\\
&(2)\ \alpha_1\otimes\alpha_2=\left\langle\left(\prod_{i=1}^{2}(1-\delta_i^q)-\prod_{i=1}^{2}(1-\varphi_i^q-\delta_i^q)\right)^{1/q},\left(1-\prod_{i=1}^{2}(1-\delta_i^q)\right)^{1/q}\right\rangle,\\
&(3)\ \lambda\alpha=\left\langle\left(1-(1-\varphi^q)^\lambda\right)^{1/q},\left((1-\varphi^q)^\lambda-(1-\varphi^q-\delta^q)^\lambda\right)^{1/q}\right\rangle,\quad \lambda>0,\\
&(4)\ \alpha^\lambda=\left\langle\left((1-\delta^q)^\lambda-(1-\varphi^q-\delta^q)^\lambda\right)^{1/q},\left(1-(1-\delta^q)^\lambda\right)^{1/q}\right\rangle,\quad \lambda>0.
\end{aligned}\tag{2}$$

Definition 3 (see [14]). For a set X, a DHq-ROFS D on X is

$$D=\left\{\langle x,h_u(x),g_u(x)\rangle \mid x\in X\right\}, \tag{3}$$

where $h_u(x)$ and $g_u(x)$ are the two sets with valued ranging from 0 to 1, are called PMD and PNMD of $x\in X$ are respectively with

$$\varphi^q+\delta^q\le 1, \tag{4}$$

where $\varphi\in h_u(x),\delta\in g_u(x)$. For convenience, $d(x)=(h_u(x),g_u(x))$ is called a dual hesitant q-rung orthopair fuzzy number (DHq-ROFN) denoted by $d=(h,g)$ with $\varphi\in h,\delta\in g,0\le\varphi,\delta\le 1$, and $\varphi^q+\delta^q\le 1$.

Definition 4 (see [14]). The score function S of DHq-ROFN $d=(h,g)$ is

$$S(d)=\frac{1}{2}\left(1+\frac{1}{\#h}\sum_{\varphi\in h}\varphi^q-\frac{1}{\#g}\sum_{\delta\in g}\delta^q\right), \tag{5}$$

where $\#h$ and $\#g$ are the numbers of the elements in h and g, respectively.

Definition 5 (see [14]). The accuracy function A of DHq-ROFN $d=(h,g)$ is

$$A(d)=\frac{1}{\#h}\sum_{\varphi\in h}\varphi^q+\frac{1}{\#g}\sum_{\delta\in g}\delta^q. \tag{6}$$

Definition 6 (see [14]). For two DHq-ROFNs $d_1=(h_1,g_1)$ and $d_2=(h_2,g_2)$, we have (i) (a) $d_1>d_2$ *for* $A(d_1)>A(d_2)$; (b) $d_1=d_2$ *for* $A(d_1)=A(d_2)$, where $S(d_1)=S(d_2)$; (ii) $d_1>d_2$ *for* $S(d_1)>S(d_2)$.

Definition 7 (see [13]). The BM is denoted as

$$\mathrm{BM}^{s,t}(\varpi_1,\varpi_2,\ldots,\varpi_n)=\left(\frac{1}{n(n-1)}\bigoplus_{i,j=1,i\ne j}^{n}\left(\varpi_i^s\otimes\varpi_j^t\right)\right)^{1/s+t}, \tag{7}$$

where $s+t>0$ for $s,t\geq 0$, and $\varpi_k\geq 0\,(k=1,2,\ldots,n)$.

Definition 8 (see [15]). The GBM is denoted as

$$\mathrm{GBM}^{s,t}(\varpi_1,\varpi_2,\ldots,\varpi_n)=\frac{1}{s+t}\left(\mathop{\oplus}_{i,j=1,i\neq j}^{n}\left(s\varpi_i\oplus t\varpi_j\right)\right)^{1/n(n-1)}, \tag{8}$$

where $s,t\geq 0$ with $s+t>0$, and $\varpi_k\geq 0\,(k=1,2,\ldots,n)$.

Definition 9 (see [17]). Let $s+t\geq 0$ where $s,t\geq 0$, and $T=(\varpi_1,\varpi_2,\ldots,\varpi_n)$ with $\varpi_k\geq 0\,(k=1,2,\ldots,n)$, which is partitioned into x distinct sorts $P_1,P_2,\ldots,P_x$, where $\bigcup_{h=1}^{x}P_h=T$. The PBM is denoted by

$$\mathrm{PBM}^{s,t}(\varpi_1,\varpi_2,\ldots,\varpi_n)=\frac{1}{d}\left(\sum_{h=1}^{x}\left(\frac{1}{|P_h|}\sum_{i\in P_h}\varpi_i^s\left(\frac{1}{|P_h|-1}\sum_{j\in P_h,j\neq i}\varpi_j^t\right)\right)^{1/s+t}\right), \tag{9}$$

where $|P_h|$ denotes the cardinality of P_h, d is the number of the partitioned sorts, and $\sum_{h=1}^{x}|P_h|=n$.

3. Some Dual Hesitant q-Rung Orthopair Fuzzy Interaction PBM Operators for DHq-ROFNs

In this part, we present some basic operational rules among the DHq-ROFSs considering the interaction.

Definition 10. Let $d_1=(h_1,g_1)$ and $d_2=(h_2,g_2)$ be two DHq-ROFNs; then,

$$(1)\; d_1\oplus d_2=\cup_{\varphi_1\in h_1,\delta_1\in g_1,\varphi_2\in h_2,\delta_2\in g_2}\left\{\left\{\left(1-\prod_{i=1}^{2}\left(1-\varphi_i^q\right)\right)^{1/q}\right\},\left\{\left(\prod_{i=1}^{2}\left(1-\varphi_i^q\right)-\prod_{i=1}^{2}\left(1-\varphi_i^q-\delta_i^q\right)\right)^{1/q}\right\}\right\}, \tag{10}$$

$$(2)\; d_1\otimes d_2=\cup_{\varphi_1\in h_1,\delta_1\in g_1,\varphi_2\in h_2,\delta_2\in g_2}\left\{\left\{\left(\prod_{i=1}^{2}\left(1-\delta_i^q\right)-\prod_{i=1}^{2}\left(1-\varphi_i^q-\delta_i^q\right)\right)^{1/q}\right\},\left\{\left(1-\prod_{i=1}^{2}\left(1-\delta_i^q\right)\right)^{1/q}\right\}\right\}, \tag{11}$$

$$(3)\; \lambda d=\cup_{\varphi\in h,\delta\in g}\left\{\left\{\left(1-\left(1-\varphi^q\right)^{\lambda}\right)^{1/q}\right\},\left\{\left(\left(1-\varphi^q\right)^{\lambda}-\left(1-\varphi^q-\delta^q\right)^{\lambda}\right)^{1/q}\right\}\right\},\quad \lambda>0, \tag{12}$$

$$(4)\; d^{\lambda}=\cup_{\varphi\in h,\delta\in g}\left\{\left\{\left(\left(1-\delta^q\right)^{\lambda}-\left(1-\varphi^q-\delta^q\right)^{\lambda}\right)^{1/q}\right\},\left\{\left(1-\left(1-\delta^q\right)^{\lambda}\right)^{1/q}\right\}\right\},\quad \lambda>0. \tag{13}$$

Then, based on interaction operations for DHq-ROFNs, we provide the DHq-ROFIPBM, DHq-ROFWIPBM, DHq-ROFIPGBM, and DHq-ROFWIPGBM operators along with their corresponding properties. Besides, we provide their special cases.

3.1. The DHq-ROFIPBM Operator

Definition 11. Let $T=(d_1,d_2,\ldots,d_n)$ be a collection of DHq-ROFNs, which is partitioned into x distinct sorts $P_1,P_2,\ldots,P_x$, where $d_i=(h_i,g_i)\,(i=1,2,\ldots,n)$ and $\bigcup_{h=1}^{x}P_h=T$. The DHq-ROFIPBM operator is defined as

$$\mathrm{DHq-ROFIPBM}^{s,t}(d_1,d_2,\ldots,d_n)=\frac{1}{x}\left(\mathop{\oplus}_{h=1}^{x}\left(\frac{1}{|P_h|}\mathop{\oplus}_{i\in P_h}\left(d_i^s\otimes\left(\frac{1}{|P_h|-1}\mathop{\oplus}_{j\in P_h,j\neq i}d_j^t\right)\right)\right)^{1/s+t}\right), \tag{14}$$

where $s, t \geq 0$, $|P_h|$ denotes the cardinality of P_h, x is the number of the partitioned sorts, and $\sum_{h=1}^{x} |P_h| = n$.

Theorem 12. *Let $d_i = (h_i, g_i)\,(i = 1, 2, \ldots, n)$ be a collection of DHq-ROFNs and $s, t \geq 0$. Then, the aggregated value of d_i obtained by DHq-ROFIPBM operator is a DHq-ROFNs, shown as follows:*

$$\begin{aligned}
&\text{DHq–ROFIPBM}^{s,t}(d_1, d_2, \ldots, d_n) = \cup_{\varphi_i \in h_i, \delta_i \in g_i, \varphi_j \in h_j, \delta_j \in g_j} \\
&\left\{ \begin{array}{c}
\left\{ \left(1 - \prod_{h=1}^{x} \left(1 - \left(\left(1 - \prod_{i \in P_h} (1 - \alpha_i^s (1 - \zeta + \gamma) + \beta_i^s \gamma)^{1/|P_h|} + \prod_{i \in P_h} (\beta_i^s \gamma)^{1/|P_h|} \right)^{1/s+t} + \left(\prod_{i \in P_h} (\beta_i^s \gamma)^{1/|P_h|} \right)^{1/s+t} \right)^{1/x} \right) \right)^{1/q} \right\}, \\
\left\{ \left(\prod_{h=1}^{x} \left(1 - \left(1 - \prod_{i \in P_h} (1 - \alpha_i^s (1 - \zeta + \gamma) + \beta_i^s \gamma)^{1/|P_h|} + \prod_{i \in P_h} (\beta_i^s \gamma)^{1/|P_h|} \right)^{1/s+t} + \left(\prod_{i \in P_h} (\beta_i^s \gamma)^{1/|P_h|} \right)^{1/s+t} \right)^{1/x} - \prod_{h=1}^{x} \left(\left(\prod_{i \in P_h} (\beta_i^s \gamma)^{1/|P_h|} \right)^{1/s+t} \right)^{1/x} \right)^{1/q} \right\}
\end{array} \right\},
\end{aligned} \tag{15}$$

where $\zeta = \prod_{j \in P_h, j \neq i} (1 - \alpha_j^t + \beta_j^t)^{1/|P_h|-1}$, $\gamma = (\prod_{j \in P_h, j \neq i} \beta_j^t)^{1/|P_h|-1}$, *and* $\alpha_i = 1 - \delta_i^q$, $\alpha_j = 1 - \delta_j^q$, $\beta_i = 1 - \varphi_i^q - \delta_i^q$, $\beta_j = 1 - \varphi_j^q - \delta_j^q$.

Proof. By equation (13), we have

$$\begin{aligned}
d_i^s &= \cup_{\varphi_i \in h_i, \delta_i \in g_i} \left\{ \left((1 - \delta_i^q)^s - (1 - \varphi_i^q - \delta_i^q)^s \right)^{1/q} \right\}, \left\{ \left(1 - (1 - \delta_i^q)^s \right)^{1/q} \right\}, \\
d_j^t &= \cup_{\varphi_j \in h_j, \delta_j \in g_j} \left\{ \left(\left(1 - \delta_j^q\right)^t - \left(1 - \varphi_j^q - \delta_j^q\right)^t \right)^{1/q} \right\}, \left\{ \left(1 - \left(1 - \delta_j^q\right)^t \right)^{1/q} \right\}.
\end{aligned} \tag{16}$$

Let $\alpha_i = 1 - \delta_i^q, \alpha_j = 1 - \delta_j^q, \beta_i = 1 - \varphi_i^q - \delta_i^q, \beta_j = 1 - \varphi_j^q - \delta_j^q$, and thus

$$\begin{aligned}
\underset{j \in P_h, j \neq i}{\oplus} d_j^t &= \cup_{\varphi_j \in h_j, \delta_j \in g_j} \left\{ \left\{ \left(1 - \prod_{j \in P_h, j \neq i} \left(1 - \alpha_j^t + \beta_j^t\right) \right)^{1/q} \right\}, \left\{ \left(\prod_{j \in P_h, j \neq i} \left(1 - \alpha_j^t + \beta_j^t\right) - \prod_{j \in P_h, j \neq i} \beta_j^t \right)^{1/q} \right\} \right\}, \\
\frac{1}{|P_h| - 1} \underset{j \in P_h, j \neq i}{\oplus} d_j^t &= \cup_{\varphi_j \in h_j, \delta_j \in g_j} \left\{ \left\{ \left(1 - \prod_{j \in P_h, j \neq i} \left(1 - \alpha_j^t + \beta_j^t\right)^{1/|P_h|-1} \right)^{1/q} \right\}, \left\{ \left(\prod_{j \in P_h, j \neq i} \left(1 - \alpha_j^t + \beta_j^t\right)^{1/|P_h|-1} - \left(\prod_{j \in P_h, j \neq i} \beta_j^t \right)^{1/|P_h|-1} \right)^{1/q} \right\} \right\}.
\end{aligned} \tag{17}$$

Let $\zeta = \prod_{j \in P_h, j \neq i} (1 - \alpha_j^t + \beta_j^t)^{1/|P_h|-1}, \gamma = (\prod_{j \in P_h; j \neq i} \beta_j^t)^{1/|P_h|-1}$, and thus

$$
\begin{aligned}
&d_i^s \otimes \left(\frac{1}{|P_h|-1} \underset{j\in P_h, j\neq i}{\oplus} d_j^t\right) = \cup_{\varphi_i\in h_i,\delta_i\in g_i,\varphi_j\in h_j,\delta_j\in g_j}\\
&\left\{\left\{\left(\alpha_i^s(1-\zeta+\gamma)-\beta_i^s\gamma\right)^{1/q}\right\},\left\{\left(1-\alpha_i^s(1-\zeta+\gamma)\right)^{1/q}\right\}\right\},\\
&\underset{i\in P_h}{\oplus}\left(d_i^s \otimes \left(\frac{1}{|P_h|-1} \underset{j\in P_h, j\neq i}{\oplus} d_j^t\right)\right) = \cup_{\varphi_i\in h_i,\delta_i\in g_i,\varphi_j\in h_j,\delta_j\in g_j}\\
&\left\{\left\{\left(1-\prod_{i\in P_h}\left(1-\alpha_i^s(1-\zeta+\gamma)+\beta_i^s\gamma\right)\right)^{1/q}\right\},\left\{\left(\prod_{i\in P_h}\left(1-\alpha_i^s(1-\zeta+\gamma)+\beta_i^s\gamma\right)-\prod_{i\in P_h}\beta_i^s\gamma\right)^{1/q}\right\}\right\},\\
&\left(\frac{1}{|P_h|}\underset{i\in P_h}{\oplus}\left(d_i^s \otimes \left(\frac{1}{|P_h|-1} \underset{j\in P_h, j\neq i}{\oplus} d_j^t\right)\right)\right)^{1/s+t} = \cup_{\varphi_i\in h_i,\delta_i\in g_i,\varphi_j\in h_j,\delta_j\in g_j}\\
&\left\{\begin{array}{c}
\left\{\left(\left(\prod_{i\in P_h}\left(1-\alpha_i^s(1-\zeta+\gamma)+\beta_i^s\gamma\right)^{1/|P_h|}+\prod_{i\in P_h}\left(\beta_i^s\gamma\right)^{1/|P_h|}\right)^{1/s+t}-\left(\prod_{i\in P_h}\left(\beta_i^s\gamma\right)^{1/|P_h|}\right)^{1/s+t}\right)^{1/q}\right\},\\
\left\{\left(1-\left(1-\prod_{i\in P_h}\left(1-\alpha_i^s(1-\zeta+\gamma)+\beta_i^s\gamma\right)^{1/|P_h|}+\prod_{i\in P_h}\left(\beta_i^s\gamma\right)^{1/|P_h|}\right)^{1/s+t}\right)^{1/q}\right\}
\end{array}\right\}.
\end{aligned}
\tag{18}
$$

Therefore, we can get

$$
\begin{aligned}
&\frac{1}{x}\left(\underset{h=1}{\overset{x}{\oplus}}\left(\frac{1}{|P_h|}\underset{i\in P_h}{\oplus}\left(d_i^s \otimes \left(\frac{1}{|P_h|-1} \underset{j\in P_h, j\neq i}{\oplus} d_j^t\right)\right)\right)^{1/s+t}\right) = \cup_{\varphi_i\in h_i,\delta_i\in g_i,\varphi_j\in h_j,\delta_j\in g_j}\\
&\left\{\begin{array}{c}
\left\{\left(1-\prod_{h=1}^{x}\left(1-\left(1-\prod_{i\in P_h}\left(1-\alpha_i^s(1-\zeta+\gamma)+\beta_i^s\gamma\right)^{1/|P_h|}+\prod_{i\in P_h}\left(\beta_i^s\gamma\right)^{1/|P_h|}\right)^{1/s+t}+\left(\prod_{i\in P_h}\left(\beta_i^s\gamma\right)^{1/|P_h|}\right)^{1/s+t}\right)^{1/x}\right)^{1/q}\right\},\\
\left\{\left(\prod_{h=1}^{x}\left(1-\left(1-\prod_{i\in P_h}\left(1-\alpha_i^s(1-\zeta+\gamma)+\beta_i^s\gamma\right)^{1/|P_h|}+\prod_{i\in P_h}\left(\beta_i^s\gamma\right)^{1/|P_h|}\right)^{1/s+t}+\left(\prod_{i\in P_h}\left(\beta_i^s\gamma\right)^{1/|P_h|}\right)^{1/s+t}\right)^{1/x}-\prod_{h=1}^{x}\left(\left(\prod_{i\in P_h}\left(\beta_i^s\gamma\right)^{1/|P_h|}\right)^{1/s+t}\right)^{1/x}\right)^{1/q}\right\}
\end{array}\right\}.
\end{aligned}
\tag{19}
$$

Hence, we have completed the proof. □

Next, we provide some basic properties of the DHq-ROFIPBM operator.

Theorem 13 (idempotency). *Let $\tilde{d}_i = (h_i, g_i)\,(i = 1, 2, \ldots, n)$ be a collection of DHq-ROFNs and $s, t > 0$. If $(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_n)$ are equal, which is $\tilde{d} = \tilde{d}_i = (h, g)$ $(i = 1, 2, \ldots, n)$, then*

$$\mathrm{DHq-ROFIPBM}^{s,t}\left(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_n\right) = \tilde{d}. \tag{20}$$

Proof. When $\tilde{d} = \tilde{d}_1 = \tilde{d}_2 = \cdots = \tilde{d}_n = (h, g)$, we can get $\alpha_i = 1 - \delta_i^q = \alpha$, $\alpha_j = 1 - \delta_j^q = \alpha$, $\beta_i = 1 - \varphi_i^q - \delta_i^q = \beta$, $\beta_j = 1 - \varphi_j^q - \delta_j^q = \beta$. Therefore, we have

$$\begin{aligned}
&\zeta = \prod_{j\in P_h, j\neq i} \left(1 - \alpha_j^t + \beta_j^t\right)^{1/|P_h|-1} = \prod_{j\in P_h, j\neq i} \left(1 - \alpha^t + \beta^t\right)^{1/|P_h|-1} = 1 - \alpha^t + \beta^t,\\
&\gamma = \left(\prod_{j\in P_h, j\neq i} \beta_j^t\right)^{1/|P_h|-1} = \left(\prod_{j\in P_h, j\neq i} \beta^t\right)^{1/|P_h|-1} = \beta^t,\\
&\mathrm{DHq-ROFIPBM}^{s,t}\left(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_n\right) = \cup_{\varphi\in h, \delta\in g}\\
&\left\{\begin{array}{c}
\left\{\left(1 - \prod_{h=1}^{x}\left(1 - \left(1 - \prod_{i\in P_h}\left(1 - \alpha^s\alpha^t + \beta^s\beta^t\right)^{1/|P_h|} + \beta^s\beta^t\right)^{1/s+t} + \left(\beta^s\beta^t\right)^{1/s+t}\right)^{1/x}\right)^{1/q}\right\},\\
\left\{\left(\prod_{h=1}^{x}\left(1 - \left(1 - \prod_{i\in P_h}\left(1 - \alpha^s\alpha^t + \beta^s\beta^t\right)^{1/|P_h|} + \prod_{i\in P_h}\left(\beta^s\beta^t\right)^{1/|P_h|}\right)^{1/s+t} + \left(\prod_{i\in P_h}\left(\beta^s\beta^t\right)^{1/|P_h|}\right)^{1/s+t}\right)^{1/x} - \prod_{h=1}^{x}\left(\left(\prod_{i\in P_h}\left(\beta^s\beta^t\right)^{1/|P_h|}\right)^{1/s+t}\right)^{1/x}\right)^{1/q}\right\}
\end{array}\right\}\\
&= \cup_{\varphi\in h, \delta\in g}\\
&\left\{\left\{\left(1 - \prod_{h=1}^{x}\left(1 - \left(1 - \left(1 - \alpha^{s+t} + \beta^{s+t}\right) + \beta^{s+t}\right)^{1/s+t} + +\left(\beta^{s+t}\right)^{1/s+t}\right)^{1/x}\right)^{1/q}\right\}, \left(\prod_{h=1}^{x}\left(1 - \left(1 - \left(1 - \alpha^{s+t} + \beta^{s+t}\right) + \left(\beta^{s+t}\right)\right)^{1/s+t} + \left(\left(\beta^{s+t}\right)\right)^{1/s+t}\right)^{1/x} - \prod_{h=1}^{x}\left(\left(\left(\beta^{s+t}\right)\right)^{1/s+t}\right)^{1/x}\right)^{1/q}\right\}\right\}\\
&= \cup_{\varphi\in h, \delta\in g}\{\{\varphi\}, \{\delta\}\} = \tilde{d}.
\end{aligned} \tag{21}$$

□

Theorem 14 (commutativity). *Let $\tilde{d}_i = (h_i, g_i)$ and $\tilde{d}'_i = (h'_i, g'_i)$ $(i = 1, 2, \ldots, n)$ be two collections of DHq-ROFNs. If $\tilde{d}'_i = (h'_i, g'_i)$ is a permutation of $\tilde{d}_i = (h_i, g_i)$, then*

$$\begin{aligned}
&\mathrm{DHq-ROFIPBM}^{s,t}\left(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_n\right)\\
&\quad = \mathrm{DHq-ROFIPBM}^{s,t}\left(\tilde{d}'_1, \tilde{d}'_2, \ldots, \tilde{d}'_n\right).
\end{aligned} \tag{22}$$

Proof. Based on equation (15), we have

$$\begin{aligned}
&\mathrm{DHq-ROFIPBM}^{s,t}\left(d_1, d_2, \ldots, d_n\right) = \cup_{\varphi_i\in h_i, \delta_i\in g_i, \varphi_j\in h_j, \delta_j\in g_j}\\
&\left\{\begin{array}{c}
\left\{\left(1 - \prod_{h=1}^{x}\left(1 - \left(1 - \prod_{i\in P_h}\left(1 - \alpha_i^s(1 - \zeta + \gamma) + \beta_i^s\gamma\right)^{1/|P_h|} + \prod_{i\in P_h}\left(\beta_i^s\gamma\right)^{1/|P_h|}\right)^{1/s+t} + \left(\prod_{i\in P_h}\left(\beta_i^s\gamma\right)^{1/|P_h|}\right)^{1/s+t}\right)^{1/x}\right)^{1/q}\right\},\\
\left\{\left(\prod_{h=1}^{x}\left(1 - \left(1 - \prod_{i\in P_h}\left(1 - \alpha_i^s(1 - \zeta + \gamma) + \beta_i^s\gamma\right)^{1/|P_h|} + \prod_{i\in P_h}\left(\beta_i^s\gamma\right)^{1/|P_h|}\right)^{1/s+t} + \left(\prod_{i\in P_h}\left(\beta_i^s\gamma\right)^{1/|P_h|}\right)^{1/s+t}\right)^{1/x} - \prod_{h=1}^{x}\left(\left(\prod_{i\in P_h}\left(\beta_i^s\gamma\right)^{1/|P_h|}\right)^{1/s+t}\right)^{1/x}\right)^{1/q}\right\}
\end{array}\right\},\\
&\mathrm{DHq-ROFIPBM}^{s,t}\left(d'_1, d'_2, \ldots, d'_n\right) = \cup_{\varphi'_i\in h'_i, \delta'_i\in g'_i, \varphi'_j\in h'_j, \delta'_j\in g'_j}\\
&\left\{\begin{array}{c}
\left\{\left(1 - \prod_{h=1}^{x}\left(1 - \left(1 - \prod_{i\in P_h}\left(1 - \alpha_i'^s\left(1 - \zeta' + \gamma'\right) + \beta'^s_i\gamma'\right)^{1/|P_h|} + \prod_{i\in P_h}\left(\beta'^s_i\gamma'\right)^{1/|P_h|}\right)^{1/s+t} + \left(\prod_{i\in P_h}\left(\beta'^s_i\gamma'\right)^{1/|P_h|}\right)^{1/s+t}\right)^{1/x}\right)^{1/q}\right\},\\
\left\{\left(\prod_{h=1}^{x}\left(1 - \left(1 - \prod_{i\in P_h}\left(1 - \alpha_i'^s\left(1 - \zeta' + \gamma'\right) + \beta'^s_i\gamma'\right)^{1/|P_h|} + \prod_{i\in P_h}\left(\beta'^s_i\gamma'\right)^{1/|P_h|}\right)^{1/s+t} + \left(\prod_{i\in P_h}\left(\beta'^s_i\gamma'\right)^{1/|P_h|}\right)^{1/s+t}\right)^{1/x} - \prod_{h=1}^{x}\left(\left(\prod_{i\in P_h}\left(\beta'^s_i\gamma'\right)^{1/|P_h|}\right)^{1/s+t}\right)^{1/x}\right)^{1/q}\right\}
\end{array}\right\}.
\end{aligned} \tag{23}$$

Since $\tilde{d}'_i = (h'_i, g'_i)$ is a permutation of $\tilde{d}_i = (h_i, g_i)$, then

$$\begin{aligned} &\mathrm{DHq-ROFIPBM}^{s,t}\left(\tilde{d}_1,\tilde{d}_2,\ldots,\tilde{d}_n\right)\\ &\quad=\mathrm{DHq-ROFIPBM}^{s,t}\left(\tilde{d}_1',\tilde{d}_2',\ldots,\tilde{d}_n'\right). \end{aligned} \tag{24}$$

Further, we give several special cases of the $\mathrm{DHq-ROFIPBM}^{s,t}$ operator by adjusting the parameters s and t.

(1) When $t \longrightarrow 0$, we can get $\zeta = \prod_{j\in P_h, j\neq i}(1-\alpha_j^t+\beta_j^t)^{1/|P_h|-1} = 1$, $\gamma = (\prod_{j\in P_h, j\neq i}\beta_j^t)^{1/|P_h|-1} = 1$. Thus, we have

$$\mathrm{DHq{-}ROFIPBM}^{s,0}(d_1,d_2,\ldots,d_n) = \cup_{\varphi_i\in h_i,\delta_i\in g_i}\left\{\begin{array}{c}\left\{\left(1-\prod_{h=1}^{x}\left(1-\left(1-\left(\prod_{i\in P_h}(1-\alpha_i^s+\beta_i^s)\right)^{1/|P_h|}+\prod_{i\in P_h}(\beta_i^s)^{1/|P_h|}\right)^{1/s}+\left(\prod_{i\in P_h}(\beta_i^s)^{1/|P_h|}\right)^{1/s}\right)^{1/x}\right)^{1/q}\right\},\\ \left\{\left(\prod_{h=1}^{x}\left(1-\left(1-\prod_{i\in P_h}(1-\alpha_i^s+\beta_i^s)^{1/|P_h|}+\prod_{i\in P_h}(\beta_i^s)^{1/|P_h|}\right)^{1/s}+\left(\prod_{i\in P_h}(\beta_i^s)^{1/|P_h|}\right)^{1/s}\right)^{1/x}-\prod_{h=1}^{x}\left(\left(\prod_{i\in P_h}(\beta_i^s)^{1/|P_h|}\right)^{1/s}\right)^{1/x}\right)^{1/q}\right\}\end{array}\right\}. \tag{25}$$

(2) When $s=1$, $t \longrightarrow 0$, we can get $\zeta = \prod_{j\in P_h; j\neq i}(1-\alpha_j^t+\beta_j^t)^{1/|P_h|-1} = 1$, $\gamma = (\prod_{j\in P_h; j\neq i}\beta_j^t)^{1/|P_h|-1} = 1$. Thus, we have

$$\mathrm{DHq-ROFIPBM}^{1,0}(d_1,d_2,\ldots,d_n) = \cup_{\varphi_i\in h_i,\delta_i\in g_i}\left\{\begin{array}{c}\left\{\left(1-\prod_{h=1}^{x}\left(\prod_{i\in P_h}(1-\varphi_i^q)^{1/|P_h|}\right)^{1/x}\right)^{1/q}\right\},\\ \left\{\left(\prod_{h=1}^{x}\left(\prod_{i\in P_h}(1-\varphi_i^q)^{1/|P_h|}\right)^{1/x}-\prod_{h=1}^{x}\left(\prod_{i\in P_h}(1-\varphi_i^q-\delta_i^q)^{1/|P_h|}\right)^{1/x}\right)^{1/q}\right\}\end{array}\right\}. \tag{26}$$

(3) When $s \longrightarrow 0$, we can get

$$\mathrm{DHq-ROFIPBM}^{0,t}(d_1,d_2,\ldots,d_n) = \cup_{\varphi_j\in h_j,\delta_j\in g_j}\left\{\begin{array}{c}\left\{\left(1-\prod_{h=1}^{x}\left(1-(1-\zeta+\gamma)^{1/t}+\gamma^{1/t}\right)^{1/x}\right)^{1/q}\right\},\\ \left\{\left(\prod_{h=1}^{x}\left(1-(1-\zeta+\gamma)^{1/t}+\gamma^{1/t}\right)^{1/x}-\prod_{h=1}^{x}\left(\gamma^{1/t}\right)^{1/x}\right)^{1/q}\right\}\end{array}\right\}. \tag{27}$$

(4) When $s=1$, $t=1$, we can get $\zeta = \prod_{j\in P_h; j\neq i}(1-\alpha_j+\beta_j)^{1/|P_h|-1}$, $\gamma = (\prod_{j\in P_h; j\neq i}\beta_j)^{1/|P_h|-1}$. Thus, we have

$$\text{DHq}-\text{ROFIPBM}^{s,t}\left(d_1,d_2,\ldots,d_n\right)=\cup_{\varphi_i\in h_i,\delta_i\in g_i,\varphi_j\in h_j,\delta_j\in g_j}$$

$$\left\{\begin{array}{c}\left\{\Bigg(1-\prod_{h=1}^{x}\Bigg(1-\Bigg(1-\prod_{i\in P_h}\left(1-\alpha_i\left(1-\zeta+\gamma\right)+\beta_i\gamma\right)^{1/\left|P_h\right|}+\prod_{i\in P_h}\left(\beta_i\gamma\right)^{1/\left|P_h\right|}\Bigg)^{1/2}+\Bigg(\prod_{i\in P_h}\left(\beta_i\gamma\right)^{1/\left|P_h\right|}\Bigg)^{1/2}\Bigg)^{1/x}\Bigg)^{1/q}\right\}, \\ \left\{\Bigg(\prod_{h=1}^{x}\Bigg(1-\Bigg(1-\prod_{i\in P_h}\left(1-\alpha_i\left(1-\zeta+\gamma\right)+\beta_i\gamma\right)^{1/\left|P_h\right|}+\prod_{i\in P_h}\left(\beta_i\gamma\right)^{1/\left|P_h\right|}\Bigg)^{1/2}+\Bigg(\prod_{i\in P_h}\left(\beta_i\gamma\right)^{1/\left|P_h\right|}\Bigg)^{1/2}\Bigg)^{1/x}-\prod_{h=1}^{x}\Bigg(\Bigg(\prod_{i\in P_h}\left(\beta_i\gamma\right)^{1/\left|P_h\right|}\Bigg)^{1/2}\Bigg)^{1/x}\Bigg)^{1/q}\right\}\end{array}\right\}. \tag{28}$$

If all the DHq-ROFNS are partitioned into one sort, the DHq-ROFIPBM operator reduces to the dual hesitant q-rung orthopair fuzzy interaction Bonferroni mean (DHq-ROFIBM) operator as follows:

$$\text{DHq}-\text{ROFIBM}^{s,t}\left(d_1,d_2,\ldots,d_n\right)=\left(\frac{1}{n(n-1)}\bigoplus_{i,j=1,i\neq j}^{n}\left(d_i^s\otimes d_j^t\right)\right)^{1/s+t}$$

$$=\cup_{\varphi_i\in h_i,\delta_i\in g_i,\varphi_j\in h_j,\delta_j\in g_j}$$

$$\left\{\begin{array}{c}\left\{\Bigg(\Bigg(1-\Bigg(\prod_{i,j=1,i\neq j}^{n}\left(1-\alpha_i^s\alpha_j^t+\beta_i^s\beta_j^t\right)^{1/n(n-1)}\Bigg)+\Bigg(\prod_{i,j=1,i\neq j}^{n}\beta_i^s\beta_j^t\Bigg)^{1/n(n-1)}\Bigg)^{1/s+t}-\Bigg(\Bigg(\prod_{i,j=1,i\neq j}^{n}\beta_i^s\beta_j^t\Bigg)^{1/n(n-1)}\Bigg)^{1/s+t}\Bigg)^{1/q}\right\}, \\ \left\{\Bigg(1-\Bigg(1-\Bigg(\prod_{i,j=1,i\neq j}^{n}\left(1-\alpha_i^s\alpha_j^t+\beta_i^s\beta_j^t\right)\Bigg)^{1/n(n-1)}+\Bigg(\prod_{i,j=1,i\neq j}^{n}\beta_i^s\beta_j^t\Bigg)^{1/n(n-1)}\Bigg)^{1/s+t}\Bigg)^{1/q}\right\}\end{array}\right\}. \tag{29}$$

□

3.2. The DHq-ROFWIPBM Operator

Definition 15. Let $T=\left(d_1,d_2,\ldots,d_n\right)$ be a collection of DHq-ROFNs, which is partitioned into x distinct sorts $P_1,P_2,\ldots,P_x$, where $d_i=\left(\varphi_i,\delta_i\right)\left(i=1,2,\ldots,n\right)$ and $\bigcup_{h=1}^{x}P_h=T$. The DHq-ROFWIPBM operator is defined as

$$\text{DHq}-\text{ROFWIPBM}^{s,t}\left(d_1,d_2,\ldots,d_n\right)=\frac{1}{x}\left(\bigoplus_{h=1}^{x}\left(\frac{1}{\left|P_h\right|\left(\left|P_h\right|-1\right)}\bigoplus_{i,j\in P_h,j\neq i}\left(\left(\omega_i d_i\right)^s\otimes\left(\omega_j d_j\right)^t\right)\right)^{1/s+t}\right), \tag{30}$$

where $s,t\geq 0$, $\left|P_h\right|$ denotes the cardinality of P_h, x is the number of the partitioned sorts, and $\sum_{h=1}^{x}\left|P_h\right|=n$, $\omega=\left(\omega_1,\omega_2,\ldots,\omega_n\right)$ is the weight vector of $\left(d_1,d_2,\ldots,d_n\right)$, $\omega_j\in[0,1]$, $j=1,2,\ldots,n$, and $\sum_{j=1}^{n}\omega_j=1$.

Theorem 16. *Let $d_i=\left(h_i,g_i\right)\left(i=1,2,\ldots,n\right)$ be a collection of DHq-ROFNs and $s,t\geq 0$. Then, the aggregated value of d_i obtained by DHq-ROFWIPBM operator is a DHq-ROFNs, shown as follows:*

$$\mathrm{DHq-ROFWIPBM}^{s,t}(d_1,d_2,\ldots,d_n)=\cup_{\varphi_i\in h_i,\delta_i\in g_i,\varphi_j\in h_j,\delta_j\in g_j}\left\{\begin{array}{c}\left\{\left(1-\prod_{h=1}^{x}\left(1-\left(1-\prod_{i,j\in P_h,j\neq i}(1-\zeta+\gamma)^{1/|P_h|(|P_h|-1)}+\prod_{i,j\in P_h,j\neq i}(\gamma)^{1/|P_h|(|P_h|-1)}\right)^{1/s+t}+\prod_{i,j\in P_h,j\neq i}(\gamma)^{1/|P_h|(|P_h|-1)(s+t)}\right)^{1/x}\right)^{1/q}\right\},\\ \left\{\left(\prod_{h=1}^{x}\left(1-\left(1-\prod_{i,j\in P_h,j\neq i}(1-\zeta+\gamma)^{1/|P_h|(|P_h|-1)}+\prod_{i,j\in P_h,j\neq i}(\gamma)^{1/|P_h|(|P_h|-1)}\right)^{1/s+t}+\prod_{i,j\in P_h,j\neq i}(\gamma)^{1/|P_h|(|P_h|-1)(s+t)}\right)^{1/x}-\prod_{h=1}^{x}\left(\prod_{i,j\in P_h,j\neq i}(\gamma)^{1/|P_h|(|P_h|-1)(s+t)x}\right)\right)^{1/q}\right\}\end{array}\right\}, \tag{31}$$

where $\zeta=(1-\alpha_i+\beta_i)^s(1-\alpha_j+\beta_j)^t$, $\gamma=\beta_i^s\beta_j^t$ *and* $\alpha_i=(1-\varphi_i^q)^{\omega_i}$, $\alpha_j=(1-\varphi_j^q)^{\omega_j}$, $\beta_i=(1-\varphi_i^q-\delta_i^q)^{\omega_i}$, $\beta_j=(1-\varphi_j^q-\delta_j^q)^{\omega_j}$.

Proof. By equation (12), we have

$$\begin{aligned}\omega_i d_i&=\cup_{\varphi_i\in h_i,\delta_i\in g_i}\left\{\left\{\left(1-(1-\varphi_i^q)^{\omega_i}\right)^{1/q}\right\},\left\{\left((1-\varphi_i^q)^{\omega_i}-(1-\varphi_i^q-\delta_i^q)^{\omega_i}\right)^{1/q}\right\}\right\},\\ \omega_j d_j&=\cup_{\varphi_j\in h_j,\delta_j\in g_j}\left\{\left\{\left(1-\left(1-\varphi_j^q\right)^{\omega_j}\right)^{1/q}\right\},\left\{\left(\left(1-\varphi_j^q\right)^{\omega_j}-\left(1-\varphi_j^q-\delta_j^q\right)^{\omega_j}\right)^{1/q}\right\}\right\}.\end{aligned} \tag{32}$$

Let $\alpha_i=(1-\varphi_i^q)^{\omega_i}$, $\alpha_j=(1-\varphi_j^q)^{\omega_j}$, $\beta_i=(1-\varphi_i^q-\delta_i^q)^{\omega_i}$, $\beta_j=(1-\varphi_j^q-\delta_j^q)^{\omega_j}$. Then, we can get

$$\begin{aligned}(\omega_i d_i)^s\otimes\left(\omega_j d_j\right)^t&=\cup_{\varphi_i\in h_i,\delta_i\in g_i,\varphi_j\in h_j,\delta_j\in g_j}\\ &\left\{\left\{\left((1-\alpha_i+\beta_i)^s\left(1-\alpha_j+\beta_j\right)^t-\beta_i^s\beta_j^t\right)^{1/q}\right\},\left\{\left(1-(1-\alpha_i+\beta_i)^s\left(1-\alpha_j+\beta_j\right)^t\right)^{1/q}\right\}\right\}.\end{aligned} \tag{33}$$

Let $\zeta=(1-\alpha_i+\beta_i)^s(1-\alpha_j+\beta_j)^t,\gamma=\beta_i^s\beta_j^t$. Then, we can get

$$\begin{aligned}&\oplus_{i,j\in P_h,j\neq i}(\omega_i d_i)^s\otimes\left(\omega_j d_j\right)^t=\cup_{\varphi_i\in h_i,\delta_i\in g_i,\varphi_j\in h_j,\delta_j\in g_j}\\ &\left\{\left\{\left(1-\prod_{i,j\in P_h,j\neq i}(1-\zeta+\gamma)\right)^{1/q}\right\},\left\{\left(\prod_{i,j\in P_h,j\neq i}(1-\zeta+\gamma)-\prod_{i,j\in P_h,j\neq i}(\gamma)\right)^{1/q}\right\}\right\},\\ &\left(\frac{1}{|P_h|(|P_h|-1)}\oplus_{i,j\in P_h,j\neq i}(\omega_i d_i)^s\otimes\left(\omega_j d_j\right)^t\right)^{1/s+t}=\cup_{\varphi_i\in h_i,\delta_i\in g_i,\varphi_j\in h_j,\delta_j\in g_j}\\ &\left\{\begin{array}{c}\left\{\left(\left(1-\prod_{i,j\in P_h,j\neq i}(1-\zeta+\gamma)^{1/|P_h|(|P_h|-1)}+\prod_{i,j\in P_h,j\neq i}(\gamma)^{1/|P_h|(|P_h|-1)}\right)^{1/s+t}-\prod_{i,j\in P_h,j\neq i}(\gamma)^{1/|P_h|(|P_h|-1)(s+t)}\right)^{1/q}\right\}\\ \left\{\left(1-\left(1-\prod_{i,j\in P_h,j\neq i}(1-\zeta+\gamma)^{1/|P_h|(|P_h|-1)}+\prod_{i,j\in P_h,j\neq i}(\gamma)^{1/|P_h|(|P_h|-1)}\right)^{1/s+t}\right)^{1/q}\right\}\end{array}\right\},\\ &\frac{1}{x}\left(\oplus_{h=1}^{x}\left(\frac{1}{|P_h|(|P_h|-1)}\oplus_{i,j\in P_h,j\neq i}\left((\omega_i d_i)^s\otimes\left(\omega_j d_j\right)^t\right)\right)^{1/s+t}\right)=\cup_{\varphi_i\in h_i,\delta_i\in g_i,\varphi_j\in h_j,\delta_j\in g_j}\\ &\left\{\begin{array}{c}\left\{\left(1-\prod_{h=1}^{x}\left(1-\left(1-\prod_{i,j\in P_h,j\neq i}(1-\zeta+\gamma)^{1/|P_h|(|P_h|-1)}+\prod_{i,j\in P_h,j\neq i}(\gamma)^{1/|P_h|(|P_h|-1)}\right)^{1/s+t}+\prod_{i,j\in P_h,j\neq i}(\gamma)^{1/|P_h|(|P_h|-1)(s+t)}\right)^{1/x}\right)^{1/q}\right\},\\ \left\{\left(\prod_{h=1}^{x}\left(1-\left(1-\prod_{i,j\in P_h,j\neq i}(1-\zeta+\gamma)^{1/|P_h|(|P_h|-1)}+\prod_{i,j\in P_h,j\neq i}(\gamma)^{1/|P_h|(|P_h|-1)}\right)^{1/s+t}+\prod_{i,j\in P_h,j\neq i}(\gamma)^{1/|P_h|(|P_h|-1)(s+t)}\right)^{1/x}-\prod_{h=1}^{x}\left(\prod_{i,j\in P_h,j\neq i}(\gamma)^{1/|P_h|(|P_h|-1)(s+t)x}\right)\right)^{1/q}\right\}\end{array}\right\}.\end{aligned} \tag{34}$$

If all the DHq-ROFNs are partitioned into one sort, the DHq-ROFWIPBM operator reduces to the dual hesitant q-rung orthopair fuzzy weighted interaction Bonferroni mean (DHq-ROFWIBM) operator as follows:

$$
\begin{aligned}
\mathrm{DHq-ROFWIBM}^{s,t}(d_1,d_2,\ldots,d_n) &= \left(\frac{1}{n(n-1)}\underset{i,j=1,j\neq i}{\overset{n}{\oplus}}\left(\left(\omega_i d_i\right)^s\otimes\left(\omega_j d_j\right)^t\right)\right)^{1/s+t}\\
&= \cup_{\varphi_i\in h_i,\delta_i\in g_i,\varphi_j\in h_j,\delta_j\in g_j}\\
&\left\{\begin{array}{c}
\left\{\Bigg(\Bigg(\Bigg(1-\prod_{i,j=1,j\neq i}(1-\zeta+\gamma)^{1/n(n-1)}+\prod_{i,j=1,j\neq i}(\gamma)^{1/n(n-1)}\Bigg)^{1/s+t}-\prod_{i,j=1,j\neq i}(\gamma)^{1/n(n-1)(s+t)}\Bigg)^{1/q}\right\},\\
\left\{\left(1-\left(1-\prod_{i,j=1,j\neq i}(1-\zeta+\gamma)^{1/n(n-1)}+\prod_{i,j=1,j\neq i}(\gamma)^{1/n(n-1)}\right)^{1/s+t}\right)^{1/q}\right\}
\end{array}\right\}.
\end{aligned}
\tag{35}
$$

□

3.3. The DHq-ROFIPGBM Operator

Definition 17. Let $T=(d_1,d_2,\ldots,d_n)$ be a collection of DHq-ROFNs, which is partitioned into x distinct sorts $P_1,P_2,\ldots,P_x$, where $d_i=(h_i,g_i)\,(i=1,2,\ldots,n)$ and $\bigcup_{h=1}^{x}P_h=T$. The DHq-ROFIPGBM operator is defined as

$$
\mathrm{DHq-ROFIPGBM}^{s,t}(d_1,d_2,\ldots,d_n)=\left(\overset{x}{\underset{h=1}{\otimes}}\left(\frac{1}{s+t}\left(\underset{i,j\in P_h,j\neq i}{\otimes}\left(\left(sd_i\right)\oplus\left(td_j\right)\right)\right)^{1/|P_h|(|P_h|-1)}\right)\right)^{1/x},
\tag{36}
$$

where $s,t\geq 0$, $|P_h|$ denotes the cardinality of P_h, x is the number of the partitioned sorts, and $\sum_{h=1}^{x}|P_h|=n$.

Theorem 18. *Let $d_i=(h_i,g_i)\,(i=1,2,\ldots,n)$ be a collection of DHq-ROFNs and $s,t\geq 0$. Then, the aggregated value of d_i obtained by DHq-ROFIPGBM operator is DHq-ROFN, shown as follows:*

$$
\begin{aligned}
&\mathrm{DHq-ROFIPGBM}^{s,t}(d_1,d_2,\ldots,d_n)=\cup_{\varphi_i\in h_i,\delta_i\in g_i,\varphi_j\in h_j,\delta_j\in g_j}\\
&\left\{\begin{array}{c}
\left\{\left(\prod_{h=1}^{x}\left(1-\left(1-\prod_{i\in P_h}(1-\alpha_i^s(1-\zeta+\gamma)+\beta_i^s\gamma)^{1/|P_h|}+\prod_{i\in P_h}(\beta_i^s\gamma)^{1/|P_h|}\right)^{1/s+t}+\left(\prod_{i\in P_h}(\beta_i^s\gamma)^{1/|P_h|}\right)^{1/s+t}\right)^{1/x}-\prod_{h=1}^{x}\left(\left(\prod_{i\in P_h}(\beta_i^s\gamma)^{1/|P_h|}\right)^{1/s+t}\right)^{1/x}\right)^{1/q}\right\},\\
\left\{\left(1-\prod_{h=1}^{x}\left(1-\left(1-\prod_{i\in P_h}(1-1-\alpha_i^s(1-\zeta+\gamma)+\beta_i^s\gamma)^{1/|P_h|}+\prod_{i\in P_h}(\beta_i^s\gamma)^{1/|P_h|}\right)^{1/s+t}+\left(\prod_{i\in P_h}(\beta_i^s\gamma)^{1/|P_h|}\right)^{1/s+t}\right)^{1/x}\right)^{1/q}\right\}
\end{array}\right\},
\end{aligned}
\tag{37}
$$

where $\zeta=\prod_{j\in P_h,j\neq i}(1-\alpha_j^t+\beta_j^t)^{1/|P_h|-1}$, $\gamma=(\prod_{j\in P_h,j\neq i}\beta_j^t)^{1/|P_h|-1}$ *and* $\alpha_i=1-\varphi_i^q$, $\alpha_j=1-\varphi_j^q$, $\beta_i=1-\varphi_i^q-\delta_i^q$, $\beta_j=1-\varphi_j^q-\delta_j^q$.

The proof is similar to Theorem 12, so we omit it.

Next, we can derive some basic properties for the DHq-ROFIPGBM operator.

Theorem 19 (idempotency). *Let* $\widetilde{d}_i = (h_i, g_i)\,(i = 1, 2, \ldots, n)$ *be a collection of DHq-ROFNs and* $s, t > 0$. *If* $(\widetilde{d}_1, \widetilde{d}_2, \ldots, \widetilde{d}_n)$ *are equal, which is* $\widetilde{d} = \widetilde{d}_i = (h, g)$, $i = 1, 2, \ldots, n$, *then*

$$\mathrm{DHq-ROFIPGBM}^{s,t}\left(\widetilde{d}_1, \widetilde{d}_2, \ldots, \widetilde{d}_n\right) = \widetilde{d}. \tag{38}$$

Theorem 20 (commutativity). *Let* $\widetilde{d}_i = (h_i, g_i)$ *and* $\widetilde{d}'_i = (h'_i, g'_i)\,(i = 1, 2, \ldots, n)$ *be two collections of DHq-ROFNs. If* $\widetilde{d}'_i = (h'_i, g'_i)$ *is any permutation of* $\widetilde{d}_i = (h_i, g_i)$, *then*

$$\mathrm{DHq-ROFIPGBM}^{s,t}\left(\widetilde{d}_1, \widetilde{d}_2, \ldots, \widetilde{d}_n\right) = \mathrm{DHq-ROFIPGBM}^{s,t}\left(\widetilde{d}'_1, \widetilde{d}'_2, \ldots, \widetilde{d}'_n\right). \tag{39}$$

The proofs of Theorems 19 and 20 are similar to Theorems 13 and 14, so we omit them.

Moreover, we give some certain situations of the DHq − ROFIPGBM^{s,t} operator by adjusting the parameters s and t.

(1) When $t \longrightarrow 0$, we can get $\zeta = 1, \gamma = 1$. Thus, we have

$$\begin{aligned}
&\mathrm{DHq-ROFIPGBM}^{s,0}\left(d_1, d_2, \ldots, d_n\right) = \cup_{\varphi_i \in h_i, \delta_i \in g_i} \\
&\left\{\left\{\left(\prod_{h=1}^{x}\left(1 - \left(1 - \prod_{i \in P_h}\left(1 - \alpha_i^s + \beta_i^s\right)^{1/|P_h|} + \prod_{i \in P_h}\left(\beta_i^s\right)^{1/|P_h|}\right)^{1/s} + \left(\prod_{i \in P_h}\left(\beta_i^s\right)^{1/|P_h|}\right)^{1/s}\right)^{1/x} - \prod_{h=1}^{x}\left(\left(\prod_{i \in P_h}\left(\beta_i^s\right)^{1/|P_h|}\right)^{1/s}\right)^{1/x}\right)^{1/q}\right\}\right\}, \\
&\left\{\left(1 - \prod_{h=1}^{x}\left(1 - \left(1 - \left(\prod_{i \in P_h}\left(1 - \alpha_i^s + \beta_i^s\right)\right)^{1/|P_h|} + \prod_{i \in P_h}\left(\beta_i^s\right)^{1/|P_h|}\right)^{1/s} + \left(\prod_{i \in P_h}\left(\beta_i^s\right)^{1/|P_h|}\right)^{1/s}\right)^{1/x}\right)^{1/q}\right\}.
\end{aligned} \tag{40}$$

(2) When $s = 1, t \longrightarrow 0$, we can get $\zeta = 1, \gamma = 1$. Thus, we have

$$\begin{aligned}
&\mathrm{DHq-ROFIPGBM}^{1,0}\left(d_1, d_2, \ldots, d_n\right) = \cup_{\varphi_i \in h_i, \delta_i \in g_i} \\
&\left\{\left\{\left(\prod_{h=1}^{x}\left(\prod_{i \in P_h}\left(1 - \delta_i^q\right)^{1/|P_h|}\right)^{1/x} - \prod_{h=1}^{x}\left(\prod_{i \in P_h}\left(1 - \varphi_i^q + \delta_i^q\right)^{1/|P_h|}\right)^{1/x}\right)^{1/q}\right\}, \left\{\left(1 - \prod_{h=1}^{x}\left(\prod_{i \in P_h}\left(1 - \delta_i^q\right)^{1/|P_h|}\right)^{1/x}\right)^{1/q}\right\}\right\}.
\end{aligned} \tag{41}$$

(3) When $s \longrightarrow 0$, we can get

$$\begin{aligned}
&\mathrm{DHq-ROFIPGBM}^{0,t}\left(d_1, d_2, \ldots, d_n\right) = \cup_{\varphi_j \in h_j, \delta_j \in g_j} \\
&\left\{\left\{\left(\prod_{h=1}^{x}\left(1 - (1 - \zeta + \gamma)^{1/t} + \gamma^{1/t}\right)^{1/x}, -\prod_{h=1}^{x}\left(\gamma^{1/t}\right)^{1/x}\right)^{1/q}\right\}, \left\{\left(1 - \prod_{h=1}^{x}\left(1 - (1 - \zeta + \gamma)^{1/t} + \gamma^{1/t}\right)^{1/x}\right)^{1/q}\right\}\right\}.
\end{aligned} \tag{42}$$

(4) When $s = 1$, $t = 1$, we can get $\zeta = \prod_{j \in P_h, j \neq i}(1 - \alpha_j + \beta_j)^{1/|P_h|-1}, \gamma = (\prod_{j \in P_h; j \neq i}\beta_j)^{1/|P_h|-1}$. Thus, we have

$$\mathrm{DHq-ROFIPGBM}^{s,t}(d_1,d_2,\ldots,d_n) = \cup_{\varphi_i\in h_i,\delta_i\in g_i,\varphi_j\in h_j,\delta_j\in g_j}$$
$$\left\{\left\{\left(\prod_{h=1}^{x}\left(1-\left(1-\prod_{i\in P_h}(1-\alpha_i(1-\zeta+\gamma)+\beta_i\gamma)^{1/|P_h|}+\prod_{i\in P_h}(\beta_i\gamma)^{1/|P_h|}\right)^{1/2}+\left(\prod_{i\in P_h}(\beta_i\gamma)^{1/|P_h|}\right)^{1/2}\right)^{1/x}-\prod_{h=1}^{x}\left(\left(\prod_{i\in P_h}(\beta_i\gamma)^{1/|P_h|}\right)^{1/2}\right)^{1/x}\right)^{1/q}\right\}\right\},$$
$$\left\{\left(1-\prod_{h=1}^{x}\left(1-\left(1-\prod_{i\in P_h}(1-\alpha_i(1-\zeta+\gamma)+\beta_i\gamma)^{1/|P_h|}+\prod_{i\in P_h}(\beta_i\gamma)^{1/|P_h|}\right)^{1/2}+\left(\prod_{i\in P_h}(\beta_i\gamma)^{1/|P_h|}\right)^{1/2}\right)^{1/x}\right)^{1/q}\right\}\Bigg\}. \tag{43}$$

If all the DHq-ROFNs are partitioned into one sort, the DHq-ROFIPGB-M operator reduces to the dual hesitant q-rung orthopair fuzzy interaction geometric Bonferroni mean (DHq-ROFIGBM) operator as follows:

$$\mathrm{DHq-ROFIGBM}^{s,t}(d_1,d_2,\ldots,d_n) = \frac{1}{s+t}\left(\underset{i,j=1,i\neq j}{\overset{n}{\otimes}}\left((sd_i)\oplus(td_j)\right)\right)^{1/n(n-1)}$$
$$= \cup_{\varphi_i\in h_i,\delta_i\in g_i,\varphi_j\in h_j,\delta_j\in g_j}$$
$$\left\{\begin{array}{c}\left\{\left(1-\left(1-\left(\prod_{i,j=1,i\neq j}^{n}\left(1-\alpha_i^s\alpha_j^t+\beta_i^s\beta_j^t\right)\right)^{1/n(n-1)}+\left(\prod_{i,j=1,i\neq j}^{n}\beta_i^s\beta_j^t\right)^{1/n(n-1)}\right)^{1/s+t}\right)^{1/q}\right\},\\ \left\{\left(\left(1-\left(\prod_{i,j=1,i\neq j}^{n}\left(1-\alpha_i^s\alpha_j^t+\beta_i^s\beta_j^t\right)\right)^{1/n(n-1)}+\left(\prod_{i,j=1,i\neq j}^{n}\beta_i^s\beta_j^t\right)^{1/n(n-1)}\right)^{1/s+t}-\left(\left(\prod_{i,j=1,i\neq j}^{n}\beta_i^s\beta_j^t\right)^{1/n(n-1)}\right)^{1/s+t}\right)^{1/q}\right\}\end{array}\right\}. \tag{44}$$

3.4. The DHq-ROFWIPGBM Operator

Definition 21. Let $T=(d_1,d_2,\ldots,d_n)$ be a collection of DHq-ROFNs, which is partitioned into x distinct sorts $P_1,P_2,\ldots,P_x$, where $d_i=(\varphi_i,\delta_i)\,(i=1,2,\ldots,n)$ and $\cup_{h=1}^{x}P_h=T$. The DHq-ROFWIPGBM operator is defined as

$$\mathrm{DHq-ROFWIPGBM}^{s,t}(d_1,d_2,\ldots,d_n) = \left(\underset{h=1}{\overset{x}{\otimes}}\left(\frac{1}{s+t}\left(\underset{i,j\in P_h,j\neq i}{\otimes}\left(s(d_i)^{\omega_i}\oplus t(d_j)^{\omega_j}\right)\right)^{1/|P_h|(|P_h|-1)}\right)\right)^{1/x}, \tag{45}$$

where $s,t\geq 0$, $|P_h|$ denotes the cardinality of P_h, d is the number of the partitioned sorts, and $\sum_{h=1}^{x}|P_h|=n$, $(\omega_1,\omega_2,\ldots,\omega_n)$ is the weight vector of $(d_1,d_2,\ldots,d_n)$, $\omega_j\in[0,1]$, $j=1,2,\ldots,n$, and $\sum_{j=1}^{n}\omega_j=1$.

Theorem 22. *Let $\{d_i=(\varphi_i,\delta_i)\}$ be a collection of DHq-ROFNs with $s,t\geq 0$ where $i=1,2,\ldots,n$. By DHq-ROFWIPGBM operator, the aggregated value of d_i is DHq-ROFN with*

$$
\begin{aligned}
&\mathrm{DHq-ROFWIPGBM}^{s,t}(d_1,d_2,\ldots,d_n) = \cup_{\varphi_i\in h_i,\delta_i\in g_i,\varphi_j\in h_j,\delta_j\in g_j}\\
&\left\{\begin{array}{c}
\left\{\left(\left(\prod_{h=1}^{x}\left(1-\left(1-\prod_{i,j\in P_h,j\neq i}(1-\zeta+\gamma)^{1/|P_h|(|P_h|-1)}+\prod_{i,j\in P_h,j\neq i}(\gamma)^{1/|P_h|(|P_h|-1)}\right)^{1/s+t}+\prod_{i,j\in P_h,j\neq i}(\gamma)^{1/|P_h|(|P_h|-1)(s+t)}\right)^{1/x}-\left(\prod_{h=1}^{x}\prod_{i,j\in P_h,j\neq i}(\gamma)^{1/|P_h|(|P_h|-1)(s+t)}\right)^{1/x}\right)^{1/q}\right\},\\
\left\{\left(1-\prod_{h=1}^{x}\left(1-\left(1-\prod_{i,j\in P_h,j\neq i}(1-\zeta+\gamma)^{1/|P_h|(|P_h|-1)}+\prod_{i,j\in P_h,j\neq i}(\gamma)^{1/|P_h|(|P_h|-1)}\right)^{1/s+t}+\prod_{i,j\in P_h,j\neq i}(\gamma)^{1/|P_h|(|P_h|-1)(s+t)}\right)^{1/x}\right)^{1/q}\right\}
\end{array}\right\},
\end{aligned}
\tag{46}
$$

where $\zeta = (1-\alpha_i+\beta_i)^s(1-\alpha_j+\beta_j)^t$, $\gamma = \beta_i^s\beta_j^t$ and $\alpha_i = (1-\delta_i^q)^{\omega_i}$, $\alpha_j = (1-\delta_j^q)^{\omega_j}$, $\beta_i = (1-\varphi_i^q-\delta_i^q)^{\omega_i}$, $\beta_j = (1-\varphi_j^q-\delta_j^q)^{\omega_j}$.

The proof is similar to Theorem 16, so we omit it.

4. MCGDM Approach Based on the Novel Proposed BM and PBM Operators

If all the DHq-ROFNs are partitioned into one sort, the DHq-ROFWIP-GBM operator reduces to the dual hesitant q-rung orthopair fuzzy weighted interaction Bonferroni mean (DHq-ROFWIGBM) operator as follows:

$$
\begin{aligned}
&\mathrm{DHq{-}ROFWIGBM}^{s,t}(d_1,d_2,\ldots,d_n) = \frac{1}{s+t}\left(\underset{i,j=1,j\neq i}{\otimes}\left(s(d_i)^{\omega_i}\oplus t(d_j)^{\omega_j}\right)\right)^{1/n(n-1)}\\
&= \cup_{\varphi_i\in h_i,\delta_i\in g_i,\varphi_j\in h_j,\delta_j\in g_j}\\
&\left\{\begin{array}{c}
\left\{\left(1-\left(1-\prod_{i,j=1,j\neq i}(1-\zeta+\gamma)^{1/n(n-1)}+\prod_{i,j=1,j\neq i}(\gamma)^{1/n(n-1)}\right)^{1/s+t}\right)^{1/q}\right\},\\
\left\{\left(\left(1-\prod_{i,j=1,j\neq i}(1-\zeta+\gamma)^{1/n(n-1)}+\prod_{i,j=1,j\neq i}(\gamma)^{1/n(n-1)}\right)^{1/s+t}-\prod_{i,j=1,j\neq i}(\gamma)^{1/n(n-1)(s+t)}\right)^{1/q}\right\}
\end{array}\right\}.
\end{aligned}
\tag{47}
$$

In the approach, the complexity of MCGDM problem is first considered, so the BM operator is used to aggregate several decision matrices to obtain a group matrix, and the correlation of some attributes is considered, so we obtain the decision result by PBM operator.

For a MCGDM problem with DHq-ROFNs, let $E = \{E_1,E_2,\ldots,E_c\}$ be a set of DMs, $A_i\,(i=1,2,\ldots,m)$ be a set of alternatives, and $G_j\,(j=1,2,\ldots,n)$ be a set of attributes. Let $\omega_c\,(c=1,2,\ldots,t)$ is weight vector of DMs, n attributes whose weight vector is $w_j\,(j=1,2,\ldots,n)$, and $w_j>0, \sum_{j=1}^{n}w_j=1$. The DMs give the decision matrix $D_p = \{h_{ij}^p, g_{ij}^p\}_{m\times n}$, which contains h_{ij}^p that indicates the MD set and g_{ij}^p that indicates the NMD set. In the following, the proposed MCGDM algorithm based on the novel BM and PBM operators is given.

Step 1. Standardizing all the decision matrices: In general, we construct the standard decision matrix by converting the cost criteria values to the benefit standard values. If there are no cost criteria values, this step can be ignored.

$$
\tilde{\xi}_{ij}^t = \begin{cases} \{h_{ij}^t, g_{ij}^t\}, & \text{for benefit criteria } G_j,\\ \{h_{ij}^t, g_{ij}^t\}^c, & \text{for cost criteria } G_j. \end{cases}
\tag{48}
$$

Step 2. To collect decision matrix D, using DHq − ROFWIBM operator or DHq − ROFWIGBM operator, we compose all the individual DHq-ROFNs decision matrices $D_p\,(p=1,2,\cdots,c)$ as follows:

$$
\begin{aligned}
\mathrm{DHq-ROFWIBM}^{s,t}(d_1,d_2,\cdots,d_n) &= \left(\frac{1}{n(n-1)}\underset{i,j=1;i\neq j}{\overset{n}{\oplus}}\left((\omega_i d_i)^s\otimes(\omega_j d_j)^t\right)\right)^{1/s+t},\\
\mathrm{DHq-ROFWIGBM}^{s,t}(d_1,d_2,\cdots,d_n) &= \frac{1}{s+t}\left(\underset{i,j=1;i\neq j}{\overset{n}{\oplus}}\left(s(d_i)^{\omega_i}\oplus t(d_j)^{\omega_j}\right)\right)^{1/n(n-1)}.
\end{aligned}
\tag{49}
$$

Step 3. By using $\mathrm{DHq-ROFWIPBM}$ operator or $\mathrm{DHq-ROFWIPGBM}$ operator, $d_i\,(i=1,\ldots,m)$ of each alternative are calculated:

$$\mathrm{DHq-ROFWIPBM}^{s,t}(d_1,d_2,\cdots,d_n)=\frac{1}{d}\left(\overset{d}{\underset{h=1}{\oplus}}\left(\frac{1}{|P_h|(|P_h|-1)}\underset{i,j\in P_h,j\neq i}{\oplus}\left((\omega_i d_i)^s\otimes(\omega_j d_j)^t\right)\right)^{1/s+t}\right),$$
$$\mathrm{DHq-ROFWIPGBM}^{s,t}(d_1,d_2,\cdots,d_n)=\left(\overset{d}{\underset{h=1}{\otimes}}\left(\frac{1}{s+t}\left(\underset{i,j\in P_h,j\neq i}{\otimes}\left(s(d_i)^{\omega_i}\oplus t(d_j)^{\omega_j}\right)\right)^{1/|P_h|(|P_h|-1)}\right)\right)^{1/d}. \tag{50}$$

Step 4. According to equations (5) and (6) of DHq-ROFNs, calculate and compare them to obtain best solution.

5. Numerical Example

In this part, we offer a concrete example to purchase strategic missiles with DHq-ROFNs.

Pakistan plans to purchase strategic missiles from China. After primary evaluation, it has decided to choose one of the four types of missiles $X=\{x_1,x_2,x_3,x_4\}$ for purchase. Currently, three military weapon experts $(E=\{e_1,e_2,e_3\})$ with weight vector $\omega=(0.3,0.4,0.3)$ conduct comprehensive evaluation on the missile from four attributes (the attribute weight vector is $w=(0.3,0.3,0.2,0.2)$), including price (A_1), accuracy (A_2), range (A_3), and speed (A_4). Further, all the attributes are partitioned into two sets $P_1=\{A_1,A_2\}$ and $P_2=\{A_3,A_4\}$ based on the interrelationship. Assume that three DMs give the decision matrices D_1,D_2,D_3 which are shown in Tables 1–3. Then, based on the DHq-ROFWIBM and DHq-ROFWIPBM operators, we make use of the proposed MCGDM method to solve the "Purchase Strategic Missiles" problem, shown as follows:

Step 1. It does not need to be normalized with regard to the decision matrices since all attributes are the benefit type.

Step 2. By using the DHq-ROFWIBM operator, compose D_1,D_2,D_3 to the integrated D, shown as Tables 4 and 5 (where $q=3$, $s=t=2$).

Step 3. According to D, by using DHq-ROFWIPBM, $d_i\,(i=1,\ldots,m)$ of each alternative are calculated, and then we obtain by equation (5) $S(d_1)=0.5072$, $S(d_2)=0.5115$, $S(d_3)=0.5155$, $S(d_4)=0.5086$.

Step 4. Compare and rank the results by using equation (3), and we can derive that the best strategic missile is A_3 since

$$A_3>A_2>A_4>A_1. \tag{51}$$

5.1. Influence of Different Parameters q on the Results. In the following, we discuss the influence of the parameter q on the ranking result of MCGDM problem based on the DHq-ROFWIBM and DHq-ROFWIPBM operators. For this, we calculate the ranking results of different q values in Table 6 (when $s=2$, $t=2$).

From Table 6, we have observed that the score functions $S(d_i)$ decrease with the increase of parameter value q. Simultaneously, the ranking result has not changed, and the best alternative is A_3. The DMs can choose the proper parameter q according to their personal preference.

For the selection of s,t parameters, we can choose the corresponding value according to the attitude of decision makers' preference for risk. The smaller the value of the parameter, the greater the decision makers' preference for risk avoidance.

5.2. Comparison with the Existing Approach. In the following, we aim to show the superiority and rationality of our method. To accomplish this, we compare our approach to three existing methods which include situation when $q=1,2,3$ found in [26, 27], and the result is shown in Table 7.

From [26], we can find that this approach cannot calculate the example data. From [27], the result is $A_3>A_2>A_4>A_1$, which is the same as the proposed approach. So, this shows the rationality of the proposed approach. In addition, we can observe that the ranking result is $A_3>A_2>A_1>A_4$ based on the $\mathrm{DHq-ROFWIGBM}$ and $\mathrm{DHq-ROFWIPGBM}$ operators. By comparison, the ranking results of the two approaches are slightly different. However, the best alternative is A_3. In the following, we analyze the merits of our method over the two methods from three different aspects.

(i) From the perspective of information data, the method in [26] is based on IFSs, and we can find that it cannot process such data; the method in [27] is based on Pythagorean fuzzy numbers, so it has few specific applications. The proposed method is based on DHq-ROFNs, and it can adjust q values according to the actual data, so it is more flexible when solving MCGDM problems.

(ii) From the perspective of operational laws, the method in [27] does not consider interaction operational, so it cannot solve the case where the MD or NMD is zero, whereas the common advantage of the method in [26] and our method is that they

Table 1: The DHq-ROFNs decision matrix D_1.

	G_1	G_2	G_3	G_4
A_1	$\{\{0.5\},\{0.3,0.2\}\}$	$\{\{0.4\},\{0.3\}\}$	$\{\{0.4\},\{0.3,0.2\}\}$	$\{\{0.6\},\{0.2\}\}$
A_2	$\{\{0.6,0.5\},\{0.2,0.1\}\}$	$\{\{0.7\},\{0.3,0.2\}\}$	$\{\{0.6\},\{0.3\}\}$	$\{\{0.5,0.4\},\{0.2\}\}$
A_3	$\{\{0.7\},\{0.4\}\}$	$\{\{0.5\},\{0.3\}\}$	$\{\{0.6\},\{0.3,0.2\}\}$	$\{\{0.7,0.6\},\{0.3\}\}$
A_4	$\{\{0.4,0.3\},\{0.1\}\}$	$\{\{0.4\},\{0.2,0.1\}\}$	$\{\{0.5\},\{0.2,0.1\}\}$	$\{\{0.4\},\{0.2\}\}$

Table 2: The DHq-ROFNs decision matrix D_2.

	G_1	G_2	G_3	G_4
A_1	$\{\{0.6\},\{0.2\}\}$	$\{\{0.5\},\{0.3,0.2\}\}$	$\{\{0.7,0.5\},\{0.3\}\}$	$\{\{0.8\},\{0.3,0.2\}\}$
A_2	$\{\{0.7\},\{0.2\}\}$	$\{\{0.6,0.5\},\{0.2\}\}$	$\{\{0.8\},\{0.1\}\}$	$\{\{0.7\},\{0.2\}\}$
A_3	$\{\{0.8,0.6\},\{0.3\}\}$	$\{\{0.8\},\{0.3,0.2\}\}$	$\{\{0.8\},\{0.3,0.1\}\}$	$\{\{0.8\},\{0.2\}\}$
A_4	$\{\{0.6,0.5\},\{0.1\}\}$	$\{\{0.7\},\{0.1\}\}$	$\{\{0.7\},\{0.2\}\}$	$\{\{0.8\},\{0.2,0.1\}\}$

Table 3: The DHq-ROFNs decision matrix D_3.

	G_1	G_2	G_3	G_4
A_1	$\{\{0.5\},\{0.3\}\}$	$\{\{0.5,0.4\},\{0.2\}\}$	$\{\{0.5\},\{0.2\}\}$	$\{\{0.5,0.3\},\{0.3\}\}$
A_2	$\{\{0.4\},\{0.1\}\}$	$\{\{0.6\},\{0.2\}\}$	$\{\{0.6,0.5\},\{0.2\}\}$	$\{\{0.5\},\{0.3\}\}$
A_3	$\{\{0.7,0.6\},\{0.3\}\}$	$\{\{0.7\},\{0.3\}\}$	$\{\{0.5\},\{0.4\}\}$	$\{\{0.6\},\{0.3,0.1\}\}$
A_4	$\{\{0.4\},\{0.2\}\}$	$\{\{0.3\},\{0.2,0.1\}\}$	$\{\{0.4,0.3\},\{0.2\}\}$	$\{\{0.5\},\{0.2,0.1\}\}$

Table 4: The DHq-ROFNs decision matrix D.

	G_1	G_2
A_1	$\{\{0.386\},\{0.193,0.172\}\}$	$\{\{0.333,0.313\},\{0.197,0.169\}\}$
A_2	$\{\{0.437,0.418\},\{0.137,0.120\}\}$	$\{\{0.455,0.432\},\{0.181,0.148\}\}$
A_3	$\{\{0.548,0.531,0.479,0455\},\{0.264,0.256\}\}$	$\{\{0.519\},\{0.234,0.203\}\}$
A_4	$\{\{0.355,0.313,0.345,0.299\},\{0.103\}\}$	$\{\{0.398\},\{0.121,0.102,0.073\}\}$

Table 5: The decision value of DHq-RONFS.

	G_3	G_4
A_1	$\{\{0.418,0.333\},\{0.206,0.197,0.179\}\}$	$\{\{0.5,0.487\},\{0.216,0.178\}\}$
A_2	$\{\{0.512,0.5\},\{0.162\}\}$	$\{\{0.43,0.418\},\{0.174\}\}$
A_3	$\{\{0.499\},\{0.254,0.242,0.216,0.199\}\}$	$\{\{0.531,0.512\},\{0.204,0.178,0.172\}\}$
A_4	$\{\{0.418,0.411\},\{0.147,0.134\}\}$	$\{\{0.478\},\{0.153,0.142,0.123,0.104\}\}$

Table 6: Ranking results for different q based on DHq-ROFWIBM and DHq-ROFWIPBM.

Methods	Score values	Ranking
$q=2$	$S(d_1)=0.5110$, $S(d_2)=0.5182$, $S(d_3)=0.5204$, $S(d_4)=0.5134$	$A_3>A_2>A_4>A_1$
$q=3$	$S(d_1)=0.5072$, $S(d_2)=0.5115$, $S(d_3)=0.5155$, $S(d_4)=0.5086$	$A_3>A_2>A_4>A_1$
$q=4$	$S(d_1)=0.5047$, $S(d_2)=0.5074$, $S(d_3)=0.5116$, $S(d_4)=0.5049$	$A_3>A_2>A_4>A_1$
$q=5$	$S(d_1)=0.5037$, $S(d_2)=0.5053$, $S(d_3)=0.5082$, $S(d_4)=0.5040$	$A_3>A_2>A_4>A_1$

consider the interaction operational rules, and the difference is that the operators in reference [26] are special cases of our new operators. So, it is more comprehensive and practical in solving MCGDM problems.

(iii) From the perspective of the AOs functions, the method in [27] uses weighted averaging operator to aggregate data without considering the complexity of MCGDM problem, whereas the method in [26] and the proposed method take account of the interrelationships between attributes or partitioned attributes, so it is more reasonable and effective to solve concrete problem.

5.3. Discussion about the Superiority of the Proposed Interaction Operational. To verify the superiority of the interaction operational of our method, in the example, we change one of the data to obtain ranking results and

Table 7: Ranking results for different q based on DHq-ROFWIBM and DHq-ROFWIPBM.

Methods	Score values	Ranking
Verma and Merigó's MCGDM method [26] (based on the GIFWIPBM operator)	Cannot be calculated	No
Wei's MCGDM method [27] (based on the DHPFWA operator)	$S(d_1) = 0.6246$, $S(d_2) = 0.6686$, $S(d_3) = 0.6981$, $S(d_4) = 0.6442$	$A_3 > A_2 > A_4 > A_1$
The novel MCGDM method		
Based on the DHq – ROFWIBM and DHq – ROFWIPBM operators	$S(d_1) = 0.5110$, $S(d_2) = 0.5182$, $S(d_3) = 0.5204$, $S(d_4) = 0.5134$	$A_3 > A_2 > A_4 > A_1$
Based on the DHq – ROFWIGBM and DHq – ROFWIPGBM operators	$S(d_1) = 0.5154$, $S(d_2) = 0.5195$, $S(d_3) = 0.5265$, $S(d_4) = 0.5146$	$A_3 > A_2 > A_1 > A_4$

Table 8: Ranking results for different q based on DHq-ROFWIBM and DHq-ROFWIPBM.

	G_1	G_2	G_3	G_4
A_1	$\{\{0.5\}, \{0.3, 0.2\}\}$	$\{\{0.4\}, \{0.3\}\}$	$\{\{0.4\}, \{0.3, 0.2\}\}$	$\{\{0.6\}, \{0.2\}\}$
A_2	$\{\{0.6, 0.5\}, \{0.2, 0.1\}\}$	$\{\{0.7\}, \{0.3, 0.2\}\}$	$\{\{0.6\}, \{0.3\}\}$	$\{\{0.5, 0.4\}, \{0.2\}\}$
A_3	$\{\{0.7\}, \{0.4\}\}$	$\{\{0\}, \{0.3\}\}$	$\{\{0.6\}, \{0.3, 0.2\}\}$	$\{\{0.7, 0.6\}, \{0.3\}\}$
A_4	$\{\{0.4, 0.3\}, \{0.1\}\}$	$\{\{0.4\}, \{0.2, 0.1\}\}$	$\{\{0.5\}, \{0.2, 0.1\}\}$	$\{\{0.4\}, \{0.2\}\}$

Table 9: Comparison of the characteristics of different MCGDM methods.

Methods	The correlation between aggregating parameters is considered	The ability to handle complex MCGDM data	The ability to handle zero data of MD or NMD	Consider categorization among attributes
Verma and Merigó's MCGDM method [26]	Yes	Strong	Yes	Yes
Liu et al.'s method [25]	Yes	Weak	Strong	Strong
Yang et al.'s method [24]	Yes	Strong	Stronger	Strong
The proposed method	Yes	Strongest	Strongest	Strongest

compare it with the original results. In general, if the ranking results are consistent, it indicates that the advantage of interaction operational can be reflected, so that the approach we proposed is less influenced by the extreme evaluation value of certain decision makers. Therefore, we reduce the MD of the evaluation value in A_3 of one of the DMs to 0, so as to verify the ability of our method to handle extreme data. We change the evaluation value $\tilde{\xi}_{32}^{1}$ from $\{\{0.5\}, \{0.3\}\}$ to $\{\{0\}, \{0.3\}\}$ by decreasing MD, shown as Table 8, and figure out that $A_3 = 0.515$, and the ranking result is $A_3 > A_2 > A_4 > A_1$; it is obvious that the ranking results have not changed, which shows that the given interaction operational rules and the proposed methods are reasonable and effective.

5.4. Comparison of the Characteristics of Different MCGDM Methods. Next, we compare our method with the methods given in [24–26] from four characteristics. As shown in Table 9, the evaluation indicators are categorized into two groups: Yes, No, Weak, Strong, Stronger, and Strongest. By comparison, we can see that

(i) These methods consider the correlations among attributes, and they use PBM operators that consider the correlation between partial attributes, whereas the proposed method adopts BM and PBM operators to handle MCGDM problem more reasonably.

(ii) The methods in [24–26] use the interaction operations of IFNs and PFNs, whereas the proposed method adopts the interaction operations DHq-ROFNs, respectively. Therefore, the method proposed in this paper can describe the uncertainty information more effectively.

In a word, the method proposed in this paper based on DHq-ROFWIBM and DHq-ROFWIPBM operator is more general and reasonable than the methods in [24, 25, 26].

6. Conclusions and Future Research

The present paper introduced novel interaction operational on DHq-ROFNs and denoted a series of AOs, which provide a suitable method for DMs to deal with a certain type of MCGDM problem.

Firstly, we have developed four different operators: DHq-ROFIPBM, DHq-ROFWIPBM, DHq-ROFIPGBM, and DHq-ROFWIPGBM, for DHq-ROFNs. In addition, we also have introduced their corresponding properties and certain situations associated with the proposed AOs. Afterward, by using the proposed AOs, we provided a novel approach to solve MCGDM problems. Finally, by comparing different characteristics with existing methods in purchasing strategic missiles problems, we demonstrated the rationality and proved the superiority of the proposed method.

The next research is as follows:

(1) To find the specific application of the method proposed in this paper.

(2) To find the improvement and extension of the method proposed in this paper, such as using the novel AOs to solve the practical MCGDM problems, and investigate more AOs to fuse dual hesitant q-rung orthopair fuzzy interaction operational, such as Heronian mean operators [14] and TOPSIS method [9].

Data Availability

No underlying data were collected or produced in this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Yabin Shao established the research direction and content. Lu Zhang and Ning Wang conducted the literature review and wrote the entire manuscript. All authors have read and agreed to the published version of the manuscript.

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