Hindawi
Journal of Mathematics
Volume 2023, Article ID 6972483, 9 pages
https://doi.org/10.1155/2023/6972483

Research Article

A Novel Study on Ordered Anti-Involution LA-Semihypergroups

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Received 1 December 2022; Revised 29 January 2023; Accepted 7 February 2023; Published 22 March 2023

Academic Editor: Tareq Al-shami

In this study, we introduce a new concept called "anti-involution" in relation to ordered LA-semihypergroups. An anti-involution is basically an involuntary automorphism, which is just a fancy term for a mathematical function that can be reversed. We looked at several fundamental results before introducing anti-involution hyperideals. We studied the anti-involution hyperideals of ordered anti-involution LA-semihypergroups using the rough set theory. In an ordered anti-involution LA-semihypergroup, the ρ-upper and ρ-lower rough approximations of anti-involution hyperideals are an anti-involution hyperideals.

1. Introduction

One of the reasons for studying hyperstructures is to understand biological inheritance and physical phenomena such as nuclear fission. Chemical and redox processes are another source of inspiration for the research of hyperstructures. Classical algebraic structures can be generalized with algebraic superstructures. A composition of two elements is an element of a classical algebraic structure, and a composition of two elements is a set of algebraic superstructures. The term "hyperstructure" originally came from Marty [1], a French mathematician who pioneered hyperstructure theory. He extended the notion of classical binary operation to binary hyperoperation in his study. Several scholars directed their studies in this way, resulting in an excess of novel notions. Many applications of hyperstructures in geometry, codes, and cryptography are described in the book produced by Corsini and Leoreanu-Fotea [2] and Vougiouklis [3].

Kazim and Naseeruddin [4], as the originators of the notion of almost semigroups, enlarged the notion of associative law to left invertive law in his study. Hila and Dine [5] provided the notion of left almost semihypergroups. Yaqoob et al. [6] also contributed to the idea of left almost semihypergroups. Yaqoob and Gulistan [7] introduced the notion of partially ordered left almost semihypergroups.

Pawlak [8] established the notion of rough sets in 1982. Biswas and Nanda [9] applied the rough sets theory to groups. Jun [10] investigated the lower and upper approximations of gamma-subsemigroups/ideals in gamma-semigroups. Qurashi and Shabir [11, 12] generalized rough fuzzy ideals in quantales and roughness in quantale modules. Shabir and Irshad [13] studied roughness in ordered semigroups. Several writers have applied the rough set theory to various algebraic hyperstructures, such as Ameri et al. [14] applied roughness to bi-hyperideals of semihypergroups. Anvariyeh et al. [15] examined the roughness of Γ-semihypergroups. The rough set theory was applied to hyperrings by Davvaz [16], Γ-semihyperrings by Dehkordi and Davvaz [17], hyperlattices by He et al. [18], hypergroups by Leoreanu-Fotea [19], and nonassociative po-semihypergroups by Zhan et al. [20].

An involution f is a transformation [21] (or unary operation) $\alpha \mapsto f(\alpha)$ that satisfies the following 3 axioms:

Axiom 1. $f(f(\alpha)) = \alpha$. An involution is its own inverse.

Axiom 2. An involution is linear: $f(\alpha_1 + \alpha_2) = f(\alpha_1) + f(\alpha_2)$ and $\lambda f(\alpha) = f(\lambda\alpha)$, where λ is a real constant.

Axiom 3. $f(\alpha_1\alpha_2) = f(\alpha_2)f(\alpha_1)$.

An anti-involution is a self-inverse transformation similar to an involution. It satisfies the Axiom 1, 2, and Axiom 4:

Axiom 4. $f(\alpha_1\alpha_2) = f(\alpha_1)f(\alpha_2)$.

Foulis [22] submitted his Ph.D thesis in 1958, in which he studied unary operations in semigroups and provided some results for the theory of involution semigroups. Following that, Scheiblich and Nordahl [23] established the concept of regular $*$-semigroups. They labeled a semigroup S with a unary operation $x \longrightarrow x^*$ a regular $*$-semigroup if the following identities were satisfied:

$$\begin{aligned} (x^*)^* &= x, \\ (xy)^* &= y^* x^*, \\ x &= x x^* x. \end{aligned} \tag{1}$$

Following the introduction of this notion, various writers studied natural structures on semigroups with involution [24–26] and analyzed some findings on regular semigroups with involution [27]. Reilly [28] introduced a new class of ordinary $*$-semigroups. Wu [29] recently studied intraregular ordered $*$-semigroups. Aburawash [30] introduced the definition of involution group rings. Baxter [31] investigated rings with proper involution. Herstein [32, 33] added various results for involution rings.

In hyperstructures, the theory of involutions is seldom studied. Feng et al. [34] investigated regular equivalence relations on ordered $*$-semihypergroups. Yaqoob et al. [35] investigated the structures of involution Γ-semihypergroups. Tang and Yaqoob [36] introduced a fuzzy set theory to hyperideals of ordered $*$-semihypergroups.

2. Preliminaries and Basic Definitions

Some basic definitions are provided in this section on ordered LA-semihypergroups.

Definition 1 (see [5, 6]). A hypergroupoid $(\Theta, \circ)$ is said to be an LA-semihypergroup if for all $\vartheta, \sigma, \varsigma \in \Theta$,

$$(\vartheta \circ \sigma) \circ \varsigma = (\varsigma \circ \sigma) \circ \vartheta. \tag{2}$$

Equation 2 is known as the left invertive law.

Definition 2 (see [7]). Let Θ be a non-empty set and "$\leq$" be an ordered relation on Θ. Then, $(\Theta, \circ, \leq)$ is called an ordered LA-semihypergroup if

(1) $(\Theta, \circ)$ is an LA-semihypergroup;

(2) $(\Theta, \leq)$ is a partially ordered set;

(3) for every $\vartheta, \sigma, \varsigma \in \Theta$, $\vartheta \leq \sigma$ implies $\vartheta \circ \varsigma \leq \sigma \circ \varsigma$ and $\varsigma \circ \vartheta \leq \varsigma \circ \sigma$, where $\vartheta \circ \varsigma \leq \sigma \circ \varsigma$ means that for every $\alpha \in \vartheta \circ \varsigma$ there exists $\beta \in \sigma \circ \varsigma$ such that $\alpha \leq \beta$.

Definition 3 (see [7]). If $\mathfrak{I}$ is a nonempty subset of $(\Theta, \circ, \leq)$, then $(\mathfrak{I}]$ is the subset of Θ defined as follows:

$$(\mathfrak{I}] = \{\tau \in \Theta: \ \tau \leq \vartheta, \text{for some } \vartheta \in \mathfrak{I}\}. \tag{3}$$

Definition 4 (see [7]). A non-empty subset $\mathfrak{I}$ of an ordered LA-semihypergroup $(\Theta, \circ, \leq)$ is called an LA-subsemihypergroup of Θ if $(\mathfrak{I} \circ \mathfrak{I}] \subseteq (\mathfrak{I}]$.

An anti-involution is an automorphism that does not change the order of the elements. The distinction between involutions and anti-involutions can exist only in algebra. The involution property does not work in left invertive structures where associativity and commutativity both do not hold.

3. Ordered Anti-InvolutionLA-Semihypergroups

In this section, we define the concept of an ordered anti-involution LA-semihypergroup and provide some results.

Definition 5. An ordered LA-semihypergroup $(\Theta, \circ, \leq)$ with an unary operation $\circledast: \Theta \longrightarrow \Theta$ is called an ordered anti-involution LA-semihypergroup if it satisfies

(1) $(\forall \alpha \in \Theta)(\alpha^{\circledast})^{\circledast} = \alpha$

(2) $(\forall \alpha, \beta \in \Theta)(\alpha \circ \beta)^{\circledast} = \alpha^{\circledast} \circ \beta^{\circledast}$

If $\alpha^{\circledast}, \beta^{\circledast} \in \Theta$, then $\alpha^{\circledast} \circ \beta^{\circledast} = \{\gamma^{\circledast}: \gamma \in \alpha \circ \beta\}$. For any nonempty subset $\mathfrak{I}$ of an ordered anti-involution LA-semihypergroup Θ,

$$\mathfrak{I}^{\circledast} = \{\vartheta^{\circledast} \in \Theta: \ \vartheta \in \mathfrak{I}\}. \tag{4}$$

If for any $\alpha, \beta \in \Theta$ with $\alpha \leq \beta$, we have $\alpha^{\circledast} \leq \beta^{\circledast}$, then $\circledast$ is called an order preserving anti-involution.

Example 1. Consider a set $\Theta = \{\vartheta, \sigma, \varsigma\}$ with the following hyperoperation "$\circ$" and the order "$\leq$" (Table1):

We give the covering relation "$\prec$" as $\prec = \{(\vartheta, \sigma), (\vartheta, \varsigma)\}$. The figure of Θ is shown in Figure 1.

Then, $(\Theta, \circ, \leq)$ is an ordered LA-semihypergroup. Now, we define the anti-involution $\circledast$ by $\vartheta^{\circledast} = \vartheta$, $\sigma^{\circledast} = \varsigma$, and $\varsigma^{\circledast} = \sigma$. Then, it is easy to check that $(\Theta, \circ, \leq)$ is an ordered anti-involution LA-semihypergroup with order preserving anti-involution $\circledast$.

$$\leq \ := \{(\vartheta, \vartheta), (\vartheta, \sigma), (\vartheta, \varsigma), (\sigma, \sigma), (\varsigma, \varsigma)\}. \tag{5}$$

Lemma 1. *Suppose Θ is an ordered anti-involution LA-semihypergroup. Then, we have the following:*

Table 1: Hyperoperation "$\circ$" defined on $\Theta = \{\vartheta, \sigma, \varsigma\}$.

$\circ$	ϑ	σ	ς
ϑ	ϑ	$\{\vartheta, \sigma\}$	$\{\vartheta, \varsigma\}$
σ	$\{\vartheta, \varsigma\}$	$\{\sigma, \varsigma\}$	ς
ς	$\{\vartheta, \sigma\}$	σ	$\{\sigma, \varsigma\}$

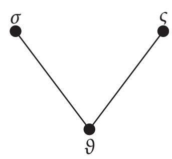

Figure 1: Figure of Θ for Example 1.

(i) $\mathfrak{I} \subseteq (\mathfrak{I}]$ *for any* $\mathfrak{I} \subseteq \Theta$

(ii) $(\mathfrak{I}] \subseteq (\mathfrak{R}]$ *for any* $\mathfrak{I}, \mathfrak{R}$ *with* $\mathfrak{I} \subseteq \mathfrak{R} \subseteq \Theta$

(iii) $(\mathfrak{I}] \circ (\mathfrak{R}] \subseteq (\mathfrak{I} \circ \mathfrak{R}]$ *for all* $\mathfrak{I}, \mathfrak{R} \subseteq \Theta$

(iv) $((\mathfrak{I}]] \subseteq (\mathfrak{I}]$ *for all* $\mathfrak{I} \subseteq \Theta$

(v) For any right (left, two-sided) hyperideal I *of* Θ, $(I] = I$

(vi) If I *and* J *are hyperideals of* Θ, *then* $(I^{\circ} J]$ *and* $I \cap J$ *are also hyperideals of* Θ

Proposition 1. *If* $\mathfrak{I}$ *and* $\mathfrak{R}$ *are nonempty subsets of an ordered anti-involution LA-semihypergroup*Θ, *then the following hold:*

(1) $(\mathfrak{I}] \subseteq (\mathfrak{R}]$ *if and only if* $(\mathfrak{I}]^{*} \subseteq (\mathfrak{R}]^{*}$;

(2) $(\mathfrak{I} \cup \mathfrak{R}]^{*} = (\mathfrak{I}]^{*} \cup (\mathfrak{R}]^{*}$;

(3) $(\mathfrak{I} \cap \mathfrak{R}]^{*} = (\mathfrak{I}]^{*} \cap (\mathfrak{R}]^{*}$.

Proof. (i) Consider $(\mathfrak{I}] \subseteq (\mathfrak{R}]$. Also, consider

$$\begin{aligned} & \tau \in (\mathfrak{I}]^{*} \\ & \Longrightarrow \tau^{*} \in (\mathfrak{I}] \subseteq (\mathfrak{R}] \\ & \Longrightarrow \tau^{*} \in (\mathfrak{R}] \\ & \Longrightarrow \tau \in (\mathfrak{R}]^{*}. \end{aligned} \tag{6}$$

Therefore, $(\mathfrak{I}]^{*} \subseteq (\mathfrak{R}]^{*}$. Conversely, consider $(\mathfrak{I}]^{*} \subseteq (\mathfrak{R}]^{*}$. Also, consider

$$\begin{aligned} & u \in (\mathfrak{I}] \\ & \Longrightarrow u^{*} \in (\mathfrak{I}]^{*} \subseteq (\mathfrak{R}]^{*} \\ & \Longrightarrow u^{*} \in (\mathfrak{R}]^{*} \\ & \Longrightarrow u \in (\mathfrak{R}]. \end{aligned} \tag{7}$$

Therefore, $(\mathfrak{I}] \subseteq (\mathfrak{R}]$.

(ii) Consider

$$\begin{aligned} & \tau \in (\mathfrak{I} \cup \mathfrak{R}]^{*} \\ & \Leftrightarrow \tau^{*} \in (\mathfrak{I} \cup \mathfrak{R}] \\ & \Leftrightarrow \tau^{*} \in (\mathfrak{I}] \text{ or } \tau^{*} \in (\mathfrak{R}] \\ & \Leftrightarrow \tau \in (\mathfrak{I}]^{*} \text{ or } \tau \in (\mathfrak{R}]^{*}. \end{aligned} \tag{8}$$

Hence, we obtain $(\mathfrak{I} \cup \mathfrak{R}]^{*} = (\mathfrak{I}]^{*} \cup (\mathfrak{R}]^{*}$.

(iii) The proof is similar to (ii). □

Proposition 2. *Let* Θ *be an ordered anti-involution LA-semihypergroup. Then,*

(1) $((\alpha \circ \Theta \circ \beta]^{*} = ((\alpha^{*} \circ \Theta \circ \beta^{*}]$, *for any*$\alpha, \beta \in \Theta$,

(2) $((\Theta \circ \alpha \circ \Theta]^{*} = ((\Theta \circ \alpha^{*} \circ \Theta]$, *for any*$\alpha \in \Theta$.

Proof. (1) Let $\tau \in (\alpha \circ \Theta) \circ \beta]^{*}$. By definition $\tau^{*} \in ((\alpha \circ \Theta \circ \beta]$, $\tau^{*} \leq (\alpha \circ \delta) \circ \beta$ for some $\delta \in \Theta$. This implies that

$$\begin{aligned} \tau & \leq ((\alpha \circ \delta) \circ \beta)^{*} = (\alpha \circ \delta)^{*} \circ \beta^{*} \\ & = (\alpha^{*} \circ \delta^{*}) \circ \beta^{*} \\ & \subseteq (\alpha^{*} \circ \Theta) \circ \beta^{*}. \end{aligned} \tag{9}$$

Therefore, $\tau \leq (\alpha^{*} \circ \Theta) \circ \beta^{*}$, that is, $\tau \in ((\alpha^{*} \circ \Theta \circ \beta^{*}]$ and we get that $((\alpha \circ \Theta \circ \beta]^{*} \subseteq ((\alpha^{*} \circ \Theta \circ \beta^{*}]$. On the other hand, if $\tau \in ((\alpha^{*} \circ \Theta \circ \beta^{*}]$, then for some $\delta \in \Theta$, we have $\tau \leq (\alpha^{*} \circ \delta) \circ \beta^{*}$. This implies that

$$\begin{aligned} \tau^{*} & \leq ((\alpha^{*} \circ \delta) \circ \beta^{*})^{*} = (\alpha^{*} \circ \delta)^{*} \circ (\beta^{*})^{*} \\ & = ((\alpha^{*})^{*} \circ (\delta)^{*}) \circ (\beta^{*})^{*} = (\alpha \circ \delta^{*}) \circ \beta \\ & \subseteq (\alpha \circ \Theta) \circ \beta. \end{aligned} \tag{10}$$

So, $\tau \leq ((\alpha \circ \Theta) \circ \beta)^{*}$, that is, $\tau \in ((\alpha \circ \Theta \circ \beta]^{*}$. Therefore, $((\alpha^{*} \circ \Theta \circ \beta^{*}] \subseteq ((\alpha \circ \Theta \circ \beta]^{*}$. Consequently, $((\alpha^{*} \circ \Theta \circ \beta^{*}] = ((\alpha \circ \Theta \circ \beta]^{*}$.

(2) The proof is similar to (1). □

Definition 6. A nonempty subset $\mathfrak{I}$ of an ordered anti-involution LA-semihypergroup Θ is called an ordered sub anti-involution LA-semihypergroup of Θ if $(\mathfrak{I} \circ \mathfrak{I}] \subseteq (\mathfrak{I}]$ and $\mathfrak{I}^{*} = \mathfrak{I}$.

Example 2. Consider a set $\Theta = \{\vartheta, \sigma, \varsigma, \kappa, \lambda\}$ with the following hyperoperation "$\circ$" and the order "$\leq$" (Table 2):

The figure of Θ is shown in Figure 2.

Then, $(\Theta, \circ, \leq)$ is an ordered LA-semihypergroup. Now, we define the anti-involution $*$ by $\vartheta^{*} = \vartheta$, $\sigma^{*} = \varsigma$, $\varsigma^{*} = \sigma$, $\kappa^{*} = \kappa$, and $\lambda^{*} = \lambda$. Then, it is easy to check that $(\Theta, \circ, \leq)$ is an ordered anti-involution LA-semihypergroup with order

Table 2: Left almost semihypergroup $(\Theta = \{\vartheta, \sigma, \varsigma, \kappa, \lambda\}, \circ)$.

$\circ$	ϑ	σ	ς	κ	λ
ϑ	ϑ	ϑ	ϑ	$\{\kappa, \lambda\}$	λ
σ	ϑ	ϑ	$\{\vartheta, c\}$	$\{\kappa, \lambda\}$	λ
ς	ϑ	$\{\vartheta, \sigma\}$	ϑ	$\{\kappa, \lambda\}$	λ
κ	$\{\kappa, \lambda\}$	$\{\kappa, \lambda\}$	$\{\kappa, \lambda\}$	$\{\kappa, \lambda\}$	$\{\kappa, \lambda\}$
λ	λ	λ	λ	$\{\kappa, \lambda\}$	λ

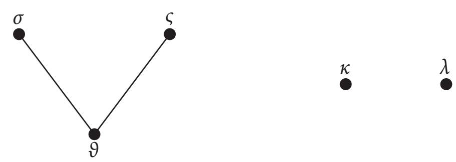

Figure 2: Figure of Θ for Example 2.

preserving anti-involution $\circledast$. Here, $\{\vartheta\}$ and $\{\vartheta, \sigma, \varsigma\}$ are sub anti-involution LA-semihypergroups of Θ. One can see that $\{\vartheta, \sigma\}\circ\{\vartheta, \sigma\}\subseteq\{\vartheta, \sigma\}$ but $\{\vartheta, \sigma\}^{\circledast} \neq \{\vartheta, \sigma\}$, so $\{\vartheta, \sigma\}$ is not a sub anti-involution LA-semihypergroup of Θ.

$$\leq \;:= \{(\vartheta, \vartheta), (\vartheta, \sigma), (\vartheta, \varsigma), (\sigma, \sigma), (\varsigma, \varsigma), (\kappa, \kappa), (\lambda, \lambda)\}. \tag{11}$$

Definition 7. A nonempty subset $\mathfrak{I}$ of an ordered anti-involution LA-semihypergroup Θ is called a right (resp., left) anti-involution hyperideal of Θ if

(1) $\mathfrak{I}\circ\Theta\subseteq\mathfrak{I}$ (resp., $\Theta\circ\mathfrak{I}\subseteq\mathfrak{I}$)

(2) If $\vartheta \in \mathfrak{I}$ and $\sigma \leq \vartheta$, then $\sigma \in \mathfrak{I}$ for every $\sigma \in \Theta$

(3) $\mathfrak{I}^{\circledast} = \mathfrak{I}$

Definition 8. A nonempty subset $\mathfrak{I}$ of an ordered anti-involution LA-semihypergroup Θ is called an anti-involution hyperideal of Θ if it is both a right and a left anti-involution hyperideal of Θ.

Table 3: Hyperoperation "$\circ$" defined on $\Theta = \{\vartheta, \sigma, \varsigma, \kappa, \lambda\}$.

$\circ$	ϑ	σ	ς	κ	λ
ϑ	σ	σ	λ	λ	λ
σ	$\{\vartheta, \sigma\}$	$\{\vartheta, \sigma\}$	λ	λ	λ
ς	λ	λ	$\{\varsigma, \kappa, \lambda\}$	κ	λ
κ	λ	λ	ς	$\{\varsigma, \kappa, \lambda\}$	λ
λ	λ	λ	λ	λ	λ

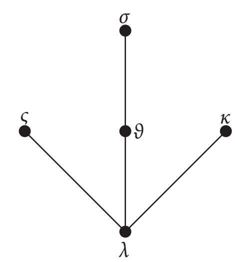

Figure 3: Figure of Θ for Example 3.

Definition 9. A nonempty subset $\mathfrak{R}$ of an ordered anti-involution LA-semihypergroup Θ is called an anti-involution bi-hyperideal of Θ if

(1) $\mathfrak{R}^{\circ}\mathfrak{R}\subseteq\mathfrak{R}$,

(2) $(\mathfrak{R}^{\circ}\Theta)\circ\mathfrak{R}\subseteq\mathfrak{R}$,

(3) If $\vartheta \in \mathfrak{R}$ and $\sigma \leq \vartheta$, then $\sigma \in \mathfrak{R}$ for every $\sigma \in \Theta$,

(4) $\mathfrak{R}^{\circledast} = \mathfrak{R}$.

Example 3. Consider a set $\Theta = \{\vartheta, \sigma, \varsigma, \kappa, \lambda\}$ with the following hyperoperation "$\circ$" and the order "$\leq$" (Table 3):

The figure of Θ is shown in Figure 3.

Then, $(\Theta, \circ, \leq)$ is an ordered LA-semihypergroup. Now, we define the anti-involution $\circledast$ by $\vartheta^{\circledast} = \vartheta$, $\sigma^{\circledast} = \sigma$, $\varsigma^{\circledast} = \kappa$, $\kappa^{\circledast} = \varsigma$, and $\lambda^{\circledast} = \lambda$. Then, it is easy to check that $(\Theta, \circ, \leq)$ is an ordered anti-involution LA-semihypergroup with order preserving anti-involution $\circledast$. Here, $\{\lambda\}$, $\{\vartheta, \sigma, \lambda\}$, and $\{\varsigma, \kappa, \lambda\}$ are anti-involution bi-hyperideals of Θ.

$$\leq \;:= \{(\vartheta, \vartheta), (\sigma, \sigma), (\varsigma, \varsigma), (\kappa, \kappa), (\lambda, \vartheta), (\lambda, \sigma), (\lambda, \varsigma), (\lambda, \kappa), (\lambda, \lambda)\}. \tag{12}$$

Definition 10. A nonempty subset Q of an ordered anti-involution LA-semihypergroup Θ is called an anti-involution quasi-hyperideal of Θ if

(1) $(Q\circ\Theta] \cap (\Theta\circ Q]\subseteq Q$,

(2) $(Q]\subseteq Q$,

(3) $Q^{\circledast} = Q$.

Proposition 3. *Let Θ be an ordered LA-semihypergroup with order preserving anti-involution $\circledast$. Then,*

(1) $\mathfrak{I}^{\circledast}$ is a left (resp., right) anti-involution hyperideal for any left (resp., right) hyperideal $\mathfrak{I}$ of Θ,

(2) $\mathfrak{R}^{\circledast}$ is an anti-involution bi-hyperideal for any bi-hyperideal$\mathfrak{R}$ of Θ,

(3) $Q^{\circledast}$ is an anti-involution quasi-hyperideal for any quasi-hyperideal$Q$ of Θ.

Proof. Assume $\mathfrak{I}$ is a left hyperideal of Θ. Since $\Theta\circ\mathfrak{I}\subseteq\mathfrak{I}$ and $\Theta^{*}=\Theta$, we have

$$\Theta\circ\mathfrak{I}^{*}=\Theta^{*}\circ\mathfrak{I}^{*}=(\Theta\circ\mathfrak{I})^{*}\subseteq\mathfrak{I}^{*}. \tag{13}$$

Now, let $\vartheta\in\mathfrak{I}^{*}$ and $\sigma\leq\vartheta$, then $\sigma^{*}\leq\vartheta^{*}\in\mathfrak{I}$. Thus, $\sigma^{*}\in\mathfrak{I}$, hence, $\sigma\in\mathfrak{I}^{*}$. Therefore, $\mathfrak{I}^{*}$ is a left anti-involution hyperideal of Θ. Similar is the case for right hyperideal $\mathfrak{I}$ of Θ.

(2) Similar to equation (1).

(3) Let Q be a quasi-hyperideal of Θ. Since $(Q\circ\Theta]\cap(\Theta\circ Q]\subseteq Q$, and $\Theta^{*}=\Theta$. By Proposition 1, we have

$$\begin{aligned}(Q^{*}\circ\Theta]\cap(\Theta\circ Q^{*}]&=(Q^{*}\circ\Theta^{*}]\cap(\Theta^{*}\circ Q^{*}]\\&=(Q\circ\Theta]^{*}\cap(\Theta\circ Q]^{*}\\&=(Q\circ\Theta\cap\Theta\circ Q]^{*}\\&\subseteq Q^{*}.\end{aligned} \tag{14}$$

Now, let $\vartheta\in Q^{*}$ and $\sigma\leq\vartheta$, then $\sigma^{*}\leq\vartheta^{*}\in Q$. Thus, $\sigma^{*}\in Q$, hence, $\sigma\in Q^{*}$. This shows that Q^{*} is an anti-involution quasi-hyperideal of Θ. □

Theorem 1. *Let Θ be an ordered LA-semihypergroup with order preserving anti-involution $*$. Let $\{\mathfrak{I}_i^{*}\colon i\in I\}$ be a family of*

(1) Left (resp., right) anti-involution hyperideals of Θ. Then, the intersection $\cap_{i\in I}\mathfrak{I}_i^{*}\neq\varnothing$ is a left (resp., right) anti-involution hyperideal of Θ.

(2) Anti-involution bi-hyperideals of Θ. Then, the intersection $\cap_{i\in I}\mathfrak{I}_i^{*}\neq\varnothing$ is an anti-involution bi-hyperideal of Θ.

(3) Anti-involution quasi-hyperideals of Θ. Then, the intersection $\cap_{i\in I}\mathfrak{I}_i^{*}\neq\varnothing$ is an anti-involution quasi-hyperideal of Θ.

Proof. Straightforward. □

Proposition 4. *Let Θ be an ordered LA-semihypergroup with order preserving anti-involution $*$ and $\mathfrak{I}$ be any anti-involution hyperideal of Θ. For any $\tau\in\Theta$, if $\tau\in(((\Theta\circ\tau^{*})\circ\Theta)\circ\tau^{*})\circ\Theta$, then $\mathfrak{I}=\mathfrak{I}^{*}\circ\mathfrak{I}^{*}$.*

Proof. Let $\vartheta\in\mathfrak{I}^{*}\circ\mathfrak{I}^{*}$. Then, $\vartheta\in a_1\circ a_2$ for some $a_1,a_2\in\mathfrak{I}^{*}$. Let $a_1\in(((\Theta\circ a_1^{*})\circ\Theta)\circ a_1^{*})\circ\Theta$. Then, $a_1\in(((p_1\circ a_1^{*})\circ p_2)\circ a_1^{*})\circ p_3$ for some $p_1,p_2,p_3\in\Theta$. Similarly, $a_2\in(((q_1\circ a_2^{*})\circ q_2)\circ a_2^{*})\circ q_3$ for some $q_1,q_2,q_3\in\Theta$. Consequently, we have

$$\begin{aligned}\vartheta&\in a_1\circ a_2\\&\subseteq((((p_1\circ a_1^{*})\circ p_2)\circ a_1^{*})\circ p_3)\circ((((q_1\circ a_2^{*})\circ q_2)\circ a_2^{*})\circ q_3)\\&=(((a_1^{*}\circ p_2)\circ(p_1\circ a_1^{*}))\circ p_3)\circ((((q_1\circ a_2^{*})\circ q_2)\circ a_2^{*})\circ q_3)\\&=((p_3\circ(p_1\circ a_1^{*}))\circ(a_1^{*}\circ p_2))\circ((((q_1\circ a_2^{*})\circ q_2)\circ a_2^{*})\circ q_3)\\&=((p_3\circ a_1^{*})\circ((p_1\circ a_1^{*})\circ p_2))\circ((((q_1\circ a_2^{*})\circ q_2)\circ a_2^{*})\circ q_3)\\&\subseteq((\Theta\circ\mathfrak{I})\circ\Theta)\circ\Theta\\&\subseteq\mathfrak{I}.\end{aligned} \tag{15}$$

Hence, $\mathfrak{I}^{*}\circ\mathfrak{I}^{*}\subseteq\mathfrak{I}$. On the other hand, let $\vartheta\in\mathfrak{I}$. Then, we have $\vartheta\in(((r_1\circ\vartheta^{*})\circ r_2)\circ\vartheta^{*})\circ r_3$ for some $r_1,r_2,r_3\in\Theta$, because $\vartheta\in(((\Theta\circ\vartheta^{*})\circ\Theta)\circ\vartheta^{*})\circ\Theta$. Clearly,

$$\begin{aligned}(\vartheta&\in((r_1\circ\vartheta^{*})\circ r_2)\circ\vartheta^{*})\circ r_3\\&\subseteq(((\Theta\circ\mathfrak{I}^{*})\circ\Theta)\circ\mathfrak{I}^{*})\circ\Theta\\&=(\mathfrak{I}^{*}\circ\mathfrak{I}^{*})\circ\Theta\\&=(\Theta\circ\mathfrak{I}^{*})\circ\mathfrak{I}^{*}\\&\subseteq\mathfrak{I}^{*}\circ\mathfrak{I}^{*}.\end{aligned} \tag{16}$$

Thus, $\mathfrak{I}\subseteq\mathfrak{I}^{*}\circ\mathfrak{I}^{*}$. Hence, $\mathfrak{I}=\mathfrak{I}^{*}\circ\mathfrak{I}^{*}$. □

Definition 11. An ordered anti-involution LA-semihypergroup Θ is called a regular ordered anti-involution LA-semihypergroup, if $\alpha\in(\alpha\circ\alpha^{*})\circ\alpha$, for all $\alpha\in\Theta$.

Proposition 5. *Let Θ be an ordered LA-semihypergroup with order preserving anti-involution $*$. If Θ is regular and has left identity, then*

$$((\Theta\circ\alpha)\circ\alpha^{*})\circ\Theta=(\Theta\circ(\alpha\circ\alpha^{*}))\circ\Theta, \tag{17}$$

for any $\alpha\in\Theta$.

Proof. Let $\alpha\in\Theta$. Since Θ is regular, we have $\alpha\in(\alpha\circ\alpha^{*})\circ\alpha$. Then,

$$\begin{aligned}((\Theta\circ\alpha)\circ\alpha^{*})\circ\Theta&=((\Theta\circ((\alpha\circ\alpha^{*})\circ\alpha))\circ\alpha^{*})\circ\Theta\\&=(((\alpha\circ\alpha^{*})\circ(\Theta\circ\alpha))\circ\alpha^{*})\circ\Theta\\&\subseteq(((\alpha\circ\alpha^{*})\circ\Theta)\circ\alpha^{*})\circ\Theta\\&=((\alpha^{*}\circ\Theta)\circ(\alpha\circ\alpha^{*}))\circ\Theta\\&\subseteq(\Theta\circ(\alpha\circ\alpha^{*}))\circ\Theta.\end{aligned} \tag{18}$$

Thus, $((\Theta\circ\alpha)\circ\alpha^{*})\circ\Theta\subseteq(\Theta\circ(\alpha\circ\alpha^{*}))\circ\Theta$. On the other hand, we have

$$\begin{aligned}(\Theta\circ(\alpha\circ\alpha^*))\circ\Theta &= (\Theta\circ(((\alpha\circ\alpha^*)\circ\alpha)\circ\alpha^*))\circ\Theta\\ &= (\Theta\circ((\alpha^*\circ\alpha)\circ(\alpha\circ\alpha^*)))\circ\Theta\\ &= ((\alpha^*\circ\alpha)\circ(\Theta\circ(\alpha\circ\alpha^*)))\circ\Theta\\ &= ((\alpha^*\circ\Theta)\circ(\alpha\circ(\alpha\circ\alpha^*)))\circ\Theta\\ &= (((\alpha\circ(\alpha\circ\alpha^*))\circ\Theta)\circ\alpha^*)\circ\Theta\\ &= (((\Theta\circ(\alpha\circ\alpha^*)\circ\alpha\circ\alpha^*)\circ\Theta\subseteq((\Theta\circ\alpha)\circ\alpha^*)\circ\Theta.\end{aligned} \tag{19}$$

Thus, $(\Theta\circ(\alpha\circ\alpha^*))\circ\Theta\subseteq((\Theta\circ\alpha)\circ\alpha^*)\circ\Theta$. Hence, $((\Theta\circ\alpha)\circ\alpha^*)\circ\Theta = (\Theta\circ(\alpha\circ\alpha^*))\circ\Theta$. □

4. Rough Anti-Involution Hyperideals

In this section, we applied rough set theory to anti-involution hyperideals of ordered anti-involution LA-semihypergroups.

Definition 12. A relation ρ on an ordered anti-involution LA-semihypergroupΘ is called a pseudohyperorder if

(1) $\leq\ \subseteq\rho$

(2) ρ is transitive, that is, $(\vartheta,\sigma), (\sigma,\varsigma)\in\rho$ implies $(\vartheta,\varsigma)\in\rho$ for all $\vartheta,\sigma,\varsigma\in\Theta$.

(3) ρ is compatible, that is, if $(\vartheta,\sigma)\in\rho$ then, $(\vartheta\circ\alpha,\sigma\circ\alpha)\in\rho$ and $(\alpha\circ\vartheta,\alpha\circ\sigma)\in\rho$ for all $\vartheta,\sigma,\alpha\in\Theta$.

(4) for all $(\vartheta,\sigma)\in\rho$, we have $(\vartheta,\sigma)^* = (\vartheta^*,\sigma^*)\in\rho$.

Definition 13. A pseudohyperorder relation ρ on an ordered anti-involution LA-semihypergroupΘ is said to be complete if $\rho N(\alpha\circ\beta) = \rho N(\alpha)\circ\rho \mathrm{N}(\beta)$.

Definition 14. [20] Let α be a nonempty set and ρ be a binary relation on X. By $\wp(X)$, we mean the power set of X. For all $\mathfrak{I}\subseteq X$, we define ρ^- and ρ^+: $\wp(X)\longrightarrow\wp(X)$ by

$$\begin{aligned}\rho^-(\mathfrak{I}) &= \{\alpha\in X:\ \rho N(\alpha)\subseteq\mathfrak{I}\},\\ \rho^+(\mathfrak{I}) &= \{\alpha\in X:\ \rho N(\alpha)\cap\mathfrak{I}\neq\varnothing\},\end{aligned} \tag{20}$$

where $\rho N(\alpha) = \{\beta\in X:\ \alpha\rho\beta\}$. $\rho^-(\mathfrak{I})$ and $\rho^+(\mathfrak{I})$ are called the lower approximation and the upper approximation operations, respectively.

Lemma 2. *Let ρ be a pseudohyperorder on an ordered anti-involution LA-semihypergroupΘ. Then, for any $\alpha\in\Theta$, $(\rho N(\alpha))^* = \rho N(\alpha^*)$.*

Proof. Consider $\tau\in(\rho N(\alpha))^*$. Then, $\tau^*\in\rho N(\alpha)$, this implies that $(\alpha,\tau^*)\in\rho$. By definition of pseudohyperorder relation $(\alpha,\tau^*)^*\in\rho$, this implies that $(\alpha^*,\tau)\in\rho$. So, $\tau\in\rho N(\alpha^*)$. Thus, we get $(\rho N(\alpha)))^*\subseteq\rho N(\alpha^*)$. Conversely, consider $\tau\in\rho N(\alpha^*)$, then $(\alpha^*,\tau)\in\rho$. By definition of pseudohyperorder relation $(\alpha^*,\tau)^*\in\rho$, this implies that $(\alpha,\tau^*)\in\rho$. So, $\tau^*\in\rho N(\alpha)$, this implies that $\tau\in(\rho N(\alpha))^*$. Thus, we obtain $\rho N(\alpha^*)\subseteq(\rho N(\alpha))^*$. Hence, $(\rho N(\alpha))^* = \rho N(\alpha^*)$. □

Theorem 2. *Let ρ be a pseudohyperorder on an ordered LA-semihypergroupΘ. If $\mathfrak{I}$ and $\mathfrak{R}$ are nonempty subsets of Θ, then*

$$\rho^+(\mathfrak{I})\circ\rho^+(\mathfrak{R})\subseteq\rho^+(\mathfrak{I}\circ\mathfrak{R}). \tag{21}$$

Proof. The proof is straightforward. □

Theorem 3. *Let ρ be a complete pseudohyperorder on an ordered LA-semihypergroupΘ. If $\mathfrak{I}$ and $\mathfrak{R}$ are nonempty subsets of Θ, then*

$$\rho^-(\mathfrak{I})\circ\rho^-(\mathfrak{R})\subseteq\rho^-(\mathfrak{I}\circ\mathfrak{R}). \tag{22}$$

Proof. The proof is straightforward. □

Definition 15. Assume ρ is a pseudohyperorder on an ordered anti-involution LA-semihypergroupΘ. Then, a nonempty subset $\mathfrak{I}$ of Θ is called a ρ-upper rough LA-subsemihypergroup (resp., left anti-involution hyperideal, right anti-involution hyperideal, anti-involution hyperideal, and anti-involution bi-hyperideal) of Θ if $\rho^+(\mathfrak{I})$ is an LA-subsemihypergroup (resp., left anti-involution hyperideal, right anti-involution hyperideal, anti-involution hyperideal, and anti-involution bi-hyperideal) of Θ.

Theorem 4. *Let ρ be a pseudohyperorder on an ordered anti-involution LA-semihypergroupΘ and $\mathfrak{I}$ an LA-subsemihypergroup (resp., left anti-involution hyperideal, right anti-involution hyperideal, anti-involution hyperideal, and anti-involution bi-hyperideal) of Θ. Then, $\rho^+(\mathfrak{I})$ is an LA-subsemihypergroup (resp., left anti-involution hyperideal, right anti-involution hyperideal, anti-involution hyperideal, and anti-involution bi-hyperideal) of Θ.*

Proof. Let $\mathfrak{I}$ be an anti-involution bi-hyperideal of Θ.

(1) By Theorem 2, we have

$$\rho^+(\mathfrak{I})\circ\rho^+(\mathfrak{I})\subseteq\rho^+(\mathfrak{I}\circ\mathfrak{I})\subseteq\rho^+(\mathfrak{I}). \tag{23}$$

(2) By Theorem 2, we have

$$\begin{aligned}(\rho^+(\mathfrak{I})\circ\Theta)\circ\rho^+(\mathfrak{I}) &= (\rho^+(\mathfrak{I})\circ\rho^+(\Theta))\circ\rho^+(\mathfrak{I})\\ &\subseteq\rho^+((\mathfrak{I}\circ\Theta)\circ\mathfrak{I})\\ &\subseteq\rho^+(\mathfrak{I}).\end{aligned} \tag{24}$$

(3) Let $\vartheta \in \rho^{+}(\mathfrak{I})$ and $\sigma \in \Theta$ such that $\sigma \leq \vartheta$. Then, there exists $\beta \in \mathfrak{I}$, such that $\vartheta \rho \beta$ and $\sigma \rho \vartheta$. Since ρ is transitive, so $\sigma \rho \beta$ implies $\sigma \in \rho^{+}(\mathfrak{I})$.

(4) Consider

$$\begin{aligned}
\tau \in \left(\rho^{+}(\mathfrak{I})\right)^{*} &\Longleftrightarrow \tau^{*} \in \rho^{+}(\mathfrak{I}) \\
&\Longleftrightarrow \rho N\left(\tau^{*}\right) \cap \mathfrak{I} \neq \varnothing \\
&\Longleftrightarrow (\rho N(\tau))^{*} \cap \mathfrak{I} \neq \varnothing \,(\text{using Lemma 2}) \\
&\Longleftrightarrow (\rho N(\tau))^{*} \cap \mathfrak{I}^{*} \neq \varnothing \left(\text{because } \mathfrak{I}^{*}=\mathfrak{I}\right) \\
&\Longleftrightarrow (\rho N(\tau) \cap \mathfrak{I})^{*} \neq \varnothing \,(\text{using Proposition 1 (3)}) \\
&\Longleftrightarrow \rho N(\tau) \cap \mathfrak{I} \neq \varnothing \\
&\Longleftrightarrow \tau \in \rho^{+}(\mathfrak{I}).
\end{aligned} \tag{25}$$

Thus, $(\rho^{+}(\mathfrak{I}))^{*}=\rho^{+}(\mathfrak{I})$. Hence, $\rho^{+}(\mathfrak{I})$ is an anti-involution bi-hyperideal of Θ. The other cases can be seen in a similar way. □

Definition 16. Assume ρ is a complete pseudohyperorder on an ordered anti-involution LA-semihypergroupΘ. Then, a nonempty subset $\mathfrak{I}$ of Θ is called a ρ-lower rough LA-subsemihypergroup (resp., left anti-involution hyperideal, right anti-involution hyperideal, anti-involution hyperideal, and anti-involution bi-hyperideal) of Θ if $\rho^{-}(\mathfrak{I})$ is an LA-subsemihypergroup (resp., left anti-involution hyperideal, right anti-involution hyperideal, anti-involution hyperideal, and anti-involution bi-hyperideal) of Θ.

Theorem 5. *Assume ρ is a complete pseudohyperorder on an ordered anti-involution LA-semihypergroupΘ and $\mathfrak{I}$ an LA-subsemihypergroup (resp., left anti-involution hyperideal, right anti-involution hyperideal, anti-involution hyperideal, and anti-involution bi-hyperideal) of Θ. Then, $\rho^{-}(\mathfrak{I})$ is an LA-subsemihypergroup (resp., left anti-involution hyperideal, right anti-involution hyperideal, anti-involution hyperideal, and anti-involution bi-hyperideal) of Θ.*

Proof. Let $\mathfrak{I}$ be an anti-involution bi-hyperideal of Θ.

(1) By Theorem 3, we have

$$\rho^{-}(\mathfrak{I}) \circ \rho^{-}(\mathfrak{I}) \subseteq \rho^{-}(\mathfrak{I} \circ \mathfrak{I}) \subseteq \rho^{-}(\mathfrak{I}). \tag{26}$$

(2) By Theorem 3, we have

$$\begin{aligned}
\left(\rho^{-}(\mathfrak{I}) \circ \Theta\right) \circ \rho^{-}(\mathfrak{I}) &= \left(\rho^{-}(\mathfrak{I}) \circ \rho^{-}(\Theta)\right) \circ \rho^{-}(\mathfrak{I}) \\
&\subseteq \rho^{-}((\mathfrak{I} \circ \Theta) \circ \mathfrak{I}) \\
&\subseteq \rho^{-}(\mathfrak{I}).
\end{aligned} \tag{27}$$

(3) Let $\vartheta \in \rho^{-}(\mathfrak{I})$ and $\sigma \in \Theta$ such that $\sigma \leq \vartheta$. Then, there exists $\beta \in \mathfrak{I}$, such that $\vartheta \rho \beta$ and $\sigma \rho \vartheta$. Since ρ is transitive, so $\sigma \rho \beta$ implies $\sigma \in \rho^{-}(\mathfrak{I})$.

(4) Consider

$$\begin{aligned}
\tau \in \left(\rho^{-}(\mathfrak{I})\right)^{*} &\Longleftrightarrow \tau^{*} \in \rho^{-}(\mathfrak{I}) \\
&\Longleftrightarrow \rho N\left(\tau^{*}\right) \subseteq \mathfrak{I} \\
&\Longleftrightarrow (\rho N(\tau))^{*} \subseteq \mathfrak{I} \,(\text{using Lemma 2}) \\
&\Longleftrightarrow (\rho N(\tau))^{*} \subseteq \mathfrak{I}^{*} \left(\text{because } \mathfrak{I}^{*}=\mathfrak{I}\right) \\
&\Longleftrightarrow \rho N(\tau) \subseteq \mathfrak{I} \\
&\Longleftrightarrow \tau \in \rho^{-}(\mathfrak{I}).
\end{aligned} \tag{28}$$

Thus, $(\rho^{-}(\mathfrak{I}))^{*}=\rho^{-}(\mathfrak{I})$. Hence, $\rho^{-}(\mathfrak{I})$ is an anti-involution bi-hyperideal of Θ. The other cases can be seen in a similar way. □

The following example shows that the converse of Theorems 4 and 5, for the case of anti-involution bi-hyperideal, but does not hold in general.

Example 4. Let $\Theta=\{\vartheta, \sigma, \varsigma, \kappa, \lambda\}$ be an ordered LA-semihypergroup with the following multiplication table and the order "$\leq$." (Table 4)

The figure of Θ is shown in Figure 4.

Clearly, Θ is an ordered LA-semihypergroup. We define the anti-involution $*$ by $\vartheta^{*}=\vartheta$, $\sigma^{*}=\sigma$, $\varsigma^{*}=\varsigma$, $\kappa^{*}=\lambda$, and $\lambda^{*}=\kappa$. Then, it is easy to check that $(\Theta, \circ, \leq)$ is an ordered anti-involutionLA-semihypergroup with order preserving anti-involution $*$. Now, let

$$\rho=\{(\vartheta, \vartheta),(\vartheta, \sigma),(\sigma, \sigma),(\varsigma, \varsigma),(\varsigma, \kappa),(\varsigma, \lambda),(\kappa, \varsigma),(\kappa, \kappa),(\kappa, \lambda),(\lambda, \varsigma),(\lambda, \kappa),(\lambda, \lambda)\}, \tag{29}$$

be a complete pseudohyperorder on Θ, such that

$$\begin{aligned}
&\rho N(\vartheta)=\{\vartheta\}, \rho N(\sigma)=\{\vartheta, \sigma\}, \\
&\rho N(\varsigma)=\rho N(\kappa)=\rho N(\lambda)=\{\varsigma, \kappa, \lambda\}.
\end{aligned} \tag{30}$$

Now, for $\mathfrak{I}=\{\vartheta, \sigma, \lambda\} \subseteq \Theta$,

$$\begin{aligned}
&\rho^{-}(\{\vartheta, \sigma, \lambda\})=\{\vartheta, \sigma\}, \\
&\rho^{+}(\{\vartheta, \sigma, \lambda\})=\{\vartheta, \sigma, \varsigma, \kappa, \lambda\}.
\end{aligned} \tag{31}$$

It is clear that $\rho^{-}(\{\vartheta, \sigma, \lambda\})$ and $\rho^{+}(\{\vartheta, \sigma, \lambda\})$ are both anti-involution bi-hyperideals of Θ but $\{\vartheta, \sigma, \lambda\}$ is not an anti-involution bi-hyperideal of Θ.

Table 4: LA-semihypergroup ($\Theta = \{\vartheta, \sigma, \varsigma, \kappa, \lambda\}, \circ$).

$\circ$	ϑ	σ	ς	κ	λ
ϑ	ϑ	ϑ	ϑ	ϑ	ϑ
σ	ϑ	σ	σ	σ	σ
ς	ϑ	σ	ς	κ	λ
κ	ϑ	σ	λ	$\{\kappa, \lambda\}$	$\{\kappa, \lambda\}$
λ	ϑ	σ	κ	$\{\kappa, \lambda\}$	$\{\kappa, \lambda\}$

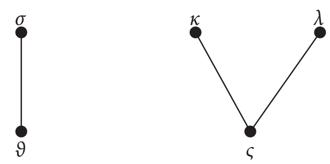

Figure 4: Figure of Θ for Example 4.

$$\leq \; := \; = \{(\vartheta, \vartheta), (\vartheta, \sigma), (\sigma, \sigma), (\varsigma, \varsigma), (\varsigma, \kappa), (\varsigma, \lambda), (\kappa, \kappa), (\lambda, \lambda)\}. \tag{29}$$

5. Conclusion

Involutions can be applied to associative algebraic structures. This theory fails for noncommutative and nonassociative structures (left invertive structures). Here, we chose a noncommutative and nonassociative structure (ordered left almost semihypergroup) and then applied the concept of involutions. However, the involution theory failed on the ordered LA-semihypergroup because of its noncommutative and nonassociative nature. Then, we applied anti-involutions to an ordered LA-semihypergroup and provided some results and examples. We constructed some results on roughness in an ordered anti-involution LA-semihypergroup.

In the future, the following topics might be considered for further studies:

(i) Regular and intraregular ordered anti-involution LA-semihypergroups

(ii) Prime and weakly prime hyperideals in ordered anti-involution LA-semihypergroups

(iii) Fuzzy ordered anti-involution LA-semihypergroups

(iv) Soft ordered anti-involution LA-semihypergroups

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This research project was funded by the Deanship of Scientific Research, Princess Nourah Bint Abdulrahman University, through the Program of Research Project Funding After Publication, grant no. (43-PRFA-P-12).

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